Productive demand, sectoral comovement, and total capacity utilization

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Abstract

We develop a three-sector model in which goods market frictions impart a causal effect of demand shocks on measured productivity and estimate it using Bayesian techniques. For identification, we make novel use of total capacity utilization in nondurables and durables, which in the model depend on a weighted average of shopping effort and variable capital utilization. In a simple version of the model, the use of these observables greatly improves the precision of estimated shopping-related parameters and implies a strong demand channel on productivity. In the general model, unanticipated and news shocks to shopping effort explain a major part of the forecast error variance decomposition of output, the Solow residual, utilization, and other variables. Capacity utilization accounts for over 80% of the Solow residual variance. Search demand shocks are essential for generating positive comovement of the utilization series, and variable capital utilization helps generate positive autocorrelation as in the data.

Keywords: goods market frictions, capacity utilization, sectoral comovement, endogeneity of Solow residual, Bayesian estimation

JEL Classification: D10; E21; E22; E32; E37

1. Introduction

How important are demand shocks for explaining business cycle fluctuations, sectoral comovement, and movements in the Solow residual? What role do goods market frictions play, and what do they imply for capacity utilization?

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Output, consumption, labor supply, and TFP comove positively in the data. Hence, the contribution of non-technology shocks to business cycle fluctuations depends on their ability to influence the Solow residual–the endogeneity of TFP. Pioneering work by Basu, Fernald, and Kimball (2006) 'purifies' the Solow residual for various non-technological influences and find that the extracted technology process is about half as volatile as TFP, appears permanent, is generally uncorrelated with output. Gali (1999) provides evidence from a structural VAR identified with long run restrictions that technology shocks induce a decline in labor hours on impact. Francis and Ramey (2005) show that the results of of Gali (1999) are robust using several long-run restrictions and controlling for capital income tax rates.

These findings undoubtedly place a greater responsibility on non-technology shocks to generate a positive correlation among the series. The introduction of goods market frictions by Diamond (1982) presents a promising avenue for demand shocks to influence total factor productivity. Bai, Rios-Rull, and Storesletten (2024), hereafter BRS, develop and estimate a two-sector neoclassical DSGE model with matching frictions. Output in this model depends by firms' technology, inputs, and their efficiency in matching with customers. Increases in shopping effort, whether due to exogenous factors or as a response to other economic shocks, enhance the matching process, leading to higher output and measured total factor productivity. The disparity between potential output and matched output (value added) aligns with the essence of Keynes (1936) and reflects a reversal of causality in comparison to a standard TFP shock. One possible interpretation of this gap is that search effort is an omitted input in production.

The causal role of demand shocks on productivity depends crucially on key structural parameters pertaining to (1) the matching technology, (2) shopping disutility, and (3) stochastic processes of demand shocks. Given the novelty and structural significance of the mechanism, identification and precise estimation of these parameters is fundamental. The estimation by BRS relies on two datasets, one which features shopping time from the American Time Use Survey as a proxy for effort.[2] Additionally, they use shopping-time data to calibrate the elasticity of the matching function, denoted ϕ, and the elasticity of disutility, denoted η. Specifically, they employ cross-sectional price dispersion for identical goods and the elasticity of shopping time with respect to expenditure to calibrate ϕ and η.

[2]The other dataset uses the relative price of investment instead. Both datasets include output, investment, and labor productivity.

While leveraging identified micro moments to derive ϕ and η is generally compelling, using shopping time as a proxy for effort raises at least two concerns. First, as discussed by BRS, fluctuations in shopping effort should be interpreted in a broader sense to encompass changes in match efficiency, rather than solely focusing on time. Second, leisure activities can potentially contaminate shopping time. For instance, spending more time browsing a store for goods can be attributed to either genuine effort or simply engaging in window shopping. An increased desire to find a particular item may lead to a shift towards actively searching for it and away from mere window shopping, resulting in an overall change in shopping time that reflects a combination of both factors.

We explore an alternative way to discipline the goods market frictions. Our analysis incorporates disaggregated total capacity utilization and sectoral data as observable series in estimation. The Federal Reserve Board constructs total capacity utilization as the ratio of an output index to capacity index for manufacturing, mining, and electric and gas utilities. The objective of the measure is to capture the maximum level of output that a plant can sustain given its available resources. Relative to a major alternative, the quarterly measure of utilization developed by Fernald (2014), it does not require constant returns to scale and zero profits. This is appealing because goods market frictions and competitive search generally require decreasing returns to scale. We encouragingly find, however, a close correspondence. If one defines Fernald utilization as the difference in cyclical components of total factor productivity and its utilization-adjusted counterpart, then it comoves closely with total capacity utilization.

Qiu and Ríos-Rull (2022) are the first to carefully define capacity utilization within the model in terms of the ratio of an output index to a capacity index, analogous to the empirical measure. They show that in a setting with goods market frictions, capacity utilization depends on a weighted sum of variable capital utilization and shopping effort, with the former magnified by the share of fixed costs. We define capacity utilization in the model the same way except at a sectoral level.[3]

We show that, absent fixed costs in production, cyclical movements in the Solow residual comprise technology, utilization, and mismeasurement of input shares. The last of these results from misspecifying constant returns to scale in capital and labor and imposing perfectly compet-

[3] In general, few papers in the literature use capacity utilization, and some equate it with variable capital utilization (Born, Peter, and Pfeifer (2013), Christiano, Eichenbaum, and Trabandt (2016)).

itive labor markets.[4] Thus, total capacity utilization is a sufficient statistic for the contribution of demand shocks (and variable capital utilization) to the Solow residual. Furthermore, mismeasurement of input shares depends on sectoral movements in labor supply, providing additional justification for using this series as an additional observable variable.

Sectoral labor data, when combined with output data, effectively encapsulates labor productivity data within each sector, thus disciplining the model mechanism. Specifically, within the BRS framework, the ratio of shopping effort in the consumption and investment sectors corresponds to the ratio of labor input. In a more general setting, we demonstrate that the shopping-effort ratio depends on the ratio of labor income across sectors and the ratio of the marginal disutility of shopping between these sectors. The use of sectoral data is thus informative about relative shopping effort.

Capacity utilization data relies on a measurable capacity index and is thus not available economy-wide. Notably, it is unavailable for services. Hence, we disaggregate consumption in the model into nondurables and services. We define sector-specific equivalents of utilization, and map utilization measures for durables and nondurables to their empirical analogues in the measurement equations of the Kalman filter. Interestingly, the empirical utilization measures have a strong positive correlation. By using these measures, along with sectoral labor and output, we extend the range of comovement facts used to discipline the model.

The set of observables we use for Bayesian estimation extends that of Katayama and Kim (2018) with the utilization measures but drops aggregate wages. Specifically, we target demeaned growth rates of consumption, investment, labor hours in consumption, labor hours in investment, utilization in nondurable goods, utilization in durable goods, and the relative price of investment to consumption.[5]

We introduce contemporaneous and news shocks to various components, including a stochastic trend in technology, stationary neutral technology, investment-specific technology, neutral shopping effort cost, investment-specific shopping effort cost, and wage markups. Given that state-of-the-art DSGE models often highlight the significance of news shocks, it is natural for us to examine their importance in the presence of goods market frictions.

[4] This term would be absent if the econometrician knew the exact production technology.

[5] Whereas BRS use labor productivity as an observable, the use of sectoral data on inputs and outputs means we effectively target labor productivity in each sector and also proxy for relative shopping effort.

The model's components and shock structure build upon the framework introduced by BRS while integrating key elements from Jaimovich and Rebelo (2009), Schmitt-Grohé and Uribe (2012), and Katayama and Kim (2018). Specifically, the model incorporates convex adjustment costs for investment, allows for greater capital utilization subject to higher depreciation, and accounts for imperfect substitution of labor between the consumption and investment sectors by households. Furthermore, preferences over consumption, shopping effort, and labor supply are nonseparable, with a parameter capturing the degree of short-run wealth effects on labor supply. The consumption good is a CES aggregate of nondurables and services.

Our model nests BRS as a special case, allowing us to use it as a laboratory to examine the alternative identification strategy. The exercise is in the spirit of Guerron-Quintana (2010), who investigates the choice of observable variables on estimated parameters in the context of a rich New Keynesian model. We drop shopping time as an observable and estimate ϕ and η directly using the dataset with the relative price of investment. We find that the posterior 90% probability band of ϕ ranges from 0.00 to 0.20, and the importance of shopping-disutility shocks in the variance decomposition drops significantly relative to BRS. Next, we estimate the same model but include capacity utilization as an observable series. Remarkably, the posterior probability band of ϕ changes to $(0.85, 0.90)$, and the contribution of demand shocks to the variance decomposition rises dramatically. Additionally, the standard deviation of capacity utilization rises ten-fold in this case compared to the former, similar to the empirical value.

The general model appreciably builds upon this exercise and reveals the following insights. Search demand shocks account for nearly two thirds of the forecast error variance of output and nearly 50% for the Solow residual. Additionally, these shocks explain a significant share of the relative price of investment and labor supply. The variance of utilization represents over 80% of the variance of the Solow residual, and the model matches the comovement of consumption and investment, utilization series, and labor reasonably well. Removing fixed costs reduces model fit but does not alter the qualitative findings, but excluding variable capital utilization prevents the model in generating autocorrelation of the utilization growth rates. However, the exclusion of search demand shocks leads to a drastic deterioration in the marginal likelihood. The volatility of output exceeds the empirical value by several orders of magnitude. This fact holds even though the same model is able to fit non-utilization data well if the utilization series are dropped from the set of observables. However, it implies not only a low volatility for the utilization series, but also a strong negative correlation between the two, counterfactual to the data. We thus conclude that utilization data stringently tests models above and beyond other series, and that search demand shocks are indispensable to capture all the comovement patterns.

For reasons of pedagogy and exposition, we focus on a neoclassical environment with competitive search. This choice facilitates comparison with BRS and Katayama and Kim (2018), who also abstract from nominal rigidity, and emphasizes the role of demand shocks operating via goods market frictions with well-functioning submarkets. Additionally, this choice ensures that conclusions follow without using monetary variables such as inflation or nominal interest rates.[6]

Though our work aligns most closely with Bai, Rios-Rull, and Storesletten (2024), it is also greatly inspired by Michaillat and Saez (2015), who model and argue for a prominent role for aggregate demand on unemployment and idle time operating through goods market frictions. Similar to our approach, they regard rates of operation in the economy and their business cycle comovement as fundamental outcome variables in their own right. However, they do not formally discipline the model using time series data. Moreover, they model matching costs in terms of additional expenditures rather than effort. Appendix E carefully compares the two specifications and shows that it does not matter for the essence of the transmission mechanism but that it does affect the labor share of income, which is relevant for the Solow residual.

Section 2 provides key background facts on utilization and sectoral comovement. Section 3 lays out the model environment. Section 4 characterizes key equilibrium relationships. It also decomposes the growth rate of the Solow residual into structural forces and relates these to capacity utilization. Section 6 examines the informative role of capacity utilization in the nested BRS version of the model. Section 7 estimates the full model. It decomposes the forecast error variance decomposition and shows that crucial parameters related to goods market frictions and shocks are precisely estimated. Section 8 concludes. The appendices describe the data construction, derivation of equilibrium, calibration strategy, the identification of key parameters by estimating the model on artificial data, and the role of the matching costs. We sometimes omit time indices in describing static relationships to economize on notation.

[6] An extension with sticky prices and monetary policy transmission is of great interest. Sticky prices affect the transmission of technology shocks, particularly on hours. Moreover, this setting would permit the use of inflation and interest-rate data as observables, which would provide additional identification.

2. Background and stylized facts on utilization and sectoral comovement

The early real business cycle literature treated the Solow residual as a pure measure of technology, but subsequent analysis found that it contained important components unrelated to technology. To address this issue, Basu, Fernald, and Kimball (2006) purify the Solow residual by removing aggregation effects, variation in capital and labor utilization, non-constant returns to scale, and imperfect competition. They find that the purified technology process is about half as volatile as TFP, appears to be permanent, and is generally uncorrelated with output. Building on these findings, Fernald (2014) constructs a quarterly measure of TFP adjusted for utilization. Figure 1 plots detrended utilization-adjusted TFP alongside standard TFP. The Fernald series not only leads the Solow residual but also exhibits less volatility. Moreover, these series occasionally diverge significantly, most notably during the pandemic shock in 2020Q1, the Great Recession, and the recession of the early 1980's. For what follows, we define Fernald utilization as the difference between cyclical TFP and its utilization-adjusted counterpart.

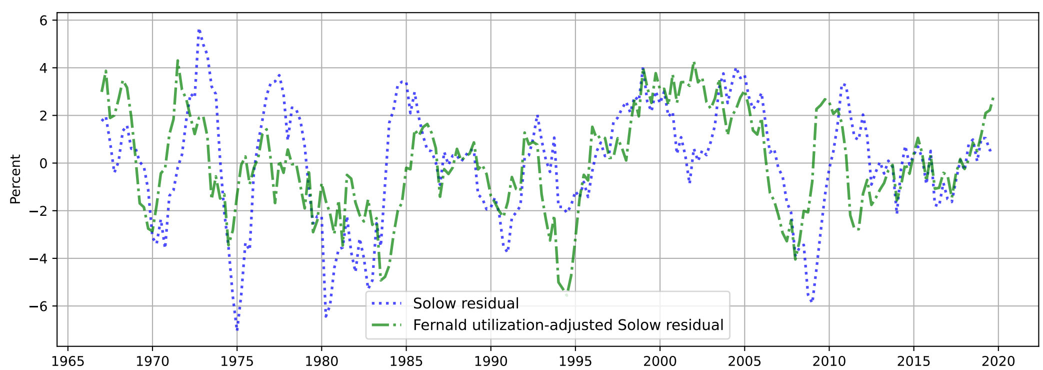

Figure 1: Time series of the Solow residual and its utilization-adjusted counterpart, following the methodology in Fernald (2014). Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$)

Next, we turn to total capacity utilization. This measure is provided by the Federal Reserve Board and encompasses 89 detailed industries (71 in manufacturing, 16 in mining, 2 in utilities).[7] These industries primarily correspond to the 3 or 4-digit North American Industry Classification System (NAICS) codes. Importantly for our purposes, estimates are available for durable and nondurable goods. In manufacturing, most capacity indices are based on responses to the

[7]This data can be downloaded at https://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17.

Census Bureau's Quarterly Survey of Plant Capacity. The census is conducted quarterly at the establishment level. The probability that each establishment is selected is proportional to the value of shipments within each industry.

Figure 2 compares cyclical total capacity utilization with the Fernald measure. The two series comove closely with each other and output, with total capacity utilization being more volatile. The volatility difference likely reflects the greater cyclical sensitivity of manufacturing. However, it also shows that cyclical sensitivity cannot be accounted for entirely by variation in labor inputs.

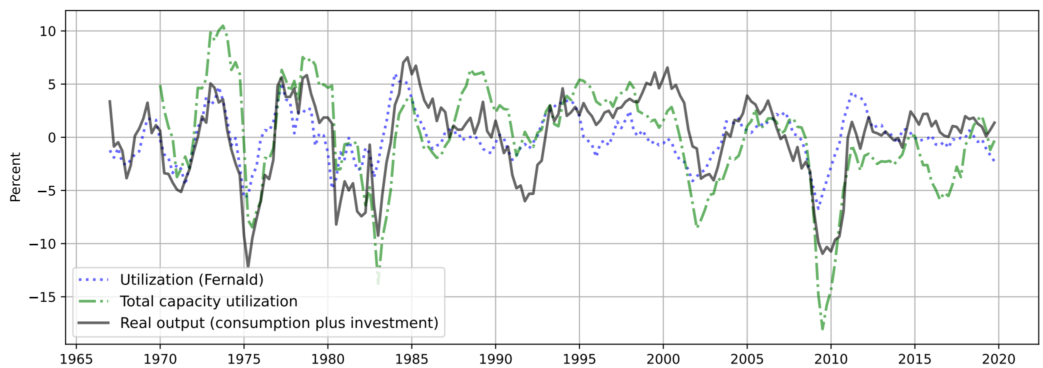

Figure 2: Time series of total capacity utilization; Fernald utilization, following the methodology in Fernald (2014); and output (here defined as consumption plus investment). Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Next, we disaggregate total capacity utilization into subcomponents for nondurables and durables. Consistent with Katayama and Kim (2018) and standard practice, the former is part of consumption and latter is part of investment. Figure 3 plots the two measures side by side. We see that utilization for durables tracks and slightly leads that of non-durables and exhibits greater volatility.

