Bond Flotation with Exotic Commodity Collateral

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Abstract

Exotic high tech metals such as rare earth oxides, titanium and nickel wire are increasingly important to semi-conductor, aerospace and high end defence technology R & D and production. As a result – while not market traded and therefore sporadic order dependent and highly volatile – prices of these commodities have been rising on average over the past decade. The metals trading subsidiary of an international group acquired over 6 M metres of nickel wire which at the current market price of about 300 EUR per metre is worth over 1.6 B EUR. The firm wished to raise from 300 M to 1B EUR in the capital markets in order to purchase a variety of these high tech metals to take advantage over the medium term (5 to 10 years) of generally rising prices. After a brief description of the current state of the exotic high tech metals markets, this paper treats the technical pricing and default risk analysis of an example 350 M EUR 7 year amortized corporate bond issue backed by a nickel wire inventory and subsequent high tech metal trading as collateral. Topics covered include security price modelling with high tech metal collateral, the design of 100% risk free securities with third party derivatives and security pricing and trading methodology. The complex stochastic and Monte Carlo simulation analyses presented are based in part on specially developed modelling of the nickel wire catalogue price and third party price projections for rare earth oxides and titanium. This analysis is based on 10 year (2008—2017) daily market data and supports an optimistic view in that after accounting for all ongoing costs we find a zero default probability for the bond issue – a situation seldom seen to accompany its stipulated 12% internal rate of return.

2 January 2020

Introduction and Background

This paper undertakes the fair pricing and default risk assessment of a 350 M EUR bond flotation developed as a project of Hanover Square Capital (UK) Ltd (Hanover) for an international client.Backed by nickel wire collateral, this is a 7 year amortized EUR bond issue with semi-annual coupons which we will assume to pay a basic 6% per annum.From the bond proceeds the issuer is assumed to immediately purchase in the open market a 225 M EUR portfolio of rare earth oxides (REO) and titanium (Ti) as a second form of collateral.
ries is a suitable proxy for the nickel wire catalogue price. The modelling of REO/Ni collateral prices in this paper is based on 78 months of historical data from November 2009 and price projections to bond maturity supplied by Argus Media (Argus, 2017a). Both types of high tech metal collateral for the bond issue can be sold to semiconductor, aerospace and high end defence technology companies, but do not trade on any of the exchanges (Argus, 2017a,b).

The trustee and the collateral manager, Hanover, assumed for this bond issue are also assumed to require ongoing support in the form of fair bond prices and default probability at each reset date through to the maturity of the bond issuance.The price source used for nickel wire is Alfa Aesar Switzerland, product number #40672[1]. STE SpA, an Italian Ministry of Defence and NATO security cleared defence contractor, are assumed to sell both the nickel wire collateral and the REO/Ti collateral purchased initially from the bond proceeds and to facilitate price discovery quarterly on a best efforts basis. Nickel metal supply and demand, expenditures on the global defence industry – particularly on stealth and advanced drone technology , the price of oil, and technology push in the electronics sector all influence the nickel wire catalogue price. The higher quantity of nickel stock currently in storage at LME approved warehouses corroborates the present lower LME Nickel prices. The bond issue with nickel wire collateral alone could be expected to have effectively *zero* default probability owing to nickel wire price mean reversion.

The next section briefly describes the markets for high tech nickel wire, rare earth oxides and titanium. See Argus (2017a,b) for more details. This is followed in Section 3 by an overview of the models employed to semi-annually sell collateral to meet liabilities and to price the bond and evaluate its default probability accordingly. An appendix describes these mathematical models in more detail and the numerical results corresponding to Section 3 are presented in the Section 4. To evaluate the robustness of the Take Profit trading strategy employed to price and risk assess the bond, the penultimate section of the paper contains various comparisons and an analysis of alternative collateral selling strategies. Section 6 concludes.

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[1]Thermo Fisher Scientific Inc. (NYSE: TMO) acquired Alfa Aesar, part of Johnson Matthey PLC, for £256 million (or approximately $405 million) in cash in September 2015. Thermo Fisher Scientific Inc. is the world leader in serving science, with revenues of $17 billion and approximately 50,000 employees in 50 countries. The company offers analytical instruments, laboratory equipment, software, services, consumables, reagents, chemicals, and supplies to pharmaceutical and biotech companies, hospitals and clinical diagnostic labs, universities, research institutions, and government agencies (`www.alfa.com`).

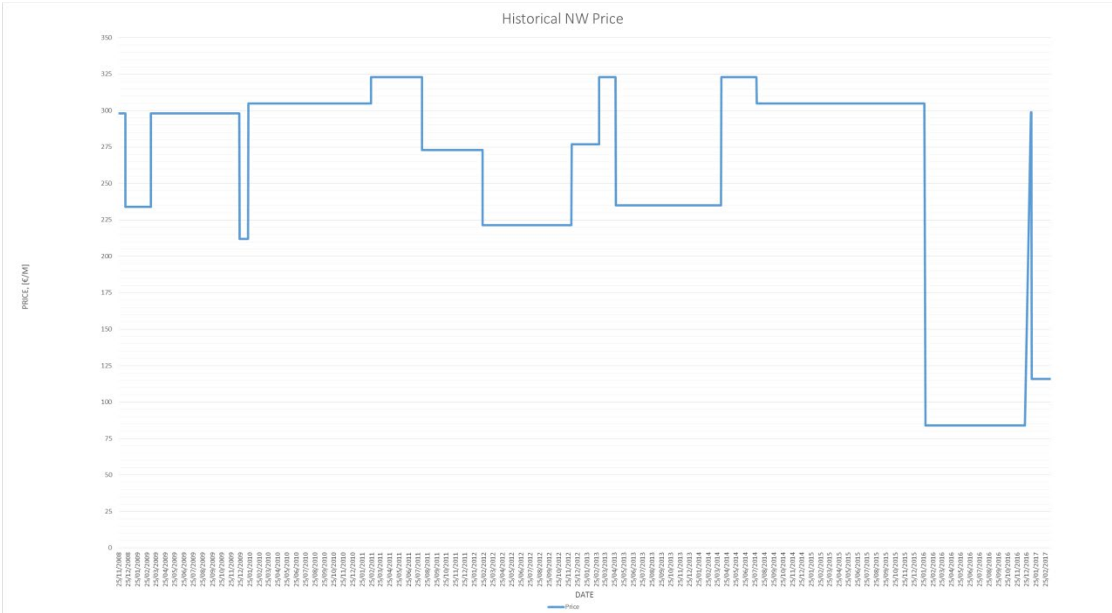
2. High Tech Metal Markets

The term *high-tech metal* or *technology metal* is used to refer to rare earth oxide metals, more well known metals such as titanium and molybdenum and the high-tech processing of common metals such as nickel and copper. All these metals go through advanced chemical and physical processing and are used as components in industries such as additive manufacturing, aerospace, defence, nuclear, electric cars and microelectronics. The metals employed in these industries are required to ensure high performance levels without any kind of disruption.

Nickel wire

The nickel collateral for the bond issue is not simply the mineral used to make stainless steel alloy or other large scale applications, but rather it is high purity nickel *wire*. The added value of this material comes from its level of purity – exceeding 99.9% -- and from an industrial manufacturing process that achieves an extremely thin cross section – 0.025mm, thinner than a human hair. This process requires highly sophisticated technology in order to maintain in large-production batches the material's characteristics such as: corrosion resistance, high tensile strength, ductility, thermal and radiation resistance and good electrical conductivity.

