

Consumption variety from shopping time and product creation in an estimated model

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Abstract

Shopping time lacks strong positive comovement with either output or business formation. I develop and estimate by Bayesian means a multisector model in which product diversity arises from both shopping time and firm entry and investigate its ability to match these and salient features of the aggregate data. I compare the full model to alternatives in which either shopping time or firm entry is absent. The baseline generates mildly procyclical firm entry and shopping time and flat comovement of these two series. Demand shocks induce firm entry, and shopping time is less procyclical in the baseline model than the no-entry variant.

Keywords: shopping time, firm entry, endogenous variety, labor supply, Bayesian estimation

JEL Classification: D10; E21; E22; E32; E37

1. Introduction

Intuitively, consumption diversity can rise due to greater business formation or shopping intensity. The search literature has emphasized a frictional process in which buyers and sellers match and trade goods from its inception (i.e. [Diamond \(1982\)](#)), and more recently shopping intensity plays a role in estimated DSGE models (i.e. [Bai et al. \(2012\)](#)). Moreover, a recent literature examines the effects of sluggish firm entry over the business cycle (i.e. [Bilbiie et al. \(2012\)](#), [Offick and Winkler \(2019\)](#), [Lewis and Stevens \(2015\)](#), and other successor papers). In a setting with both firm entry and shopping time, agents can raise consumption diversity with either margin. We can thereby examine how the relative response of these margins

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depends on the type of shock and compare their comovement to the data. As we will see, shopping time and firm entry do not comove strongly in the data. Therefore, if either variable is procyclical conditional on certain shocks, the shocks must be composed in such a way that largely negates their overall correlation.

The model features households and firms. The former consume, shop, work a variable amount of hours, accumulate capital and invest in firm shares; the latter produce a unique variety given labor and capital. Firms must pay a sunk entry cost in terms of labor to initiate production the following period, following [Bilbiie et al. \(2012\)](#). There is, therefore, a tradeoff between allocating labor to the production of existing goods or new goods. In this sense, the model has multiple sectors and does not feature an aggregate production technology. The environment also incorporates shopping time similarly to [Laing et al. \(2007\)](#) and [Huo and Ríos-Rull \(2016\)](#). Matches between households and products depend on a technology with shopping time and firms as inputs. Consequently, consumption diversity depends on a weighted average of jump and predetermined variables. Moreover, habit formation induces households to smooth consumption with less curvature in preferences, and congestion effects in entry create a friction that prevents too many firms from entering at once.

Investment in this model takes the form of product creation and capital and thus directly pertains to the tradeoff between quantity and diversity studied by [Dixit and Stiglitz \(1977\)](#). The share of product creation in output depends on the elasticity of substitution, rate of time preference, and rate of product destruction. Whereas the (intratemporal) elasticity of substitution and markup only affect dynamics in a small-scale New Keynesian model through their interaction with nominal rigidities, here they are fundamental for investment and shopping.²

To understand how the responses depend on the type of disturbance, first consider a positive preference shock. Consumers spend more on existing varieties and shop more to expand their basket of goods. Increased demand for labor pushes up wages and labor-intensive entry costs. Provided the shock is sufficiently persistent, the discounted value of firm profits rises enough to promote entry. Aggregate firm profits increase from both higher

²Indeed, it is well-known that if a standard real business cycle model is augmented with CES monopolistic competition, then the elasticity of substitution does not affect the first-order dynamics. [Bilbiie et al. \(2007\)](#) make a related point.

sales of incumbents and entry. In the absence of shopping time, consumption variety expands by less, so that households consume relatively more of fewer goods. Technology shocks, however, induce a strong response in business formation but actually reduce shopping time due to wealth effects. That is, with a lower marginal utility of consumption, the rate of return on shopping falls. Therefore, the composition of shocks plays an important role in the comovement between shopping time, output, and entry. To better understand the role of each ingredient, I also compare the forecast error variance decomposition of the baseline model to separately estimated alternative models in which either ingredient is absent.

The results depend on two delicate challenges. First, [Bilbiie et al. \(2007\)](#) consider sticky prices but flexible wages. Under a standard calibration, a reduction in nominal interest rates boosts consumption demand but also sharply raises the real wage and entry costs. On net, entry declines. [Lewis \(2009\)](#) recommends nominal wage rigidity as a means to flip the sign of the response. Similar issues apply to the response of firm entry under a preference shock. However, in this setting the estimated demand shock is sufficiently persistent as to generate positive entry. Second, under standard additively separable preferences between consumption and labor, the response of shopping time is ambiguous. As wealth rises, households raise consumption, which increases the incentive to diversify the basket of goods. However, the falling marginal utility of consumption actually reduces the incentive to shop. It turns out that if wealth effects exceed substitution effects (the inverse intertemporal elasticity of substitution exceeds 1), then the second effect predominates and households shop less. By allowing for habit formation, as in [Lewis and Stevens \(2015\)](#), households act as if they are more risk averse. Therefore, they smooth consumption with more modest curvature in preferences and hence less pronounced wealth effects. In fact, shopping time remains procyclical if demand shocks account for most of its variation, which is borne out by the forecast error variance decomposition.³

³There are, of course, a number of other modifications of preferences that weaken wealth effects in the literature. The GHH preferences from [Greenwood et al. \(1988\)](#) eliminate wealth effects altogether. The preferences from [Jaimovich and Rebelo \(2009\)](#) are time inseparable and nest those of [King et al. \(1988\)](#). However, the nature of the time inseparability seems difficult to interpret economically, and it raises the concern of whether the weaker wealth effects should be regarded as an intertemporal vs. and intratemporal phenomenon. Home production models, such as [Benhabib et al. \(1991\)](#), weaken wealth effects by inducing by the substitution of work at the market and work at home. As labor supply rises, market consumption rises

I estimate the model by Bayesian means to data on consumption, investment, output, labor supply, real wages, and firm entry, which include all the real series used by [Lewis and Stevens \(2015\)](#).⁴ The choice of these series is intuitively reasonable for the following reasons. Consumption and output/investment are important series for disentangling demand and technology shocks. The reason is that demand shocks trigger a bigger response of consumption to output, or, equivalently, that technology shocks induce stronger investment in new goods relative to output. Data on investment directly disciplines the size of both product creation and development of physical capital. Labor supply is also an important amplification mechanism but responds differently depending on the type of shock. Consistent with [Basu et al. \(2006\)](#), I find that demand shocks account for a much larger response in labor supply. Firm entry is a proxy for product creation and disciplines the investment margin in the model. Though shopping time data would be ideal, the construction of this variable from the American Time Use Survey only allows for an annual frequency and insufficient years. However, this fact allows us to assess the implications of the model for a variable that was not used in estimation.

The stochastic disturbances include (intratemporal) consumption preference shocks, discount-factor shocks, shopping disutility shocks, technology shocks, and entry-cost shocks. Each shock follows an AR(1) process in logs except for that of shopping disutility, which is iid.⁵ Following [Offick and Winkler \(2019\)](#), I also include measurement errors in wages and investment.

I find that the model fits the data well, generates moderately procyclical firm entry and shopping time and a flat correlation between the two, and induces a positive response of firm entry to demand shocks. Moreover, consumption diversity is most volatile in the baseline specification. However, shopping time is less procyclical than in the model without entry.

more sharply than overall consumption. This mechanism is well-supported empirically but detracts from the purpose of building the simplest estimable model.

⁴[Lewis and Stevens \(2015\)](#) study a monetary model and thus also include data on interest rates and inflation. Moreover, [Offick and Winkler \(2019\)](#) also make use of aggregate profits. In general, a two-sector model of the vein of [Bilbiie et al. \(2012\)](#) cannot match the volatility of profits relative to output, so measurement error ends up absorbing much of the variability in profits.

⁵If one attempts to estimate κ_t as an AR(1) process in logs, then it turns out that its persistence is not identified. Moreover, in the no-entry specification even the volatility is not identified.

For each model, preference shocks explain nearly two fifths of the variation in consumption and a majority of the variation of labor supply. Moreover, technology shocks explain most variation in output, and discount-rate shocks account for about two thirds of total investment in each model. Disturbances to entry costs explain nearly all consumption diversity under firm entry even though but preference shocks are the most important source in the absence of entry.

The structure of the paper is as follows. Section 2 describes empirical evidence on firm entry and shopping time over the business cycle. Sections 3 and 4 lay out the environment and equilibrium. Section 5 discusses the quantitative results, and Section 6 concludes. The appendices describe the data sources, derive and list equilibrium conditions, and provide additional results from the estimation.

2. Empirical evidence on business formation and shopping time

There is limited evidence that consumption diversity is procyclical. [Broda and Weinstein \(2010\)](#) analyze data from the Nielsen Consume Panel Dataset and find that net product creation is procyclical and that households spend 9% of annual consumer purchased on new goods. In this model, consumption diversity arises from either both endogenous entry and shopping time. To that end, it is important to quantify how these series comove with output and each other. The primary available data on shopping time for the United States is by the American Time Use Survey (ATUS). The ATUS is a nationally representative diary time use survey available since 2003. The Bureau of Labor Statistics conducts the ATUS and draws on individuals exiting the Current Population Survey. Each wave is based on 24-hour time diaries in which respondents document the activities from the previous day in detailed time intervals. The activities are then classified into one of 400 time use categories. Thus, the ATUS represents repeated cross sections of daily time use. [Petrosky-Nadeau et al. \(2016\)](#) run cross-state and individual regressions on shopping time, income, and demographic variables from the ATUS. Comparing the periods 2005-2007 and 2008-2010, they find that shopping time fell more greatly in states in which GDP per capita contracted more.

Following [Aguilar et al. \(2013\)](#), I limit the sample to individuals between the ages of 18 and 65 and remove any observations with unclassified time.⁶ I merge shopping time data

⁶I compile the dataset using the ATUS Extract Builder database accessible from IPUMS. The American

with two types of data on firm entry. The first is the entry of establishments from the Business Dynamics Statistics(BDS) program of the Census Bureau. The BDS is compiled by the Center for Economic Studies from the Longitudinal Business Database. The BDS uses employment to identify new businesses. This dataset provides measures on job creation and destruction, establishment births and deaths, and firm startups and shutdowns. It is available annually from 1977 to 2018. The second dataset is the Business Formation Statistics (BFS), which is available after 2004. The BFS calculates business formations and applications based on the first recorded payroll tax liability. I use the dataset on business applications with planned wages because it is available through 2018 and also to verify the extent to which business applications track entry of establishments. Merging the two series with the time use data and state-level per capita output provides a state-level panel between 2003 and 2018.⁷ In general, using both variables ensures that the conclusion is not sensitive to the entry measure.

Figure 1 plots the growth rates of business applications and new establishments. The two series roughly track each other.

Time Use Survey classifies activities in the diary into 18 major categories, which is further broken down into second and third tiers. The activities at the finest level are coded as a sequence of three two-digit combinations, i.e. 07 – 01 – 04. See [Petrosky-Nadeau et al. \(2016\)](#) Appendix B1 for a description of the codes in each shopping category. [Hamermesh et al. \(2005\)](#) provide more information on the types of activities recorded in the ATUS.

⁷The Bureau of Economic Analysis provides data on state output through the Gross Domestic Product by State release. The Census Bureau provides data on the resident population for each state. Both types of data are accessible through the FRED database. Dividing these two series yields per capita output.

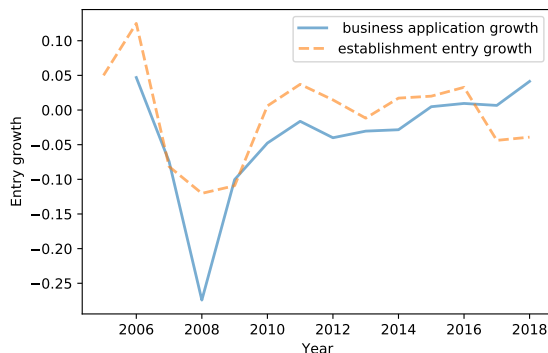


Figure 1: Growth rate of business applications and new establishments from the Business Formation Statistics and Business Dynamics Statistics databases, respectively. Entry is summed across states for each year, and then percentage differences are calculated between years.

Though my focus is on time-series properties of shopping time, I characterize basic features of the distribution at the respondent level. Figure 2 plots the empirical cumulative distribution function. Shopping time appears very lumpy: slightly over half of the observations are 0. Though over 90% of observations fall below 200 minutes, there is a small tail extending to much greater values. This lumpiness is qualitatively reminiscent of firm-level data on investment; aggregation to the state level of course dramatically smooths the distribution.

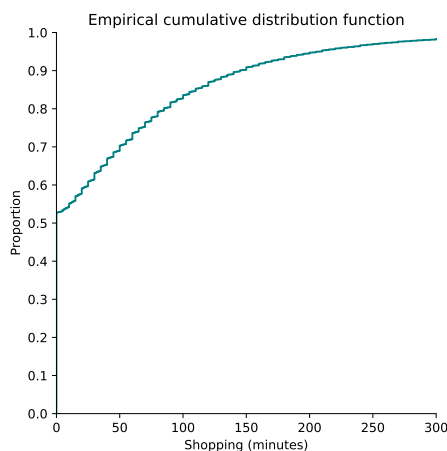


Figure 2: Cumulative distribution function of shopping time at the respondent level.

Following Aguiar et al. (2013), I use state-level variation of business cycles and smooth measurement error in time use by taking two-year averages. Specifically, the sample consists of the two-year periods 2005-2006, 2007-2008, 2009-2010, 2011-2012, 2013-2014, 2015-2016,

and 2017-2018. The first time period is lost due to differencing, leading to an effective sample size of $7 \times 50 = 350$. The state-level aggregates of the time use categories are

$$\tau_{st}^j = \frac{1}{\sum_{i=1}^{N_{st}} w_{ist}} \sum_{i=1}^{N_{st}} w_{ist} \tau_{ist}^j$$

where τ_{ist}^j denotes the daily minutes that individual i from state s during period t spends on category j , w_{ist} is the corresponding sampling weight, and N_{st} is the number of individuals from state s at time t . The sample weight represents the number of person-days in the calendar quarter that a single ATUS respondent represents. The weights adjust for two features of the sample data. First, individuals with particular characteristics (i.e. gender) are either over-or under-represented. Second, weekend days are over-represented.

Before turning to the regressions, Figure 3 shows a bubble plot of output growth with both shopping growth and shopping time from the pooled sample. Though I emphasize shopping growth in the regression exercises to remove any low-frequency trend, it is also worth inspecting against the series in levels. The size of each circle is proportional to the average state population over the sample.

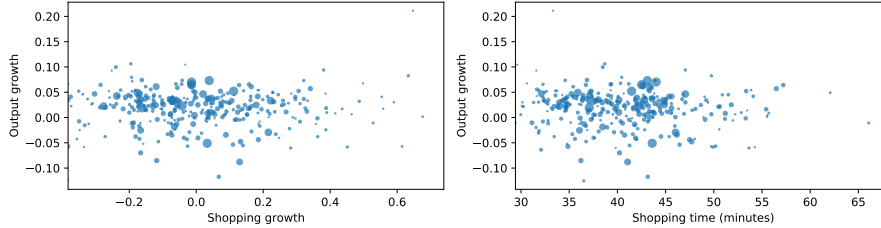


Figure 3: Bubble plot of output growth with shopping growth (left subplot) and shopping growth (right subplot) in the full sample. The size of each circle is proportional to the average state population over the sample. The horizontal axis is set to the interval between the 1 and 99 percentiles.

The plot displays no obvious pattern but provides significant evidence of outliers. The outliers are overwhelmingly from small states; thus, weighting the regressions by the state population also attenuates the influence of outliers.

We can also gain insight by examining the time trend of shopping time (in levels and growth) and output growth. Figure 4 calculates the means of each series for each pair of years weighted by the average population of the state.

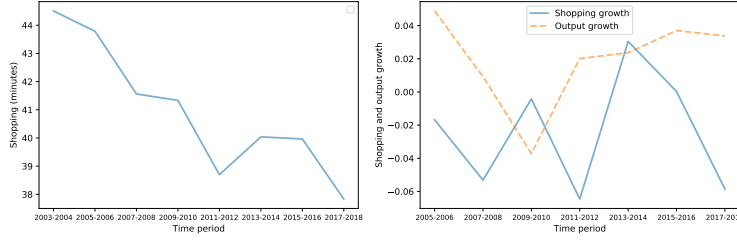


Figure 4: Means over pairs of years for shopping time, shopping growth, and output growth weighted by the average state population over the time period.

A direct comparison of shopping time and output growth through 2009-2010 suggests a negative relationship. However, shopping time decreases further through 2011-2012 even though output growth rises significantly in this period. Furthermore, shopping time only slightly recovers in 2013-2014 and does not revert to pre-crisis levels. Indeed, if we compare shopping growth and output growth, there is no obvious relationship. These figures illustrate why, following [Aguilar et al. \(2013\)](#), we need to account for trends in time use.