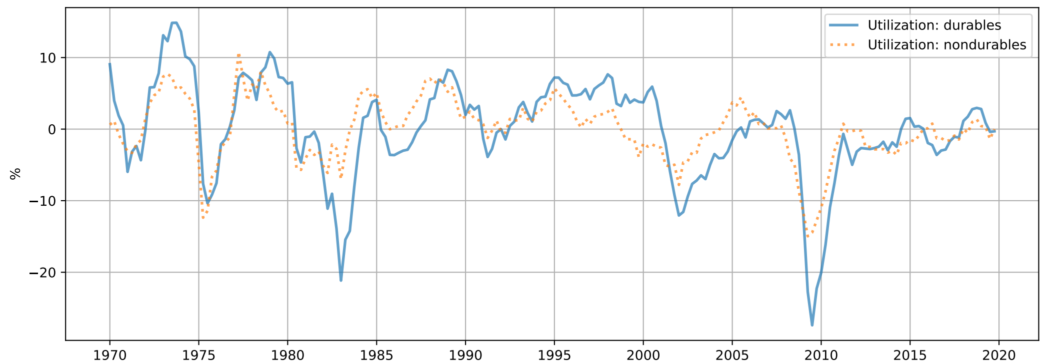

Figure 3: Total capacity utilization in nondurable and durable goods. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Finally, Figure 4 shows the detrended time series of hours in each sector alongside the aggregate measure. The series comove strongly, with hours in the investment sector being substantially more volatile. The construction of hours uses the BLS Current Employment Statistics following Katayama and Kim (2018). The data appendix provides details.

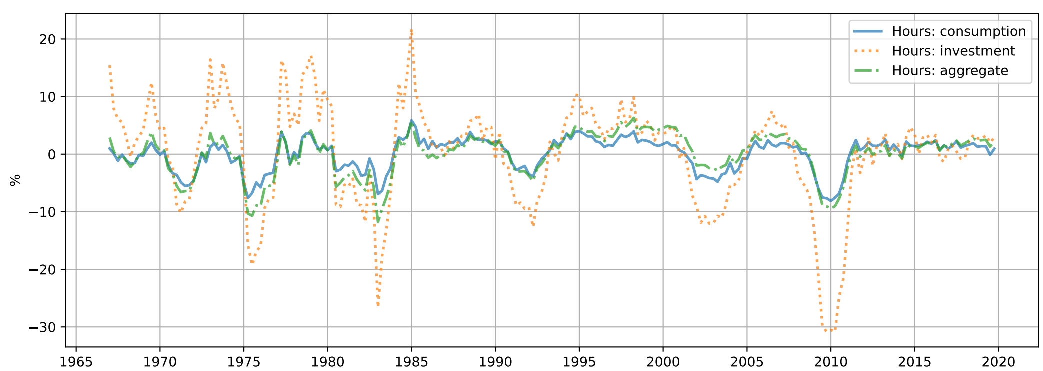

Figure 4: Sectoral and aggregate hours. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

We conclude by examining business cycle statistics of the sectoral and utilization data. Table 1 presents the second moments of the series expressed in growth rates from 1964Q1-2019Q4. The use of growth rates aligns with the treatment of data in estimation, a standard practice since Smets and Wouters (2007), and facilitates comparison with other studies. Following BRS, we define output as the sum of consumption and investment, consistent with our model framework. The findings indicate a strong correlation of 0.87 between labor hours, a moderate correlation of 0.54 between consumption and investment, and robust comovement between the utilization measures and investment, as well as labor hours in investment. Additionally, all series exhibit significant autocorrelation, except for labor productivity. Notably, investment, labor hours in investment, and utilization in durables display substantial volatility compared to consumption, labor hours in consumption, and utilization in nondurables.

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_i)	Cor(x, x_{-1})
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
n_c	0.57	0.66	0.66	0.87	0.67
n_i	1.94	2.23	0.73	1.00	0.64
Y/n	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_D$	2.27	2.61	0.69	0.84	0.55
$util_{ND}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment. We use the symbols Y for output, C for consumption, I for investment, n_c for labor supply, in consumption, n_i for labor supply in investment, Y/n for labor productivity, p_i for the relative price of investment, and $util_D$ and $util_{ND}$ for the utilization of durables and nondurables, respectively. Appendix A describes the construction of variables in detail.

3. Model environment

3.1. Technology and markets

There is a unit mass of households and a unit mass of firms within each production sector. There are three sectors, two of these for consumption (goods M_c and services S_c), and one for investment. Each sector uses capital and labor to produce output. Moreover, capital can be used at a rate h, and production involves a fixed cost ν_j.[8] The economy grows with a stochastic trend X. Its growth rate $g_t = X_t/X_{t-1}$ is a stationary process with steady state $\overline{g}$. The production function satisfies

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\} \tag{1}$$

[8] By 'fixed' we mean that the cost does not vary with the choices of inputs. The costs scales with the stochastic trend X, so that on the balanced growth the share of fixed costs to output is stationary.

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k} \tag{2}$$

The representation (1) and (2) ensures balanced growth, so that the share of fixed costs to output is stationary.

Higher utilization of capital raises depreciation according to an increasing and convex function $\delta^K(\cdot)$. We assume

$$\delta^j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2, \quad j \in \{mc, sc, i\}$$

where δ^K is an exogenous rate of depreciation. Note that $\delta(1) = \delta^K$, so that δ^K is the economy-wide steady-state depreciation rate of capital. Moreover, $\sigma_b = \delta_h(1)$ is the marginal cost of utilization in the steady state and $\sigma_a = (1)\delta_{hh}(1)/\delta_h(1)$ is the elasticity of the marginal utilization cost with respect to rate h at $h = 1$. Alternatively, $1/\sigma_a$ is the elasticity of capital utilization with respect to the rental rate. We restrict the parameter σ_b to set steady-state utilization to unity in each sector.

Investment is specific to each sector and subject to endogenous depreciation and adjustment costs:

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j \quad j \in \{mc, sc, i\},\ S_{mc} = S_{sc}$$

subject to $i = i_{mc} + i_{sc} + i_i$. The investment adjustment cost function is quadratic following Christiano, Eichenbaum, and Evans (2005):

$$S_j(x) = \frac{\Psi_j}{2}(x-1)^2$$

Extending Moen (1997), there is a competitive search protocol in which each submarket is indexed by price, market tightness, and quantity (p, q, F). The measure of matches in each submarket is given by a sector-specific constant returns to scale Cobb Douglas function

$$M_j(D, T) = A_j D^{\phi} T^{1-\phi} \tag{3}$$

of aggregate shopping effort D (in the submarket) and measure of firms T. The level parameter A_j measures sector-specific match efficiency. Given (3), the implied matching rates for households and firms are

$$\Psi_{jd}(D) = M_j/D = A_j D^{\phi-1}$$

$$\Psi_{jT}(D) = M_j/T = A_j D^{\phi}$$

using constant returns to scale and $T = 1$. Note that D measures market tightness. Once a match is formed, goods are traded at the posted price p_j per unit. A household who exerts search effort d_j purchases a real quantity of goods

$$y_j = d_j \Psi_{jd}(D) F_j \quad j \in \{mc, sc, i\}$$

3.2. Households

Households have preferences over search effort, consumption, and a labor composite following Bai, Rios-Rull, and Storesletten (2024). However, preferences also accommodate parameterized short-run wealth effects on labor supply and external habit formation. Specifically, letting θ be a vector of preference shifters, we have

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1-\sigma} \tag{4}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta} S$$

and

$$S = \left(c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta}\right)^{\gamma} S_{-1}^{1-\gamma} \tag{5}$$

Here C is aggregate consumption and $d = d_{mc} + d_{sc} + \theta_i d_i$ is total search effort. Thus, θ_i is an exogenous wedge in the search cost of investment goods relative to consumption.

The parameter γ regulates the strength of wealth effects while preserving stationarity in labor supply. Setting $\gamma = 0$ yields GHH preferences, and the further restriction that $ha = 0$ coincides with the preferences in Bai, Rios-Rull, and Storesletten (2024). Setting $\gamma = 1$ implements the general nonlinear form of King, Plosser, and Rebelo (1988) preferences. Standard additively separable preferences arise with the additional restriction $\sigma = \gamma = 1$.

Household consumption is a constant-elasticity-of-substitution aggregator of a bundle of goods y_{mc} and services y_{sc}:

$$c = [\omega_c^{1-\rho_c} y_{mc}^{\rho_c} + (1-\omega_c)^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \tag{6}$$

The elasticity of substitution is $\xi = 1/(1-\rho_c)$, and the price index satisfies

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}}$$

such that $\omega_{mc} + \omega_{sc} = 1$. Thus, p_{mc}/p_c and p_{sc}/p_c are the relative prices of nondurables and services to consumption overall.

Households have preferences with regard to the composition of labor they supply across sectors, following Horvath (2000) and Katayama and Kim (2018). Specifically, the labor composite n^a is

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{7}$$

The elasticity of substitution $1/\theta$ measures intersectoral labor mobility. The standard case of infinite marginal rate of substitution applies as $\theta \to 0$, in which case $n^a \to n_c + n_i = n$.

Figure 5 summarizes the timing of moves in the model. First, aggregate shocks occur at the beginning of each period. Second, in each sector j, a firm posts a submarket offer (p_j, D_j, F_j). Third, given the submarket choice, households choose shopping, consumption, labor supply, and capital utilization. Firms simultaneously hire labor in a competitive spot market, which determines the wage. Fourth, matching takes place. Matched firms produce and sell. Fifth, the capital stock is updated in each sector, reflecting investment adjustment costs and endogenous depreciation.

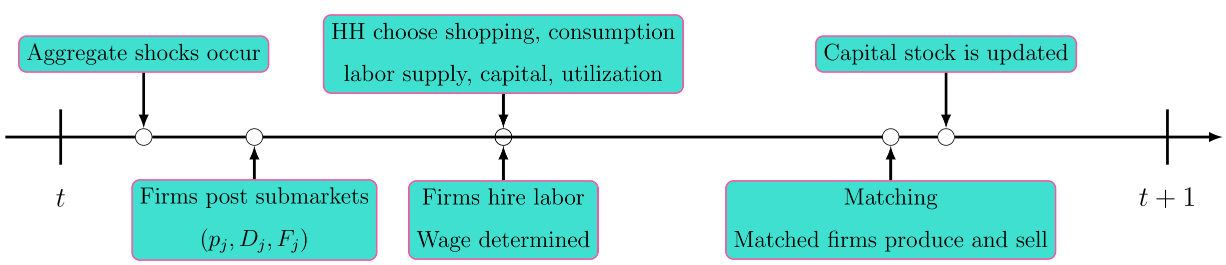

Figure 5: Timing

4. Equilibrium

4.1. Households

We start with household problem. Let $(p, D, F) = \{(p_j, D_j, F_j) | j \in \{mc, sc, i\}\}$ be the set of submarkets available to a household. Let Λ be the aggregate state and let $\hat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$

be the value of the hosehold conditional on these submarkets. Letting Φ be the set of available submarkets, then the value function is determined by the best combination of submarkets: $V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$. The household chooses search effort, labor hours, consumption, future capital, and utilization rates to solve

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_{mc}, k'_{sc}, k'_i)|\Lambda\}$$

$$\text{s.}t. \quad y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1-\delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and the consumption and labor aggregators (6) and (7).

Appendix B derives each step of the household and firm problem. Here we focus on central and innovative features of equilibrium. The presence of a goods market friction leads households to optimally balance the marginal disutility of shopping with the marginal benefit of output in both the consumption and investment sectors:

$$-u_d = u_j \phi A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{8}$$

$$-u_d \theta_i = \frac{u_{mc} p_i}{p_{mc}} \phi A_i D_i^{\phi-1} F_i \tag{9}$$

Equation (8) can be interpreted two ways. First, it states that the marginal rate of substitution between consumption and shopping effort ($-u_d/u_j$) is equal to the marginal rate of transformation, which is determined by the increase in the firm's matching probability $\Psi'_{jT}(D)$ multiplied by the quantity of output sold. Second, it states that the marginal rate of substitution is equal to the household's matching probability multiplied by the quantity of output sold and the fraction of the marginal utility of the good paid above the price. Notably, a higher value of ϕ implies a larger wedge between the marginal utility and the price. We can express this wedge using the marginal utility of wealth λ as:

$$\frac{u_j}{\lambda p_j} = \frac{1}{1-\phi} \Rightarrow \frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \tag{10}$$

or $\phi = (u_j - \lambda p_j)/u_j$. The GHH structure of preferences between consumption and shopping effort, as represented in equations (4) to (5), implies that the marginal rate of substitution is an increasing function of shopping effort: $-u_j/u_d = \theta_d d^{1/\eta}$. Combining this with equation (8), we can conclude that households increase their shopping effort in response to higher firm capacity and matching probability, as well as a lower disutility of shopping effort. The condition for investment goods in equation (9) is similar, but with the marginal disutility adjusted by θ_i and the value of output computed in consumption units, accounting for the relative price.

Given (7), households optimally divide their labor hours between consumption and investment sectors:

$$\frac{n_c}{n_i} = \frac{\omega}{1-\omega}\left(\frac{W_c^*}{W_i^*}\right)^{1/\theta}$$

so that $1/\theta$ is the elasticity of substitution.

Taking the first order condition with respect to mc and sc and combining it with (6), we derive the demand curves for nondurables and services

$$y_j = p_j^{-\xi}\omega_j C \quad j \in \{mc, sc\} \tag{11}$$

where $\xi = 1/(1-\rho_c)$ repreents the elasticity of substitution. By using (11) together with (10), we find that $\lambda = \Gamma^{-\sigma}(1-\phi)$. Here, the term $\Gamma^{-\sigma}$ captures the direct influence from the marginal utility of consumption, while the goods market frictions introduce a wedge represented by ϕ.

Furthermore, the ratio of (8) and (9) provides insight into the relative price of investment:

$$\frac{p_i}{p_j} = \theta_i \frac{A_j}{A_i}\left(\frac{D_j}{D_i}\right)^{\phi-1}\frac{z_j}{z_i}\frac{f(h_j k_j, n_j) - \nu_j X}{f(h_i k_i, n_i) - \nu_i X} \tag{12}$$

If the price p_i increases compared to p_j, with capacity held constant, it implies that investment goods become more valuable in terms of consumption, leading to an increase in D_i/D_j. Additionally, equation (12) reflects the typical mechanism where an increase in investment capacity results in a decrease in the relative price p_i/p_j.

4.2. Firms and labor unions

A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets. We introduce exogenous time-varying wage markups following the approach by Schmitt-Grohé and Uribe (2012). In this framework, a continuum of monopolistically competitive labor unions in sector j sell differentiated services, indexed by type s. The firm chooses inputs and market bundle (p_j, D_j, F_j) and ensures compliance with the participation constraint of the household.