This type of nickel wire is considered a high-tech metal due to its particular characteristics and it is widely used in electronics, thermal sensor manufacturing, aerospace and the defence industries. Today, its field of use in large volumes spans many industries and industrial processes. Given the difficulties in producing and sourcing this material, it retains a very high intrinsic value, as short-term alternatives to it do not exist due to the particular combination of suitable mechanical physical and magnetic properties (Pocci (2015), p.12).The corresponding nickel wire in the Alfa Aesar catalogue is #40672. Nickel wire *value* can be estimated by consulting the Alfa Aesar catalogue,or other major distributors' websites such as VWR and Chemie Brunschwig (Pocci (2015), p.12). Nickel wire is not currently exchange traded, so that the term "market price" applied to it is a misnomer and actual prices are determined by a relatively few OTC bi-lateral trades compared with the typical number on an exchange. In the period September 2008 to the present, the historical price range of this material (to be used here as bond collateral) is as high as €323.00 per metre, in November 2011, and as low as €83.90 per metre in February 2016 from which in January 2017 it briefly jumped back to €299.00 per metre before jumping down again to the price in summer 2017 of €116.00 – 15 price jumps, see Figure 2.



Source: Alpha Aesar Switzerland, a subsidiary of Thermo Fischer Scientific

Figure 1. Alfa Aesar catalogue wire price 2008 to 2017

The companies that analyze the market for these materials are highly regarded internationally. They include:

- Rina Services delivers services of classification, certification, testing and inspection (TIC Services) to guarantee excellence to organizations in the marine, environment and energy, infrastructures, transport and logistics, and quality, safety and agri- food sectors (www.rina.org).

- The Schloer Consulting Group is among the few players in this market that can provide an exhaustive and certified documentation. Its client portfolio includes: UN, World Bank, IMF, AT&T,GeneralDynamics, General Electric, General Motors, American Industries,Leading Systems, Aramco, and Siemens (www.schloerconsulting.com).
- Argus Media of London with offices around the world is the leading specialist in the analysis of the structure of global high tech metal markets from sourcing through to final applications (www.argusmedia.com). They specialize in rare earth oxides.

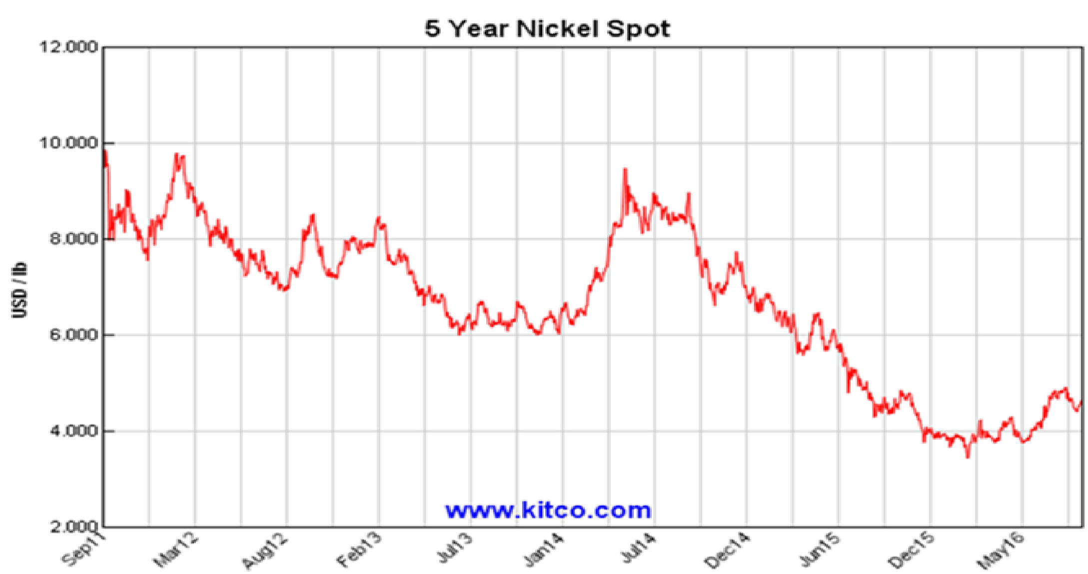
Nickel and copper

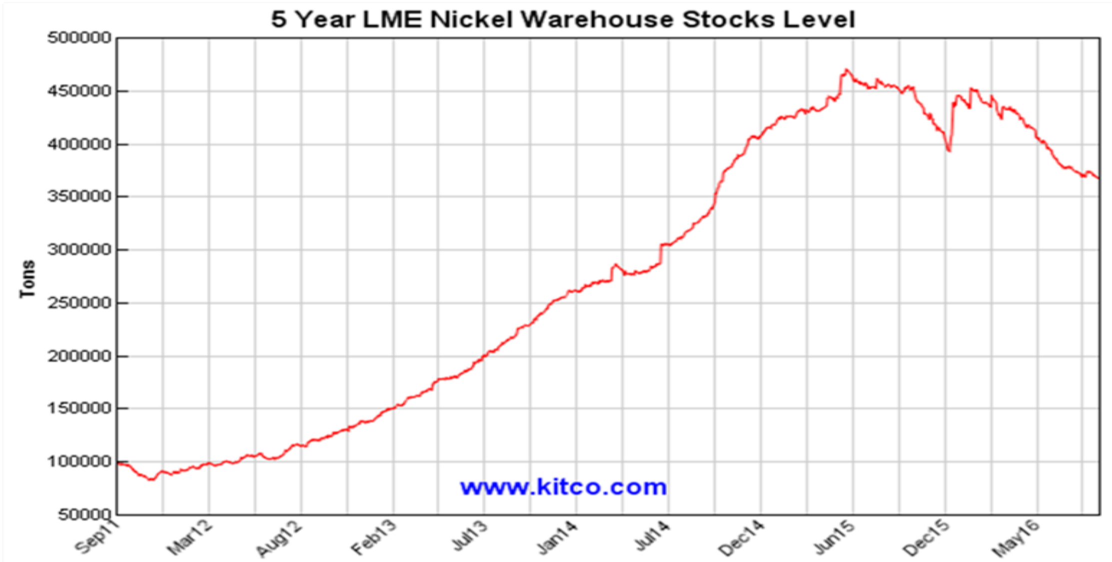
Turning to the underlying element for the nickel wire collateral, Figure 2 shows the recent evolution of the spot nickel price and inventory level on the London Metal Exchange. It indicates clearly the negative correlation between inventory levels and spot price. Warehouse stocks tend to rise with falling prices and fall with rising prices. The recent rise in nickel spot price is mainly due to market macroeconomic factors influenced by political factors in the producing countries. We shall return to this observation more generally in the sequel.

The value of the price of nickel, understood as a metal in all its applications, has changed due to a reduced demand related to the reduction of industrial production in China, which is the product's largest consumer. Most of the nickel produced is used in alloys for making stainless steels and the slowdown in recent years of Chinese production has considerably reduced the demand for the metal itself during that period.

In the recent years, the nickel price has also been affected by other aspects linked to the producing countries. At the beginning of 2014, Indonesia banned the export of unprocessed nickel, determining an upward trend in prices.In 2015, two mines were closed in the Philippines for environmental pollution reasons, which led to a subsequent reduction of the available reserves. Once the current reserves are reduced, the price of the metal should increase.Further mine closures are expected in the next few years for the same reason. In addition, the Philippine authorities intend to raise taxes for mining companies in the country, which will increase the local price of nickel directly and the global price subsequently. Independently, we have developed some model-based 7 year predictions for the nickel price which are consistent with this view.