Next, we examine the procyclicality of shopping time, market hours, entry of establishments, and applications of businesses with planned wages more formally in panel regressions. I also include market hours, which are defined using the same time use codes as [Aguilar et al. \(2013\)](#) and listed in the data appendix. Though this variable in the ATUS has been analyzed before, I include it to affirm the close relationship between market work and output directly from time use data in a state-level panel. All variables enter as growth rates. There are four specifications. The first (I) is a simple unweighted regression. The second (II) weights by population. The third (III) adds state-level fixed effects, and the (IV) adds a state-specific linear time trend. State-specific fixed effects in the growth formulation corresponds to state-specific linear trends in log levels, and a state linear time trend corresponds to nonlinear trends specific to each state. The fourth column is the preferred specification, but we do not expect a radical difference in results between II-IV as they only reflect the role of state-specific time trends. With four dependent variables and four specifications of the independent variables, there are a total of sixteen regressions. Table 1 presents the results.

Dependent variable	I	II	III	IV
Shopping growth	0.382 (0.549)	-0.0557 (0.314)	-0.0206 (0.349)	-0.0709 (0.426)
Market hours growth	0.341 (0.318)	0.634*** (0.148)	0.688*** (0.170)	0.678** (0.216)
Establishment entry growth	0.840*** (0.0866)	0.881*** (0.0692)	0.973*** (0.0738)	1.089*** (0.103)
Business application growth	0.968*** (0.124)	1.088*** (0.0673)	1.175*** (0.0831)	1.121*** (0.103)

Table 1: Per capita output growth is the independent variable in specification *I*. Column II weights by population, column III adds state-fixed effects, and column IV adds a state-specific linear time trend. Standard errors are clustered by state. Significance values follow * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The first key observation is there is no significant evidence of positive comovement between shopping time and output. The estimated coefficient is small and actually negative for specifications II-IV, and the standard errors are large.⁸ Given the imprecise estimates, we can say little other than that shopping time is unlikely to be either strongly procyclical or countercyclical. [Appendix B](#) considers a regression of log shopping time on output growth for the four specifications, and the results remain insignificant. The time use data on market work supports the evidence that labor hours are strongly procyclical. A one-unit increase in output growth rate is expected to raise labor growth by 0.678 units in column IV. Finally, both measures of firm entry are strongly procyclical, with similar estimates and standard errors. A one-unit increase in the output growth rate is associated with a 1.1-unit increase in the growth rate of both establishment entry and business applications.⁹

To further establish the strong comovement between the two entry series, we regress the growth of business applications on the growth of establishments. [Table 2](#) presents the results for specifications I-IV. Each specification gives a similar result, which has a very low standard

⁸The results are robust to excluding the lowest and highest percentiles of shopping time.

⁹The correlation between establishment entry growth rates and output growth at the state level is much higher than the correlation between output growth and firm entry growth on the aggregate data used for the estimation.

error. Column IV indicates that a one-unit increase in the growth rate of establishments is associated with a nearly 0.98 unit increase in business application growth.

Dependent variable	(I)	(II)	(III)	(IV)
Business application growth	0.926***	0.951***	0.949***	0.977***
	(0.0494)	(0.0579)	(0.0577)	(0.0584)

Table 2: The table presents regressions of business application growth on establishment entry growth. Column I is the simple, unweighted regression. Column II weights by population, column III adds state-fixed effects, and column IV adds a state-specific linear time trend. Standard errors are clustered by state. Significance values follow * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

We also examine whether there is positive comovement between shopping time and entry of establishments. Table 3 reports the result using the same specifications as in Figure 1. Each result is statistically insignificant, though the sign is positive in the main specification.

	I	II	III	IV
Shopping growth	-0.0964	0.140	0.137	0.129
	(0.340)	(0.165)	(0.170)	(0.181)

Table 3: Regression of shopping growth on establishment entry growth. The column specifications I-IV are analogous to Table 1.

3. Environment

3.1. Households

There is a unit mass of households and an endogenous mass N_t of firms. Each firm sells a differentiated product. Prices are written in nominal terms but are flexible. Households consume, shop, and supply labor.

The preference of the representative household is

$$\mathbb{E} \sum_t \beta^t b_t \left\{ \theta_t \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+1/\psi}}{1+1/\psi} - \kappa_t S_t \right\}$$

where consumption C takes the form of a Dixit-Stiglitz aggregator:

$$C_t = \left(\int_0^{\mathcal{A}} c_{i,t}^{(\varepsilon-1)/\varepsilon} di \right)^{(\varepsilon/(\varepsilon-1))} \quad (1)$$

and \mathcal{A} is the measure of the potential set of goods. The parameter ψ is the Frisch elasticity of labor supply and the parameter κ measures the cost of one more unit of shopping. Finally, the intratemporal shock θ_t affects the relative desirability of consumption. The preferences nest [Lewis and Stevens \(2015\)](#) if $\kappa = 0$ and [Bai et al. \(2012\)](#) if $h = 0$.¹⁰

I follow [Laing et al. \(2007\)](#), [Bai et al. \(2012\)](#), and [Huo and Ríos-Rull \(2016\)](#) in assuming a constant returns to scale matching function between shopping time and the measure of firms. The aggregate number of shopper-firm matches $M_t \subset \mathcal{A}$ is

$$M_t = AS_t^\phi N_t^{1-\phi}$$

where S is the aggregate shopping time devoted to searching and satisfies

$$S_t = \int_0^1 s_{i,t} di$$

for individual search units s_i .

Define the market tightness as the ratio of firms per shopping unit: $Q_t = N_t/S_t$. The measure of matches for a particular firm is $\mu_{N,t} = M_t/N_t = AQ_t^{-\phi}$ and the measure of matches for a single search unit is $\mu_S(Q_t) = M_t/S_t = AQ_t^{1-\phi}$. The matches of an individual shopper with $s_{i,t}$ search units are $s_{i,t}AQ_t^{1-\phi}$. Note that, upon aggregating, $\int_0^1 s_{i,t}AQ_t^{1-\phi} di = S_tAQ_t^{1-\phi} = M_t$. Thus, the amount of product variety that households enjoy depends on individual shopping effort and market tightness. Implicitly, shopping allows consumers to extend the range of goods. Each firm i posts price p_{it} . Households take the prices as given in deciding how much to shop.

An entrant is successful with probability $m(N_{E,t}) = \min(1, N_{E,t}^{-\eta})$, as in [Berentsen and Waller \(2009\)](#). This congestion externality captures the idea that entrants crowd each other out and affect their ability to enter the market. The successful entrants are of measure $m(N_{E,t})N_{E,t}$ and produce each period until they are hit with an obsolescence shock, which

¹⁰It is arguably more natural to embed shopping time and labor in a time budget constraint. However, I have found that quantitatively it makes little difference but has technical disadvantages. In this formulation, the only steady-state quantities necessary for the log linearized system can be computed analytically. To the best of my knowledge, with the time budget constraint one needs to solve for the steady state numerically for each parameter draw.

occurs at rate δ each period. The laws of motion for firms and capital are

$$\begin{aligned} N_t &= (1 - \delta)[N_{t-1} + m(N_{E,t-1})N_{E,t-1}] \\ K_t &= (1 - \delta_K)K_{t-1} + I_{t-1} \end{aligned}$$

Each period the household consumes a subset $\mathcal{A}_t \subset \mathcal{A}$ goods of measure $s_t A Q_t^{1-\phi}$, which depends on tightness Q_t and shopping effort s_t . To formulate a budget constraint of the household, we define the price index P :

$$P_t = \left(\int_{i \in \mathcal{A}_t} p_{i,t}^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}$$

The price index P_t satisfies $P_t C_t = \int_{i \in \mathcal{A}_t} p_{it} c_{it} di$ and can be interpreted as the minimal cost of one unit of consumption. Under a symmetric equilibrium, $1/(1-\varepsilon)$ is the elasticity of the price index with respect to the range of goods and thus measures the love of variety. The budget constraint in units of currency is

$$P_t C_t + P_t \nu_t (N_t + m(N_{E,t})N_{E,t})x_{t+1} + P_t I_t = P_t (d_t + \nu_t)N_t x_t + W_t L_t + P_t r_t^K K_t$$

Firms issue stock after successfully entering the market place but before facing the obsolescence shock. Households hold shares x_t in a mutual fund of all $N_t + m(N_{E,t})N_{E,t}$ firms. They receive income from dividends, the value of shares, and labor and capital rental. In turn, they consume and purchase new shares and capital. The omission of firm indices reflects the assumed symmetry of firms. The next section verifies that this symmetry holds.

The aggregate state is $\Omega_t = \{N_t, K_t, C_{t-1}, Z_t, \theta_t, f_t, b_t, \kappa_t\}$. The sunk entry cost and entry lag make the number of firms N_t a state variable, and habit formation makes C_{t-1} a state.

3.2. Firm entry

Prospective entrants in period t produce in period $t+1$. They compute the expected discounted stream of dividends

$$\nu_t = \mathbb{E}_t \sum_{s=t+1}^{\infty} Q_{t,s} d_s$$

where the stochastic discount factor is $Q_{t,s} = [\beta(1-\delta)]^{s-t} \lambda_{t+s}/\lambda_t$ and dividends satisfy

$$d_t = \rho_t y_t - w_t l_t - r_t^K k_t$$

Entry requires f_t/Z_t units of labor. A prospective entrant is successful with probability $m(N_{E,t})$, and enjoys the present discounted value ν_t . This formulation reflects the evidence

that entry margins rise with labor costs in [Bollard et al. \(2014\)](#). Thus, as in [Bilbiie et al. \(2012\)](#), productivity shocks Z_t are truly aggregate, affecting both the production of existing goods and the development of new ones. Labor employed in new goods L_{Et} is determined by the production function $N_{Et} = Z_t L_{Et} / f_t$. Even unsuccessful startups nevertheless require labor in setup.¹¹

4. Equilibrium

We examine the household problem, the firm problem, aggregation, and steady-state properties.

4.1. Households: consumption, shopping, labor supply, and investment

Consumers choose consumption, shopping effort, shares, and work hours to maximize utility, given the laws of motion for firms and capital specified below. The dynamic programming problem for a household is

$$v(x_t, k_t, \Omega_t) = \max_{c_{it}, L_t, s_t, x_{t+1}, k_{t+1}, I_t} b_t \left[\theta_t \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+1/\psi}}{1+1/\psi} - \kappa_t S_t \right. \\ \left. + \beta \mathbb{E} \{ v(x_{t+1}, k_{t+1}, \Omega_{t+1}) \} \right]$$

subject to

$$C_t + \nu_t(N_t + m(N_{E,t})N_{E,t})x_{t+1} + I_t = (d_t + \nu_t)N_t x_t + w_t L_t + r_t^K k_t \quad (2)$$

$$C_t = \int_0^{s_t A Q_t^{1-\phi}} \rho_{i,t} c_{i,t} di \quad (3)$$

$$k_{t+1} = (1 - \delta_K)k_t + I_t$$

where $\rho_i = p_i / P$ is the relative price. Note that, as greater availability of goods reduces the price index P , it raises ρ_i for a given nominal price p_i . Thus, we interpret ρ_i as a measure of consumption diversity. We will see that ρ_i is a function of the range of goods that the household actually consumes.

¹¹This congestion externality is related to work on research and development races and simultaneous innovation, as by [Gabrovski \(2020\)](#).

Let λ denote the Lagrangian multiplier on the budget constraint (2). Substituting (3), then the optimality conditions for consumption and shopping time are

$$[C] \quad b_t \theta_t (C_t - hC_{t-1})^{-\sigma} = \lambda_t \quad (4)$$

$$[c_i] \quad c_{i,t} = \rho_t^{-\varepsilon} C_t \quad (5)$$

$$[s] \quad b_t \theta_t (C_t - hC_{t-1})^{-\sigma} (\varepsilon/(\varepsilon - 1)) C_t^{1/\varepsilon} \mu_{S,t} C_{i,t}^{(\varepsilon-1)/\varepsilon} - \kappa_t = \lambda_t \mu_{s,t} \rho_t c_{i,t} \quad (6)$$

$$[L_t] \quad w_t = \frac{\chi L_t^{1/\psi}}{\theta_t (C_t - hC_{t-1})^{-\sigma}} \quad (7)$$

We differentiate (1) with respect to c_i to calculate individual consumption demand.¹² The Euler equations with respect to next-period shares x_{t+1} and capital are

$$1 = \beta(1 - \delta) \mathbb{E} \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{d_{t+1} + \nu_{t+1}}{\nu_t} \right\} \quad (8)$$

$$1 = \beta \mathbb{E} \left\{ \frac{\lambda_{t+1}}{\lambda_t} [1 - \delta_K + r_{t+1}^K] \right\} \quad (9)$$

Equation (4) equates the marginal utility of market consumption to the marginal utility of wealth. Equation (5) characterizes the demand curve for an individual good, which is a constraint on the firm problem. Equation (6) says that the benefit of extra search equals the foregone leisure value. The benefit of extra search is the net utility from switching expenditure from existing goods to new goods. Equation (7) equates the wage to the marginal rate of substitution between consumption and leisure. Equations (8) and (9) equate the marginal utility of consumption to the discounted expected marginal utility next period of consumption adjusted for the rate of return on shares and capital, respectively.

Using $\rho_t = (C_t/c_{i,t})^{1/\varepsilon}$ and substituting for λ_t yields the shopping optimality condition:

$$\frac{\theta_t (C_t - hC_{t-1})^{-\sigma} C_t}{\varepsilon - 1} = \kappa_t S_t \quad (10)$$

Equation (10) notes that shopping time rises with taste for diversity for a given consumption level. Consider the special case of no habit formation: $h = 0$. In that case, if $\sigma > 1$, then consumption and shopping time vary inversely unless θ_t rises.

¹²The first order condition with respect to c_i yields

$$\frac{\partial C}{\partial c_i} = s A Q^{1-\phi} \rho_i$$

Since $\frac{\partial C}{\partial c_i} = s A Q^{1-\phi} \left(\frac{C}{c_i} \right)^{1/\varepsilon}$ from (1), it follows that $(C/c_i)^{1/\varepsilon} = \rho_i$, or $c_i = \rho_i^{-\varepsilon} C$.

4.2. Firms: entry, input choices, and price setting

Firms enter until the value equals entry cost:

$$m(N_{E,t})\nu_t = \frac{w_t f_t}{Z_t}$$

A firm matches with $AQ_t^{-\phi}$ consumers and produces $y_t = AQ_t^{-\phi}c_t$ units of output. Each firm produces $Z_t k_t^\alpha l_t^{1-\alpha}$ given l_t units of labor and k_t units of capital. Consider the problem of minimizing the cost of producing y_t units of output, taking the factor prices w_t and r_t^K as given:

$$\begin{aligned} \min_{l,k} \quad & w_t l_t + r_t^K k_t \quad \text{s.t.} \\ & Z_t k_t^\alpha l_t^{1-\alpha} \geq y_t \end{aligned} \tag{11}$$

Let mc_t denote the marginal cost (Lagrangian multiplier on (11)). The first order conditions yield

$$\begin{aligned} w_t &= (1 - \alpha) \frac{y_t}{l_t} mc_t \\ r_t^K &= \alpha \frac{y_t}{k_t} mc_t \end{aligned}$$

Profit maximization implies the familiar price setting rule

$$\rho_t = \frac{\varepsilon}{\varepsilon - 1} mc_t$$

Multiplying both numerator and denominator by N_t and using the pricing rule yields

$$\begin{aligned} w_t &= \frac{(1 - \alpha) Y_t^C}{\frac{\varepsilon}{\varepsilon - 1} L_{Ct}} \\ r_t^K &= \frac{\alpha Y_t^C}{\frac{\varepsilon}{\varepsilon - 1} K_t} \end{aligned}$$

where L_{Ct} is the total labor involved in goods production.

4.3. Aggregation

Aggregate retail output in the economy satisfies

$$\begin{aligned} Y_t^C &= N_t \rho_t y_t \\ &= \rho_t Z_t K_t^\alpha (L_{Ct})^{1-\alpha} \end{aligned}$$

The profit of a firm satisfies $d_t = \rho_t y_t / \varepsilon$. Multiplying by N_t and rearranging yields the profit equation for the household sector:

$$d_t = \frac{Y_t^C}{\varepsilon N_t}$$

Hence, aggregate profits are $D_t = N_t d_t = Y_t^C / \varepsilon$. With constant markups, retail output and aggregate profits exhibit the same dynamics.

We aggregate (2) across households, using $x_t = 1$ for all t and the law of motion for firms to yield

$$Y_t = C_t + I_t + \nu_t m(N_{E,t}) N_{E,t} = w_t L_t + N_t d_t + r^K K_t \quad (12)$$

Equation (12) says that total expenditures (consumption and both types of investment) equals the sum of labor income, profits, and rental income.