The problem of the firm is

$$\max_{k_j,n_j,p_j,D_j,F_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j$$
$$\text{s.}t. \quad \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p_j, D_j, F_j) \geq V(\Lambda, k_{mc}, k_{sc}, k_i)$$
$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$
$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

The conditional demand for labor type s in sector j is

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

and the corresponding wage index is

$$W_j = \left[\int_0^1 w_j(s)^{1/(1-\mu_j)} ds \right]^{1-\mu_j}$$

The labor union charges the firm a wage $W_j(s)$ and pays W_j^* to its members. It maximizes earnings subject to the conditional labor demand of the firm. The problem of the union is thus

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{13}$$

The solution to (13) is $W_j(s) = \mu_j W_j^*$. Within sector j, labor unions pay the same wage and firms choose identical quantities of labor within j: $W_j(s) = W_j, n_j(s) = n_j$ for all s. Labor unions provide additional earnings to households in the form of a wage rebate. Consequently, $W_j(s) - W_j^* = (\mu_j - 1) W_j^*$ represents a fixed component of the wage from the perspective of workers.[9]

The factor demand curves for the firm are

$$(1-\phi) \frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc} \tag{14}$$
$$(1-\phi) \frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\} \tag{15}$$

The demand for inputs increases with technology, through capacity F_j and matching probability $A_j D_j^{\phi}$, and decreases with real factor prices (W_j/p_j or R_j/p_j). An interesting addition,

[9] Labor unions here are a mechanism here designed entirely for the benefit of workers. Thus, the earnings rebated to the workers count as labor income, which matters for the mapping between model and data.

distinguishing it from a typical multisector growth model, is the inclusion of the matching function elasticity ϕ as an independent factor. One intuitive interpretation is that when the firm hires more inputs and produces additional output, it relaxes the participation constraint of households, thereby expanding the feasible combinations of price and tightness. This effectively reduces the associated labor cost by $1-\phi$.

To provide an alternative characterization of the relative price of investment, we take the ratio of (14) for sectors i and $j \in \{mc, sc\}$:

$$\frac{p_i}{p_j} = \frac{n_i W_i}{n_j W_j} \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi} \frac{z_j f(h_j k_j, n_j)}{z_i f(h_i k_i, n_i)} \tag{16}$$

When D_j/D_i increases, while holding inputs and technology constant, it becomes easier to sell nondurables or services to customers, resulting in an increase in p_i/p_j. Equation (16) also takes into account the standard relationship where p_i/p_j decreases as investment-specific technology z_i/z_j rises.

Relationships (12) and (16) represent distinct curves that connect the relative investment price p_i/p_j to the relative shopping effort D_i/D_j. However, there is a slight complication in comparison, as fixed costs are present in (12) but not in (16). In the case of zero fixed costs, mutual consistency requires the following relationship:

$$\frac{D_i}{D_j} = \frac{1}{\theta_i} \frac{n_i W_i}{n_j W_j} \tag{17}$$

Relative shopping effort is determined by relative labor income and the variation in shopping disutility. Over the business cycle, the level of sectoral comovement influences n_i/n_j and thus provides information about relative shopping effort.

The final three equilibrium conditions encompass Tobin's Q, optimal utilizations, and Euler equations pertaining to the selection of future capital. These conditions incorporate investment adjustment costs and depreciation resulting from utilization:

$$\begin{aligned}
\frac{p_i}{1-\phi} &= Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda} Q_j' S_j'(x_j')(x_j')^2 \quad j \in \{mc, sc, i\} \\
\delta_h(h_j)Q_j &= R_j \quad j \in \{mc, sc, i\} \\
Q_j &= \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j' h_j'\right] \quad j \in \{mc, sc, i\}
\end{aligned}$$

The variable Q_j represents the relative price of capital in sector j in terms of consumption. The presence of investment adjustment costs introduces a disparity between Q_j and $p_i/(1-$

ϕ). Households determine the level of utilization such that the value of depreciated capital $\delta_h(h_j)Q_j$ is equal to the marginal product of capital R_j. Finally, households decide on the capital level that equates the marginal cost of foregone consumption Q_j to the anticipated discounted return. The expected return comprises the marginal product of capital in addition to the value of undepreciated capital, and the stochastic discount factor $\beta\theta_b\mathbb{E}\lambda'/\lambda$ transforms returns into current marginal utility.

4.3. Inducing stationarity

The specification of technology (1) implies that output, consumption, wages, and capital have the same stochastic trend as technology X_t, characterized by the growth rate $g_t = X_t/X_{t-1}$. The next section shows that the trend growth rate of the Solow residual is $g_t^{1-\tau}$ for labor share τ. Preferences regarding labor supply imply zero long-run wealth effects and hence ensure stationarity of labor supply. We adjust GHH preference weights to ensure stationarity of shopping effort. To focus on equilibrium fluctuations around stochastic trends, we divide each trending variable other than capital by the stochastic trend X_t. For the capital stock, we instead divide by X_{t-1} to maintain its predetermined nature.

4.4. The sector-specific Solow residual and capacity utilization

We construct the Solow residual for a specific sector in the model and relate it to capacity utilization and other structurally interesting components. Begin by expressing sectoral output as follows:

$$Y_{jt} = A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} - \nu_j X_t)$$

Let $\nu_j^R = \nu_j X/F_j$ be the fixed cost share of capacity. Then note that $\nu_j X/(z_j f(h_j k_j, n_j)) = \nu_j^R/(1+\nu_j^R)$, so that

$$Y_{jt} = \frac{A_j D_j^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n})}{1+\nu_{jt}^R}$$

Fernald (2014) constructs the sectoral Solow residual under the assumptions of constant returns to scale Cobb-Douglas technology in capital and labor, no fixed costs, and perfectly competitive factor markets. Accordingly, we define the Solow residual in sector j as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k - 1 + \tau} n_{jt}^{\alpha_n - \tau})}{1+\nu_{jt}^R} \tag{18}$$

where τ represents the steady-state labor income share. To express (18) in terms of growth rates, we introduce the symbol $dx_t = \Delta \log x_t$ and rewrite as

$$dSR_{jt} = \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} \\ +(\alpha_n - \tau)dn_{jt} - d(1+\nu_{jt}^R) \tag{19}$$

From (19) we note that the trend net growth rate of the Solow residual is

$$(1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dX_t = \tau \log g_t$$

which implies that the Solow residual grows at the rate of output multiplied by the labor share of income. By introducing the log deviation $\tilde{\nu}_j^R = \log(\nu_j^R/\nu_{ss}^R)$, we can rewrite (19) as[10]:

$$dSR_{jt} = \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt} - \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta\tilde{\nu}_{jt}^R \tag{20}$$

Expression (20) decomposes the growth rate of the Solow residual into structural forces. It comprises a demand component ϕdD_{jt}, a capital utilization component $\alpha_k dh_{jt}$, a technology component $dz_{jt}+(1-\alpha_k)dX_t$, an input share mismeasurement component $(\alpha_k-1+\tau)dk_{jt}+(\alpha_n-\tau)dn_{jt}$, and a change in the fixed cost share component $[\nu_{ss}/(1+\nu_{ss}^R)]\Delta\tilde{\nu}_{jt}^R$. The first component reflects the direct effect of goods market frictions, and there is also a general equilibrium feedback between higher shopping effort and the other components. Additionally, the calibration strategy establishes a relationship between the coefficients α_k and α_n in relation to ϕ. It is worth noting that the growth rate of cyclical labor productivity $d(Y_{jt}/n_{jt})$ has the same expression as (20), except that τ is replaced by 1. Therefore, $d(Y_{jt}/n_{jt}) = dSR_{jt} + (1-\tau)(dk_{jt} - dn_{jt})$. In general, we find that the Solow residual and labor productivity behave similarly in cyclical terms, and choose to emphasize the former because of its significance in the literature.

We next turn to capacity utilization and relate it to the Solow residual. Following Qiu and Ríos-Rull (2022), we define capacity in sector j as

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

[10]Calculate

$$\log(1+\nu_j^R) \approx \log(1+\nu_{ss}^R) + \frac{1}{1+\nu_{ss}^R}(\nu_{jt}^R - \nu_{ss}^R) \approx \log(1+\nu_{ss}^R) + \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\tilde{\nu}_{jt}^R$$

Hence, $d(1+\nu_{jt}^R) = \Delta\log(1+\nu_{jt}^R) \approx \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta\tilde{\nu}_{jt}^R$

Consistent with the definition from the Federal Reserve Board, capacity utilization in sector j is the ratio of output to capacity:

$$\begin{aligned} util_{jt} \equiv \frac{Y_{jt}}{cap_{jt}} &= \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt} X_t)}{z_{jt} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} X_t^{1-\alpha_k} - \nu_{jt} X_t} \\ &= \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} (k_{jt}/X_t)^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt})}{z_{jt}(k_{jt}/X_t)^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt}} \end{aligned} \tag{21}$$

Capacity utilization is stationary since k_j grows at the same rate g as X on the balanced growth path. Expressing (21) in growth rates yields

$$dutil_{jt} = \phi dD_{jt} + (1 + \nu_{ss}^R)\alpha_k dh_{jt} \tag{22}$$

The growth rate of utilization equals that of shopping effort scaled by ϕ and capital utilization scaled by $(1 + \nu_{ss}^R)\alpha_k$. Therefore, higher fixed costs amplify the relative weight of capital utilization to shopping effort.

By comparing (22) and (20), we see that shopping effort enters with the same weight ϕ but that the weight of capital utilization differs due to the presence of fixed costs. In the special case of zero fixed costs, the Solow residual growth rate simplifies into the sum of growth rates of utilization, technology, and mismeasurement of input shares.

$$dSR_{jt}|_{\nu_j=0} = dutil_{jt} + dz_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt} \tag{23}$$

Our definition of the sectoral Solow residual follows the methodology outlined by Fernald (2014). This approach mitigates potential additional composition bias that may arise from employing an aggregate production technology. Furthermore, it aligns sensibly with the concept of utilization, which is only applicable to specific industries. Accordingly, the aggregate Solow residual and capacity utilization can be defined as the output-weighted average of sectoral values, as consistent with Fernald (2014):

$$SR = \sum_j \frac{Y_j}{Y} SR_j, \quad util = \sum_j \frac{Y_j}{Y} util_j$$

To a first-order approximation, the linearized expressions (20), (22), and (23) also apply to their respective aggregates. This allows us to quantify the proportion of Solow residual variance explained by the utilization component, $Var(dutil)/Var(dSR)$.

We have discussed the Solow residual and capacity utilization in terms of growth rates to facilitate comparison with empirical practice (e.g., Fernald (2014)) and to maintain consistency with the form of variables used in the observation equations and for business cycle statistics. In Appendix D, we provide a similar comparison between the cyclical deviations of the Solow residual and capacity utilization.

5. Effect of demand shocks in simplified static model

This section presents a highly simplified static model to illustrate the impact of demand shocks on capacity, output, labor, and the Solow residual. We consider a scenario with a single consumption good produced using only labor ($\xi \to \infty, \alpha_k \to 0$). Households have GHH preferences without habit formation ($\gamma = ha = 0$) and sell homogeneous labor to firms in competitive spot markets ($\mu_c = \mu_i = 1$).

Equilibrium is a tuple (C, D, W, n) satisfying optimal shopping, consistency of output, labor supply, and labor demand.

$$\theta_d D^{\frac{1}{\eta}} = \phi \frac{C}{D} \tag{24}$$

$$C = AD^{\phi} z n^{\alpha_n} \tag{25}$$

$$\theta_n N^{\frac{1}{\zeta}} = (1-\phi)W \tag{26}$$

$$(1-\phi)W = \alpha_n \frac{C}{n} \tag{27}$$

Equation (24) characterizes shopping effort D as a concave function of consumption C. The curve $C = AD^{\phi} z n^{\alpha_n}$ in equation (25) represents shopping effort D as a convex function of consumption C. In the space of labor supply and wage (n, W), equation (26) illustrates an upward-sloping supply curve, while equation (27) depicts a downward-sloping convex demand curve for labor. The labor share of income $\tau \equiv wN/C = \alpha_n/(1-\phi)$ using (27). Hence, The Solow residual is $SR \equiv C/(zN^{\tau}) = AD^{\phi} z n^{\alpha_n - \tau} = AD^{\phi} z n^{-\alpha_n \phi/(1-\phi)}$. The Solow residual thus depends on technology, shopping effort, and mismeasurement of labor component.

Figure (6) depicts equilibrium using two graphs. The figure on the right shows the determination of search effort and consumption, for a given level of capacity F, as the intersection between (24) and (25). The figure on the left shows the determination of hours and wages given consumption C.

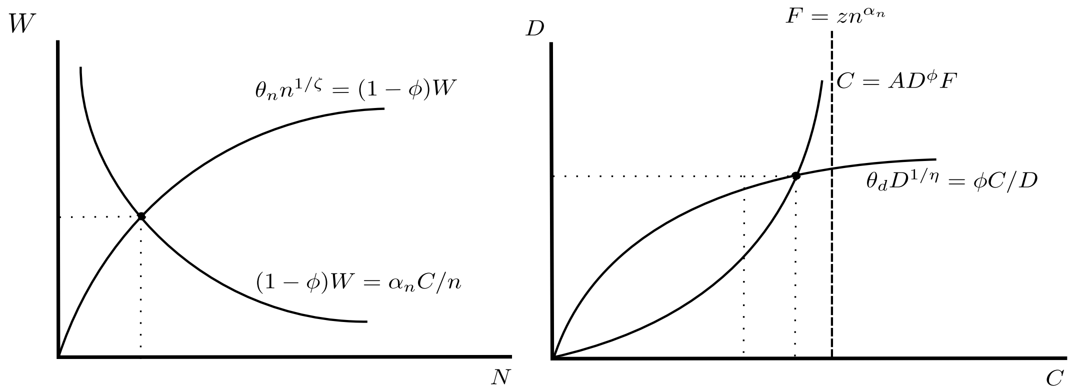

Figure 6: Equilibrium of static model

Now, let us consider a negative shock to the shopping disutility θ_d (Figure 7). The marginal cost of exerting shopping effort falls, which induces households to shop more intensely. Hence, the shopping curve shifts to the upper right. More shopping effort increases firms' matching rate and therefore boosts total production. This effect constitutes a movement along the consumption curve from point 1 to point 2. To satisfy higher production levels, firms demand more workers, boosting labor hours and wages. Finally, more labor hours expands the productive capacity of firms, so the consumption curve curve shifts rightward. This higher capacity further spurs shopping effort, represented by movement along the shopping curve from point 2 to point 3. The Solow residual therefore reflects both the initial increase in shopping effort from the demand shock followed by a further increase in shopping effort as households respond to increased capacity of firms. However, the rise of the Solow residual is slightly dampened by the mismeasurement of input shares. Notice that the demand shock to θ_d induces positive comovement across all variables in the economy and therefore resembles a standard technology shock to z.