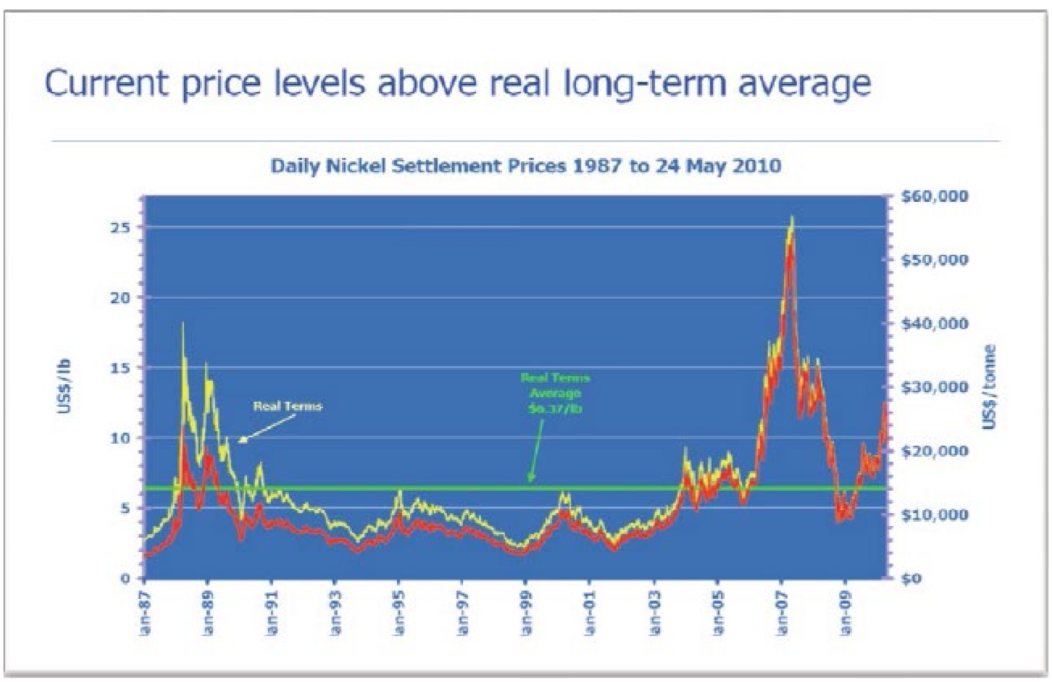
Figure 3 takes a longer term view of the spot nickel price and shows unmistakably the price effects of Chinese demand pre-crisis and of the collapse of global demand post-crisis.





Source: Kitco (2016)

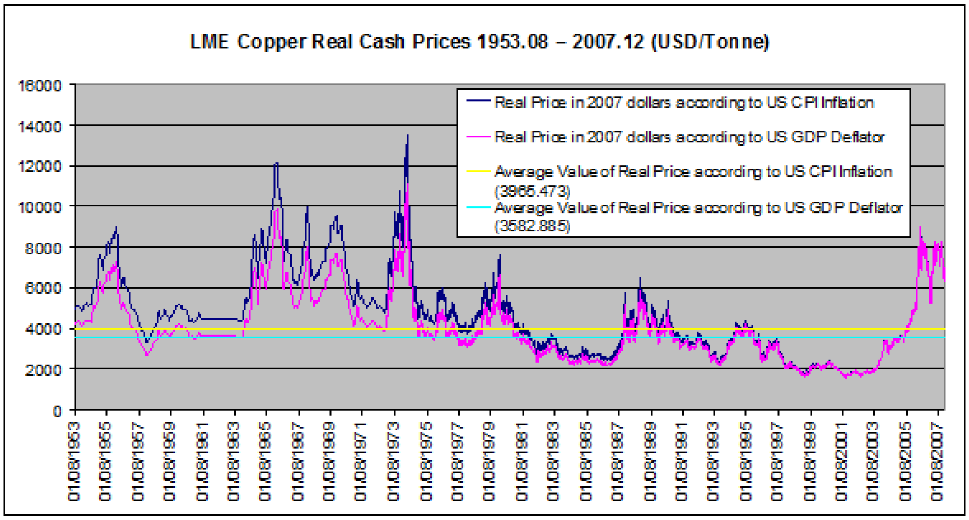
Figure 2. 5 year nickel spot prices and nickel warehouse stock levels 2011-2016



Source: Schloer Consulting Group (2010)

Figure 3. LME Nickel spot prices 1987 to 2010

The above average excursions of nickel spot price in the figure are remarkably similar to those shown for copper over a longer period in Figure 4.

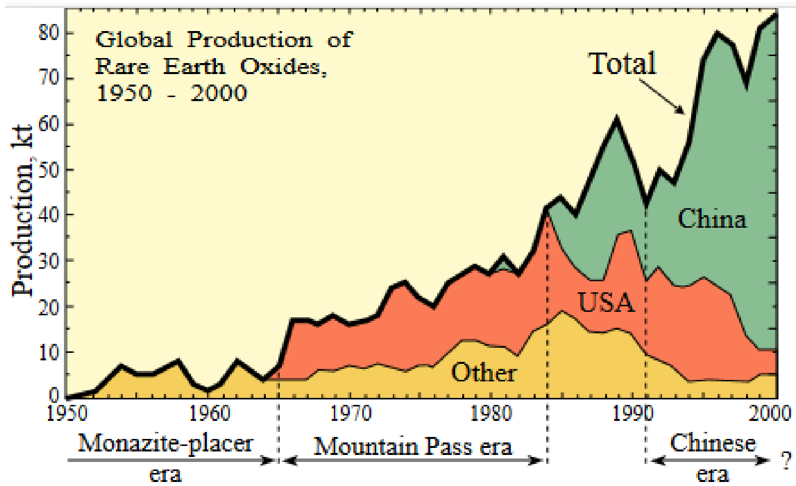


Source: CSA (2008)

Figure 4. LME Copper spot prices 1953 to 2008

For the years 1987 to 2008 in which the two price series overlap the pattern of agreement is even more remarkable with a correlation over 90% as result of geopolitical influence on the global economy and the demand for metals. Recently, a leading equity investment manager has noted that shocks from geopolitics represent the greatest long run multi-asset investment risks stating that “”Economies and capital markets are an outgrowth of what happens geopolitically. The big macro factors – economic growth, inflation, and interest rates – are each a result of what happens geopolitically. The trick is separating important trends and changes from shorter term noise.” (Franklin Templeton, 2019). A new book by a highly successful investor (Hawley & Lukomnik, 2020) discusses the mitigation of such long term risks.

However, regarding long run behaviour of REO exotic metals which constitute the second form of collateral for the bond analysed here, Figure 5 shows the growth of rare earth metal production from 1950 to 2000 as a rising trend which can be expected to continue well into the future. Note that while global demand is fluctuating all short term down trends reverse themselmes within a five year period. Since it is demanded by the same industries for related purposes the production of nickel wire must follow a similar pattern.



Source: Wikipedia (2019)

Figure 5. Global rare earth metal production 1950- 2000

Figure 6 graphs three potentially related price series in USD from 2008 to 2016: LME mid-price spot nickel (USD/tonne), nickel wire (Swiss price USD/m) and Thermo Fisher Scientific stock price (USD). The three series are all normalized to the sample mean and volatility of the LME spot nickel price.