4.4. Steady state

I highlight a few key steady state results but defer a sequential calculation to [Appendix E.2](#). The Euler equation implies that the gross return on shares satisfies $1 + \frac{d}{\nu} = \frac{1+r}{1-\delta}$, or that the dividend value ratio is $d/\nu = (r + \delta)/(1 - \delta)$. The number of new entrants replaces those firms which are exogenously destroyed: $mN_E = \delta N/(1 - \delta)$. The Euler equation on capital implies $r^K = r + \delta_K$. The shares of profits and the extensive margin of investment to retail output is

$$\begin{aligned} \frac{dN}{Y^C} &= \frac{1}{\varepsilon} \\ \gamma \equiv \frac{\nu m N_E}{Y^C} &= \frac{\nu}{d} \frac{\delta}{1 - \delta} \frac{Nd}{Y^C} \\ &= \frac{\delta}{(r + \delta)\varepsilon} \end{aligned}$$

The share of (firm) investment and profit income to output are

$$\begin{aligned} \frac{\nu m N_E}{Y} &= \frac{\gamma}{1 + \gamma} = \frac{\delta}{\delta + \varepsilon(r + \delta)} \\ \frac{dN}{Y} &= \frac{dN}{\nu m N_E} \frac{\nu m N_E}{Y} = \frac{r + \delta}{\delta} \frac{\gamma}{1 + \gamma} = \frac{r + \delta}{\delta + \varepsilon(r + \delta)} \end{aligned}$$

The income approach to output implies $Y = wL + r^K K + Nd$, so that the joint share of rental and labor income to output is therefore $1 - Nd/Y = 1 - \frac{r + \delta}{\delta + \varepsilon(r + \delta)}$. As discussed by [Bilbiie et al. \(2012\)](#), the model is consistent with a constant share of profits in firm capital

$dN/(\nu N) = (r + \delta)/(1 - \delta)$ and a high correlation between the profit share and investment share of output.

Using these formulas and the capital share of retail output from the appendix, we can calculate the steady state shares of physical capital and firm entry in investment. Note that

$$\begin{aligned}\frac{I}{TI} &= \frac{\delta_K K}{\delta_K K + \nu m N_E} \\ &= \frac{\delta_K K / Y^C}{\delta_K K / Y^C + \nu m N_E / Y^C} \\ &= \frac{\delta_K \alpha}{\delta_K \alpha + \frac{\delta(r + \delta_K)}{(r + \delta)(\varepsilon - 1)}}\end{aligned}$$

Hence, the investment share in new businesses is

$$\frac{\nu m N_E}{TI} = \frac{\frac{\delta}{(r + \delta)(\varepsilon - 1)}}{\frac{\delta_K \alpha}{r + \delta_K} + \frac{\delta}{(r + \delta)(\varepsilon - 1)}}$$

The investment share in new goods depends positively on δ , negatively on the elasticity of substitution ε , and negatively on $\delta_K \alpha$.

Since the labor can be used in both for producing existing goods and developing new ones sold at a markup, the labor share of income reflects more than just technology. The appendix shows that the ratio of labor income to rental income satisfies $wL/(r^K uK) = [(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta]/[\alpha(\varepsilon - 1)(r + \delta)]$. Let $\tilde{\alpha}$ denote the capital share of income. Using this ratio and the total share of labor and rental income of output, we find that the labor share of income is

$$\frac{wL}{Y} \equiv 1 - \bar{\alpha} = \frac{(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta}{\delta + \varepsilon(r + \delta)}$$

The labor share of income is a function of the elasticity of retail output with respect to labor $1 - \alpha$, the elasticity of substitution between goods ε , the rate of time preference r , and the firm destruction rate δ . Applying limits helps to better understand the contribution of each ingredient. As $\varepsilon \rightarrow \infty$, the labor share converges to $1 - \alpha$, which coincides with a neoclassical Cobb-Douglas production economy. Interestingly, as $\varepsilon \rightarrow 1$, the lower bound, the labor share of income approaches $1/(2 + r/\delta)$. Given consumers' preferences, it is profit maximizing to devote all resources to producing new firms and infinitely little to production. Since essentially no resources are used for production, the coefficient α does not matter. However, setting up new firms requires labor. The amount of labor employed varies inversely with r/δ . In a steady state, a higher depreciation rate requires a bigger rate of firm entry to

keep the mass of firms fixed. However, a higher discount rate lowers the present discounted profits and leads to fewer firms operating overall. Finally, if $r \rightarrow \infty$ or $\delta \rightarrow 0$, then the labor share of income satisfies $(1 - \alpha)(\varepsilon - 1)/\varepsilon$. That is, the labor share of income equals the neoclassical analogue scaled down by the gross markup. For the reasons discussed, I consider the labor share to be an important target and choose α to satisfy the mean value in the data. The appendix discusses in closer detail the relationship between the labor income share and markups. We finally note that the share of income in physical capital satisfies

$$\frac{r^K K}{Y} = \frac{\alpha(\varepsilon - 1)(r + \delta)}{\delta + \varepsilon(r + \delta)}$$

which approaches $\alpha(\varepsilon - 1)/\varepsilon$ in a one-sector model (as $\delta \rightarrow 0$).

4.5. The relative price and consumption diversity

The relative price ρ_t is the willingness to pay divided by the welfare-consistent price index. In log deviations, it satisfies

$$\tilde{\rho}_t = \frac{\phi \tilde{S}_t + (1 - \phi) \tilde{N}_t}{\varepsilon - 1} \quad (13)$$

There are four key objects of interest. The numerator reflects the amount of products depends, which are a weighted average of shopping time and firm entry, \tilde{S}_t and \tilde{N}_t . The total stock of firms is a state and only adjusts sluggishly, but this also provides a source of persistence in consumption variety. On the other hand, households can adjust shopping time quickly, but there is no built-in persistence. The parameter ϕ defines the weight of these two components. Finally, the elasticity of substitution ε maps numerical diversity of goods into a measure of consumption diversity $\tilde{\rho}_t$. Equation (13) synthesizes the role of goods market frictions and search intensity, frictional firm entry, and imperfect competition; the model sub-types differ in the variation of shopping time and entry and the estimates of ϕ and ε .

5. Quantitative analysis

5.1. Data used for estimation

Firm entry data from the Business Formation Statistics and Business Dynamics Statistics are not suitable for use as observable variables in estimation. The BFS does not start until 2004, and the BDS is only available annually. Instead, I follow [Offick and Winkler \(2019\)](#)

and combine new business incorporations from the Survey of Current Business and private sector establishment births from the Bureau of Labor Statistics into an entry series.¹³

The observables used for estimation are output, consumption, investment, wages, firm entry, and labor supply. Below, I plot the comovement of output with the other series after detrending with Hamilton’s regression filter with a horizon of 8 quarters. For the actual estimation, though, I transform nonstationary variables using growth rates as [Smets and Wouters \(2007\)](#) to link observables to their model analogues.¹⁴ The quantitative analysis later explicitly compares the second moments of the data to each model.

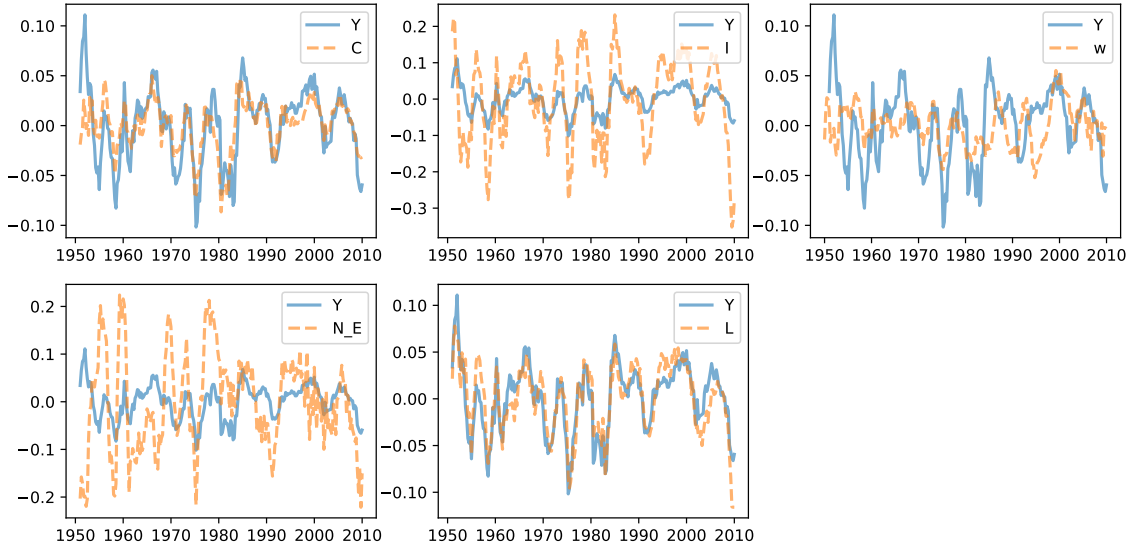


Figure 5: Business cycle component of major variables. Each variable ranges from 1950Q4 – 2009Q4. These dates reflect the initial loss of observations due to the forecast horizon. The symbols Y, C, I, w, N_E , and L are shorthand for output, consumption, investment, real wages, firm entry, and labor hours. Given series x , the Hamilton regression filter is the residual of series x_{t+h} on series x_t, x_{t-1}, x_{t-2} , and x_{t-3} , where I let the horizon $h = 2$ years following Hamilton’s recommendation.

¹³New business incorporations are discontinued after 1994, so I use private sector establishment births from 1995 onward.

¹⁴[Hamilton \(2018\)](#) recommends the use of the regression filter over the Hodrick-Prescott filter because it avoids spurious autocorrelation, the endpoint bias, and the arbitrary choice of the smoothing parameter. However, demeaned growth rates are more standard in the literature for linking observables to model variables in estimation.

As is well-known, consumption comoves closely with output but is slightly less volatile and labor exhibits similar volatility. Investment is about 3.5 times more volatile than output and more correlated than consumption. Firm entry is slightly less volatile than investment and comoves with output. Each series is highly autocorrelated.

5.2. Bayesian estimation

A quantitative evaluation of the model requires us to remove the variety effects of variables denominated in consumption units. The reason is that the consumer price index (CPI) data does not adjust for the availability of new products as in the welfare-consistent price index. Hence, for each variable in units of the consumption basket, we follow [Bilbiie et al. \(2012\)](#) and define $X_{R,t} = X_t/\rho_t$. The use of Bayesian estimation is natural for three reasons. First, with important exceptions, there are few direct ways of identifying the shocks.¹⁵ Estimating the relative contributions of the shocks is an important objective and is implementable via the forecast error variance decomposition. Second, there are several parameters which are very important for the transmission mechanism but uninformed by prior studies, especially the matching function elasticity ϕ and the congestion parameter η . Third, we can quantify parameter uncertainty by incorporating probability bands in the impulse responses.

I discuss the procedure very briefly as [An and Schorfheide \(2007\)](#) and [Herbst and Schorfheide \(2015\)](#) provide detailed expositions. First, I set a joint prior distribution $P(\Theta)$. Level parameters do not affect the first-order dynamics, and thus are excluded from Θ . I also fix several parameters. I set $\beta = 0.99$, consistent with an annual real interest rate of 4%, $\delta_K = 0.025$, which is consistent with 10% annual depreciation of physical capital. Moreover, I set $\delta = 0.025$, which approximates an average product destruction rate of 9% from [Bernard et al. \(2010\)](#). I also set α so that it matches a labor share of income of 62%.¹⁶ Since the elasticity of substitution ε is a random variable from a Bayesian perspective, α varies each

¹⁵Major exceptions include the approach of [Basu et al. \(2006\)](#) for technology shocks, that of [Greenwood et al. \(1988\)](#) for investment-specific productivity shocks, and substantial work in identifying monetary policy, government spending, and news shocks. A few key references are [Romer and Romer \(2004\)](#) and [Swanson \(2015\)](#) for monetary policy shocks, [Blanchard and Perotti \(2002\)](#) and [Ramey \(2011\)](#) for government spending shocks, and [Barsky and Sims \(2011\)](#) for news shocks, but there are many more.

¹⁶I measure labor share of income using the FRED code LABSHPUSA156NRUG. The average between 1948 and 2009 is 62%.

iteration to be consistent with the labor income share. Thus, in the estimation, the parameter ε balances competing demands of a prior on the elasticity of substitution, the labor share of income, and fluctuations.

Parameter	Value	Interpretation
β	0.99	Discount factor
δ	0.025	Firm exit rate
δ_K	0.025	Capital depreciation rate
α	$1 - \frac{0.62(\delta + \varepsilon(r + \delta)) - \delta}{(\varepsilon - 1)(r + \delta)}$	Elasticity of retail output with respect to capital

Table 4: Calibrated parameters. Here α is implicitly a random variable, since ε is a random variable and α varies with ε so as to match a labor income share of 62%.

We include iid measurement errors to wages and investment. This choice is primarily to prevent stochastic singularity. We need as many shocks as observables to calculate the likelihood function, as explained by [Ruge-Murcia \(2007\)](#).¹⁷ Additionally, aggregate wage data is known to be noisy, and firm entry is a proxy for product creation. Removing the parameters which do not affect the first-order dynamics and those calibrated directly and including the standard deviations of measurement errors yields

$$\Theta = (\psi, h, \sigma, \phi, \varepsilon, \eta, \rho_Z, \rho_\theta, \rho_f, \rho_b, \sigma_Z, \sigma_\theta, \sigma_f, \sigma_b, \sigma_\kappa, \sigma_{w,ME}, \sigma_{TI,ME})'$$

[Appendix F.3](#) plots the prior distributions for each parameter and also provides a table of 95% probability intervals and the means and standard deviation for each parameter. For several parameters, I use tight priors. For the Frisch elasticity of labor supply, the prior mean is 0.72 and the standard deviation is 0.4. This choice is based on the results of [Heathcote et al. \(2010\)](#), who account for the hours worked for both men and women in a setting in which husbands and wives form households. The prior distribution for ε has a mean of 3.8

¹⁷To grasp the problem of stochastic singularity, consider a simple real business cycle model with an unobserved technology series and consumption and output used as observables. In the reduced form VAR(1) solution, the shocks in each equation are just multiples of each other. This finding, in turn, implies that certain ratios of observed variables are constant. Thus, one observable can be inferred deterministically from the other. However, this relationship does not hold in the data, so another stochastic disturbance is necessary.

with a prior standard deviation of 0.5. In contrast, the prior distributions for ϕ and η are very diffuse.

The next step is to recast the model in linear state space form. Accordingly, Table E.11 summarizes the log linear system. For a linearized model, the likelihood function can be computed using the Kalman filter, which generates optimal predictions and updates of the unobservable variables given the data. We first maximize the posterior density, and then use the Metropolis Hastings algorithm to sample the posterior distribution. I simulate 250,000 draws with a burn-in of 20%, which suffices given the rapid convergence to the posterior distribution. Appendix F.2 traces the posterior density and its moving average across iterations, which looks stationary.

5.3. Posterior distribution and identification

Figure 6 shows the marginal densities from the posterior and prior distribution. Plotting the densities together allows us to visually assess the information imparted by the data and the identification of each parameter. Each parameter looks well-identified. The posterior mean on the Frisch elasticity of labor supply is 1.19, which is slightly high for microeconomic estimates but low for macroeconomic applications. The estimated elasticity of matching with respect to shopping time ϕ is has a posterior mean of 0.29 and standard deviation of 0.12, indicating moderate importance of shopping time to consumption diversity. The entry congestion externality η has a similar posterior mean as the prior mean but is much more concentrated. The shock autocorrelation coefficients are very compressed and high. Later, we verify that each shock plays an important role in the variance decomposition, albeit for different variables.