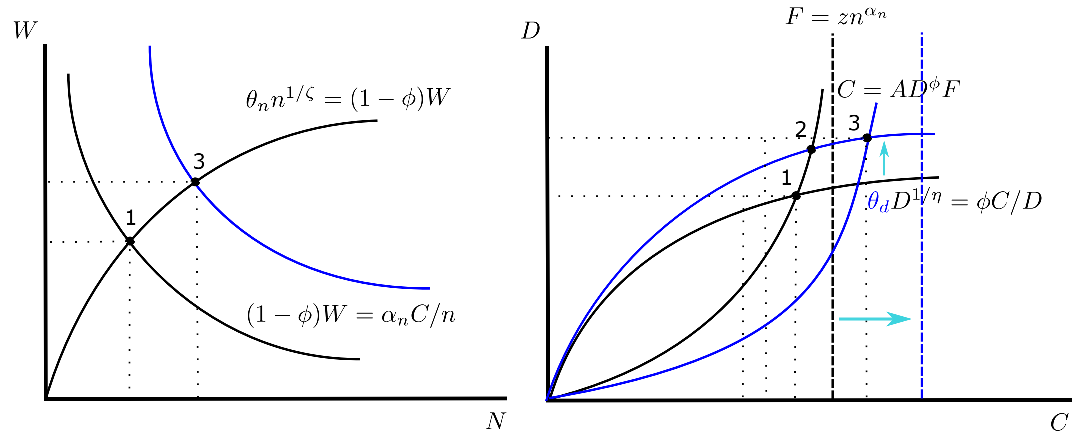

Figure 7: Reduction in shopping disutility in static model

Similarly, we can examine the impact of a fall in labor disutility θ_n. This shocks shift the labor supply curve and increases capacity. The consumption curve shifts rightward and triggers a movement along the shopping curve, as before. In Appendix E, we also examine equilibrium in a static setting in which matching costs arise from expenditure à la Michaillat and Saez (2015). The causal effect of demand on output and productivity is essentially the same, but the labor share of income is α_n, and hence there is no input share mismeasurement in the Solow residual.

6. Role of capacity utilization in estimation of simple BRS model

The baseline model by Bai, Rios-Rull, and Storesletten (2024) is a very special instance of the one outlined in the previous section. First, to obtain their preferences remove wealth effects and habit formation: $\gamma = 0$ and $ha = 0$. Second, allow for perfect mobility of labor ($\theta = 0$) and make the consumption sector unitary ($\rho_c \to 1$). Remove fixed costs in the production technology: $\nu_j = 0$ for all j. Also fix the utilization of capital ($\sigma_b \to \infty$) and remove investment adjustment costs ($\Psi_c = \Psi_i = 0$). We write the equilibrium in the detrended variables using the stochastic growth rate $g_t = X_t/X_{t-1}$.

For this section we consider the same set of shocks as BRS but also add stationary neutral technology shocks. We generally follow the same calibration strategy and targets but now fix the risk aversion parameter $\beta = 0.99$, $\sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$. We estimate the model and focus on the shopping-related parameters.[11] Next we add estimate the same model

[11]BRS also fix $\zeta = 0.72$ but they use $\sigma = 1$ and $\beta = 0.997$. We have also estimated the model with $\phi = 0.32$

but add total capacity utilization as an observable series. In the absence of fixed costs and variable capital utilization, capacity utilization in each sector is $util_j = A_j D_j^{\phi}$, and aggregate capacity utilization is $(C/Y)util_c + (I/Y)util_i$.

First, in Table 2 we report the prior distributions used for both specifications. In addition to ϕ and η, we specify distributions for the persistence parameters of nonstationary neutral technology, stationary neutral technology, investment-specific technology, labor supply, and shopping effort. We use the same prior distribution for the conditional standard deviation and persistence of the stationary shocks. These conditional standard deviations have an inverse gamma distribution with mean 0.01 and standard deviation 0.1, an the persistence parameters have prior mean 0.6 and standard deviation 0.2.

Table 2: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 2: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{z, z_I, \theta_n, \theta_d\}$.

Table 3 compares the posterior means and 90% probability bands of the key shopping-related parameters. In the former, ϕ imprecisely estimated with a lower posterior mean. In fact, the 90% probability band includes essentially a null effect. By contrast, when we add total capacity utilization, the posterior mean increases substantially to 0.88 and the estimate is precise. Estimates are similarly more precise for the shopping cost elasticity η. Generally, estimates of ρ_d and σ_d are more precise as well, though the properties differ. With total capacity utilization, demand shocks exhibit greater persistence, but their innovations become less volatile.

and $\eta = 0.2$ as by BRS and obtained a similar variance decomposition as that paper.

Table 3: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_D	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
e_D	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 3: Estimation of baseline BRS model with to sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

The first block of Table 4 compares the standard deviations at the posterior mean of shocks θ_d, shopping effort D, and utilization $util$, where the last two are expressed in growth rates. The main result is that total capacity utilization is ten times more volatile even though shopping-effort shocks are less volatile and shopping effort has similar volatility. The key difference lies in the transmission of shopping effort to utilization through ϕ. The second block highlights the role of these varying parameter estimates for the forecast error variance fraction attributable to demand shocks. It is very small in the former case but large in the latter, accounting for about two thirds of output, almost a third of labor productivity, and about half the Solow residual.

Table 4: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Volatility		
θ_d	9.84	2.00
D	1.54	1.69
$util$	0.15	1.49
FEVD		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 4: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The second sub-table shows the fraction of the unconditional variance decomposition attributable to the demand shock θ_D. See Table 3.

These two exercises sharply illustrate the informative role of total capacity utilization. Not only are shopping-related parameters more precisely estimated with the additional data series, but also the volatility of total capacity utilization in the model rises ten-fold, much closer to the empirical value.

Yet there are significant caveats to this analysis. First, in the absence of variable capital utilization, only shopping can influence total capacity utilization. Second, total capacity utilization is inappropriate as an economy-wide target since it is only constructed for specific industries. In particular, it is not measured for consumption services, a large part of the economy. Third, this model abstracts from various frictions and shocks which the DSGE literature from Smets and Wouters (2007) and Schmitt-Grohé and Uribe (2012) have demonstrated to be important. These include especially investment adjustment costs and imperfect competition in the labor market, encompassing wage markup shocks. In fact, with the introduction of wage markup shocks, labor supply shocks become unimportant.

Finally, the model struggles with key aspects of sectoral comovement. For instance, in the second specification, though the correlation of labor in each sector is not too far below the data (0.58), the autocorrelation is 0.18 for n_c and -0.01 for n_i. Labor market comovement is important for the transmission mechanism operating via goods market frictions. The relationship

(17) implies that the relative shopping effort equals the relative labor allocation:

$$\frac{D_c}{D_i} = \frac{n_c}{n_i} \tag{28}$$

Combining (28) with the shopping optimality conditions then implies

$$\frac{n_c}{n_i} = \frac{C}{p_i I}$$

The variables C, I, and p_i are observables in estimation and thus determine n_c/n_i. Trying to use n_c and n_i–or even just their ratio–as observables in estimation would induce stochastic singularity. Though one could sidestep this issue by avoiding the of labor variables, they play an important role in the transmission mechanism. The use of these series versus the relative price of investment becomes arbitrary. A more satisfactory route is to build a richer model and incorporate both pieces of data to discipline the transmission mechanism.

7. Main quantitative analysis

7.1. Stochastic processes

The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs, as Bai, Rios-Rull, and Storesletten (2024):

$$\log g_t = (1 - \rho_g) \log \overline{g} + \rho_g \log g_{t-1} + e^0_{g,t} + e^4_{g,t-4}$$

where $e^0_{g,t} \sim N(0, \sigma^0_g)$ and $e^4_{g,t} \sim N(0, \sigma^4_g)$. The shock $e^0_{g,t}$ is contemporaneous and $e^4_{g,t}$ is a 4-period-ahead news shock anticipated at time t. In the special case $\rho_g = 0$ with no news shock, $\log X_t$ follows a random walk with drift modified by anticipated noise.

We also consider a stationary neutral shock z_c and an investment-specific shock z_i. We let $z_i \equiv z_c z_I$ where z_I is independent of z_c. Finally, there are disturbances to general shopping disutility θ_b, investment-specific shopping disutility θ_i, the discount factor θ_d, labor supply θ_n, and wage markups μ_c and μ_i. We do not include consumption preference shocks because they can be replicated by sequences of labor supply, shopping-disutility, and discount-factor shocks.

Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e^0_{v,t} + e^4_{v,t-4}$$

where $e^0_{v,t} \sim N(0, \sigma^0_v)$ and $e^4_{v,t} \sim N(0, \sigma^4_v)$. We also impose $e^4_{\theta_n,t-4} = 0$ for all t, given that labor supply shocks turn out to play only a minor role in the general model. Relative to Schmitt-Grohé and Uribe (2012) and Katayama and Kim (2018), we are more parsimonious about anticipated shocks.

7.2. Bayesian estimation

The Bayesian framework allows us to incorporate prior (e.g.) microeconomic evidence, quantify parameter uncertainty, decompose the forecast error variance of each shock, and compare the fit of models via the marginal likelihood. The marginal likelihood also implicitly penalizes parameter complexity.[12]

Along these lines, we estimate the general model using Bayesian techniques with quarterly data from 1964Q1 to 2019Q4. The likelihood of the data sample Y given the estimated parameters Θ is denoted as $L(Y|\Theta)$. By incorporating the prior parameter distribution $P(\Theta)$, the posterior density is proportional to $L(Y|\Theta)P(\Theta)$. We employ the random walk Metropolis Hastings algorithm, which is a standard practice for drawing from the posterior distribution of Θ. We use the following observables expressed in growth rates: consumption C, investment I, labor hours n_c and n_i, sectoral utilization $util_{ND}$ and $util_D$, and the relative price of investment p_i. This dataset is similar to Katayama and Kim (2018), but we include the utilization variables and exclude wages.

The vector of estimated parameters Θ consists of the persistence and conditional standard deviations for the anticipated and unanticipated shocks, the risk aversion parameter σ, the habit formation parameter ha, the parameter ζ closely related to the Frisch elasticity of labor supply, the parameter γ related to the wealth effects of labor supply, the fixed cost share parameter of potential output ν^R, the elasticity of depreciation with respect to capital utilization σ_{ac} and σ_{ai}, the investment adjustment cost parameters Ψ_c and Ψ_i, the inverse of the intersectoral elasticity of labor supply θ, and the elasticity of substitution between nondurables and services ξ. Of particular interest are the elasticity of the matching function with respect to shopping effort ϕ and the elasticity of shopping cost η.

To calibrate the remaining parameters, we use long-run targets, normalizations, and a subset Θ_R of the estimated parameters. The fixed exogenous parameters include the discount factor β, average growth rate $\overline{g}$, gross wage markup μ, the share ω of labor hours in consumption, and

[12]If the expansion of the parameter space is irrelevant for fitting the data, then this reduces the prior probability mass of parameters that do help fit the data and thereby lowers the marginal likelihood.

the share of services in consumption. Following the approach of Katayama and Kim (2018) and standard practice, we set $\beta = 0.99$, $\overline{g} = 0.45\%$, $\mu = 1.15$, and $\omega = 0.8$. We pin down the weight of services ω_{sc} in the consumption aggregator as the average share of services in consumption, $\omega_{sc} = p_{sc}y_{sc}/C = 0.65$ over the sample.

The second set of parameters Θ_R is estimated and used to calibrate other parameters. These are the parameters of risk aversion σ, labor supply ζ, elasticity of the matching function ϕ, elasticity of shopping effort cost η, fixed cost share ν_R, and habit persistence ha.

The third set of parameters determines the choice of units but does not impact the cyclical behavior of the economy. We normalize output and the relative price of services and investment to unity, effectively determining the level parameters of technology for each sector. Additionally, we set the fraction of time allocated to work as 30%, which, in conjunction with other parameters, specifies the value of θ_n. To achieve a target capacity utilization of 81% in each sector, we adjust the level parameters A_j of the matching function accordingly. Finally, by setting the capital utilization rate to 1, we obtain the value for σ_b.

The fourth set of parameters are determined through long-run targets and the estimated parameters in the second group. The long-run targets includes those chosen by Bai, Rios-Rull, and Storesletten (2024). These are an investment-share of output of 20%, an annual capital-to-output ratio of 2.75, and a labor share of income of 67%. These in turn pin down the parameters δ, α_k and α_n. Appendix C discusses the calibration in detail.

Targets	Value	Parameter	Calibrated value/posterior mode
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.6
Labor supply	–	ζ	1.97
Elasticity of matching function	–	ϕ	0.84
Elasticity of shopping effort cost	–	η	0.65
Fixed cost share of capacity	–	ν_R	0.42
Habit persistence	–	ha	0.40
Third group: normalizations			
SS output	1	z_{mc}	0.45
Relative price of services	1	z_{sc}	0.69
Relative price of investment	1	z_i	0.36
Fraction time spent working	0.30	θ_n	3.85
Capacity utilization of nondurables	0.81	A_{mc}	2.51
Capacity utilization of services	0.81	A_{sc}	1.49
Capacity utilization of investment sector	0.81	A_i	3.33
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	0.014
Physical capital to output ratio	2.75	α_k	0.242
Labor share of income	0.67	α_n	0.074

Table 5: Calibration targets and parameter values. Here we calibrate a subset of parameters using long-run targets and the posterior mode of the estimated parameters $\sigma, \zeta, \phi, \eta, \nu_R$ and ha. For illustration, we specify the calibrated values corresponding to the posterior mode of these latter parameters.

Table 6 presents the posterior estimates along with the prior distributions. Of particular interest is the posterior mean of the matching function elasticity ϕ, which is estimated to be 0.86. This suggests that the search-based demand channel plays a significant role in the model. The posterior mean values of σ (1.8) and ha (0.42) are consistent with previous findings in the literature.

The inverse of the elasticity of substitution of labor, θ, has a posterior mean of 1.5, which is moderate and about half the size reported by Katayama and Kim (2018). This difference can be attributed to weaker short-run wealth effects and the use of search demand shocks, which naturally induce complementarity. The wealth effects parameter γ has a posterior mean of 0.32, higher than the near zero estimate obtained by Schmitt-Grohé and Uribe (2012). Thus, the estimates do not entirely support the GHH specification once one incorporates limited factor mobility and demand shocks operating through goods market frictions. The elasticity of substitution ξ between nondurables and services has a posterior mean of 0.92, which is fairly close to the prior mean, and is somewhat more concentrated compared to the prior distribution. The fixed cost share ν_R has a posterior mean of 0.33, somewhat higher than the prior mean. Relative to Qiu and Ríos-Rull (2022), we need a somewhat higher fixed cost share to fit the disaggregated data.[13]

Regarding investment adjustment costs, we find moderate costs with no strong evidence of differences by sector, unlike the findings of Katayama and Kim (2018). The estimated elasticities of the marginal cost of capital utilization are higher for consumption than for investment, which aligns with the greater volatility of investment and capacity utilization in durable goods. However, the estimated values are lower than those reported in Katayama and Kim (2018), likely due to the role of the overall volatile utilization series.

The posterior probability bands of the standard deviations of shocks indicate a limited role for permanent technology shocks, labor supply shocks, anticipated shopping-effort shocks, and unanticipated investment-specific shopping-effort shocks. However, a more transparent analysis of the business cycle contribution of shocks can be obtained by examining the forecast error variance decomposition, which we explore in the next section.

[13] Abraham, Bormans, Konings, and Roeger (2021) estimate the fixed cost share of output using Belgian firm-level panel data at 23.4%.