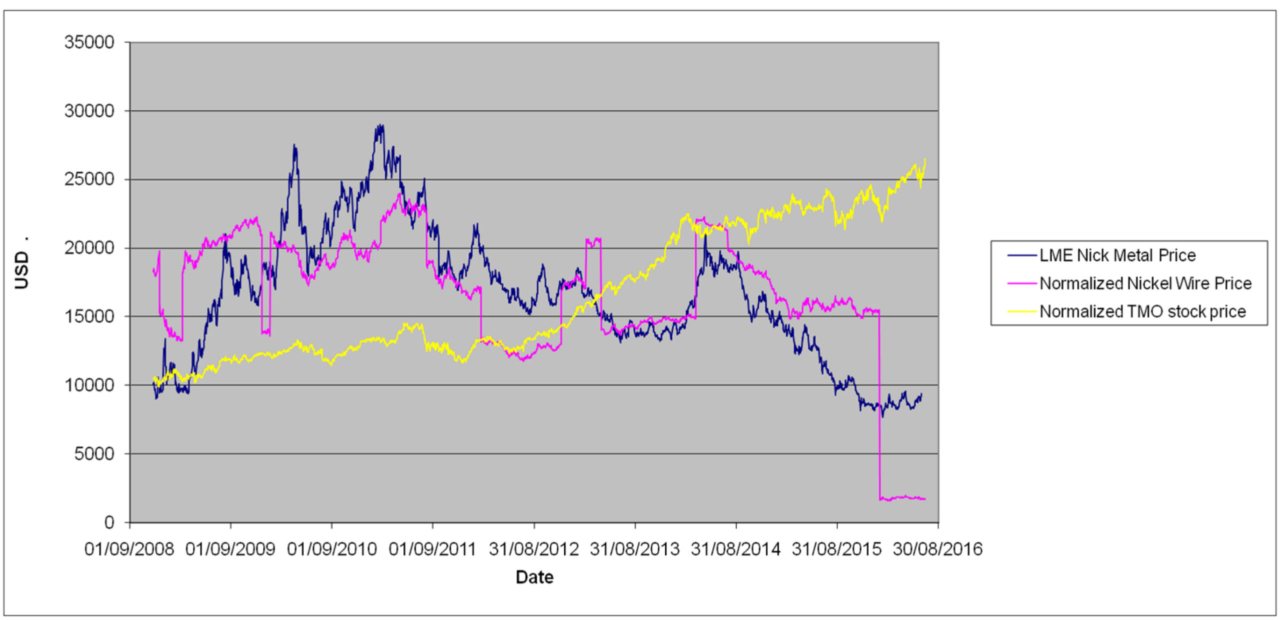


Figure 6. Graphs of 3 related price series in USD 2008-2016: LME mid-price spot nickel (USD/tonne), nickel wire (Swiss price USD/m) and Thermo Fisher Scientific stock price (USD)

Some important inferences can be drawn from this figure. At the outset of our research it was thought that the nickel price and the Thermo Fisher Scientific stock price, separately or in conjunction as factors, could serve as proxies for the catalogue wire price for the purposes of derivative pricing and hedging and/or investment. We see from the figure that this is not the case.

First, the Thermo Fisher Scientific stock price evolution over the period reflects only the general rise in US stocks over the post-crisis period. As the stock price of a large and important US based conglomerate, this is not in retrospect surprising.

Secondly, although the correlation between the LME metal price and the catalogue wire price series is 63.2%, and the two series can be seen to generally move together until early 2016, their association may be close enough for risky investment purposes, but certainly not for use as a wire price proxy for derivative pricing and hedging purposes.

Thirdly, nickel wire of 99.99% purity is achieved through technological processing. It is through this technical processing that it acquires its higher value. Generally the weight of nickel in nickel wire is very small.

Figure 6 also raises the following question: Why did the wire price not return over 2016 to its previous normalized level?[2] In fact, for the nickel wire product identified with Alfa Aesar code 40672 owned by the issuer, this is because the situation is different from that for nickel metal. The wire material has a strategic military application, and is thus related primarily to military industrial production. In recent years, particularly in Russia, military industrial has been reduced as a general budget reduction for this sector due to Western sanctions and has also. the global oil price fall. For different reasons, China has reduced its strategic materials consumption in recent years and these two elements strongly affect the wire price. However these impacts have recently reversed and throughout have been offset by increased defence expenditure by NATO countries. In fact the wire price returned to nearly 300 EUR per metre early in 2018 (outside our data sample period) where it remains at the present time.

Rare earth oxides and titanium (REO/Ti)

As well as nickel wire, the bond issuer focusses on other high tech metals including rare earths. In two recent reports (Argus, 2017a,b), Argus Media Consulting Services produced in-depth studies of the markets for, respectively, some specific rare earth oxides and titanium and for rare earths more generally. We will give a brief overviewof these markets sourced from the Argus reports,and then concentrate on their aspects relevant to modelling the profitable selling of the elements potentially chosen for of the second collateral portfolio.

The term *rare earths* refers to a group of seventeen unique chemical elements – the *lanthanides* – which are comprised of fifteen elements, plus *scandium* and *yttrium* which are

[2]Although nickel wire is priced in EUR/m, in Figure 6 the nickel wire price series was converted to USD at the historical USD/EUR rates and normalized to the mean of the metal price.

grouped alongside the lanthanides because of their similar physical and chemical properties. Rare earth elements may be separated into two sub-groups based on atomic weight. The first of these sub-groups, the *light rare earth elements*, are comprised of *lanthanum*, *cerium*, *praseodymium*, *neodymium* and *samarium* (atomic numbers 57 to 62). The second sub-group, the *heavy rare earth elements*, is comprised of the lanthanides with an atomic number ranging from 63 to 71: *europium*, *gadolinium*, terbium, *holmium*, *erbium*, *thulium*, *ytterbium*, *lutetium*, as well as *scandium* and *yttrium* (atomic numbers 21 and 39).

Permanent magnets incorporating praseodymium, samarium, neodymium, and dysprosium oxides are used in the manufacture of critical components for computers, data storage, as well as smart phones and other consumer electronics. Similarly, rare earth phosphors are used to light up high-tech plasma and liquid crystal display (LCD) screens. Lanthanum has traditionally been used as a fluid cracking catalyst (FCC) in the oil refinery process. Rare earths are also utilised in key applications across the defence industry, including missile guidance microchips and night-vision goggles.

Due to their unique chemical and physical properties, certain rare earth elements are considered to be critical inputs for a number of rapidly evolving markets in industrial and military applications. Moreover, viable substitutes for rare earth elements are not feasible without compromising critical advantages in terms of energy efficiency and overall performance. For example, there are currently no practical alternatives to rare earth permanent magnets that allow for the same level of technology miniaturisation while retaining the necessary energy yields. With very few practical alternatives available in the short-term, worldwide demand for rare earth oxides is set to strengthen well into the foreseeable future. Additive manufacturing is expected to be a driving force in expanding current fields of application of REO powders through Industry 4.0. A leading driver for overall REO demand is the expanding global use of rare earth permanent magnets. While permanent rare earthmagnets do not incorporate lanthanum and cerium, their growing use is fueling demand for neodymium, praseodymium, samarium, gadolinium, terbium and dysprosium.

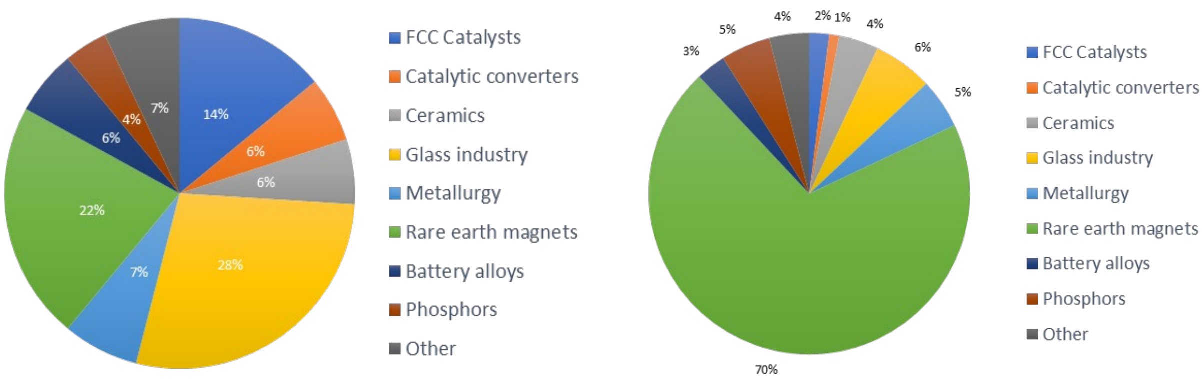
Target rare earth	Symbol	Main applications
▪ **Cerium**	Ce	Polishing compounds, catalytic converters, oxidants, phosphors, pigments, mischmetal
▪ **Dysprosium**	Dy	Magnets, discharge lamps, nuclear control rods
▪ **Europium**	Eu	Phosphors, nuclear control rods ,quantum memory chips, superconductors
▪ **Neodymium**	Nd	Permanent magnets, lasers, glass, catalysts
▪ **Praseodymium**	Pr	Magnets, mischmetal, carbon arc electrodes, pigments for glasses, enamels and glazes, high strength magnesium alloys
▪ **Terbium**	Tb	Dopants, phosphors, actuators, sonar, sensors
▪ **Yttrium**	Y	Phosphors, catalysts, synthetic garnet production, lasers, superconductors
▪ **Scandium**	Sc	Superalloys, fuel cells, lighting, ceramics, glass industry
▪ **Titanium Alloy (Grade5)^**	Ti6Al4V	Airframe and aero engine parts, naval applications, oil & gas, chemical industry, power generation, desalination, medical implants, consumer sports good (cycles, golf clubs etc.)