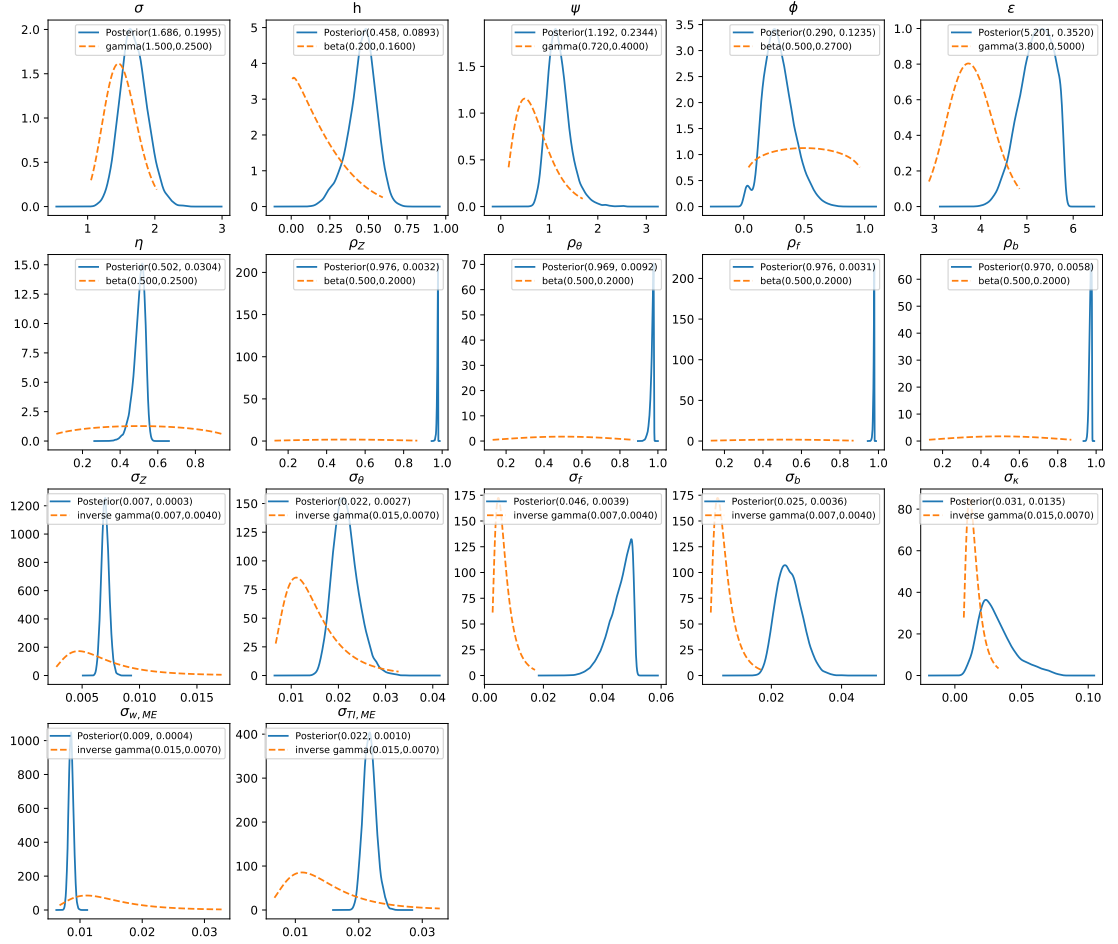


Figure 6: Marginal prior and posterior densities. The prior densities are set according to standard univariate distributions and the posterior distribution is approximated using the random walk Metropolis-Hastings algorithm.

5.4. Second moments and impulse responses

We next compare second moments between the model and data. Table 5 presents the second moments of output, consumption, labor supply, investment, and net firm entry in the data. The top panel reports moments of the series used as observations; Appendix C describes the exact transformation of the raw data. The bottom panel compares moments for the Hamilton-filtered level series of the model and data. The model implied second-moments are derived from a simulation of size 100,000.

	$SD(x)$		RSD		$Cor(x, Y)$		$Cor(x, x_{-1})$	
	Data	Model	Data	Model	Data	Model	Data	Model
Estimation								
ΔY_R	1.00	1.02	1.00	1.00	1.00	1.00	0.37	0.00
ΔC_R	0.62	0.66	0.62	0.64	0.48	0.28	0.31	0.25
ΔTI_R	3.77	3.69	3.77	3.61	0.84	0.89	0.34	-0.07
Δw_R	0.76	0.78	0.76	0.76	0.17	0.40	0.03	0.02
ΔN_E	3.60	3.72	3.60	3.63	0.22	0.25	0.15	-0.02
L	4.20	3.62	4.21	3.53	0.09	0.16	0.97	0.96
Hamilton filter								
Y_R	3.63	2.76	1.00	1.00	1.00	1.00	0.91	0.88
C_R	2.46	2.33	0.68	0.85	0.74	0.53	0.89	0.90
TI_R	11.36	8.55	3.13	3.10	0.82	0.81	0.91	0.86
w_R	2.06	2.26	0.57	0.82	0.23	0.53	0.88	0.87
N_E	10.08	9.24	2.77	3.35	0.11	0.23	0.89	0.87
L	3.67	2.50	1.01	0.90	0.86	0.65	0.90	0.87
C	—	2.47	—	0.89	—	0.56	—	0.90
S	—	5.31	—	1.92	—	0.28	—	0.60
ρ	—	0.52	—	0.19	—	0.26	—	0.74
$Cor(S, N_E) = 0.019$								

Table 5: Second moments of model and data. Model moments are based on series of 100,000 periods. The top panel examines the series for output, consumption, investment, wages, firm entry, and labor supply, consistent with the detrending procedure; and the bottom panel filters the model and empirical variables using the Hamilton regression filter.

We first examine the top panel. The standard deviations of the model match the data extremely well except for labor supply, which is slightly lower. However, even the fit of labor hours is reasonably good considering the that the prior Frisch elasticity of labor supply is clustered around a low mean relative to what macroeconomic models typically require. The contemporaneous correlations with output are also very close to the data. The autocorre-

lations of the variables in growth rates are generally difficult to match, but the model does reasonably well for consumption and wages. Any further improvement requires additional frictions and adjustment costs. Importantly, firm entry is procyclical and has similar volatility as the data. The Hamilton-filtered analogue has a remarkably similar autocorrelation to the data.

Some other features stand out from the bottom panel. The model matches the empirical autocorrelations well. Shopping time is moderately procyclical. Welfare-based consumption has similar properties as data-based consumption but is slightly more volatile.¹⁸

We are particularly concerned with the comovement of shopping time with output and firm entry. In contrast to the other series, shopping time is not an observable in the estimation. From the Hamilton-filtered data, we see that these correlations are 0.28 and 0.019, respectively. Therefore, shopping time is only moderately procyclical and is essentially uncorrelated with firm entry. We will see that entry-cost shocks and technology shocks induce a negative comovement between shopping time and the other two series, which counterbalances the positive comovement from preference and discount-rate shocks.

5.4.1. *Productivity shocks*

Figure 7 considers a one-standard deviation positive technology shock. Higher productivity raises profit expectations and demand for all goods. Free entry drives the firm value equal to the entry cost. The rate of return on investing in new varieties is high both because of expectations that firm shares will appreciate and profits will rise. Hence, entry expands on impact and gradually raises the number of products. Data-consistent firm value rises due to the congestion externality of entry. Labor supply rises initially in both sectors. Here, the presence of both forms of investment plays a key role. Setting up new firms requires labor, but raising physical capital also enhances the marginal product of labor in the production of existing goods. However, labor supply eventually dips below the steady state value and converges slowly from below. The intuition is that wealth effects increase demand for leisure and that less labor is necessary for entry once the stock of firms has been built up. Note

¹⁸The correlation between Hamilton-filtered firm entry and output is, admittedly, rather low. As we saw, in growth rates, which is used for the filter, we obtain a correlation of 0.22. The use of the HP-filter generates a higher correlation of 0.34. However, because of the arguments given by [Hamilton \(2018\)](#), I opt to describe the data by the regression filter, though I emphasize that this does not affect the observables used in estimation.

that the change in consumption variety is relatively small relative to output. In general, the change in $\tilde{\rho}_t$ relative to $\tilde{Y}_{R,t}$ is much higher in [Bilbiie et al. \(2012\)](#). Moreover, wealth effects cause a decline in shopping time, which implies that consumption variety initially falls before rising due to the buildup of firms.

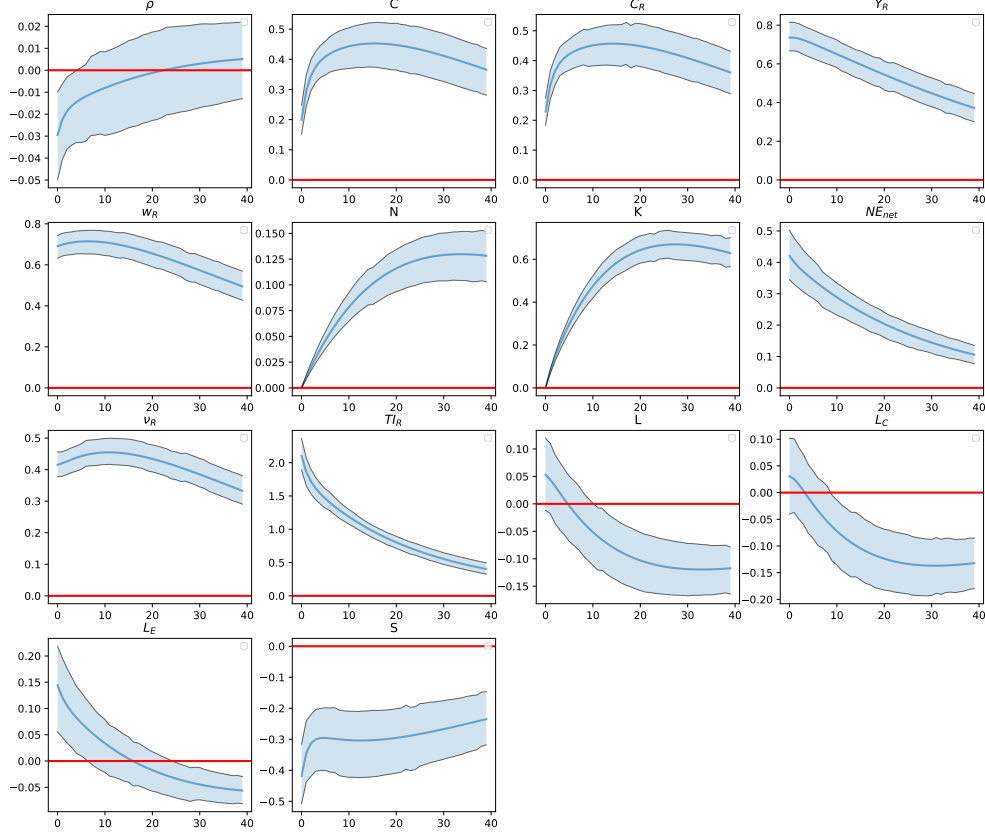


Figure 7: A positive one standard-deviation technology shock. All variables are in percentage deviations. The units of the horizontal axis are quarters following the shock. The bold line represents the mean impulse response, and the shaded region represents the 90% probability bands. The horizontal line denotes an impulse of 0 for reference.

5.4.2. Preference shocks

Figure 7 considers a one-standard deviation positive preference shock.

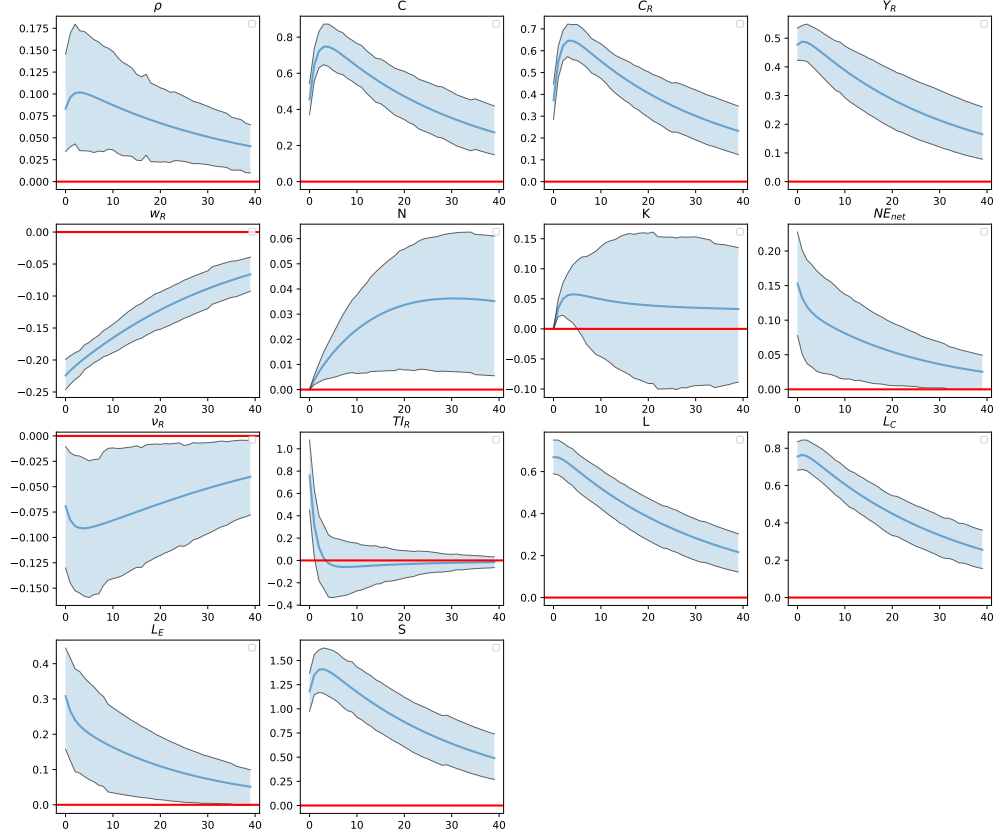


Figure 8: A positive one standard-deviation preference shock. All variables are in percentage deviations. The horizontal axis is in a quarterly frequency. The bold line represents the mean impulse response, and the shaded region represents 90% probability bands. The dashed horizontal line denotes an impulse of 0 for reference.

A positive one-standard deviation demand shock generates an immediate rise in shopping time of over 1.3%, which increases the relative price ρ by 0.1% at the mean. However, the posterior confidence bands on consumption variety are very large. Greater consumption demand boosts both the production of existing goods and new varieties. Shopping time and firm entry both raise consumption diversity, but the former is relatively more important. Compared to a technology shock, firm entry peaks at 0.15% compared to 0.4% and exhibits greater variability. Higher consumption demand initially crowds in investment. A noteworthy feature of positive demand shocks is that labor in both existing existing and new products remains higher than the steady state throughout and converges monotonically. However, the

change in consumption variety $\tilde{\rho}_t$ remains relatively small.

5.5. *Entry cost shock*

Figure 9 examines the effects of a one-standard deviation negative shock to the entry cost. This shock triggers a reallocation of resources away from consumption and toward business formation. Firm value declines alongside the entry cost, reaches a minimum, and then rises monotonically. The congestion externality dampens the fall in firm value. Labor in entry rises dramatically at 2.5% on impact, though it falls by over 0.4% in the production of existing goods. Intuitively, it is cheaper to produce new goods than existing ones, and households' desire for goods remains unchanged. Overall, labor demand rises by just about 0.15%. The rise in output is the mirror image of data-consistent consumption, and aggregate profits fall with lower expenditure. The dynamics of shopping time are a scaled negative of those of welfare-based consumption. The posterior mean of consumption variety rises eventually to 0.17%, nearly double the amount associated with preference shocks.

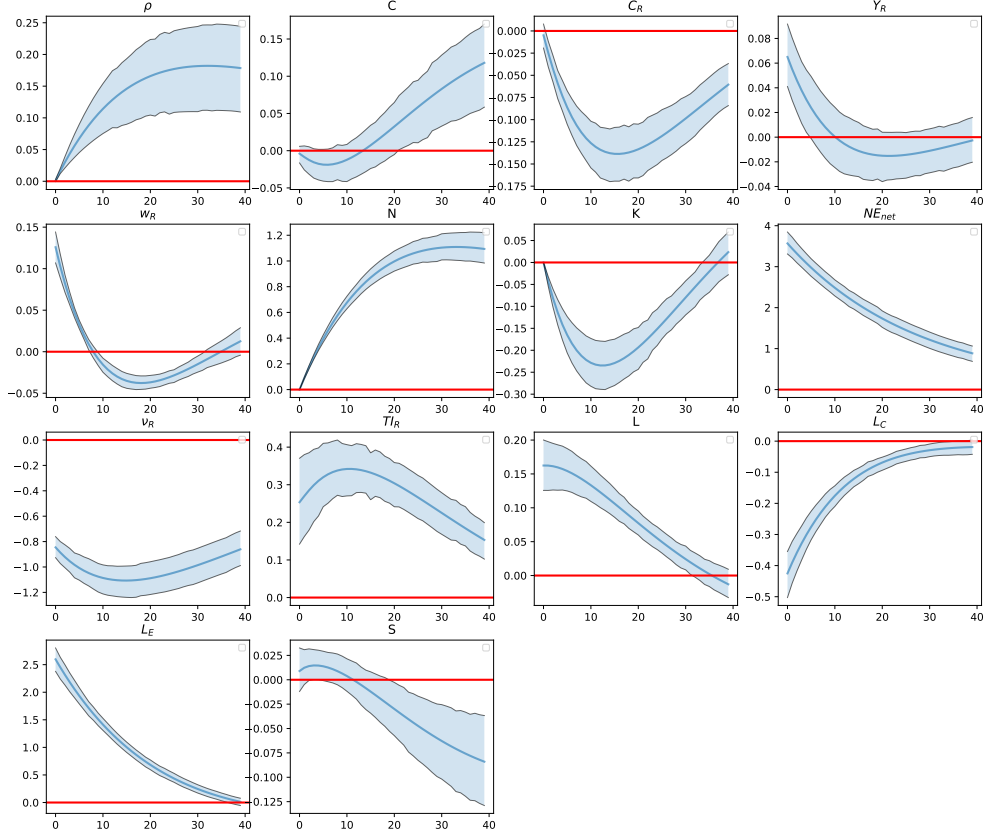


Figure 9: A negative one standard-deviation entry cost shock. All variables are in percentage deviations. The horizontal axis is in a quarterly frequency. The bold line represents the mean impulse response, and the shaded region represents 90% probability bands. The dashed horizontal line denotes an impulse of 0 for reference.