Table 6: Bayesian estimation of baseline model

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5%	HPD 95%
σ	beta	1.500	0.2500	1.808	0.1763	1.5813	2.0856
ha	beta	0.500	0.2000	0.424	0.0486	0.3528	0.5045
ζ	gamm	0.720	0.2500	1.845	0.1277	1.6394	2.0000
γ	beta	0.500	0.2000	0.317	0.0414	0.2496	0.3818
ϕ	beta	0.320	0.2000	0.858	0.0419	0.7941	0.9304
η	gamm	0.200	0.1500	0.563	0.1168	0.3777	0.7266
ξ	gamm	0.850	0.1000	0.918	0.0625	0.8201	1.0233
ν_R	beta	0.200	0.1000	0.328	0.0854	0.1726	0.4438
σ_{ac}	invg	1.000	1.0000	1.370	0.3430	0.7120	1.8836
σ_{ai}	invg	1.000	1.0000	0.542	0.1498	0.3302	0.7284
Ψ_c	gamm	4.000	1.0000	4.816	0.3461	4.2633	5.3963
Ψ_i	gamm	4.000	1.0000	4.176	0.7375	3.1244	5.3114
θ	gamm	1.000	0.5000	1.545	0.4977	0.9333	2.3192
ρ_g	beta	0.100	0.0500	0.398	0.1006	0.2333	0.5629
ρ_Z	beta	0.600	0.2000	0.685	0.0766	0.5639	0.8202
ρ_{ZI}	beta	0.600	0.2000	0.930	0.0265	0.8915	0.9699
ρ_N	beta	0.600	0.2000	0.796	0.2183	0.4259	0.9999
ρ_D	beta	0.600	0.2000	0.936	0.0164	0.9108	0.9638
ρ_{DI}	beta	0.600	0.2000	0.995	0.0052	0.9887	0.9999
ρ_b	beta	0.600	0.2000	0.851	0.0486	0.7667	0.9286
$\rho_{\mu c}$	beta	0.600	0.2000	0.982	0.0147	0.9628	1.0000
$\rho_{\mu i}$	beta	0.600	0.2000	0.982	0.0100	0.9691	0.9997
e_g	gamm	0.010	0.0100	0.003	0.0022	0.0000	0.0061
$e_{g,-4}$	gamm	0.010	0.0100	0.009	0.0018	0.0059	0.0118
e_Z	gamm	0.010	0.0100	0.004	0.0007	0.0032	0.0054
$e_{Z,-4}$	gamm	0.010	0.0100	0.004	0.0012	0.0021	0.0059
e_{ZI}	gamm	0.010	0.0100	0.011	0.0010	0.0097	0.0128
$e_{ZI,-4}$	gamm	0.010	0.0100	0.002	0.0014	0.0001	0.0040
e_N	gamm	0.010	0.0100	0.003	0.0024	0.0001	0.0057

e_D	gamm	0.010	0.0100	0.040	0.0081	0.0276	0.0523
$e_{D,4}$	gamm	0.010	0.0100	0.007	0.0061	0.0001	0.0165
e_{DI}	gamm	0.010	0.0100	0.002	0.0014	0.0001	0.0040
$e_{DI,-4}$	gamm	0.010	0.0100	0.020	0.0011	0.0177	0.0212
e_b	gamm	0.010	0.0100	0.001	0.0009	0.0001	0.0023
$e_{b,-4}$	gamm	0.010	0.0100	0.006	0.0022	0.0025	0.0091
$e_{\mu c}$	gamm	0.010	0.0100	0.001	0.0014	0.0001	0.0035
$e_{\mu c,-4}$	gamm	0.010	0.0100	0.003	0.0022	0.0001	0.0055
$e_{\mu i}$	gamm	0.010	0.0100	0.015	0.0097	0.0006	0.0290
$e_{\mu i,-4}$	gamm	0.010	0.0100	0.023	0.0037	0.0174	0.0299

Table 6: Prior and posterior distribution.

Table 7 documents the unconditional forecast error variance decomposition of the model. Technology shocks and shopping-effort shocks are the primary drivers of forecast error variance in output, the Solow residual, investment, the relative price of investment, and variable capital utilization. Among these shocks, shopping-effort shocks have a particularly significant impact on utilization and, unsurprisingly, shopping effort itself. The only significant contribution of discount-factor, wage markup, and labor supply shocks lies in explaining portions of labor in consumption and investment.

Here our primary focus is on the Solow residual and utilization. Shopping-effort and technology shocks play similarly important roles for the former, but the search demand shocks explain over 85% of utilization. Hence, the evidence strongly supports a powerful causal channel of demand shocks into productivity operating via capacity utilization.

Table 7: Forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	35.1	0.01	64.1	0.73	0.08
SR	41.3	0.73	52.9	3.12	2.00
I	38.1	0.01	54.9	6.90	0.03
p_i	54.5	0.00	45.2	0.12	0.14
n_c	14.5	14.3	31.2	23.6	16.5
n_i	18.6	1.28	26.6	13.4	40.1
$util$	13.0	0.01	86.1	0.84	0.03
D	2.36	0.00	97.6	0.06	0.00
h	31.2	0.01	68.0	0.78	0.02

Table 7: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Table 8 compares the log marginal likelihood, parameter estimates of ϕ, variance decomposition, and second moments for various specifications of the model. The baseline model accounts for two thirds of the variance decomposition of output and nearly half of the Solow residual. The relative variance of utilization to the Solow residual is 0.87. These statistics are similar in the absence of variable capital utilization but fall somewhat without fixed costs. This suggests significant complementarity between the demand channel and fixed costs.

Table 8: Comparison of model specification

	Data	Baseline	Remove			
			Fixed cost	VCU	SDS	SDS and utilization data
LML	–	4531.0	4516.9	4470.9	4202.2	–
Δ LML	–	0	-14.1	-60.1	-328.8	–
90% HPDI band ϕ	–	(0.8, 0.94)	(0.84, 0.96)	(0.2467, 0.3452)	(0.69, 0.72)	(0.56, 0.70)
FEVD(Y, SDS)	–	64.1	58.7	54.01	–	–
FEVD(SR, SDS)	–	52.9	36.3	54.2	–	–
Var(util)/Var(SR)	–	0.87	0.65	0.77	1.49	0.11
std(Y)	0.87	1.62	1.63	2.00	60.5	0.6
std($util_{ND}$)	1.26	1.15	1.1	1.27	47.9	0.27
std($util_D$)	2.27	2.98	3.25	2.44	85.6	1.18
std(n_c)	0.57	0.53	0.63	0.53	17.3	0.48
std(n_i)	1.94	1.83	1.92	1.76	39.6	1.66
Cor(C, I)	0.54	0.63	0.55	0.58	0.99	0.26
Cor($util_{ND}, util_D$)	0.75	0.57	0.53	0.62	1.00	-0.71
Cor(n_c, n_i)	0.87	0.77	0.81	0.84	1.00	0.82
Cor($util_{ND}, util_{ND,-1}$)	0.51	0.36	0.40	-0.040	0.999	0.17
Cor($util_D, util_{D,-1}$)	0.55	0.55	0.69	0.043	0.999	0.42

Table 8: Comparison of log marginal likelihood, parameter estimates of ϕ, variance decomposition, and second moments for various specifications of the model. The Laplace approximation is used for the marginal likelihood. The first column describes relevant empirical moments and the second column corresponds to the baseline model. The third, fourth, and fifth columns respectively present estimates in which fixed costs, variable capital utilization, and search demand shocks (SDS) are removed. The sixth column also removes the utilization series from the set of observables.

Next, consider second moments. The analysis of second moments provides further insights into the model's fit to the data. The baseline model tends to overestimate the volatility of output but fits the volatility of the utilization series and labor hours quite well. Additionally, it captures the correlation between consumption and investment, as well as the correlations of the utilization series and labor hours, reasonably accurately. Finally, the model does a reasonable job with respect to the autoocorrelation of the utilization series, matching that of durables and coming close to the autocorrelation of nondurables.

Removing fixed costs causes some deterioriation in performance but removing variable capital utilization is far more detrimental: the log marginal likelihood falls by about 60. We can obtain intuition by noting that the model loses flexibility in explaining utilization and output in the absence of variable capital utilization. There is more excess volatility of output in this case, and the implied autocorrelation of the utilization variables is roughly zero in the model, compared to over 0.5 in the data.

Removing demand shocks, even while preserving the goods market frictions, generates by far the greatest deterioration in model fit: the log marginal likelihood falls by over 328. In this case, the moments exceed the empirical values by nearly two orders of magnitude, and the correlations approach unity. To gain a better understanding of why the model fails to explain the data, the analysis removes the utilization series from the set of observables. The model without demand shocks can explain the other series quite well, as it incorporates the framework proposed by Katayama and Kim (2018). We can then identify the counterfactual implications for utilization in this setting. The volatility of nondurables utilization is substantially below the empirical value, echoing the finding we obtained with respect to the BRS specification. Moreover, the autocorrelation of nondurables utilization falls to 0.17. Most strikingly, this specification implies a strong negative correlation (-0.71) between the two utilization series. That is, fitting standard macroeconomic series, including sectoral labor market data, comes at the expense of fitting the volatility, comovement, and autocorrelation of the utilization series.

To better interpret these results, we examine impulse responses of consumption, investment, their respective labor inputs, and utilization in nondurables and durables from the baseline model with the parameters set to the posterior mean. In line with the analysis presented thus far, the impulse responses are depicted in terms of growth rates to ensure consistency.

Figure 8 plots the impulse response to a unit standard deviation reduction in shopping effort. This shock prompts households to increase their shopping effort, leading to a boost in matching and utilization. As a result, firms experience a higher demand for labor in both sectors, thanks to their improved ability to match. Consequently, the shock generates positive comovement in the growth rates of sectoral output, sectoral input, and utilization in the nondurables and services sectors. As expected, the Solow residual rises on impact.

Figure 9 plots the impulse response to a unit standard deviation discount-factor shock. Households are more patient, which raises the desire to consume in the future relative to the present. As a result, consumption falls while investment rises. Additionally, there is an increase

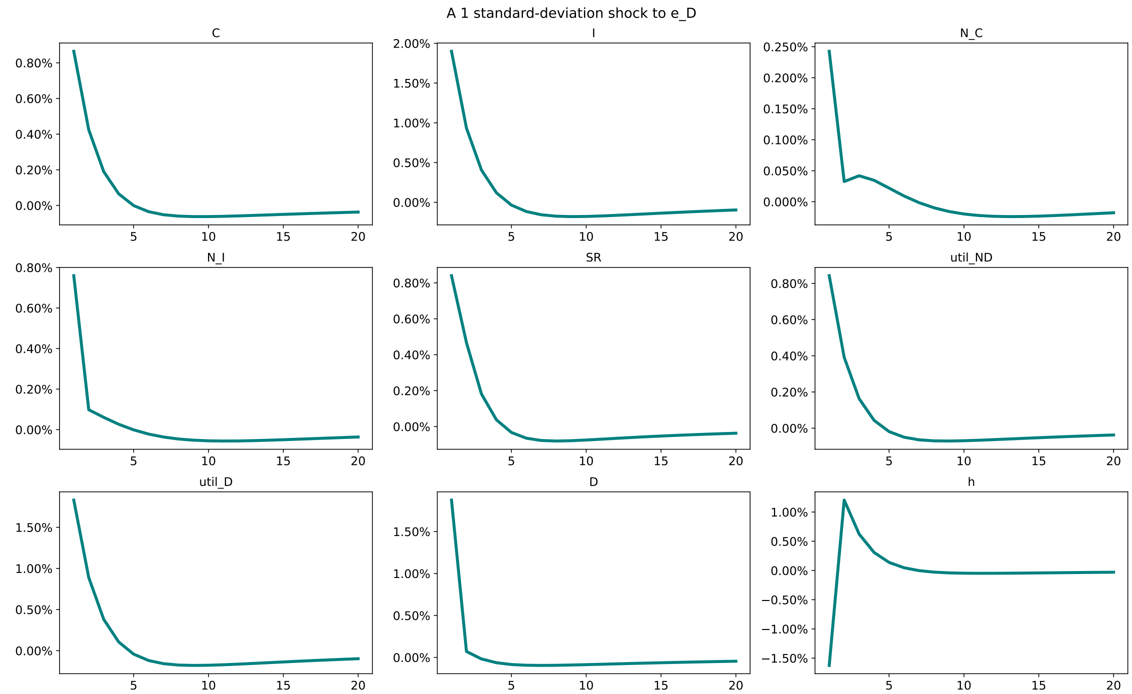

Figure 8: A unit standard deviation negative shock e_d to shopping effort in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

in utilization in the durables sector but a decrease in utilization in the nondurables sector. It is worth noting that due to limited factor mobility, labor in the consumption sector does not decrease, but the rise in labor in the investment sector is more pronounced. Hence, search demand shocks are unique in producing positive comovement in the growth rates of all series.

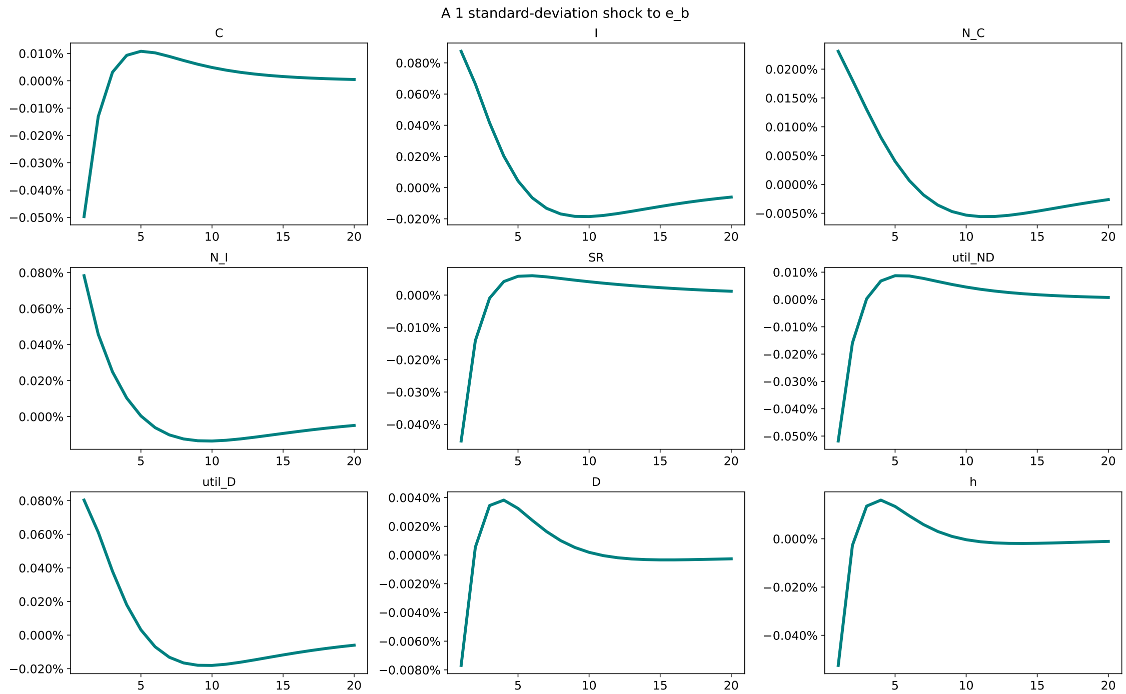

Figure 9: A unit standard deviation negative shock e_b to the discount factor in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

What about technology shocks? It may appear that technology shocks can generate all the comovement properties as search demand shocks. To that end, Figure 10 plots the impulse response to a unit standard deviation neutral stationary technology shock. The Solow residual rises about half as much on impact as under a demand shock. The shock generates positive comovement in consumption and investment, as well as in the labor input of each sector. Thus, a positive technology shock is consistent with sectoral comovement as described by Christiano and Fitzgerald (1998) and Katayama and Kim (2018). Limited factor mobility and moderate short-run wealth effects play a role in generating this feature. However, utilization in nondurables, part of the consumption sector, actually falls before rising. The technology boost increases the expected return on investment, thereby incentivizing an immediate rise in utilization in the durable sector. Only after the effects of the technology shock subside and households enjoy greater resources does utilization in nondurables respond positively.