Source: CMTI

Table 7. Industrial uses of rare earth oxides and titanium alloy targeted for the second collateral portfolio

Production of high tech metals is in two stages: purification of mined ore to a high level of the basic element and then transformation into forms, such as powder, foil, wire or rods, used in the production processes of high tech industries with applications similar to those described above for nickel wire. More specifically, Table 7 lists some industrial uses of the high tech metals considered for the second collateral portfolio. With the exception of titanium alloy, all of these are in powder form.

Argus estimates global REO production at approximately 165,000T units output in 2016. China’s official and unlicensed production accounts for around 150,000T, or 91% of global REO output, well ahead of the rest of the world with15,000T, or 9%, consisting primarily of producers in Australia, Thailand, Malaysia and Russia. When determining the breakdown of consumption by the value of the rare earth products used, rare earth permanent magnets, which rely on high-value rare earths such as neodymium, dysprosium, terbium and praseodymium, account for approximately 70% of overall value. As one of the fastest growing applications for rare earth elements, it is these particular elements that are expected to prove most lucrative for emerging rare earth projects. Accounting for a considerably smaller proportion of total consumption by value is the glass industry (6%), the phosphors industry (5%), the metallurgy industry (5%), the ceramics industry (4%) and battery alloys (3%). Figure 8 gives a breakdown by product area and illustrates these market shares.



Source: Argus Media 2017

Figure 8. Estimated REOs consumption share and market share by application area for 2016

Although reserving the right to delete REOs and to add extra REOs to the second (traded) collateral portfolio, after due deliberation the bond issuer decided to initially purchase the 9 high tech metals whose normalized USD price projections are shown in Figure 8. The figure shows that a rare earth oxide, *europium*, and *scandium* have prices that are expected in general to decline, while others have a more complex forecast evolution.

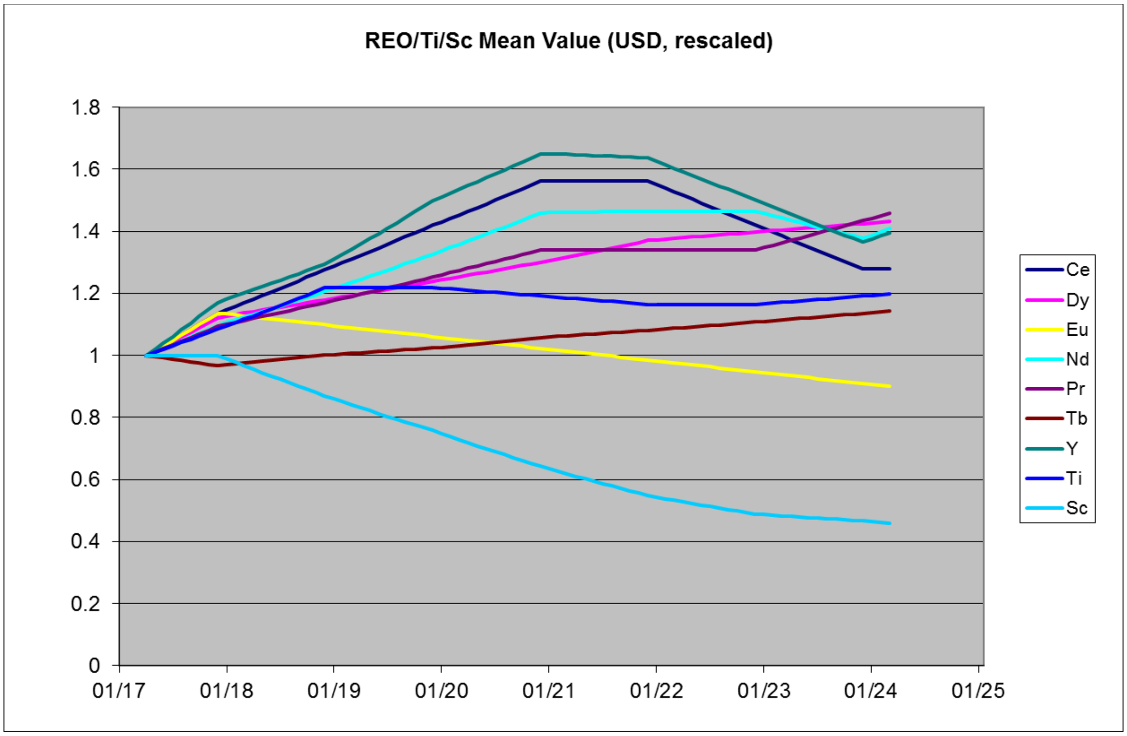


Figure 9. Argus REO/Ti/Sc normalized USD price projections

Using the semi-annual historical data from 2010 to 2017, Table 10 gives the interaction between these prices in terms of the normalized variance covariance (correlation off diagonal) matrix of the 7 REOs, Ti alloy and Scandium. Note that due to the nature of the data supplied we have been forced to estimate Scandium independently of the other price processes, which is why its correlations are zero. The correlation between each REO and Ti alloy also turns out to be quite small, while the volatilities (vols) are all fairly large relative to the usual range of equity vols (Sc is 25% p.a.) with some REO vols huge, e.g. Cerium at 63% and Europium at 56% (note that the diagonal elements of theVCV matrix are vol squared).

Ce	Dy	Eu	Nd	Pr	Tb	Y	Ti6Al4V	Sc
0.634	0.117	0.132	0.153	0.146	0.109	0.266	0.025	0
0.117	**0.4**	0.194	0.113	0.095	0.184	0.106	0.021	0
0.132	0.194	**0.312**	0.119	0.104	0.208	0.117	0.030	0
0.153	0.113	0.119	**0.126**	0.107	0.115	0.115	0.020	0
0.146	0.095	0.104	0.107	**0.113**	0.084	0.120	0.019	0
0.109	0.184	0.208	0.115	0.084	**0.217**	0.096	0.032	0
0.266	0.106	0.117	0.115	0.120	0.096	**0.275**	0.026	0
0.025	0.029	0.030	0.019	0.019	0.032	0.026	**0.056**	0
0	0	0	0	0	0	0	0	**0.065**

Table 10. Normalized variance covariance matrix of the REO/Ti/Sc price series

Table 11 lists price volatility, market share (a measure of *liquidity*) and initial portfolio weight for the 9 high tech metals proposed for the initial second collateral portfolio.

Element	Volatility % p.a.	Market Share %	Portfolio Weight %	Revised Weight %
Ce	63.5	5.0	7.22	10.83
Dy	45.1	6.5	7.22	10.83
Eu	***55.9***	3.0	7.22	***0***
Nd	35.5	6.5	7.22	10.83
Pr	33.6	6.5	7.22	10.83
Tb	46.6	4.0	7.22	10.83
Y	52.4	4.5	7.22	10.83
Sc	***25.5***	4.5	7.22	***0***
Ti	23.7	9.0	35	35

Table 11. Rare Earth Oxide/Titanium/Scandium Statistics

As a result of the analysis of this table, it was decided to drop Europium (lowest market share, second highest price volatility, declining price forecast) and Scandium (low market share, declining price forecast) from the225 M EUR initial second collateral portfolio.