Generally speaking, disturbances to entry costs lead to a far stronger response in firm entry compared to technology shocks. However, they reduce firm value and investment in physical capital. These effects cause the impact on overall investment to be small.

5.5.1. Discount rate shock

Figure 10 examines a one standard-deviation expansionary discount rate shock. That is, the discount rate falls and consumers become more patient. Intertemporal smoothing implies an immediate drop in consumption, which converges then rises gradually and eventually exceeds the steady state. Investment in physical capital rises far more than in new products. There is, however, a modest rise in consumption variety ρ due to both entry and increased

shopping. Labor increases in both sectors, which also helps promote a rise in output.

Whereas firm value rises with a discount rate shock (welfare-based wages and entry costs increase), it falls with an entry-cost shock. In general, discount-rate shocks trigger a larger initial rise in shopping time but a much smaller effect on firm entry and consumption variety overall.

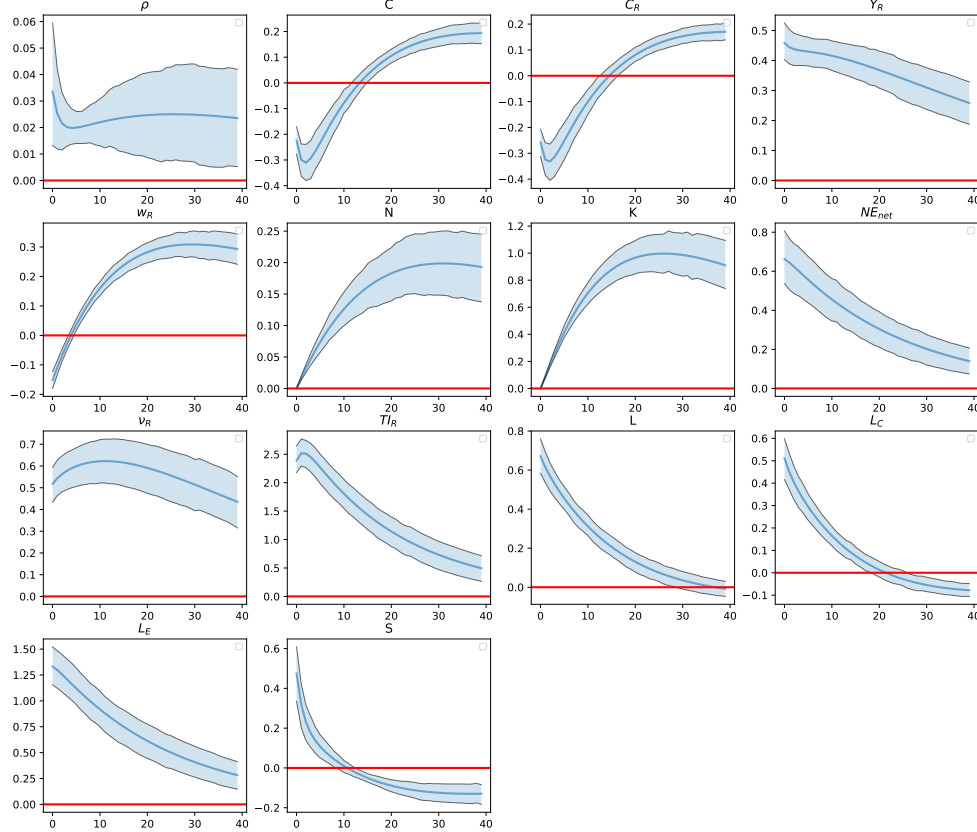


Figure 10: A negative one standard-deviation discount rate shock. All variables are in percentage deviations. The horizontal axis is in a quarterly frequency. The bold line represents the mean impulse response, and the shaded region represents 90% probability bands. The dashed horizontal line denotes an impulse of 0 for reference.

5.6. Alternate models

The model without shopping time arises by shutting down the role of shopping in matching: $\phi = 0$. The shopping time equation vanishes, and the log-linearized variety effects condition is $(\varepsilon - 1)\tilde{\rho}_t = \tilde{N}_t$. The model is otherwise the same. [Appendix F.5](#) plots the marginal posterior distributions for the no-shopping model, which are extremely similar to

the baseline. It also shows the impulse responses to a technology shock; consumption variety rises unambiguously and has narrower probability bands.

Table F.14 calculates the second moments of the model and compares to the data analogues under both the transformation used for estimation and the Hamilton filter. The second moments are similar to the full model with shopping time. However, consumption diversity is only half as volatile, and welfare-based consumption is slightly less volatile. Thus, shopping time and consumption diversity are less variable with only firm entry. This fact is plausible given that households can change shopping time immediately as economic conditions change, whereas it takes time to build the stock of products. Additionally, the full model fits the data on consumption and investment slightly better.

Figure 11 shows the impulse response to a unit standard deviation preference shock. The responses are generally similar to the baseline model, except that consumption variety rises far less than in the baseline. This result is consistent with the fact that consumption variety is less volatile without shopping time.

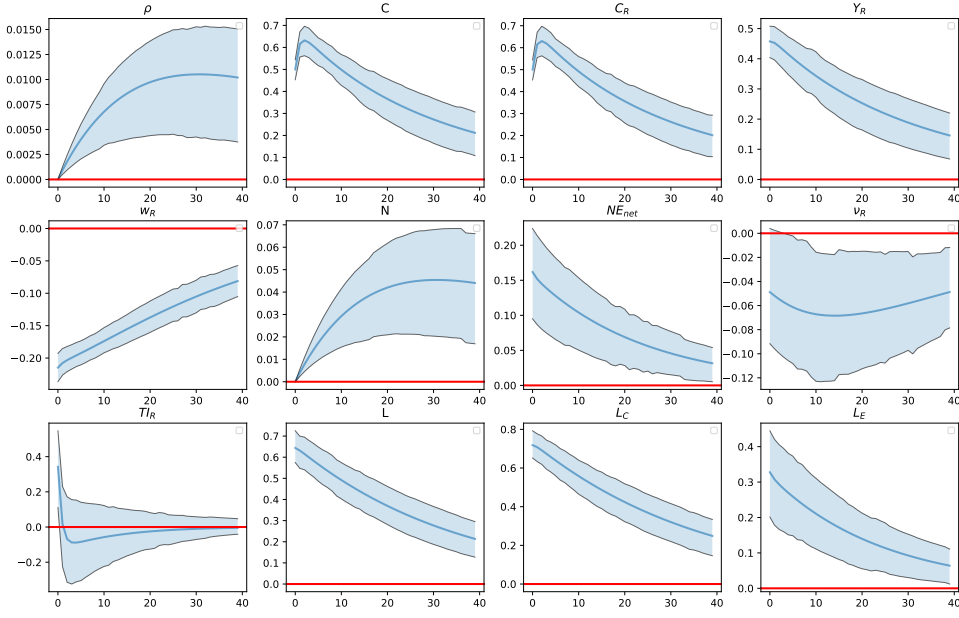


Figure 11: A positive one standard-deviation preference shock. All variables are in percentage deviations. The units of the horizontal axis are quarters following the shock. The bold line represents the impulse response, and the shaded region represents 90% probability bands. The horizontal line denotes an impulse of 0 for reference.

We now turn to the no-entry model with shopping time. Formally, we normalize $N_t = 1$, so that $N_{Et} = 0$. This specification implies $L_{Et} = 0$, $Y_t = Y_t^C$, and $TI_t = I_t$. That is, all labor is used for production of goods and investment is entirely in terms of physical capital. There is still a goods market friction between firms and shoppers, but now the tightness is just $Q_t = 1/S_t$. Implicitly, $\delta = 0$, so that the labor share of income is $(1 - \alpha)(\varepsilon - 1)/\varepsilon$. Equating this quantity to 0.62, we set $\alpha = 1 - 0.62\varepsilon/(\varepsilon - 1)$. Moreover, profits are a share $1/\varepsilon$ of output, and hence move one-for-one with the latter. It turns out that, without entry, even iid shopping disutility shocks are no longer identified, so we exclude them. Entry and shopping time are thus complementary from a methodological point of view: product creation helps us identify disturbances to shopping time.

[Appendix E.4](#) lists the log linearized conditions for the no-entry model. We lose the law of motion of firms, the Euler equation in shares, aggregate expenditure, and total investment.

As far as estimation is concerned, there is one less observable (firm entry) and two fewer shocks (entry cost and shopping disutility). We have just as many shocks as observables, so stochastic singularity does not arise.

Table F.15 calculates the second moments of the model and compares to the data analogues under both the transformation used for estimation and the Hamilton filter. Hamilton-filtered shopping time has a correlation of 0.39 with output compared to 0.28 in the baseline model. Thus, firm entry reduces the procyclicality of shopping time as consumers have alternative means of diversifying consumption. Thus, entry helps bring the model closer to the evidence suggested from the American Time Use Survey.

Firm entry plays an important role in fitting data more generally. Investment is too volatile, which is also the case in the basic model without capital examined by Bilbiie et al. (2012). Thus, it turns out that the presence of both entry and physical capital matter for matching aggregate investment. Otherwise, the model matches output, consumption, and wages well. The volatility of consumption variety, 0.29, is significantly below that of the baseline model.

Figure 12 shows the impulse response of a positive one standard deviation technology shock. Shopping falls due to wealth effects, so that, in the absence of firm entry, consumption variety declines as well. However, the magnitude is small and diffuse; the response of welfare-based and data-consistent consumption is nearly identical. Compared to the response of a productivity shock in the standard model, total investment rises by more. The response of output is similar.

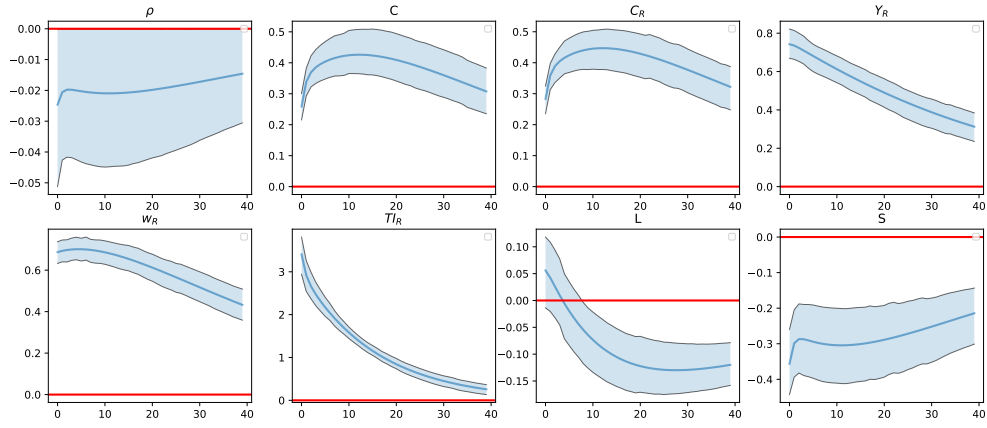


Figure 12: A positive one standard-deviation technology shock. All variables are in percentage deviations. The units of the horizontal axis are quarters following the shock. The bold line represents the mean impulse response, and the shaded region represents 90% probability bands. The horizontal line denotes an impulse of 0 for reference.

Figure 13 shows the impulse response of a positive one standard-deviation preference shock. Consumption variety rises by 0.10% in the posterior mean with a large spread, and in general the response of welfare-based consumption exceeds significantly that of data-consistent consumption. The rise in consumption variety is similar to that of the baseline model.

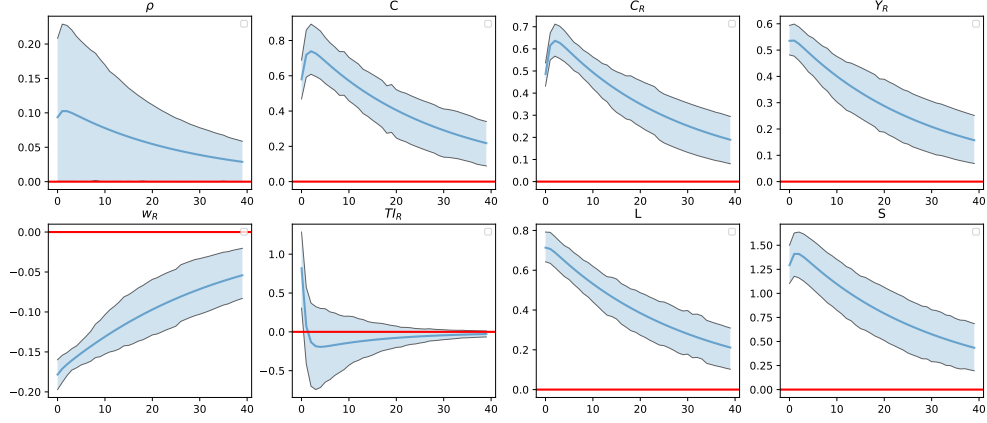


Figure 13: A positive one standard-deviation preference shock. All variables are in percentage deviations. The units of the horizontal axis are quarters following the shock. The bold line represents the mean impulse response, and the shaded region represents 90% probability bands. The horizontal line denotes an impulse of 0 for reference.

5.6.1. Forecast error variance decomposition

Table 6 examines the fraction of the unconditional forecast error variance explained by each shock for the three models. The focus is on data-consistent consumption, output, labor hours, total investment, shopping time, and net product creation. Intratemporal preference shocks explain nearly 40% of the variation in consumption and over 55% of the variation in labor supply across the board. Moreover, for each model, technology shocks explain a majority of the variation in output and nearly half of consumption; discount-rate shocks explain about a quarter of output in each case. Discount-rate shocks, however, account for at least two thirds of total investment in each model. Perhaps the most striking result is that shocks to entry costs explain nearly all variability of firm entry *and* consumption variety in the baseline and no-shopping models. In the no-entry model, by contrast, preference shocks explain over 80% of the variation in shopping time and consumption variety. In general, adding firm entry with entry-cost shocks changes the forecast error variance decomposition dramatically, whereas adding shopping time has a much milder effect. This finding is related to the fact that the number of firms, which directly affects the relative price, is a state variable that responds sluggishly. Though this limits the immediate impact, it also means that the

number of products continue to affect the relative price even as the effects of shopping time have abated.

Finally, the presence of firm entry actually increases the contribution of technology shocks to total investment by facilitating the buildup of firms. This effect occurs even though now there is an additional shock that can also contribute to investment.

Note that entry cost shocks can explain nearly all variation in entry while explaining only a small portion of investment. The explanation arose in the discussion of the impulse response. An expansionary entry-cost shock reduces barriers to entry and hence lowers firm value. Additionally, investment is diverted from physical capital. By contrast, a positive technology shock raises firm value and induces investment in physical capital, though the impact on firm entry is much lower.

	Baseline					No shopping				No entry		
	Z	θ	f	b	κ	Z	θ	f	b	Z	θ	b
C_R	47.5	39.7	2.7	10.5	0.2	48.3	39.6	3.7	10.8	48.2	38.5	13.3
Y_R	58.3	16.3	0.2	27.0	0.0	57.8	14.5	0.2	28.3	53.7	19.7	26.2
L	6.4	60.7	5.3	26.1	0.2	8.1	55.4	6.7	27.8	6.9	70.9	21.3
TI_R	31.4	0.5	2.4	67.3	0.6	31.4	0.1	2.5	66.3	22.0	0.4	76.7
ρ	0.2	9.1	85.6	1.8	2.0	1.1	0.1	97.2	2.7	10.4	86.0	3.1
S	9.1	70.3	1.4	2.6	17.8	—	—	—	—	10.4	86.0	3.1
NE_{net}	1.3	0.1	94.9	3.3	0.0	1.1	0.2	96.1	3.1	—	—	—

Table 6: Unconditional forecast error variance decomposition for the baseline, no-shopping, and the no-entry models. The numbers do not add up to 100 due to non-zero correlation of simulated shocks in small samples, as well as rounding.