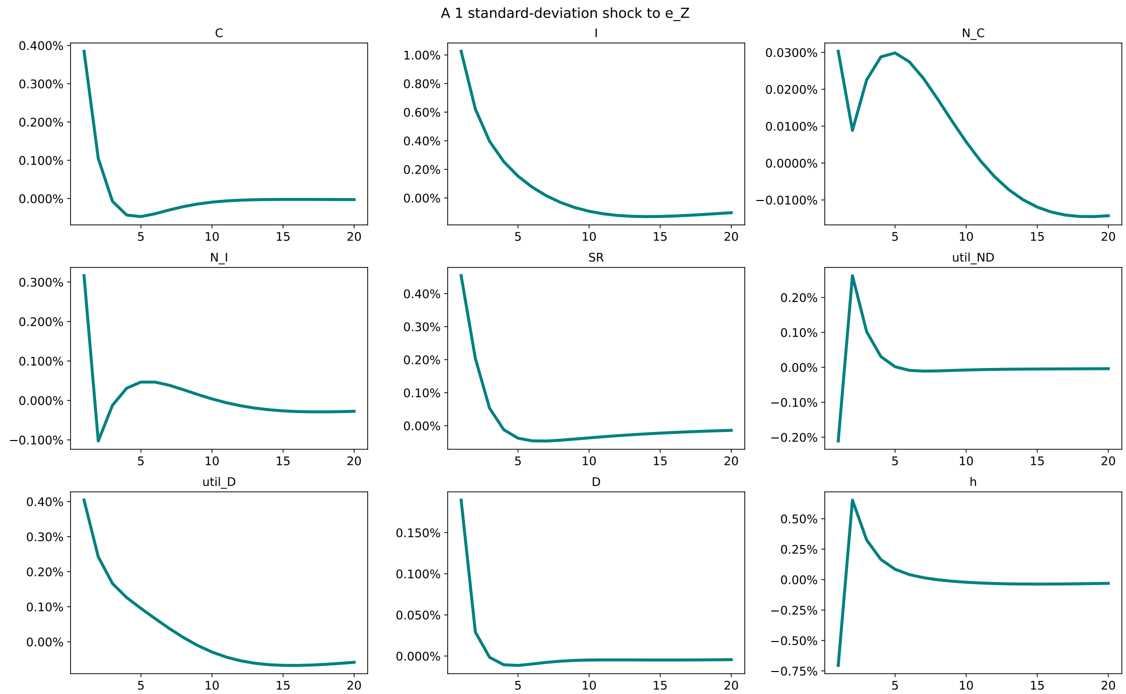

Figure 10: A unit standard deviation negative shock e_z to technology in the baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

8. Conclusion

We use Bayesian methods to estimate a three-sectoral DSGE model which features a demand channel operating via goods market frictions: value added is generally below capacity. Demand shocks generate fluctuations in capacity utilization which influence the Solow residual. To estimate the model, we adopt a novel approach that utilizes sectoral data on utilization in both the nondurables and durables sectors, along with data on labor hours and output in the consumption and investment sectors. This unique combination of data allows us to implicitly incorporate information on sectoral productivity while also subjecting the model to a rigorous test. We ask the model to not only fit dynamics of capacity utilization overall but also to capture the comovement of utilization between different sectors of the economy.

Even without making use of shopping time, we estimate reasonably high and precise values of the elasticity of the matching function with respect to shopping effort and associated shocks. We show that we can approximately recover the key parameters for the model mechanism by estimating the model on synthetic data obtained form the model. Moreover, shocks to shopping effort and its news component explain a major part of the forecast error variance decomposition of output, the Solow residual, the relative price of investment, hours, and utilization. In terms of empirical fit, the model explains comovement in labor input, output, and utilization well; as well as the volatility of the utilization series.

The role of different model ingredients in capturing the dynamics of the data is crucial to understanding the model's fit. Eliminating fixed costs reduces model fit, but the main findings remain unchanged. On the other hand, excluding variable capital utilization has a more significant impact, as it eliminates the model's ability to match the autocorrelation of the utilization series. However, the most detrimental effect is observed when search-based demand shocks are removed. In this case, the model completely fails to fit the data, resulting in volatilities that are two orders of magnitude higher than the empirical series and correlations that are close to unity. The reason is that search demand shocks are unique in matching all comovement properties. Insofar as sectoral comovement is viewed as an important test of economic models, it should be interpreted more broadly to encompass the positive correlation of capacity utilization.

This setting actually tilts the playing field in favor of technology shocks by not making use of nominal rigidities or otherwise imposing the findings from the literature that technology shocks reduce labor input int he short run (Gali (1999), Basu, Fernald, and Kimball (2006), Francis and Ramey (2005)). This deliberate abstraction aims to highlight the argument in favor of the search-based demand channel, without relying on monetary policy transmission or using monetary variables in estimation.

Nevertheless, it would be extremely fruitful to apply this framework within a monetary setting that considers the implications of demand shocks for inflation and interest rates, and includes those variables as observables. This would provide a more comprehensive understanding of the interactions between goods market frictions, demand shocks, and monetary policy.

Additionally, it is very natural to integrate goods market frictions, demand shocks, and unemployment, a key motivation of papers like Michaillat and Saez (2015). Labor market frictions facilitate sectoral comovement and provide additional persistence, and unemployment is one of the major outcome variables which has historically inspired intellectual work on aggregate demand, as seen in the seminal work of Keynes (1936).

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Appendix A. Data appendix

Table A.9 provides the details on constructing the model variables, which are used for summary statistics and Bayesian estimation.

Symbol	Description	Construction
C	Nominal consumption	PCND+PCESV
I	Nominal gross private domestic investment	PCDG+PNFI+PRFI
$Deflator$	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
P_c	Price index: consumption	PCEPI
P_i	Price index: investment	INVDEV
c	Real per capita consumption	$\frac{C}{Pop*P_c}$
i	Real per capita investment	$\frac{I}{Pop*P_i}$
y	Real per capita output	$c+i$
n_c	Labor in consumption sector	Labor in nondurables and services
n_i	Labor in investment sector	Labor in construction and durables
n	Aggregate labor	n_c+n_i
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: nondurables	TCU
$util_D$	Total capacity utilization: durables	TCU
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR$_{util}$	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Table A.9: Data sources used in motivating evidence and estimation.

The construction of sectoral data follows Katayama and Kim (2018). We obtain consumption and investment as follows:

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)}\right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)}\right)$$

We use an HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates.

For labor data, we make use of the BLS Current Employment Statistics (https://www.bls.gov/ces/data). BLS Table B6 contains the number of production and non-supervisory employees by industry, and BLS Table B7 contains average weekly hours of each sector. We compute total hours for nondurables, services, construction, and durables by multiplying the relevant components of each table. Then we impute labor in consumption as sum of labor in nondurables and services. Similarly, we construct labor in investment as sum of labor in construction and durables.

We also make use of disaggregated data on total capacity utilization from the Federal Reserve Board. Estimates are available for 89 detailed industries (71 manufacturing, 16 mining, 2 utilities) and also for several industry groups. Our focus is on durables and nondurables. The data can be downloaded at https://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17.

Appendix B. Details of household and firm problem

Competitive search creates additional interconnections between the household and firm problems. A complete characterization requires solving both jointly. We start with the household problem. Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_{mc}, \mu_{sc}, \mu_i$ be the respective Lagrangian multipliers on the constraints. The first order conditions are

$$[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc}$$

$$[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc}$$

$$[i_j]: \quad -\gamma_i - \lambda p_j + \mu_j \left(1 - S_j'(x_j)x_j - S_j(x_j)\right) + \beta\theta_b \mathbb{E}\mu_j' S_j'(x_j')(x_j')^2 = 0 \tag{B.1}$$

$$[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \tag{B.2}$$

$$[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \tag{B.3}$$

$$[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \tag{B.4}$$

$$[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \tag{B.5}$$

$$[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \tag{B.6}$$

$$[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\} \tag{B.7}$$

The multipliers $\gamma_{mc}, \gamma_{sc}, \gamma_i$ reflect the value of an additional unit of traded output. In the consumption submarkets, these represent a wedge between the marginal utility of consumption and the marginal utility of wealth. For investment, the multiplier γ_i represents an analogous wedge between the marginal utility of wealth and value of the investment good. Equations (B.2) and (B.3) equate the marginal shopping disutility to the additional units obtained by search multiplied by the value of the unit. Equations (B.4) and (B.5) equate the marginal disutility of work in each sector to the (variable) wage multiplied by the marginal utility of wealth. Equation (B.6) equates the marginal cost of depreciated capital to the value of additional output generated in terms of consumption. Finally, (B.7) equates the marginal value of capital to the expected discounted rate of return, composed of the rental income and value of undepreciated capital.

We next characterize the envelope conditions:

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc, i\} \tag{B.8}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j \gamma_j \quad j \in \{mc, sc, i\} \tag{B.9}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} \gamma_j \quad j \in \{mc, sc, i\}$$

The ratio of (B.8) and (B.9) characterizes the indifference curve between price and tightness in a submarket:

$$\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = -\frac{\lambda D_j}{(\phi - 1)\gamma_j} \tag{B.10}$$

We next turn to the firm's problem. The firm chooses labor type s in sector j so as to generate an effective labor bundle n_j at the lowest possible cost. The problem is

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.} \tag{B.11}$$

$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n} \tag{B.12}$$

Take the first order condition of (B.11) and recognize W_j as the Lagrangian multiplier on constraint (B.12). Rearrange as

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

The corresponding wage index for composite labor input in sector j is

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

We can now examine the simplified firm problem. Let ι_j and ∇_j be the multipliers on participation constraint and production technology. The first order conditions are

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n \tag{B.13}$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k \tag{B.14}$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{B.15}$$

$$[D_j] \quad \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{B.16}$$

Take the ratio of first order conditions (B.15) and (B.16) to alternately characterize the indifference curve between price and tightness:

$$\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = \frac{D_j}{\phi p_j}$$

Plug in (B.10) to find

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)\gamma_j}$$

which we rearrange as

$$\gamma_j = \frac{\phi}{1-\phi} \lambda p_j$$

Since $\gamma_j = u_j - \lambda p_j$ for $j = \{mc, sc\}$, we have

$$\lambda = (1-\phi)\frac{u_j}{p_j} \tag{B.17}$$

which allows us to characterize γ_i:

$$\gamma_i = \phi \frac{u_j}{p_j} p_i \quad j \in \{mc, sc\}$$

Note that (B.17) also implies that the marginal utility relative to the price is the same in each consumption subsector. The values of γ_{mc}, γ_{sc} and λ allows us to rewrite the shopping optimality conditions and labor leisure tradeoff:

$$-u_d = \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{mc, sc\}$$
$$-u_d \theta_i = \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]$$

$$u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}$$

We next revisit the investment first order condition (B.1) and characterize Tobin's Q. For sector $j \in \{mc, sc, i\}$ we have

$$\begin{aligned} \lambda p_i + \gamma_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\ \lambda p_i + \frac{\phi}{1-\phi}\lambda p_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\ \frac{\lambda p_i}{1-\phi} &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \end{aligned}$$

Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption. Using Q_j rewrite the choice of optimal investment as

$$\frac{p_i}{1-\phi} = Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda} Q_j' S_j'(x_j')(x_j')^2$$

We also use Tobin's Q to rewrite the optimal utilization in $j \in \{mc, sc, i\}$ and the Euler equation:

$$\begin{aligned} \delta_h(h_j)Q_j &= R_j \\ Q_j &= \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j' h_j'\right] \end{aligned}$$

It remains to solve for the Lagrangian multipliers ι_j and ∇_j on the firm problem. This is straightforward given λ and γ_j. First,

$$\iota_j = \frac{A_j q_j^{\phi} F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

Second,

$$\begin{aligned} \nabla_j &= p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j} \\ &= p_j A_j D_j^{\phi} + \frac{A_j D_j^{\phi} \gamma_j}{\lambda} \\ &= p_j A_j D_j^{\phi} + A_j D_j^{\phi} \frac{\phi}{1-\phi} p_j \\ &= A_j D_j^{\phi}\left(p_j + \frac{\phi}{1-\phi} p_j\right) \\ &= \frac{p_j A_j D_j^{\phi}}{1-\phi} \end{aligned}$$

The value of additional production capacity ∇_j exceeds the additional sales $p_j A_j D_j^{\phi}$. This is because the additional sales also relax the participation constraint of households. Finally, the value of these multipliers enables us to characterize the factor demands for the firms. Substitute for ∇_j in (B.13) to find

$$\begin{aligned}(1-\phi)\frac{W_j}{p_j} &= A_j(D_j)^{\phi} z_j \frac{\partial f(h_j k_j, n_j)}{\partial n} \\ &= \frac{\alpha_n}{n_j} A_j D_j^{\phi} z_j f(h_j k_j, n_j) \\ &= \frac{\alpha_n}{n_j} A_j D_j^{\phi} \left(\frac{y_j}{A_j D_j^{\phi}} + \nu_j \right) \\ &= \frac{\alpha_n}{n_c} (y_j + A_j D_j^{\phi} \nu_j) \\ &= \frac{\alpha}{n_c} y_j (1+\nu^R)\end{aligned}$$

where we use $\nu_j^R = \nu_j \Psi_T / y_j$. We can simplify the capital demand (or rental rate) (B.14) using ratios as

$$\frac{W_j}{R_j} = \frac{\alpha_n}{\alpha_k} \frac{h_j k_j}{n_j}$$

Aggregating across sectors, the steady-state labor labor of income is $\alpha_n(1+\nu^R)/(1-\phi)$ and the capital share of income is $\alpha_k(1+\nu^R)/(1-\phi)$.

Appendix C. Calibration

In general, we determine some (fixed) parameters from long-run targets, estimate the parameter set Θ described in the main text, and back out the remaining (dependent parameters) given draws from Θ and long-run targets. The dependent parameters are thus random variables. Here we use the term calibration more broadly to characterize the determination of dependent parameters as a function of both estimated parameters and long-run targets.

Several key targets used for calibration are investment-to-output $p_i I/Y$, capital-to-output $p_i k/Y$, the labor share of income, the unconditional growth rate $\overline{g}$, and share of services S_c in consumption. In terms of model variables at quarterly frequency, we have

$$\kappa \equiv p_i I/Y = 20\%,\ p_i k/Y = 2.75(4) = 11,\ \overline{g} = 0.45\%,\ \tau \equiv \frac{nW}{Y} = 67\%,\ S_{sc} \equiv \frac{p_{sc} y_{sc}}{C} = 65\%$$

The first two targets are identical to Bai, Rios-Rull, and Storesletten (2024), and the third corresponds to 1.8% per capital annual growth, which is very close to the average over the data sample. Capital accumulation (ignoring adjustment costs) in transformed variables [14] is given by

$$g\widehat{k}' = (1-\delta)\widehat{k} + g\widehat{I}$$

Balanced growth, in terms of original variables, implies a steady state in terms of $\widehat{k}$, such that

$$\delta = 1 - \overline{g} + \frac{I}{k} \approx 1.37\%$$

Next, we characterize α_n, α_k and σ_b. Labor demand (14) for each sector implies

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1 + \nu_j^R)$$

where $\nu_j^R = \nu_j X / F_j$. The steady state labor share is thus

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi} \frac{C + p_i I}{Y} (1 + \nu_{ss}^R) = \frac{\alpha_n}{1-\phi} (1 + \nu_{ss}^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1 + \nu_{ss}^R)$.

In steady state, the rate of return on capital in each sector is equal, so we let R denote the common value: $R = R_j$ for all j. It is helpful to use the interest rate r on an illiquid bond as the value which satisfies $\beta \overline{g}^{-\sigma} = 1/(1+r)$.

The Euler equations in the steady state imply

$$\begin{aligned} Q &= \beta \overline{g}^{-\sigma}[(1-\delta)Q + R] \Rightarrow \\ (1+r)Q &= (1-\delta)Q + R \\ (r+\delta)Q &= R \end{aligned}$$

Given that capital utilization $h_j = 1$ for all j in the steady state, the parameter σ_b satisfies

$$\sigma_b = \frac{R}{Q} = r + \delta$$

Combining with Tobin's Q, $p_i/(1-\phi) = Q$, we have

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

[14] Investment is divided by the stochastic trend $\hat{I}_t = I_t / X_t$ while the capital stock is divided by the lagged stochastic trend $\hat{K}_t = K_t / X_{t-1}$ to maintain its status as a predetermined variable.