3. Security Modelling

This section sets out in a natural order an overview of the methodology and mathematical models needed to support the bond issue, based on market data available up to the example valuation date of 28 April 2017. More detailed mathematical and statistical descriptions may be found in the Appendix.

Bond structure

As a basis for the comparison analysis of the effects of the two types of collateral we will value the amortized bond structure with all relevant payments and both types of collateral individually and together. The issue form of the bond with full collateral will be senior secured notes in the amount of up to 350M EUR – approximately half the current value of the nickel wire collateral – and we assume the full 350M EUR in the sequel.

For simplicity, we shall work with a bond *issue,* or a bond *face value* or *principal*, of 100 EUR.This convention has the advantage of being interpreted as 100% and allowing calculated bond prices to be some lower or higher percentages. This means that these prices only need to be multiplied by the issue amount in EUR divided by 100 to become EUR amounts, and all that must be reconciled initially is the corresponding wire amount for the potential issue size. Thus for exploratory calculations only one parameter needs to be changed for a different issue amount, which is derived by dividing the initial wire inventory in metres by the issue size in EUR multiplied by 100 EUR to obtain the number of metres of wire corresponding to the 100 EUR bond. For example, for an initial wire inventory of 6,034,483 metres and a 350 M EUR issue size, the number of metres of wire corresponding to the 100 EUR bond is 1.72 metres.

Moreover, in developing the issue structure, it is easier to think of relative figures in percentage terms rather than in EUR amounts. This approach has proved to be an efficient way of handling the natural considerable evolution of the research from initiation.

For the bond issue the bond holder receives a payment on each of the dates
t_i,i = 1,...,14, where

t_0 = 2017-07-01
t_1 = 2018-01-01
t_2 = 2018-07-01
t_3 = 2019-01-01
t_4 = 2019-07-01
t_5 = 2020-01-01
t_6 = 2020-07-01
t_7 = 2021-01-01
t_8 = 2021-07-01
t_9 = 2022-01-01
t_10 = 2022-07-01
t_11 = 2023-01-01
t_12 = 2023-07-01
t_13 = 2024-01-01
t_14 = 2024-07-01 .

The basic semi-annual *senior coupon (*coupon_rate) is assumed to be 6% and the bond is amortized after a two-year grace period with an annual amortisation schedule of 10% capital repayment in year 3, 15% in year 4, 20% in year 5, 25% in year 6 and 30% in year 7 with an option for the issuer to redeem the bond in part or in full on each coupon date.

The day count is 30/360.

For the 100 EUR bond, a *junior coupon* payment of 3.6260 EUR is made at t_i for i=5,…,14 for a total of 36.26 EUR to give an assumed target *internal rate of return* (IRR) of 12%.

Collateral prices

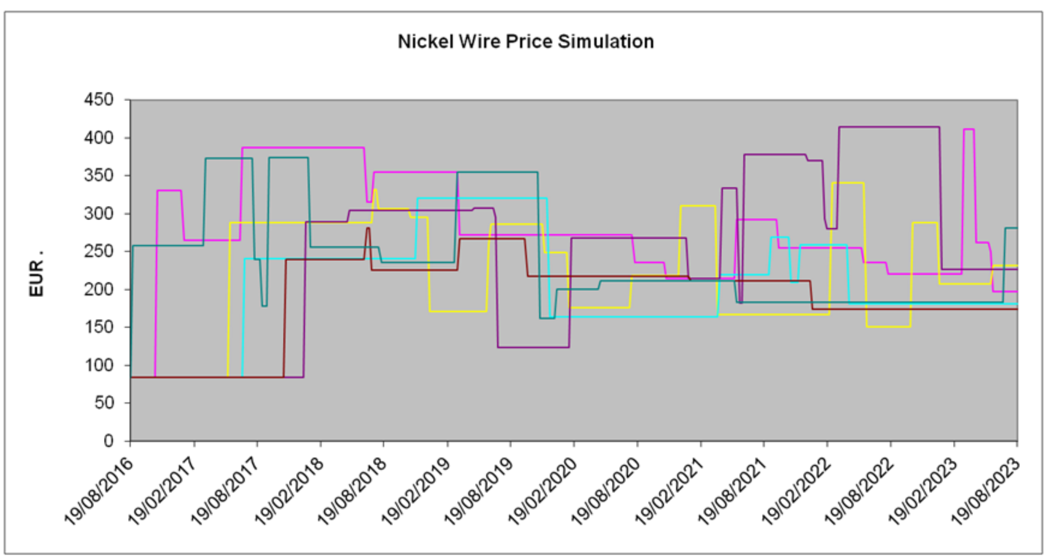
Nickel wire price

Market measure

Figure 2 in Section 2 shows not only the pure jump nature of the catalogue nickel wire price, but its mean reverting stationarity. This is a real world (indicative) price which is the closest approximation to a *market price* available for nickel wire. In this illiquid market the price jumps up or down due to occasional changes in demand for the wire and the corresponding purchases.

The model we have developed to fit the mean reverting (piecewise constant) pure jump nickel wire catalogue price actual realization shown in Figure 2 is a *mean reverting* pure jump model with a Poisson jump time process jumping to a realization of a ***fixed*** *log Gaussian* wire price distribution whose parameters are estimated from the data (CSA, 2016).

Figure 12 shows a sample of 5 simulated paths from our fixed log Gaussian mean reverting model all starting from the nickel wire price of €83.90 per metre on 19th August 2016.



Source: CSA (2016)

Figure 12. Simulated paths of the wire price process for 7 years (19.8.16 – 19.8.23) under P

Alternative pure jump wire price models were fit to the daily data with 13 jumps over 8 years from 2008 to 2016, a subset of the data depicted in Figure 2 with 15 jumps over 9 years, using both maximum likelihood and the method of moments. The model used here is a good fit to the data to 28th April 2017, and was chosen in part for its fast simulation capability (see the Appendix for more details).

Figure 13 shows the evolution of the forecast nickel wire catalogue price to bond maturity based on a 100,000 scenario Monte Carlo simulation with daily time step.

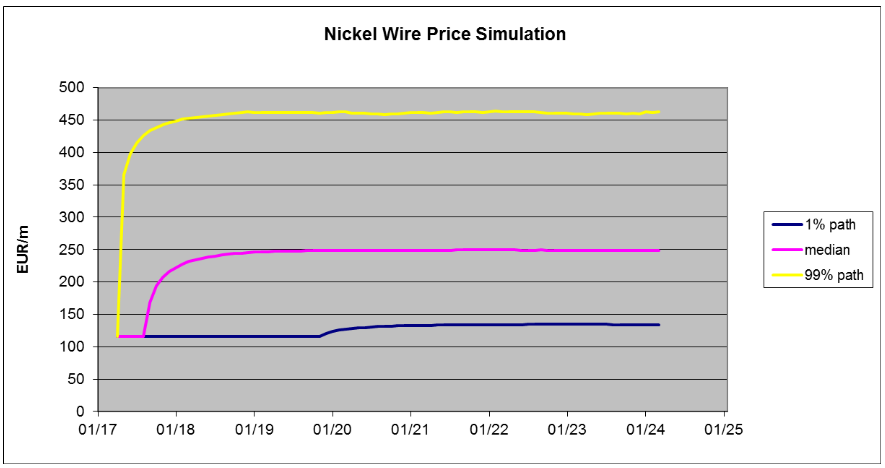


Figure 13. Nickel wire median catalogue price over 7 years with 1% and 99% confidence levels (100,000 scenarios under the market measure P)

Pricing measure

In order to price and consistently assess the default probability of the bond, we must change from the *market measure*, which includes the market's *premium for risk*, to a *pricing measure* in a *risk-neutral* world in which all securities return the instantaneous short rate of interest. After suitable discounting in such a world, all discounted stochastic cashflows are simply random fluctuations about a 0 mean path – a *martingale* (see the Appendix for the exact specification).