5.7. Interpretation: shopping time versus effort

We have seen that the limited data from the American Time Use Survey does not favor strong procyclicality and can partially account for this finding in a model with wealth effects and a mixture of technology and demand-type shocks. Business formation also allows households to raise their consumption diversity with a given level of shopping. However, papers such as [Huo and Ríos-Rull \(2016\)](#) stress shopping *effort* rather than time. They argue that

shopping effort should be procyclical even if time is not. The point is apt and analogous to the distinction between working hours and variable labor effort. However, in building testable theory, it is important to utilize as much information from observable variables (i.e. time) to inform unobservable ones (i.e. effort). For instance, [Basu et al. \(2006\)](#) show that under mild conditions working hours can proxy effort. This paper finds that the baseline model does not require very high procyclicality of shopping time even without distinguishing effort from time. Further progress on this requires a more careful treatment of the relationship between these two variables.

6. Conclusion

Even though shopping time is a prominent feature of models with goods market frictions, the available evidence does not favor strong procyclicality. Time spent shopping also does not comove significantly with entry of establishments, which is an alternate means of increasing consumption diversity.

This paper investigates the ability of a multisector business cycle model in which product diversity depends on both (intertemporal) frictional business formation and (intratemporal) shopping time of households to fit both these facts and other features of the aggregate data. There are two investment margins (new goods and physical capital), labor in production and entry, congestion externalities of entrants, and habit formation so that households smooth consumption without very strong wealth effects. I estimate the model by Bayesian means on output, consumption, investment, labor supply, wages, and firm entry. To sort out the role of firm entry and shopping time, I separately estimate versions in which one of these ingredients is absent.

The baseline model fits the aggregate data very well and produces only mild procyclicality of shopping time and essentially zero comovement between shopping time and firm entry. Moreover, shopping time is less procyclical in the presence of firm entry, showcasing how business formation provides households an alternate means to diversify consumption.

These results rely on the fact that the comovement between shopping time and output depend on the type of shock. Whereas preference shocks generate positive comovement, technology shocks induce negative comovement, and the correlation is mixed for other types of shocks. By contrast, business formations and output positively comove more consistently

across shocks.

The forecast error variance decomposition shows that entry-cost shocks explain nearly all variation in firm entry and most consumption diversity even though they explain a much smaller amount of other series. Entry also enhances the relative contribution of technology shocks to investment. The volatility of shopping disutility shocks cannot be identified without entry. Preference shocks explain large shares of consumption and labor supply in each model. They also play an important role in generating the procyclicality of shopping time but can only explain a small amount of consumption diversity with entry.

There are additional issues to address in follow-up work. The first regards identification. It would be very useful to partially identify the elasticity of the matching function ϕ using microeconomic estimates and thereby reduce dependence on macroeconomic data. It may be possible to estimate a version of (13) by merging the proprietary Nielsen Consumer Panel Dataset with the shopping time and firm entry data used in this paper in a state-year panel. A slightly richer model may also permit identification of the persistence of shopping disutility shocks in addition to the conditional volatility.

Second, there are a number of papers which exploit shopping time (or search effort) in the goods market to explain productivity (Bai et al. (2012), Huo and Ríos-Rull (2016)). The idea is that shopping reduces the share of idle inputs by firms; shopping time would thereby provide additional amplification of demand shocks. The results of this article lead me to strongly recommend the inclusion of both shopping time and frictional firm entry in such applications. A reasonable follow-up is to incorporate endogenous utilization as in Huo and Ríos-Rull (2016) and estimate the model with the Solow residual alongside the series used in this paper.

References

- Aguiar, M., Hurst, E., Karabarbounis, L., 2013. Time use during the Great Recession. *American Economic Review* 103, 1664–96.
- An, S., Schorfheide, F., 2007. Bayesian analysis of DSGE models. *Econometric reviews* 26, 113–172.
- Bai, Y., Rios-Rull, J.V., Storesletten, K., 2012. Demand shocks as productivity shocks. Federal Reserve Board of Minneapolis .
- Barsky, R.B., Sims, E.R., 2011. News shocks and business cycles. *Journal of Monetary Economics* 58, 273–289.
- Basu, S., Fernald, J.G., Kimball, M.S., 2006. Are technology improvements contractionary? *American Economic Review* 96, 1418–1448.
- Benhabib, J., Rogerson, R., Wright, R., 1991. Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy* 99, 1166–1187.
- Berentsen, A., Waller, C.J., 2009. Optimal stabilization policy with endogenous firm entry. FRB of St. Louis Working Paper No .
- Bergin, P.R., Corsetti, G., 2008. The extensive margin and monetary policy. *Journal of Monetary Economics* 55, 1222–1237.
- Bernard, A.B., Redding, S.J., Schott, P.K., 2010. Multiple-product firms and product switching. *American Economic Review* 100, 70–97.
- Bilbiie, F.O., Ghironi, F., Melitz, M.J., 2012. Endogenous Entry, Product Variety, and Business Cycles. *Journal of Political Economy* 120, 304–345. doi:[10.1086/665825](https://doi.org/10.1086/665825).
- Bilbiie, F.O., Ghironi, F., Melitz, M.J., Midrigan, V., Rotemberg, J.J., 2007. Monetary policy and business cycles with endogenous entry and product variety [with comments and discussion]. *NBER Macroeconomics Annual* 22, 299–379.
- Blanchard, O., Perotti, R., 2002. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *Quarterly Journal of Economics* 117, 1329–1368.

- Bollard, A., Klenow, P.J., Li, H., 2014. Entry costs rise with development. Stanford Center for International Development Working Paper 518, 1–35.
- Broda, C., Weinstein, D., 2010. Product creation and destruction: Evidence and price implications. *American Economic Review* 100, 691–723.
- Diamond, P.A., 1982. Aggregate demand management in search equilibrium. *The Journal of Political Economy* , 881–894.
- Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. *The American Economic Review* 67, 297–308.
- Gabrovski, M., 2020. Simultaneous innovation and the cyclicity of R&D. *Review of Economic Dynamics* 36, 122–133.
- Greenwood, J., Hercowitz, Z., Huffman, G.W., 1988. Investment, capacity utilization, and the real business cycle. *The American Economic Review* , 402–417.
- Hamermesh, D.S., Frazis, H., Stewart, J., 2005. Data watch: The american time use survey. *Journal of Economic Perspectives* 19, 221–232.
- Hamilton, J.D., 2018. Why you should never use the Hodrick-Prescott filter. *Review of Economics and Statistics* 100, 831–843.
- Heathcote, J., Storesletten, K., Violante, G.L., 2010. The macroeconomic implications of rising wage inequality in the United States. *Journal of Political Economy* 118, 681–722.
- Herbst, E.P., Schorfheide, F., 2015. Bayesian estimation of DSGE models. Princeton University Press.
- Huo, Z., Ríos-Rull, J.V., 2016. Financial frictions, asset prices, and the great recession. CEPR Discussion Paper No. DP11544 .
- Jaimovich, N., Rebelo, S., 2009. Can news about the future drive the business cycle? *American Economic Review* 99, 1097–1118.
- King, R.G., Plosser, C.I., Rebelo, S.T., 1988. Production, growth and business cycles. *Journal of Monetary Economics* 21, 196–232.

- Laing, D., Li, V.E., Wang, P., 2007. Inflation and productive activity in a multiple-matching model of money. *Journal of Monetary Economics* 54, 1949–1961.
- Lewis, V., 2009. Business cycle evidence on firm entry. *Macroeconomic Dynamics* 13, 605–624.
- Lewis, V., Stevens, A., 2015. Entry and markup dynamics in an estimated business cycle model. *European Economic Review* 74, 14 – 35. doi:<https://doi.org/10.1016/j.euroecorev.2014.11.005>.
- Offick, S., Winkler, R.C., 2019. Endogenous firm entry in an estimated model of the us business cycle. *Macroeconomic Dynamics* 23, 284–321.
- Petrosky-Nadeau, N., Wasmer, E., 2015. Macroeconomic dynamics in a model of goods, labor, and credit market frictions. *Journal of Monetary Economics* 72, 97–113.
- Petrosky-Nadeau, N., Wasmer, E., Zeng, S., 2016. Shopping time. *Economics Letters* 143, 52–60.
- Petrosky-Nadeau, N., Wasmer, E., Zheng, S., 2014. Shopping time. Working Paper .
- Ramey, V.A., 2011. Identifying government spending shocks: It’s all in the timing. *The Quarterly Journal of Economics* 126, 1–50.
- Romer, C.D., Romer, D.H., 2004. A new measure of monetary shocks: Derivation and implications. *American Economic Review* 94, 1055–1084.
- Ruge-Murcia, F.J., 2007. Methods to estimate dynamic stochastic general equilibrium models. *Journal of Economic Dynamics and Control* 31, 2599–2636.
- Smets, F., Wouters, R., 2007. Shocks and frictions in us business cycles: A bayesian DSGE approach. *American economic review* 97, 586–606.
- Swanson, E.T., 2015. Measuring the effects of unconventional monetary policy on asset prices. Technical Report. National Bureau of Economic Research.

Appendix A. Shopping time statistics by state

Table A.7 presents the number of observations and mean and standard deviation of shopping time by state. California has the most observations (13,565), and Wyoming has the least (264). The means and standard deviations only vary moderately by state.

	count	mean	std
California	13565	49.0	79.9
Texas	9924	48.5	83.2
New York	7281	46.4	78.1
Florida	6954	46.5	79.5
Illinois	5666	48.8	81.3
Pennsylvania	5579	45.5	77.2
Ohio	5184	46.8	76.8
Michigan	4807	44.7	76.0
North Carolina	4112	47.5	80.1
Virginia	3932	47.1	79.4
Georgia	3859	46.4	83.4
New Jersey	3614	50.6	81.9
Washington	3190	47.8	76.3
Minnesota	3121	43.6	76.1
Indiana	3048	47.0	82.1
Wisconsin	3025	46.0	77.8
Missouri	2900	47.1	78.1
Massachusetts	2854	48.4	81.0
Tennessee	2694	44.7	78.1
Colorado	2603	44.2	75.5
Maryland	2589	49.7	84.5
Arizona	2517	51.4	82.7
Alabama	2282	41.5	73.1
Kentucky	2267	41.5	75.5
South Carolina	2200	41.8	75.8

Oregon	2021	44.7	72.6
Louisiana	1928	43.7	81.6
Oklahoma	1864	45.6	83.8
Iowa	1804	41.4	77.7
Kansas	1638	39.4	70.2
Connecticut	1562	49.2	77.1
Mississippi	1462	37.3	73.9
Arkansas	1425	43.4	82.4
Utah	1424	37.6	68.6
Nevada	1205	43.8	71.1
New Mexico	1042	46.5	83.7
Nebraska	985	45.5	78.2
West Virginia	884	45.2	88.1
Idaho	837	43.2	75.5
New Hampshire	638	44.4	71.0
Maine	619	43.8	76.4
Montana	519	36.3	70.8
Rhode Island	479	54.3	95.5
South Dakota	467	37.9	78.5
North Dakota	419	40.8	83.8
Delaware	403	47.6	79.4
Hawaii	395	51.7	85.8
District of Columbia	348	42.5	76.9
Vermont	302	47.2	83.0
Alaska	287	38.5	71.2
Wyoming	264	42.2	90.7

Table A.7: Number of observations and mean and standard deviation of shopping time (unweighted) for each state

Appendix B. Additional regression results

We regress log shopping time (weighted by the ATUS sampling weight) on output growth following the four specifications detailed on the text. Column I is the simple unweighted regression. Column II weights by the average state population over the sample. Column III adds fixed effects, and Column IV adds a state-specific linear trend. The results switch sign in IV but remain insignificant.

	I	II	II	IV
Log shopping time	-0.153	-0.0130	-0.0305	0.149
	(0.246)	(0.213)	(0.242)	(0.246)

Table B.8: Regression of log shopping time on output growth. Standard errors are clustered by state and in parentheses. The significance levels are $p < 0.05$, $** p < 0.01$, $*** p < 0.001$

Appendix C. Data appendix

All data is accessible from the St. Louis Fed Economic database other than the shopping time data, which is available from the American Time Use Survey, and the entry data, which comes from several sources described below. The construction of specific shopping-time variables follows [Petrosky-Nadeau et al. \(2016\)](#). The Bureau of Economic Analysis provides data on state output through the Gross Domestic Product by State release. The Census Bureau provides data on the resident population for each state. I omit the list of codes for brevity. Table [C.9](#) describes the raw data sources used in both the motivation and estimation. Table [C.10](#) describes the transformations used to link the data and model in estimation. Each series used as an observable ranges from 1948Q4 to 2009Q4. I construct firm entry using new business incorporations up to 1994Q4 and private establishment births after 1995Q1.

ID	Description	Source
LABSHPUSA156NRUG	Labor share of income	University of Groningen
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPDEF	GDP Deflator	BEA
GDPIC1	Real gross private domestic investment	BEA
CNP160V	Civilian non-institutional population	BLS
<i>estabs_{entry}</i>	Entry of establishments	BDS, Census
<i>firms</i>	Number of firms	BDS, Census
BAWBA	Business applications with planned wages	BFS, Census
NBI	New business incorporations	SCB, BEA
ESTB	Private sector establishment births	BLS
-	State resident population	Census
-	State GDP	BEA
-	Shopping categories	ATUS

Table C.9: Data sources used in motivating evidence and estimation. The shopping time variables are constructed from the American Time Use Survey (<https://www.atusdata.org>) as by [Petrosky-Nadeau et al. \(2016\)](#), and the market time is constructed as [Aguiar et al. \(2013\)](#). See the description below.

I also list the time use codes for market work and shopping time:

1. **Market work.** Codes 05-01, 05-02, 05-99, 18-05-1, 18-05-02, and 18-05-99.
2. **Shopping time.** Comprised of the following subcategories:
 - (a) Consumer goods and services shopping other than groceries, gas, and food.
 - i. Shopping for consumer goods: 07-01-04, 07-01-05, 07-01-99, 07-99
 - ii. Researching goods and services: 07-02
 - iii. Waiting time: 08-01-02, 08-02-03, 08-03-02, 08-04-03, 08-05-02, 08-06-02, 08-07-02, 09-01-04, 09-02-02, 09-03-02, 09-04-02, 09-05-02, 12-05-04.
 - (b) Purchasing groceries, gas, and food:
 - i. Groceries: 07-01-01

- ii. Gas: 07-01-02
- iii. Food: 07-01-03
- (c) Travel time associated with shopping: 18-07, 18-08, 18-09, 18-12-04

Time series	Construction	Description
$dl(Y_t)$	$dl\left(\frac{GDPC1_t}{CNP160V_t}\right)$	growth rate of real per capita GDP
$dl(C_t)$	$dl\left(\frac{PCND_t+PCESV_t}{CNP160V_t \times CPIAUCSL_t}\right)$	growth rate of per capita consumption
$dl(w_t)$	$dl(COMPRNFB_t)$	growth rate of real wage
$l(L_t)$	$l\left(\frac{HOANBS}{CNP160V_t}\right)$	logarithm of per capita hours worked
$dl(INV_t)$	$dl\left(\frac{FPI_t+PCDGT+CBI_t}{CNP160V_t \times GDPDEF_t}\right)$	growth rate of per capita investment
$dl(NE_{net,t})$	$\begin{cases} dl\left(\frac{NBI_t}{CNP160V_t}\right) & t \leq 1994Q4 \\ dl\left(\frac{ESTB_t}{CNP160V_t}\right) & t > 1995Q1 \end{cases}$	growth rate of per capita firms

Table C.10: Construction of data series. The function l and dl denote the demeaned logarithm and demeaned log-difference, respectively.

Appendix D. Derivation of key results within text

Appendix D.1. Shopping time equation

$$\begin{aligned}
\lambda_t \frac{\varepsilon}{\varepsilon - 1} \rho_t \mu_{st} c_t - \lambda_t \rho_t \mu_{st} c_t &= \kappa_t b_t \\
\lambda_t \frac{1}{\varepsilon - 1} \rho_t \mu_{st} c_t &= \kappa_t b_t \\
\frac{\lambda_t S_t \rho_t \mu_{st} c_t}{\varepsilon - 1} &= \kappa_t b_t S_t \\
\frac{\lambda_t C_t}{\varepsilon - 1} &= \kappa_t b_t S_t \\
\frac{\theta_t (C_t - h C_{t-1})^{-\sigma}}{\varepsilon - 1} C_t &= \kappa_t S_t
\end{aligned}$$

which coincides with (10).