Now, turn to the firm demand for capital (15):

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{k_j}(1+\nu^R)$$

An immediate corollary is that $Y_j/k_j = Y/k$ for all k and hence

$$r+\delta = \alpha_k \frac{Y}{k}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{k}{Y}$$

We pin down the weight of services ω_{sc} as the empirical measure $S_c = p_{sc}Y_{sc}/C$ and set $S_c = 0.65$. The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

and plug in S_c:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

so that $p_{mc} = p_{sc}$. Since we normalize $p_{sc} = 1$ and have also normalized the consumption price index to unity, we have $p_{mc} = p_{sc} = p_c = 1$.

Given the target for capacity utilization $\Psi_{T,j}$, we wish to find the corresponding level coefficient $A_j = \Psi_{T,j}/D_j^{\phi}$. This entails solving for each D_j. We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Now, take the ratio of the shopping conditions rearrange for relative shopping effort:

$$\frac{D_{mc}}{D_{sc}} = \frac{p_{mc}}{p_{sc}}\frac{Y_{mc}}{Y_{sc}} = \frac{1-S_c}{S_c} \tag{C.1}$$

Similarly,

$$\frac{D_j}{D_i} = S_j \frac{1 - I/Y}{I/Y} \tag{C.2}$$

Now, we put (C.1) and (C.2) together to characterize shopping effort in each sector:

$$D_{mc} = (1 - S_c)(1 - I/Y)D$$
$$D_{sc} = S_c(1 - I/Y)D$$
$$D_i = (I/Y)D$$

Appendix D. Cyclical deviations of Solow residual and total capacity utilization

In the main text we analyze the relationship between the Solow residual and capacity utilization in growth rates. Here we compare them in terms of cyclical deviations. Using (18), the cyclical component of the Solow residual is

$$\hat{SR}_j \equiv \frac{SR_j}{X^\tau} = \frac{A_j D_j^\phi z_j h_j^{\alpha_k} g^{1-\alpha_k-\tau} \hat{k}_j^{\alpha_k - 1 + \tau} n_j^{\alpha_n - \tau}}{1 + \nu_j^R} = g^{1-\tau} \frac{\hat{Y}_j}{\hat{k}_j^{1-\tau} n_j^\tau}$$

The log linear representation is

$$\widetilde{\hat{SR}}_j = \phi \widetilde{D}_j + \widetilde{z}_j + \alpha_k \widetilde{h}_j + (1 - \alpha_k - \tau)\widetilde{g} + (\alpha_k - 1 + \tau)\widetilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j - \frac{\nu_{ss}^R}{1 + \nu_{ss}^R} \widetilde{\nu}_j^R$$

and note that $\widetilde{g}_t = \log g_t - \log \overline{g}$ which is first-order equivalent to X^{obs}. Log linearizing (21) yields

$$\widetilde{util}_j = \phi \widetilde{D}_j + (1 + \nu_{ss}^R)\alpha_k \widetilde{h}_j$$

Thus, in the absence of fixed costs, we have

$$\widetilde{\hat{SR}}_j|_{\nu_j=0} = \widetilde{util}_j + \tilde{z}_j + (1 - \alpha_k - \tau)(\log g_t - \log \overline{g}) + (\alpha_k - 1 + \tau)\widetilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j$$

Given the detrending, the coefficient on nonstationary technology is $1 - \alpha_k - \tau$ rather than $1 - \alpha_k$. Otherwise, the relationship between cyclical components of the Solow residual and utilization has the same form as the one in growth rates.

The relationship between the cyclical form and growth rate form is

$$dSR_t = \Delta \log SR_t$$

$$= \log \hat{SR}_t + \tau \log X_t - (\log \hat{SR}_{t-1} + \tau \log X_{t-1})$$

$$= \Delta \widetilde{\hat{SR}}_{jt} + \tau \log g_t$$

The growth rate of the Solow residual equals the growth rate of cyclical deviations plus the log deviation of the stochastic trend growth rate relative to the unconditional mean multiplied by the labor share.

Appendix E. Shopping costs in the form of expenditure

Michaillat and Saez (2015) also use matching frictions in the goods market and emphasizes the impact of aggregate-demand shocks on output and employment. At first glance, it is difficult to compare the two settings because Michaillat and Saez (2015) specify the matching frictions differently, formalize matching costs in terms of expenditure rather than disutility, and also incorporate money demand via money in the utility. Accordingly, we represent matching costs in terms of expenditures in a static form of BRS and show that the same key logic holds. However, the labor share of income turns out to be different since expenditure shows up in the national income accounts, but effort does not.

As in the static model in the main text, each firm has a location production function $F = zn^{\alpha_n}$ using just labor. Each unit of search requires an expenditure ρ. In terms of national income accounting, these expenditures are part of consumption, but they yield no utility to households. The remaining part of consumption, c^e, does directly yield utility.

Household preferences take the form $u(c^e, n) = U(\Gamma)$ where U is increasing, strictly concave, and differentiable

$$\Gamma = c^e - \theta_n \frac{n^{1+1/\zeta}}{1 + 1/\zeta}$$

Thus, there are zero wealth effects on labor supply (GHH).

The link between effective consumption and overall consumption satisfies

$$c^e = C - d\rho$$

$$= d(\Psi_d F - \rho)$$

The necessary units of shopping to consume one service are $1/(\Psi_d F - \rho)$. The associated expenditures are thus

$$T(D) = \frac{\rho}{\Psi_d F - \rho} \tag{E.1}$$

The expression for T in (E.1) differs from the analogue in Michaillat and Saez (2015) only by the fact that the Ψ_d is multiplied by capacity F, which is a consequence of one unit traded per match in their setup.

A household who chooses a particular submarket (p, D) has expenditure $pc^e(1+T(D)) = pC$ and associated income $\pi + nW$, where π denotes firms' profits.

The problem of the household in submarket (p, D) is

$$\max u(c^e, n) \quad s.t.$$
$$pc^e(1 + T(D)) = \pi + nW$$

The first order conditions with respect to c and n yield the following labor-leisure or labor supply condition:

$$\theta_n n^{1/\zeta} = \frac{W/p}{1 + T(D)}$$

The search wedge $1/(1+T(D))$ reduces the return to working, analogous to a consumption tax or labor income tax.

We next solve the problem of the firm. To keep customers from deviating to another submarket, it must post a combination of price and tightness (p, D) such that $p(1 + T(D)) \leq H$ for some H. The problem is

$$\max_{n,p,D} p\Psi_T(D)zn^{\alpha_n} - nW \quad s.t.$$
$$p(1 + T(D)) \leq H$$

The first order condition for n is

$$\alpha_n \frac{\Psi_T F}{n} = W$$

Aggregate consumption satisfies $C = \Psi_T F$, so that $nW/C = \alpha_n$. Hence, the labor share of income is α_n. By contrast, if the matching costs were in terms of disutility, then the corresponding labor share of income would be $\alpha_n/(1 - \phi)$.

The problem over the price-tightness pair (p, D) can be simplified by substituting for the constraint in the objective as

$$\frac{\Psi_T(D)}{1 + T(D)} = \frac{\Psi_T}{\Psi_D}(\Psi_d F - \rho) = \frac{D}{F}(AD^{\phi-1}F - \rho)$$

Differentiating with respect to D yields

$$\rho = \phi \Psi_D F$$

or, in closed form,

$$D = \left(\frac{\phi A z n^{\alpha_n}}{\rho} \right)^{1/(1-\phi)} \tag{E.2}$$

Notice that (E.2) depends not only on both the parameters of matching technology ϕ, A and cost ρ but also on z and n.

Thus, we normalize $p = 1$ and define equilibrium as a tuple (D, C, c^e, n, W) satisfying

$$\begin{aligned}
\rho &= \phi \Psi_D \\
C &= A D^{\phi} z n^{\alpha_n} \\
c^e &= \frac{C}{1 + T(D)} \\
W &= \frac{\alpha_n C}{n} \\
\theta_n n^{1/\zeta} &= \frac{W}{1 + T(D)}
\end{aligned}$$

Compared to the baseline setup, the wedge on labor supply is now $1/(1 + T(D))$ instead of $1 - \phi$ and the labor share of income is α_n. Moreover, the cost of shopping is linear, which is analogous to letting $\eta \to \infty$ in the BRS specification.

A key difference in the labor share of income is that purchased shopping services are still part of GDP. Thus, the Solow residual is $SR = C/n^{\alpha_n} = AD^{\phi} z$. Both matching frictions and technology enter into GDP, but, unlike BRS, there is no input share mismeasurement.

Michaillat and Saez (2015) argue that the effect of aggregate demand shocks on output and employment depends on sticky prices. The reason is that the demand shocks they consider–consumption preference or money supply–do not affect *efficient* level of market tightness. Under competitive search, tightness is necessarily at the efficient level, so some deviation would thus be necessary for such demand shocks to matter.

However, under the matching setup considered here, the efficient level of market tightness also depends on labor hours and technology. It follows that any demand shock that affects labor demand also raises D and the Solow residual. In the current bare-bones setup, a reduction in θ_n stimulates labor demand, which raises shopping and tightness. Additionally, we included money as Michaillat and Saez (2015), then a consumption preference shock or shock to the level of money supply would also affect labor and hence tightness.

In general, the influence of labor hours on the efficient level of tightness holds provided that the expenditure ρ does not scale one-for-one with capacity. If the cost of a shopping were ρF instead of ρ, then we would instead have $T = \rho/(\Psi_d - \rho)$ and D would be determined by $\rho = \phi\Psi_d$. The efficient level of tightness would just depend on ϕ, A, and ρ. We believe it plausible a priori that shopping expenditure costs scale less than one-for-one with firm capacity, though of course parsing these micro-level features require more granular data and research.

Appendix F. Equilibrium in basic BRS model

Given initial states $\{k_{c0}, k_{i0}\}$ and $\{g_0, \theta_{d0}, \theta_{n0}, z_{c0}, z_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{ct}, R_{it}, W_t\}_{t=0}^{\infty}$ and quantities $\{k_{ct}, k_{it}, k_t, C_t, I_t, D_{ct}, D_{it}, D_t, n_{ct}, n_{it}, n_t, g_t, \theta_{dt}, \theta_{nt}, z_{ct}, z_{It}\}_{t=0}^{\infty}$ which solve the following system given the realization of shocks $\{e_{gt}, e_{vt}\}_{t=0}^{\infty}$:

$$\theta_{nt} n_t^{1/\nu} = (1-\phi) W_t$$

$$\theta_{dt} D_t^{1/\eta} = \phi \frac{C_t}{D_{ct}}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{it} \frac{I_t}{D_{it}}$$

$$\Gamma_t = C_t - \theta_{dt} \frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt} \frac{(n_t)^{1+1/\zeta}}{1+1/\zeta}$$

$$\Gamma_t^{-\sigma} p_{it} = \beta \mathbb{E}\left\{[(1-\phi) R_{c,t+1} + p_{i,t+1}(1-\delta)](\Gamma_{t+1} g_{t+1})^{-\sigma}\right\}$$

$$\mathbb{E}(R_{c,t+1} - R_{i,t+1}) = 0$$

$$C_t = A_c (D_{ct})^{\phi} z_{ct} g_t^{-\alpha_k} k_{ct}^{\alpha_k} n_{ct}^{\alpha_n}$$

$$I_t = A_i (D_{it})^{\phi} z_{it} g_t^{-\alpha_k} k_{it}^{\alpha_k} n_{it}^{\alpha_n}$$

$$I_t g_t = (k_{c,t+1} + k_{i,t+1}) g_t - (1-\delta)(k_{ct} + k_{it})$$

$$(1-\phi) \frac{W_t}{p_t} = \alpha_n \frac{C_t}{n_{ct}} \quad j \in \{c, i\}, \quad \text{with} \quad p_{ct} = 1$$

$$\frac{W_t}{R_{jt}} = \frac{\alpha_n}{\alpha_k} \frac{k_{jt}}{n_{jt}} \quad j \in \{c, i\}$$

$$n_t = n_{ct} + n_{it}, k_t = k_{ct} + k_{it}, D_t = D_{ct} + D_{it}$$

$$\log g_t = (1-\rho_g)\overline{g} + \rho_g \log g_{t-1} + e_{gt}$$

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, v \in \{\theta_d, \theta_n, z_c, z_I\}$$

$$\log z_{it} = \log z_{ct} + \log z_{It}$$

Appendix G. Equilibrium of baseline model

Given initial states $\{k_{mc0}, k_{sc0}, k_0\}$ and $\{g_0, \theta_{b0}, \theta_{d0}, \theta_{i0}, \theta_{n0}, z_{c0}, z_{I0}, \mu_{c0}, \mu_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{jt}, Q_{jt}, W_{ct}, W_{it}\}_{t=0}^{\infty}$ and quantities $\{k_{jt}, i_{jt}, Y_{jt}, C_t, D_{jt}, n_t^a, n_{jt}, n_{ct}, n_t, g_t, \theta_{bt}, \theta_{dt}, \theta_{it}, \theta_{nt}, z_{ct}, z_{It}, \mu_{ct}, \mu_{it}\}_{t=0}^{\infty}$ for $j \in \{mc, sc, i\}$ that solves the following system given the realization of shocks $\{e_{gt}^0, e_{gt}^4, e_{vt}^0, e_{vt}^4\}_{t=0}^{\infty}$:

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{ct}}{n_t^a}\right)^{\theta} \omega^{-\theta} = (1-\phi)\frac{W_{ct}}{\mu_{ct} S_t}$$

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{it}}{n_t^a}\right)^{\theta} (1-\omega)^{-\theta} = (1-\phi)\frac{W_{it}}{\mu_{it} S_t}$$

$$n_t^a = \left[\omega^{-\theta} n_{ct}^{1+\theta} + (1-\omega)^{-\theta} n_{it}^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

$$S_t = \left(C_t - haC_{t,-1} - \theta_{dt}\frac{D_t^{1+1/\eta}}{1+1/\eta}\right)^{\gamma} S_{t-1}^{1-\gamma}$$

$$\Gamma_t = C_t - \theta_{dt}\frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt}\frac{(n_t)^{1+1/\zeta}}{1+1/\zeta}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{jt}\frac{Y_{jt}}{D_{jt}} \quad j \in \{mc, sc\}$$

$$\theta_{it}\theta_{dt} D_t^{1/\eta} = \phi p_{it}\frac{I_t}{D_{it}}$$

$$\frac{p_{it}}{1-\phi} = Q_{jt}[1 - S_j'(x_{jt})x_{jt} - S_j(x_{jt})] + \beta\theta_b \mathbb{E}t\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma} Q_{j,t+1} S_j'(x_{j,t+1})(x_{j,t+1})^2 \quad j \in \{mc, sc, i\}$$

$$Q_{jt} = \beta\theta_{bt}\mathbb{E}t\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma}\left[(1-\delta_j(h_{j,t+1}))Q_{j,t+1} + R_{j,t+1}h_{j,t+1}\right] \qquad j \in \{mc, sc, i\}$$

$$C_t = [\omega_c^{1-\rho_c} Y_{mc,t}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc,t}^{\rho_c}]^{1/\rho_c}$$

$$Y_{jt} = p_{jt}^{-1/(1-\rho_c)}\omega_j C_t \quad j \in \{mc, sc, i\}$$

$$C_t = p_{mc,t}Y_{mc,t} + p_{sc,t}Y_{sc,t}$$

$$\delta_h(h_{jt})Q_{jt} = R_{jt}, \quad j \in mc, sc, i$$

$$Y_{jt} = A_j(D_{jt})^{\phi}(z_{jt}g_t^{-\alpha_k}(h_{jt}k_{jt})^{\alpha_k}(N_{jt})^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\}$$

$$k_{j,t+1}g_t = (1-\delta_j(h_{jt}))k_{jt} + [1 - S_j(x_{jt})]I_{jt}g_t \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{W_{jt}}{p_{jt}} = \alpha_n\frac{Y_{jt} + A_j D_{jt}^{\phi}\nu_j}{n_{jt}} \quad j \in \{mc, sc, i\}$$

$$\frac{W_{jt}}{R_{jt}} = \frac{\alpha_n}{\alpha_k}\frac{h_{jt}k_{jt}}{n_{jt}} \quad j \in \{mc, sc, i\}$$

$$n_{ct} = n_{mct} + n_{sct}, n_t = n_{ct} + n_{it}, D_t = D_{mct} + D_{sct} + D_{it}$$

$$k_t = k_{mct} + k_{sct} + k_{it}, I_t = I_{mct} + I_{sct} + I_{it}$$

$$\log g_t = (1-\rho_g)\log \bar{g} + \rho_g \log g_{t-1} + e^0_{g,t} + e^4_{g,t-4}$$

$$\log v_t = \rho_f \log v_{t-1} + e^0_{v,t} + e^4_{f,t-4}, \quad v \in \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}, \quad e^4_{\theta_n,t-4} = 0\,\forall t$$

Appendix H. The forecast error variance decomposition for specific demand and technology shocks

Here we decompose the variance decomposition of demand and technology shocks into the contribution of its subcomponents. The main takeaway from Table H.10 is that the unanticipated component of neutral search demand shocks matter the most overall. However, the anticipated component of investment-specific search demand shocks is very important for the relative price of investment and also matters a significant amount for investment and its labor component.