Figure 14 shows the evolution of the forecast nickel wire *risk-neutral price* from 1st July 2017 to bond maturity based on a 100,000 scenario Monte Carlo simulation with daily time step. Note that since the short rate at which all assets evolve in the risk neutral world is -33 bps, the median risk-neutral price is declining.

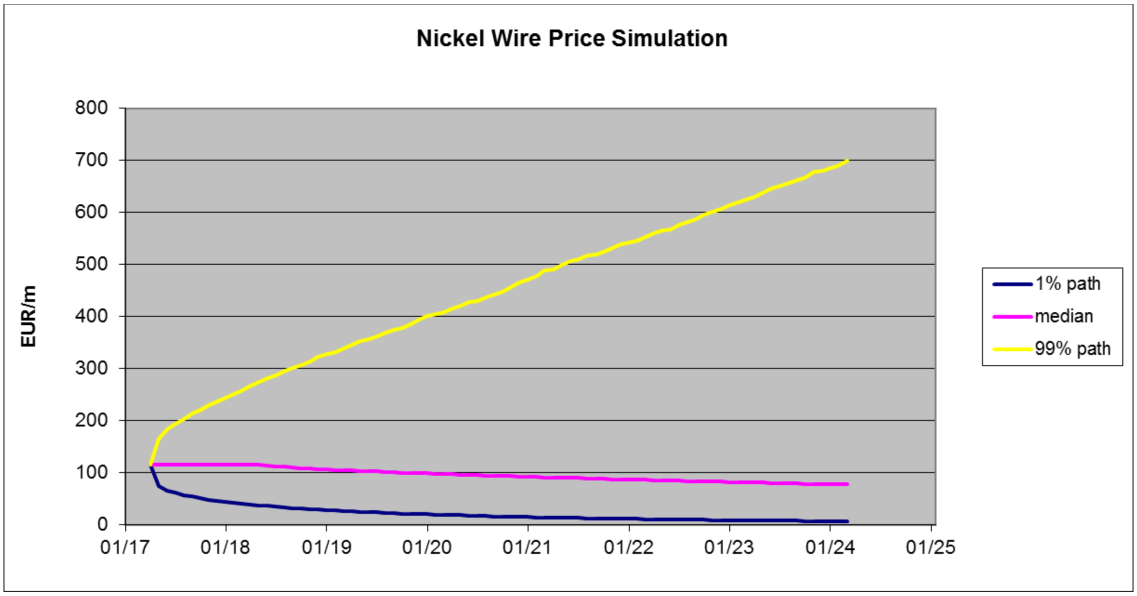


Figure 14. Nickel wire median risk-neutral price over 7 years with 1% and 99% confidence levels (100,000 scenarios under the pricing measure Q)

REO/Ti prices

Argus Media (Argus, 2017a,b) have produced annual forecasts from 2016 over the 7 year life of the bond for the rare earth oxides and titanium whose normalized forecast paths are depicted in Figure 9. We use these forecasts and monthly historical data for each REO and Ti from November 2009 to April 2017 to analyse and model the 9market price processes of these materials for forward simulation to bond maturity T under both P and Q using *geometric Brownian motion* (GBM)[3].

Market measure

As a representative example, Figure 15 presents the Monte Carlo simulation of the forward evolution to bond maturity of the market price of the rare earth oxide Cerium based on 100,000 scenarios using our model in which the drift path is the Argus forecast shown in Figure 9.

It should be noted from Table 11 that, at 63.9% per annum, the Cerium market price has the highest estimated volatility of all the REO/Ni portfolio elements. This accounts for the very high 99% path and the median path being well below the Argus forecast mean path in the figure.

[3] Preliminary investigation showed that the alternative *geometric Orstein-Uhlenbeck*(GOU) process would not be appropriate for incorporating the Argus projections.

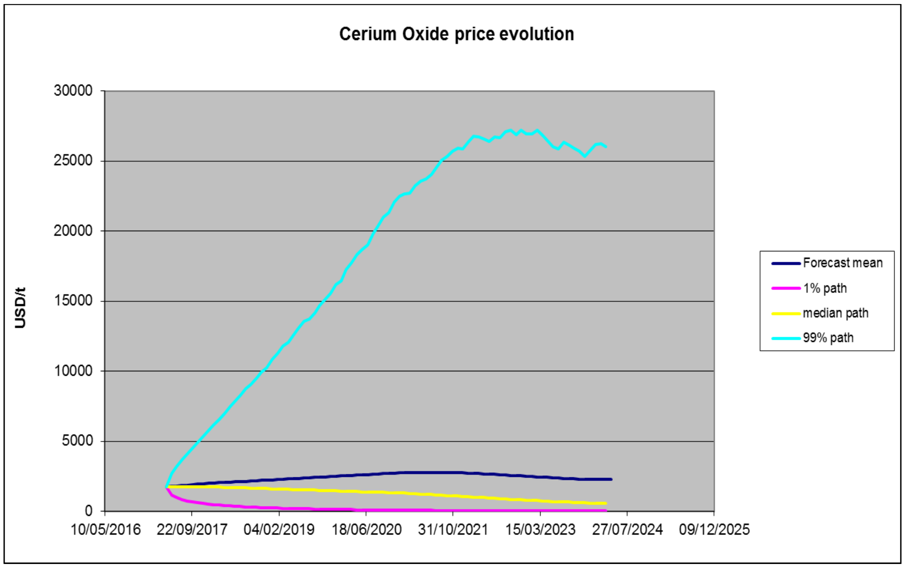


Figure 15. Cerium oxide market price evolution to bond maturity

Individual market price forecasts for each of the REO elements and Tiare given in Argus (2017a). Here we construct a portfolio of the 7 elements chosen in Table 10 with the weights given in the last column of the table and simulate the portfolio price from 1st July 2017 to bond maturity with a monthly time step.

Figure 16 depicts the simulated evolution of the portfolio market value under P to bond maturity in terms of market prices rescaled to an initial value of 1. Note the rising portfolio value and the considerable upside achieved.

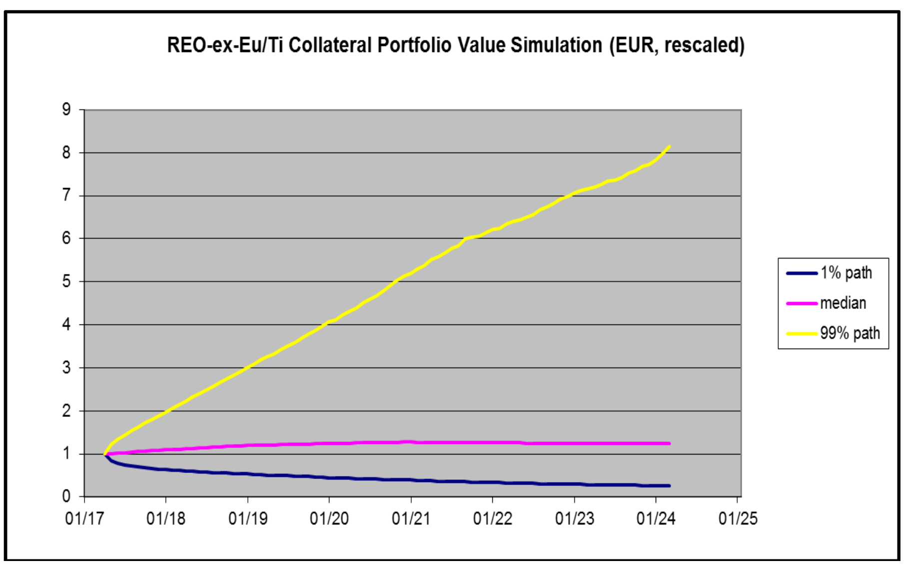


Figure 16. Rescaled evolution of the median portfolio market value with 1% and 99% confidence levels (100,000 scenarios under the market measure P)

Collateral portfolios

First we describe precisely all aspects of collateral management with the necessary assumptions to make these well defined in model terms. This is followed by a brief description of the two collateral portfolios and their combination in preparation for the results presented in Sections 5 of this report.