Appendix D.2. Pricing rule

The firm matches with $AQ^{-\phi}$ consumers and produces $y = AQ^{-\phi}c$ units. Since cost minimization implies that $wl + r^K k = mc y$, the firm problem can be written as

$$\begin{aligned}
&\max_c \rho(c) AQ^{-\phi} c - mc AQ^{-\phi} c \\
&\Leftrightarrow \max_c AQ^{-\phi} c [\rho(c) c - mcc] \\
&\Leftrightarrow \max_c [\rho(c) c - mcc]
\end{aligned}$$

which gives rise to the first order condition

$$\frac{\rho'(c)c + \rho(c)}{\rho(c)} = \frac{mc}{\rho(c)}$$

Substituting $\rho'(c)c/\rho(c) = -1/\varepsilon$, we find that

$$\rho = \frac{\varepsilon}{\varepsilon - 1} mc$$

The fact that search frictions do not affect the pricing rule arises from the fact that search does not affect the share of overhead inputs, unlike in [Bai et al. \(2012\)](#).

Appendix D.3. The labor share of income and elasticity of substitution

As shown in the text, the labor share of income satisfies

$$\frac{wL}{Y} \equiv 1 - \bar{\alpha} = \frac{(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta}{\delta + \varepsilon(r + \delta)}$$

The last limits motivate us to examine the effects of the elasticity of substitution ε on the labor share of income more generally. As we have seen, if all production was composed of the retail sector, then a higher gross markup depresses the labor share of income. In fact, this result justifies the use of labor income share to proxy markups in several studies. However, in the multisector economy, as ε falls and goods become more differentiated, this both depresses the income of retail workers and induces the creation of new firms, which requires labor. Differentiation of $1 - \bar{\alpha}$ with respect to ε shows that the labor income share rises with the elasticity of substitution provided that $\delta(1 - 2\alpha) + r(1 - \alpha) > 0$. It turns out that the labor income share rises provided that $\alpha < \alpha_S$, where

$$\alpha_S = \frac{1 + r/\delta}{2 + r/\delta} > 1/2$$

Therefore, a sufficient—but not necessary—condition for the labor income share to rise with product substitutability is $\alpha < 1/2$, which is satisfied under a reasonable parameterization. Thus, markups and the labor income share are generally inversely related in the steady state, though not as strongly as in a one-sector model.

Appendix E. Details on equilibrium

Appendix E.1. Characterization of equilibrium (nonlinear)

Relative price	$\rho_t = \frac{\varepsilon}{\varepsilon - 1} mc_t$
Variety effect	$\rho_t = (AS_t^\phi N_t^{1-\phi})^{1/(\varepsilon-1)}$
Profits	$d_t = \frac{Y_t^C}{\varepsilon N_t}$
Congestion externality	$m_t = N_{E,t}^{-\eta}$
Labor intratemporal optimality	$w_t = \frac{\chi L_t^{1/\psi}}{\theta_t (C_t - hC_{t-1})^{-\sigma}}$
Free entry	$m_t \nu_t = w_t \frac{f_t}{Z_t}$
Firm law of motion	$N_t = (1 - \delta) (N_{t-1} + m_{t-1} N_{E,t-1})$
Capital accumulation	$K_t = (1 - \delta_K) K_{t-1} + I_{t-1}$
Shopping intratemporal optimality	$S_t = \frac{\theta_t (C_t - hC_{t-1})^{-\sigma} C_t}{\kappa_t (\varepsilon - 1)}$
Consumption marginal utility	$\lambda_t = b_t \theta_t (C_t - hC_{t-1})^{-\sigma}$
Euler equation (shares)	$\lambda_t = \beta (1 - \delta) \mathbb{E} \left\{ \lambda_{t+1} \frac{d_{t+1} + \nu_{t+1}}{\nu_t} \right\}$
Euler equation (capital)	$1 = \beta \mathbb{E} \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \delta_K + r_{t+1}^{K'} \right] \right\}$
Produced output	$Y_t^C = C_t + I_t$
Aggregate accounting	$Y_t^C + \nu_t m_t N_{E,t} = w_t L_t + N_t d_t + r_t^K K_t$
Total investment	$TI_t = I_t + \nu_t m_t N_{E,t}$
Labor in entry	$N_{Et} = \frac{Z_t L_{Et}}{f_t}$

Appendix E.2. Sequential computation of steady state

The key ratios can be solved for directly in terms of parameters. It is straightforward to solve for many variables as a function of consumption. Equilibrium consumption can be determined as a root of the loss function whose output is the distance between a hypothesized quantity of labor and that level which is consistent with intratemporal optimality.

First, in the steady state, $Z = \theta = f = b = \kappa = 1$. The Euler equation for shares implies $d/\nu = (r + \delta)/(1 - \delta)$. Rewrite the aggregate accounting relationship as follows:

$$\begin{aligned}
Y^C + \frac{\delta N}{1 - \delta} \nu &= wL + r^K K + \frac{Y^C}{\varepsilon} \\
Y^C \left(\frac{\varepsilon - 1}{\varepsilon} \right) + \frac{\delta N}{1 - \delta} \nu &= wL + r^K K \\
Y^C \left(\frac{\varepsilon - 1}{\varepsilon} \right) + \frac{\delta N}{1 - \delta} \frac{\nu}{d} d &= wL + r^K K \\
Y^C \left(\frac{\varepsilon - 1}{\varepsilon} \right) + \frac{\delta N}{1 - \delta} \frac{1 - \delta}{r + \delta} d &= wL + r^K K \\
Y^C \left(\frac{\varepsilon - 1}{\varepsilon} \right) + \frac{\delta N}{r + \delta} d &= wL + r^K K \\
Y^C \left(\frac{\varepsilon - 1}{\varepsilon} \right) + \frac{\delta}{r + \delta} \frac{Y^C}{\varepsilon} &= wL + r^K K \\
Y^C \left(\varepsilon - 1 + \frac{\delta}{r + \delta} \right) &= (wL + r^K K) \varepsilon
\end{aligned} \tag{E.1}$$

Using the rental rate of capital and the composition of retail output we find

$$\frac{Y^C}{K} = \frac{\frac{\varepsilon}{\varepsilon - 1} r^K}{\alpha} \tag{E.2}$$

Using (E.1) and (E.2) yields the ratio of labor income to capital income:

$$\begin{aligned}
\frac{wL}{r^K K} &= \left(\frac{\frac{\varepsilon}{\varepsilon - 1}}{\alpha} \right) \left(\frac{\varepsilon - 1}{\varepsilon} + \frac{\delta}{\varepsilon(r + \delta)} \right) - 1 \\
&= \frac{(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta}{\alpha(\varepsilon - 1)(r + \delta)}
\end{aligned} \tag{E.3}$$

Taking the ratio of the real wage and rental rate conditions and using (E.3) pins down the ratio of production labor to total labor L_C/L :

$$\begin{aligned}
\frac{L}{L_C} &= \frac{\alpha}{1 - \alpha} \frac{wL}{r^K K} \\
\frac{L}{L_C} &= \frac{(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta}{(1 - \alpha)(\varepsilon - 1)(r + \delta)}
\end{aligned} \tag{E.4}$$

Now, given a guess for C , we can find shopping time from the first order condition:

$$S = \frac{\theta C^{1 - \sigma} (1 - h)^{-\sigma}}{\kappa(\varepsilon - 1)}$$

Using (E.1), K can be pinned down in terms of consumption:

$$K = \frac{\alpha C}{\frac{\varepsilon}{\varepsilon - 1} r^K - \delta_K \alpha}$$

Given K , investment in physical capital satisfies $I = \delta_K K$. Retail output satisfies $Y^C = C + I$. Labor can be found by rearranging the labor supply equation (7):

$$\frac{wL}{r^K K} = \frac{\chi L^{1+1/\psi}}{\theta C^{-\sigma} (1-h)^{-\sigma}} \frac{1}{r^K K}$$

Using the ratio of labor income to capital income, we can solve for L as a function of C :

$$L = \left[\frac{\theta C^{-\sigma} (1-h)^{-\sigma} r^K K}{\chi} \frac{(1-\alpha)(\varepsilon-1)(r+\delta) + \delta}{\alpha(\varepsilon-1)(r+\delta)} \right]^{1/(1+1/\psi)}$$

The intratemporal condition, given L and C , can be also be used to find w . Labor in production follows from (E.4). We can now find ρ from the production function: $\rho = Y^C / (ZK^\alpha (L_C)^{1-\alpha})$. The variety effects condition then pins down N given ρ and S : $N = (\rho / (AS^\phi))^{1/(1-\phi)}$. Finally, the loss is the discrepancy in market clearing: $wL + r^K K + Nd - Y^C - \nu N_E$.

This procedure implicitly defines a loss function: $\mathcal{L}(C) : \mathbb{R}^+ \rightarrow \mathbb{R}$. A modified bisection method can be used to find a zero \mathcal{C} .¹⁹

Appendix E.3. Derivation of select log linearized equations

1. Consumption marginal utility

Let $J_t = (C_t - hC_{t-1})^{-\sigma}$. Apply logs to both sides and then take a first-order approximation:

$$\begin{aligned} -\sigma \log(C_t - hC_{t-1}) &= \log J_t \\ -\sigma \frac{C_t - \bar{C} - h(C_{t-1} - \bar{C})}{\bar{C}(1-h)} &= \frac{J_t - \bar{J}}{\bar{J}} \\ -\sigma \frac{\tilde{C}_t - h\tilde{C}_{t-1}}{1-h} &= \tilde{J}_t \end{aligned}$$

Using this expression, we log linearize the consumption marginal utility:

$$\begin{aligned} b_t \theta_t (C_t - hC_{t-1})^{-\sigma} &= \lambda_t \\ \tilde{b}_t + \tilde{\theta}_t - \sigma \frac{\tilde{C}_t - h\tilde{C}_{t-1}}{1-h} &= \tilde{\lambda}_t \end{aligned}$$

¹⁹A robust choice is the function *brentq* from the optimization library of SciPy in Python.

2. Law of motion for firms

$$\begin{aligned}
N_t &= (1 - \delta)(N_{t-1} + m(N_{E,t-1})N_{E,t-1}) \\
\bar{N}e^{\tilde{N}_t} &= (1 - \delta)(\bar{N}e^{\tilde{N}_{t-1}} + \bar{m}\bar{N}_E e^{\tilde{m}_{t-1} + \tilde{N}_{E,t-1}}) \\
\bar{N}\tilde{N}_t &= (1 - \delta)\bar{N}\tilde{N}_{t-1} + (1 - \delta)\frac{\bar{m}\bar{N}_E}{\bar{N}}(\tilde{m}_{t-1} + \tilde{N}_{E,t-1}) \\
\bar{N}\tilde{N}_t &= (1 - \delta)\bar{N}\tilde{N}_{t-1} + (1 - \delta)\bar{m}\bar{N}_E(\tilde{m}_{t-1} + \tilde{N}_{E,t-1}) \\
\tilde{N}_t &= (1 - \delta)\tilde{N}_{t-1} + \delta(\tilde{m}_{t-1} + \tilde{N}_{E,t-1}) \\
\tilde{N}_t &= (1 - \delta)\tilde{N}_{t-1} + \delta(1 - \eta)\tilde{N}_{E,t-1}
\end{aligned}$$

3. Euler equation for shares

Let $R_{t+1} = (d_{t+1} + \nu_{t+1})/\nu_t$ be the one-period rate of return on holding a share in a mutual fund.

$$\begin{aligned}
\lambda_t &= \beta(1 - \delta)\mathbb{E}\{\lambda_{t+1}R_{t+1}\} \\
1 &= \beta(1 - \delta)\mathbb{E}\left\{\bar{R}e^{\tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \tilde{R}_{t+1}}\right\} \\
1 &= \beta(1 - \delta)\mathbb{E}\left\{\bar{R}e^{\tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \tilde{R}_{t+1}}\right\} \\
\tilde{\lambda}_t &= \mathbb{E}(\tilde{\lambda}_{t+1} + \tilde{R}_{t+1})
\end{aligned}$$

upon substituting the steady state relationship and rearranging. We next find an expression for the rate of return on shares R_{t+1} .

$$\begin{aligned}
R_{t+1} &= \frac{d_{t+1} + \nu_{t+1}}{\nu_t} \\
&= \frac{\bar{d}e^{\tilde{d}_{t+1}} + \bar{\nu}e^{\tilde{\nu}_{t+1}}}{\bar{\nu}e^{\tilde{\nu}_t}} \\
&= \frac{\bar{d}}{\bar{\nu}}e^{\tilde{d}_{t+1} - \tilde{\nu}_t} + e^{\tilde{\nu}_{t+1} - \tilde{\nu}_t}
\end{aligned}$$

so that

$$\begin{aligned}
(\bar{d} + \bar{\nu})\tilde{R}_{t+1} &= \bar{d}(\tilde{d}_{t+1} - \tilde{\nu}_t) + \bar{\nu}(\tilde{\nu}_{t+1} - \tilde{\nu}_t) \\
\tilde{R}_{t+1} &= \frac{\bar{d}}{\bar{d} + \bar{\nu}}(\tilde{d}_{t+1} - \tilde{\nu}_t) + \frac{\bar{\nu}}{\bar{d} + \bar{\nu}}(\tilde{\nu}_{t+1} - \tilde{\nu}_t)
\end{aligned}$$

From the steady state Euler equation, $\bar{d}/(\bar{d} + \bar{\nu}) = (r + \delta)/(1 + r)$ and $\bar{\nu}/(\bar{d} + \bar{\nu}) =$

$(1 - \delta)/(1 + r)$. Hence,

$$\begin{aligned}\tilde{\lambda}_t &= \mathbb{E} \left\{ \tilde{\lambda}_{t+1} + \frac{(r + \delta)(\tilde{d}_{t+1} - \tilde{v}_t) + (1 - \delta)(\tilde{v}_{t+1} - \tilde{v}_t)}{1 + r} \right\} \\ \tilde{\lambda}_t + \tilde{v}_t &= \mathbb{E} \left\{ \tilde{\lambda}_{t+1} + \frac{(r + \delta)\tilde{d}_{t+1} + (1 - \delta)\tilde{v}_{t+1}}{1 + r} \right\}\end{aligned}$$

4. Aggregate income

We decompose aggregate income and apply steady-state ratios:

$$\begin{aligned}Y_t &= N_t d_t + w_t L_t + r_t^K K_t \\ \tilde{Y}_t &= \frac{\overline{Nd}}{\overline{Y}}(\tilde{N}_t + \tilde{d}_t) + \frac{\overline{wL}}{\overline{Y}}(w_t \tilde{L}_t) + \frac{\overline{r^K K}}{\overline{Y}}r_t^K \tilde{K}_t \\ \tilde{Y}_t &= \frac{(r + \delta)(\tilde{N}_t + \tilde{d}_t) + [(1 - \alpha)(\varepsilon - 1)(r + \delta) + \delta](\tilde{w}_t + \tilde{L}_t) + \alpha(\varepsilon - 1)(r + \delta)(\tilde{r}_t^K + \tilde{K}_t)}{\delta + \varepsilon(r + \delta)}\end{aligned}$$

Appendix E.4. Log linearized system of baseline and no-entry models

Table E.11 describes the log linearized system for the baseline model.

Label	Equation
Relative price	$\rho = mc$
Variety effects	$(\varepsilon - 1)\rho = \phi S + (1 - \phi)N$
Profits	$d = Y^C - N$
Firm value	$\nu + m = w + f - Z$
Labor intratemporal	$L = \psi(w + \theta - \frac{\sigma}{1-h}(C - hC_{-1}))$
Number of firms	$N = (1 - \delta)N_{-1} + (1 - \eta)\delta N_{E,-1}$
Capital accumulation	$K = (1 - \delta_K)K_{-1} + \delta_K I_{-1}$
Shopping time	$S = \theta + C - \frac{\sigma}{1-h}(C - hC_{-1}) - \varepsilon_\kappa$
Consumption multiplier	$\lambda = b + \theta - \frac{\sigma}{1-h}(C - hC_{-1})$
Euler equation (shares)	$\lambda + \nu = \mathbb{E} \left\{ \lambda' + \frac{(r+\delta)d' + (1-\delta)\nu'}{1+r} \right\}$
Euler equation (capital)	$\lambda = \mathbb{E} \lambda' + \beta(r + \delta_K)(Y^{C'} - K')$
Production function	$Y^C = \rho + Z + \alpha K + (1 - \alpha)L_C$
Produced output	$Y^C = \frac{[\frac{\varepsilon}{\varepsilon-1}(r+\delta_K) - \delta_K \alpha]C + \delta_K \alpha I}{\frac{\varepsilon}{\varepsilon-1}(r+\delta_K)}$
Aggregate expenditure	$Y = \frac{\varepsilon(r+\delta)}{\delta + \varepsilon(r+\delta)} Y^C + \frac{\delta}{\delta + \varepsilon(r+\delta)} (\nu + N_E + m)$
Aggregate income	$Y = \frac{(r+\delta)(N+d) + [(1-\alpha)(\varepsilon-1)(r+\delta) + \delta](w+L) + \alpha(\varepsilon-1)(r+\delta)(Y^C)}{\delta + \varepsilon(r+\delta)}$
Total investment	$TI = \frac{\delta_K \alpha (r+\delta)(\varepsilon-1)I + \delta(r+\delta_K)(\nu+m+N_E)}{\delta_K \alpha (r+\delta)(\varepsilon-1) + \delta(r+\delta_K)}$
Real wage	$w = Y^C - L_C$
Labor in entry	$L_E = N_E + f - Z$
Stochastic processes	$x = \rho_x x_{-1} + \varepsilon_x \quad \text{for } x \in \{Z, \theta, f, b\}$

Table E.11: Log linearized system of baseline model. The table omits the symbol \sim , which denotes log deviations from steady state and abuse notation by using the equality sign $=$ rather than the approximation sign \approx for first-order approximations.