Table H.10: Forecast error variance decomposition

	e_D	$e_{D,\text{news}}$	e_{DI}	$e_{DI_{\text{news}}}$
Y	93.61	1.14	0.08	5.16
SR	92.91	1.06	0.11	5.92
I	77.04	0.85	0.35	21.76
p_i	6.12	0.12	0.98	92.77
n_c	80.37	1.76	0.21	17.66
n_i	70.78	1.08	0.23	27.91
$util$	93.91	1.14	0.08	4.88
D	98.20	1.49	0.00	0.30
h	90.95	1.72	0.05	7.28

Table H.10: Contribution of components to forecast error variance decomposition of search shocks.

In a similar vein, H.11, dissects the various constituent elements of technology shocks. Notably, the least significant factors by a considerable margin are an unanticipated shock to the stochastic trend growth or a news shock pertaining to investment-specific technological change. Conversely, when considering the Solow residual and output, the most pivotal contributors are news shocks associated with the stochastic trend growth and unanticipated shocks to stationary neutral technology. However, both the anticipated and unanticipated components of stationary neutral productivity shocks play a crucial role in elucidating variations in utilization.

Table H.11: Forecast error variance decomposition

	e_g	$e_{g_{\text{news}}}$	e_Z	$e_{Z_{\text{news}}}$	e_{ZI}	$e_{ZI_{\text{news}}}$
Y	4.30	33.78	35.27	19.91	6.50	0.24
SR	4.94	38.72	31.14	16.70	8.23	0.27
I	0.89	6.83	42.13	20.60	28.54	1.01
p_i	0.01	0.07	23.26	15.94	57.85	2.86
n_c	2.59	23.97	18.96	19.74	33.09	1.64
n_i	1.75	16.13	20.72	19.43	39.37	2.60
$util$	0.22	4.27	39.98	33.81	20.19	1.53
D	1.94	23.11	42.21	26.15	6.17	0.42
h	0.51	3.03	46.53	41.13	8.16	0.64

Table H.11: Contribution of components to forecast error variance decomposition of technology shocks.

Appendix I. Estimation on artificial data and identification of parameters

To assess the identifiability of key parameters, we conduct an analysis employing artificial data inspired by Schmitt-Grohé and Uribe (2012). This involves setting the parameters at their posterior mean values and following the calibration strategy outlined in Section Appendix C. We generate an artificial dataset comprising 223 observations for each observable variable. Subsequently, we estimate the model using this artificial data, employing the same estimation techniques as in the baseline model, and incorporating the same prior distributions.

Table I.12 plots the true value used in generating the artificial data alongside the 5th, 50th, and 95th percentiles of the posterior distribution for each parameter value. We find that the highest posterior density intervals usually contain and are often even centered around the true parameter value. In particular, the posterior median for ϕ, 0.866, is very close to 0.864. The parameters associated with search demand shocks $\rho_D, \rho_{DI}, e_{D,-4}, e_{DI,-4}$ are also well identified. Interestingly, the persistence of stationary technology shocks is well-identified (true value 0.708 compared to posterior median 0.714) but that of permanent technology shocks substantially less so (true value 0.435 compared to posterior median 0.243).

Table I.12: Estimation on artificial data

		Posterior distribution		
Parameter	True value	Median	5%	95%
σ	1.903	1.810	1.601	2.215
ha	0.431	0.390	0.356	0.427
ζ	1.856	1.419	1.132	1.760
γ	0.305	0.307	0.268	0.349
ϕ	0.864	0.866	0.828	0.903
η	0.560	0.643	0.559	0.725
ξ	0.905	0.847	0.730	0.967
ν_R	0.346	0.303	0.216	0.369
σ_{ac}	1.618	1.429	1.053	1.876
σ_{ai}	0.567	0.455	0.309	0.608
Ψ_c	4.948	4.766	3.979	5.704
Ψ_i	4.910	3.642	2.751	4.331
θ	1.492	1.680	1.366	1.992
ρ_g	0.435	0.243	0.103	0.306
ρ_Z	0.708	0.714	0.598	0.830
ρ_{ZI}	0.922	0.903	0.844	0.952
ρ_N	0.834	0.699	0.549	0.820
ρ_D	0.933	0.931	0.893	0.969
ρ_{DI}	0.995	0.967	0.942	0.999
ρ_b	0.861	0.719	0.577	0.873
$\rho_{\mu c}$	0.975	0.834	0.746	0.921
$\rho_{\mu i}$	0.981	0.930	0.892	0.972
e_g	0.00277	0.00358	1.93e-07	0.00882
$e_{g,-4}$	0.00843	0.00818	0.00232	0.0103
e_Z	0.00440	0.00505	0.00393	0.00621
$e_{Z,-4}$	0.00384	0.00388	0.00227	0.00531
e_{ZI}	0.0109	0.00992	0.00878	0.0111
$e_{ZI,-4}$	0.00252	0.00165	0.000100	0.00361
e_N	0.00296	0.00215	0.000100	0.00380

e_D	0.0411	0.0353	0.0304	0.0406
$e_{D,-4}$	0.00520	0.00425	0.000100	0.0111
e_{DI}	0.00188	0.00310	0.000100	0.00707
$e_{DI,-4}$	0.0192	0.0194	0.0173	0.0212
e_b	0.00118	0.00250	0.000100	0.00558
$e_{b,-4}$	0.00171	0.00170	0.000100	0.00384
$e_{\mu c}$	0.00163	0.00206	0.000100	0.00496
$e_{\mu c,-4}$	0.00163	0.00206	0.000100	0.00496
$e_{\mu i}$	0.00222	0.00307	0.000100	0.00688
$e_{\mu I,-4}$	0.00222	0.00307	0.000100	0.00688

Table I.12: We generate artificial data from the model with parameter values equal to the posterior mean of the Bayesian estimation on the actual data, in tandem with the calibration strategy. We then use this artificial data as observables in estimation. The posterior median, 5th percentile, and 95th percentile from the posterior distribution are compared alongside the true values.

Appendix J. Estimation using cross-sectional evidence on price dispersion

Kaplan and Menzio (2016) emphasize the link between cross-sectional price dispersion of identical goods and search frictions. BRS incorporate such information, alongside shopping time, to calibrate the novel parameters ϕ and η.

Here we modify the baseline specification to match the microeconomic evidence on price dispersion. To derive the relationship between prices and expenditure, consider a static economy with J types of agents who differ in their consumption expenditure $y_j \in \{1, \dots, J\}$. Given a consumption price $P = 1$, aggregate expenditure satisfies $\sum_j y_j = C$. There is a unit measure of firms with production function $f(k, n) = k^{\alpha_k} n^{\alpha_n}$. BRS show that all firms supply the same capacity F.

Lemma 1. *All firms supply the same capacity of consumption goods:* $F = zf(k_c, n_c)$

Then conjecture that there are J different markets open. Let D_j and T_j be the aggregate search effort and mass of firms in each submarket j. Each submarket has the same capacity F but differ in the price-tightness pair (p_j, q_j). By lack of arbitrage, each submarket must have the same expected profit $\overline{\pi}$:

$$p_j = \frac{\overline{\pi}}{F} q_j^{-\phi} = \frac{\overline{\pi}}{F} \left(\frac{D_j}{T_j} \right)^{-\phi}$$

The matching function implies $c_j = D_j^{\phi} T_j^{1-\phi} F$. Hence, the income of a household is

$$y_j = p_j c_j = \overline{\pi} T_j$$

Since the number of firms adds up to 1, $\int T_j d_j = 1$, aggregate revenue equals aggregate expenditure: $\overline{\pi} = \int y_j dj = C$.

Hence, $T_j = y_j / C$, so that

$$\begin{aligned} p_j &= \frac{C}{F} \left(\frac{D_j}{y_j / C} \right)^{-\phi} \\ &= \frac{C^{1-\phi}}{F} y_j^{\phi} D_j^{-\phi} \end{aligned} \tag{J.1}$$

Optimal search effort satisfies

$$\theta_d d_j^{1/\eta} = \phi D_j^{\phi - 1} y_j^{1-\phi} F$$

Using $d_j = D_j$ and applying logs, we have

$$(1 + 1/\eta - \phi) \log D_j = (1 - \phi) \log y_j + \log \left(\frac{\phi F}{\theta_d} \right) \tag{J.2}$$

Substitute (J.2) into (J.1) to obtain

$$\log p_j = (1 - \phi) \log C - \log F + \frac{\phi}{\eta(1 - \phi) + 1} \log y - \frac{\eta \phi}{\eta(1 - \phi) + 1} \log(\phi F / \theta_d)$$

Hence, the search parameters can be linked to empirical moments using

$$m \equiv \frac{std(\log(p_j))}{std(\log(y_j))} = \frac{\phi}{\eta(1 - \phi) + 1} \tag{J.3}$$

We use the same target as BRS. Kaplan and Menzio (2016) estimate cross-sectional standard deviation of household price indices of 15% using the Kielts-Nielsen Consumer Panel Data. Heathcote, Storesletten, and Violante (2010) estimate the standard deviation of log consumption expenditures on services and nondurables equal to 0.524. Hence, $m = 0.15/0.524 = 0.286$. Thus, with the value of m fixed, (J.3) characterizes a curve between ϕ and η. Given a draw η from the posterior distribution, it pins down the value of ϕ.

We thus drop ϕ from the set of estimated parameters and use (J.3) to obtain values of ϕ given draws of η. The prior distribution of η is the same as the baseline.

Table J.13 reports the estimates of the structural parameters and shock processes related to shopping effort. Most posterior means and probability bands are similar to the baseline, but some important differences emerge. The posterior mean of η is 1.70, much higher than the value 0.563 in the baseline. We can use (J.3) to map the posterior probability band of η to ϕ. Doing so we obtain the tight interval $(0.49, 0.56)$. Though this range is not as high as the baseline estimates, it nevertheless indicates substantial evidence of goods market frictions, is substantially higher than the calibrated value in BRS, and is very tight.

The posterior mean of γ, 0.595, significantly exceeds the baseline value of 0.317, and thus makes preferences lean more closely toward KPR. The posterior mean of θ, 1.05 is significantly lower than baseline value of 1.55, indicating more substitutability of labor between sectors. The persistence parameters of shopping-effort shocks are similar.

Table J.13: Bayesian estimation of model with cross-sectional evidence on price dispersion

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	beta	1.500	0.2500	1.464	0.1678	1.2235	1.7395
ha	beta	0.500	0.2000	0.312	0.0199	0.2830	0.3422
ζ	gamm	0.720	0.2500	1.827	0.1185	1.6420	1.9999
γ	beta	0.500	0.2000	0.595	0.0564	0.5123	0.6973
η	gamm	0.200	0.1500	1.698	0.2304	1.4060	2.1357
ξ	gamm	0.850	0.1000	0.844	0.0748	0.7031	0.9416
ν_R	beta	0.200	0.1000	0.403	0.0900	0.2912	0.5000
σ_{ac}	invg	1.000	1.0000	1.199	0.2720	0.7942	1.6163
σ_{ai}	invg	1.000	1.0000	0.514	0.1257	0.3185	0.7216
Ψ_c	gamm	4.000	1.0000	3.478	0.8189	2.2389	4.9814
Ψ_i	gamm	4.000	1.0000	3.731	0.7652	2.6089	4.8856
θ	gamm	1.000	0.5000	1.052	0.6126	0.3357	1.8254
ρ_D	beta	0.600	0.2000	0.933	0.0217	0.8980	0.9693
ρ_{DI}	beta	0.600	0.2000	0.991	0.0080	0.9798	0.9999

e_D	gamm	0.010	0.0100	0.033	0.0042	0.0236	0.0374
$e_{D,-4}$	gamm	0.010	0.0100	0.011	0.0069	0.0001	0.0198
e_{DI}	gamm	0.010	0.0100	0.002	0.0017	0.0001	0.0049
$e_{DI,-4}$	gamm	0.010	0.0100	0.026	0.0017	0.0231	0.0285

Table J.13: Posterior and prior distributions of model with cross-sectional evidence on price dispersion

Table J.14 describes the unconditional forecast error variance decomposition of the model imposing the price dispersion target. There is a small drop in the role of shopping-effort shocks, but they remain extremely important. The biggest effect is on labor in the consumption sector. Discount-factor shocks play a much bigger role, at the expense of shopping-effort and wage-markup shocks.

Table 7: Forecast error variance decomposition: imposing cross-sectional evidence on price dispersion

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	43.49	0.65	53.87	1.77	0.22
SR	45.66	1.03	48.73	4.38	0.20
I	43.23	0.42	42.00	14.14	0.21
p_i	56.63	0.04	38.60	2.53	2.20
n_c	4.21	50.93	4.32	39.46	1.08
n_i	19.21	5.85	17.34	37.65	19.95
$util$	17.82	0.29	78.95	2.81	0.12
D	4.77	0.09	94.87	0.26	0.01
h	39.13	0.58	55.56	4.59	0.14

Table J.14: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories. The model is identical to the baseline except that we impose the price-dispersion target $m = 0.286$. We thus drop ϕ from the estimated parameters and instead recover it using (J.3) given a draw of η.

Table J.15: Comparison of model with cross-sectional evidence on price dispersion

	Data	Baseline	Price dispersion
LML	–	4531.0	4505.9
Δ LML	–	0	-25.1
Var(util)/Var(SR)	–	0.79	0.62
std(Y)	0.87	1.63	1.77
std($util_{ND}$)	1.26	1.14	1.28
std($util_D$)	2.27	3.00	2.44
std(n_c)	0.57	0.53	0.54
std(n_i)	1.94	1.8	1.80
Cor(C, I)	0.54	0.62	0.68
Cor($util_{ND}, util_D$)	0.75	0.57	0.63
Cor(n_c, n_i)	0.87	0.78	0.68
Cor($util_{ND}, util_{ND,-1}$)	0.51	0.36	0.41
Cor($util_D, util_{D,-1}$)	0.55	0.55	0.48

Table J.15: Comparison of log marginal likelihood, unconditional variance decomposition, and second moments. The first column describes relevant empirical moments and the second column corresponds to the baseline model. The third column presents estimates that imposes the price-dispersion target $m = 0.286$. We thus drop ϕ from the estimated parameters and instead recover it using (J.3) given each draw of η.