Table E.12 describes the log-linearized system for the no-entry model.

Label	Equation
Relative price	$\rho = mc$
Variety effects	$(\varepsilon - 1)\rho = \phi S$
Labor intratemporal	$L = \psi(w + \theta - \frac{\sigma}{1-h}(C - hC_{-1}))$
Capital accumulation	$K = (1 - \delta_K)K_{-1} + \delta_K I_{-1}$
Shopping time	$S = \theta + C - \frac{\sigma}{1-h}(C - hC_{-1})$
Consumption multiplier	$\lambda = b + \theta - \frac{\sigma}{1-h}(C - hC_{-1})$
Euler equation (capital)	$\lambda = \mathbb{E}\lambda' + \beta(r + \delta_K)(Y^{C'} - K')$
Production function	$Y = \rho + Z + \alpha K + (1 - \alpha)L$
Produced output	$Y = \frac{[\frac{\varepsilon}{\varepsilon-1}(r+\delta_K)-\delta_K\alpha]C+\delta_K\alpha I}{\frac{\varepsilon}{\varepsilon-1}(r+\delta_K)}$
Real wage	$w = Y - L$
Stochastic processes	$x = \rho_x x_{-1} + \varepsilon_x \quad \text{for } x \in \{Z, \theta, b\}$

Table E.12: Log linearized system of no-entry model. The table omits the symbol $\tilde{\cdot}$, which denotes log deviations from steady state and abuse notation by using the equality sign $=$ rather than the approximation sign \approx for first-order approximations.

Appendix F. Bayesian estimation

Appendix F.1. Details on Bayesian estimation

The log linearized system admits the matrix representation

$$\Gamma_0 \tilde{\beta}_t = \Gamma_1 \tilde{\beta}_{t-1} + \Psi \epsilon_t + \Pi \eta_t$$

where $\tilde{\beta}_t$ consists of all the variables (including the expectational variables). In this form, the model can be solved using Chris Sims' algorithm, which relies on the Schur decomposition and produces a VAR(1) representation

$$\beta_t = F \beta_{t-1} + g \epsilon_t \tag{F.1}$$

where β_t consists of all the non-expectational variables of $\tilde{\beta}_t$. Moreover, $\text{var}(\epsilon_t)$ is a matrix with the shock variances on the diagonal. The reduced form (F.1) is the transition equation of the state space representation. The measurement equation is

$$\mathcal{Y}_t = H \beta_t$$

where \mathcal{Y}_t is the vector of observable variables.

To calculate the (logarithmic) posterior distribution, we use Bayes' rule:

$$\log p(\Theta|Y) = \log p(Y|\theta) + \log p(\Theta) - \log p(Y)$$

where the likelihood $p(Y|\theta)$ is a distribution of the data given the parameters and the posterior $g(\theta|Y)$ is the distribution of the parameters given the data.²⁰

Appendix F.2. Convergence diagnostics and table of prior distributions

Convergence of the posterior distribution implies that the sample properties Markov Chain Monte Carlo iterations have stabilised. In particular, the moments of parameters as well as posterior density should be stable after iterations. Figure (F.14) plots the posterior density

²⁰The constant is

$$p(Y) = \int p(Y|\theta)p(\theta)d\theta$$

and is known as the marginal likelihood. This quantity integrates the likelihood with respect to the prior density. It ensures that the posterior density integrates to one but is immaterial for the shape of the posterior distribution. Thus, we discard $p(Y)$ for characterizing the posterior distribution.

across the MCMC draws and the 20,000-period moving average.²¹ The posterior density looks fairly stationary.

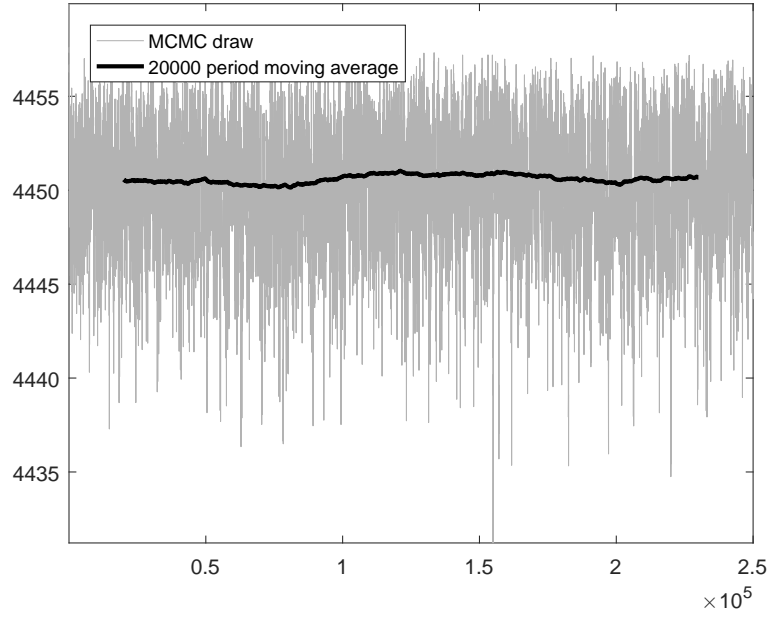


Figure F.14: Trace plot of posterior density.

Figure F.15: Trace plot of the posterior density for each of the 250,000 Metropolis Hastings draws and a 10,000-period moving average.

²¹Given a series $\{x_t\}_{t=1}^T$, where T is the number of simulations, the moving average curve is defined as $x_j = (1/(2q+1)) \sum_{k=-q}^q x_{j+k}$.

Appendix F.3. Table of prior distributions

Parameter	Distribution(μ, σ)	95% interval
σ	gamma(1.500,0.250)	1.051 .. 2.028
h	beta(0.200,0.160)	0.007 .. 0.593
ψ	gamma(0.720,0.400)	0.161 .. 1.690
ϕ	beta(0.500,0.270)	0.040 .. 0.960
ε	gamma(3.800,0.500)	2.884 .. 4.841
η	beta(0.500,0.250)	0.061 .. 0.939
ρ_Z	beta(0.500,0.200)	0.129 .. 0.871
ρ_θ	beta(0.500,0.200)	0.129 .. 0.871
ρ_f	beta(0.500,0.200)	0.129 .. 0.871
ρ_b	beta(0.500,0.200)	0.129 .. 0.871
σ_Z	inverse gamma(0.007,0.004)	0.003 .. 0.017
σ_θ	inverse gamma(0.015,0.007)	0.007 .. 0.033
σ_f	inverse gamma(0.007,0.004)	0.003 .. 0.017
σ_b	inverse gamma(0.007,0.004)	0.003 .. 0.017
σ_κ	inverse gamma(0.015,0.007)	0.007 .. 0.033
$\sigma_{w,ME}$	inverse gamma(0.015,0.007)	0.007 .. 0.033
$\sigma_{TI,ME}$	inverse gamma(0.015,0.007)	0.007 .. 0.033

Table F.13: Table of prior distributions. Each row contains the parameter symbol; the prior distribution family, mean, and standard deviation; and 95% coverage interval.

Appendix F.4. Second moments of alternate models

Tables F.14 and F.15 document the second moments of the no-shopping and no-entry models, respectively.

	$SD(x)$		RSD		$Cor(x, Y)$		$Cor(x, x_{-1})$	
	Data	Model	Data	Model	Data	Model	Data	Model
Estimation								
ΔY_R	1.00	1.02	1.00	1.00	1.00	1.00	0.37	-0.01
ΔC_R	0.62	0.68	0.62	0.66	0.48	0.43	0.31	0.25
ΔTI_R	3.77	3.43	3.77	3.35	0.84	0.88	0.03	0.03
Δw_R	0.76	0.76	0.76	0.75	0.17	0.43	0.03	0.03
ΔN_E	3.60	3.72	3.60	3.63	0.22	0.24	0.15	-0.02
L	4.20	3.68	4.21	3.59	0.09	0.16	0.97	0.96
Hamilton filter								
Y_R	3.63	2.70	1.00	1.00	1.00	1.00	0.91	0.87
C_R	2.46	2.26	0.68	0.84	0.74	0.52	0.89	0.90
TI_R	11.36	8.41	3.13	3.11	0.82	0.81	0.91	0.87
w_R	2.06	2.25	0.57	0.83	0.23	0.54	0.88	0.87
N_E	10.08	9.22	2.77	3.42	0.11	0.21	0.89	0.87
L	3.67	2.48	1.01	0.92	0.86	0.62	0.90	0.87
C	–	2.29	–	0.85	–	0.53	–	0.89
ρ	–	0.26	–	0.10	–	0.14	–	0.84

Table F.14: Second moments of model with no shopping time. Model moments are based on a series of 100,000 periods. The top panel examines the series for output, consumption, investment, wages, firm entry, and labor supply, consistent with the detrending procedure; and the bottom panel filters the model and empirical variables using the Hamilton regression filter.

	$SD(x)$		RSD		$Cor(x, Y)$		$Cor(x, x_{-1})$	
	Data	Model	Data	Model	Data	Model	Data	Model
Estimation								
ΔY_R	1.00	1.05	1.00	1.00	1.00	1.00	0.37	-0.00
ΔC_R	0.62	0.70	0.62	0.66	0.48	0.44	0.31	0.27
ΔTI_R	3.77	6.43	3.77	6.15	0.84	0.83	0.34	-0.01
Δw_R	0.76	0.74	0.76	0.71	0.17	0.42	0.03	0.02
L	4.20	3.47	4.21	3.32	0.09	0.16	0.97	0.96
Hamilton filter								
Y_R	3.63	2.76	1.00	1.00	1.00	1.00	0.91	0.88
C_R	2.46	2.33	0.68	0.84	0.74	0.58	0.89	0.90
TI_R	11.36	15.03	3.13	5.44	0.82	0.75	0.91	0.87
w_R	2.06	2.16	0.57	0.78	0.23	0.53	0.88	0.88
L	3.67	2.42	1.01	0.88	0.86	0.65	0.90	0.87
C	—	2.48	—	0.90	—	0.58	—	0.90
S	—	3.95	—	1.43	—	0.39	—	0.88
ρ	—	0.29	—	0.10	—	0.39	—	0.88

Table F.15: Second moments of model with no entry. Model moments are based on series of 100,000 periods. The top panel examines the series for output, consumption, investment, wages, and labor supply, consistent with the detrending procedure; and the bottom panel filters the model and empirical variables using the Hamilton regression filter.

Appendix F.5. Additional results of Bayesian estimation

Figure (F.16) plots the marginal prior and posterior densities for the no-shopping model. The posterior distributions are similar, but the estimated amount of external habit formation is lower.

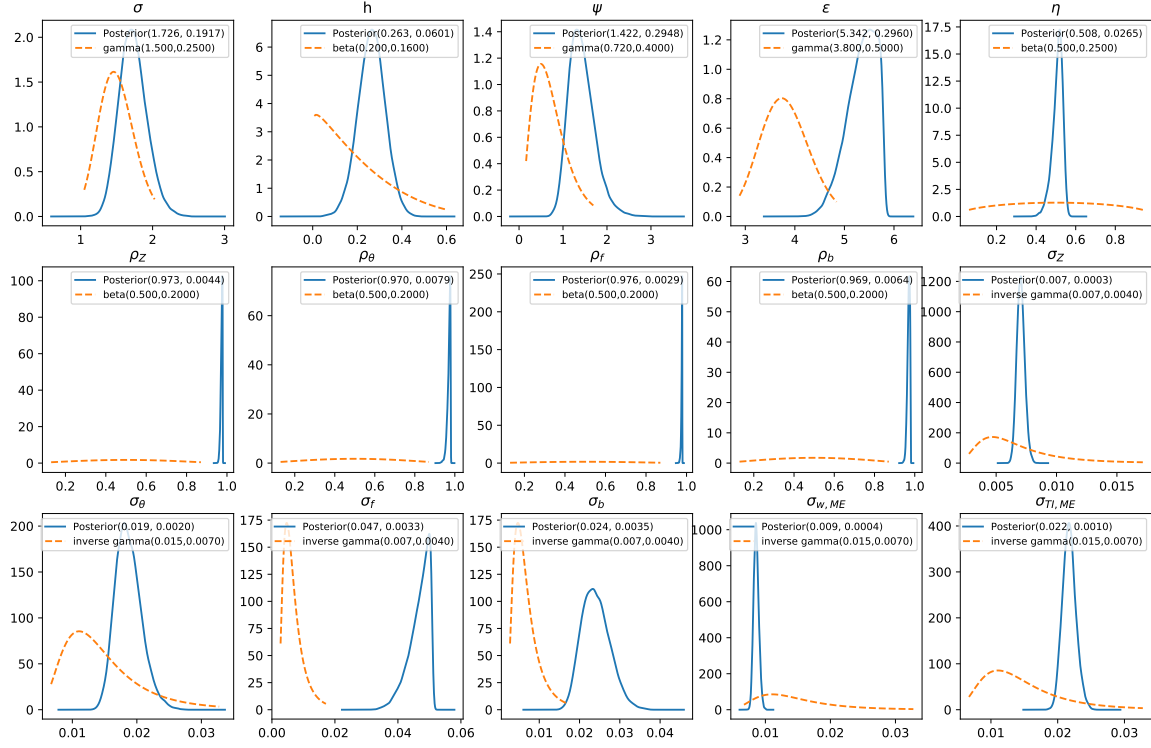


Figure F.16: Marginal prior and posterior densities for no-shopping model. The prior densities are set according to standard univariate distributions and the posterior distribution is approximated using the random walk Metropolis-Hastings algorithm.

Figure F.17 shows the impulse responses to a unit standard deviation positive technology shock. The responses are similar to the baseline model except that consumption variety unambiguously rises and has narrower confidence bands. Even this effect is quantitatively small, however.

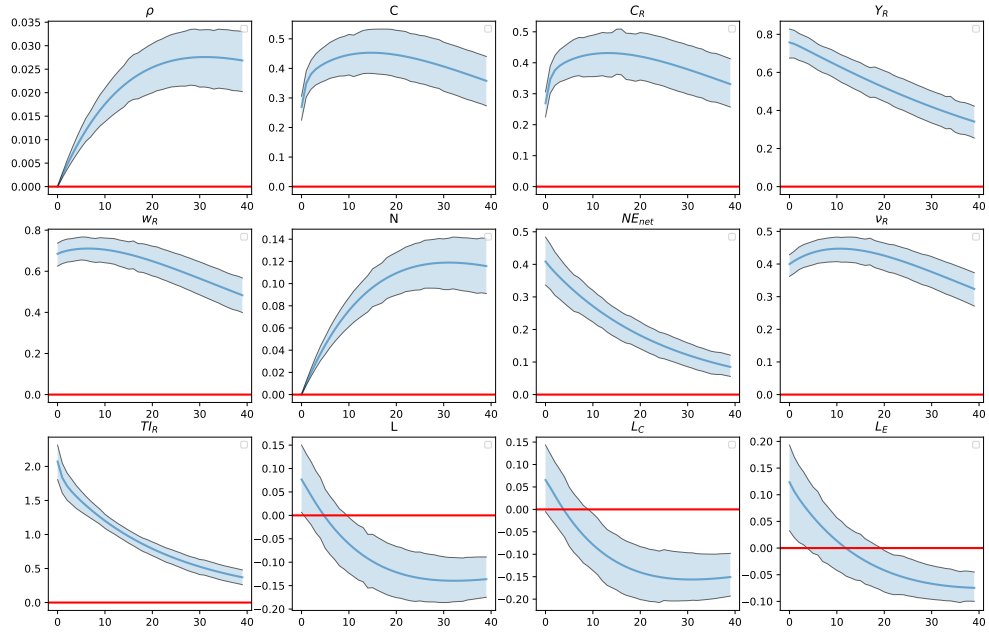


Figure F.17: A positive one standard-deviation technology shock in the no-shopping model. All variables are in percentage deviations. The units of the horizontal axis are quarters following the shock. The bold line represents the mean impulse response, and the shaded region indicates the 90% probability bands. The horizontal line denotes an impulse of 0 for reference.