Collateral manager's model

This model describes how the cash flows charged to collateral by the Collateral Manager (Hanover) on behalf of the bond issuer vary over time as bond payments are made. Put simply, at each six-monthly *reset* date the *minimum* amount of collateral is sold to meet immediate obligations. After two years, as well as making coupon and principal- reducing amortization payments at specific reset dates, from then on to maturity a fixed payment is made to bondholders at each reset date in order to deliver the advertised 12% IRR on the bond overall. If there is not enough collateral to do meet obligations at current prices, all collateral held is sold and some later payments will *not* be made in full.

In practice, nickel wire collateral will be sold whenever possible and the proceeds escrowed. The elements of the REO/Ni collateral have much more liquid markets and will in general be sold first at reset dates when necessary.

The collateral manager must ensure that there is enough escrowed cash or sales proceeds to make all necessary payments and to maintain the required escrowed cash level going forward, whenever this is possible.

Cash outflows

If there is plenty of collateral, the trustee will be able to make all the payments from escrowed cash or sales of collateral. We now describe these cash outflows under the assumption there is always enough escrowed cash or collateral to meet the required obligation.

Bond holder payments

Bond holders receive three types of payments:*senior interest payments*, *principal payments* and *junior deferrable coupons*.The issue date of the bond is 2017-07-01, and each bond of face value100 EUR entitles the holder to the following payments (all in EUR). These may be thought of as percentages of the bond principal, so that to obtain the *actual payments* in millions of EUR one simply multiplies the figures shown by 3.5.

Period	Payment Date	Interest	Principal	Junior Interest	Total Payment
1	2018-01-01	3	0	0	3
2	2018-07-01	3	0	0	3
3	2019-01-01	3	0	0	3
4	2019-07-01	3	0	0	3
5	2020-01-01	3	5	3.62604	11.626
6	2020-07-01	2.85	5	3.62604	11.476
7	2021-01-01	2.7	7.5	3.62604	13.826
8	2021-07-01	2.475	7.5	3.62604	13.601
9	2022-01-01	2.25	10	3.62604	15.876
10	2022-07-01	1.95	10	3.62604	15.576
11	2023-01-01	1.65	12.5	3.62604	17.776
12	2023-07-01	1.275	12.5	3.62604	17.401
13	2024-01-01	0.9	15	3.62604	19.526
14	2024-07-01	0.45	15	3.62604	19.076

Senior interest payments are made in arrears based on a 6% p.a. coupon rate with amortizing principal, while junior interest payments are made in arrears from the third year to maturity and are chosen here to yield the target rate of return of 12% per annum overall.

Fee payments

We assume that in each payment period the Trustees also make a total payment of 2.5M EUR in fees for services from various parties (i.e. 5M EUR annually), but these are made *in advance* (in M EUR) as follows.

Period	Payment Date	Fee Payment
"0"	2017-07-01	2.5
1	2018-01-01	2.5
2	2018-07-01	2.5
3	2019-01-01	2.5
4	2019-07-01	2.5
5	2020-01-01	2.5
6	2020-07-01	2.5
7	2021-01-01	2.5
8	2021-07-01	2.5
9	2022-01-01	2.5
10	2022-07-01	2.5
11	2023-01-01	2.5
12	2023-07-01	2.5
13	2024-01-01	2.5
14	2024-07-01	0

Note the notional "Period 0" with Payment Date 2017-07-01 corresponds to the bond issue date. Only fees are paid on this date, there is no bond holder payment. No payments are made to bond holders until the end of Period 1 on 2018-01-01.

Escrow

Between payment dates, enough cash should be held in escrow to ensure the bond holders receive full payments of senior interest at each of the next four payment dates. Since the Trustee must also make a fee payment at thestart of each payment period, in general between payment dates there needs to be enough cash for the next four bond holder payments and three fee payments.

For the issue of size 350M, there are 3.5M bonds of face value 100 EUR, so the fee payment per-bond is 2,500,000 / 3,500,000 = 0.714 EUR/bond. Hence at the bond issue date, before the initial 2.5M EUR fee is paid, we require an initial escrow level of 4*0.714 + 4*3 = 14.857 EUR/bond. After the initial fee of 0.714 EUR/bond is made we will leave 14.142 EUR/bond in escrow between the issue date (2017-07-01) and the first bond payment date (2018-01-01).

At later payment dates, after the bondholder payment for the payment period just ended and the fee for the payment period about to start have been made, the amount of cash which needs to be left in escrow is given by the following table (in EUR/bond).

Period	Payment Date	Fee Payment	Bondholder payment	Escrow Level
0	2017-07-01	0.714	0	14.142
1	2018-01-01	0.714	3	22.7689
2	2018-07-01	0.714	3	31.2449
3	2019-01-01	0.714	3	42.071
4	2019-07-01	0.714	3	52.672
5	2020-01-01	0.714	11.626	56.922
6	2020-07-01	0.714	11.476	61.022
7	2021-01-01	0.714	13.826	64.972
8	2021-07-01	0.714	13.601	68.772
9	2022-01-01	0.714	15.876	72.422
10	2022-07-01	0.714	15.576	75.922
11	2023-01-01	0.714	17.776	57.4317
12	2023-07-01	0.714	17.401	39.3164
13	2024-01-01	0.714	19.526	19.076
14	2024-07-01	0	19.076	0

For example, at 2023-07-01, the end of period 12, we make the bond holder payment for period 12 and the fee payment in advance for period 13, then we need to leave enough cash in escrow for the two remaining bond holder payments and for the one remaining fee payment: 19.526 + 19.076 + 0.714 = 39.316.

Priority of Trustee payments

To ensure that escrow works as described, at the end of a payment period, including the notional period "0", the collateral manager must perform three steps in the following order:

1) Determine how much collateral must be sold to bring the escrowed cashup to the level required to: a) authorize the trustee to make the bondholders payment, b) authorize the trustee to make the fee payment, c) leave enough cash in escrow for future payments and d) meet the loan to value test. If there is not enough collateral, even if all has been sold to bring the collateral level to this point, the bond will eventually default, and there will be no more collateral from this point forward.

2) Pay the bondholders, except at the bond issue date when bondholders receive no payment. This must happen before the fee payment for the nextperiod ismade as escrow generally contains four bondholderpayments but only three fee payments. The fees are however paid inadvance, so the fee payment for each period automatically takes priorityover the bondholder payment for the same period since it is made sixmonths earlier.

3) Pay the fee, except for the final payment date when no payment is made since fees are paid in advance.

Collateral trading strategy

To raise cash from selling collateral when it is required, we assume that positions in the REO/Ti portfolio are reduced first until that portfolio is entirely exhausted, when nickel wire is sold. In order to understand the relative contributions of the two collateral portfolios to bond valuation and default probability, before reporting the full results we will assume that the bond is only backed by each single collateral separately. This is easily accomplished by restricting the valuation software appropriately.

Nickel wire collateral (EUR)

For a bond principal of 350 M EUR, this is initially the current collateral of 6,112,126 metres of nickel wire valued at 709,006,616 EUR (or 116 EUR per metre) and only requires keeping track of the current inventory level of nickel wire and its value in each scenario after sales at each reset date.

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REO/Ti collateral (USD)

This is assumed to be a 225 M EUR portfolio of 6 rare earth oxides and titanium purchased from the 350 M EUR bond proceedswith the individual position weights given in the last column of Table 10. We shall see in the next section that not surprisingly the default rate of the bond with *only* this collateral will be high in the market measure.

We consider five possible strategies fordeciding which positions in the REO/Ti portfolio should be liquidated first:

Strategy 1: Each position is reduced by the same fraction. Thus to raise xEUR in cash when the current value of the portfolio is y EUR, we sell a fraction (x/y) of each position. This strategy pays no attention to the forecast return on different assets in the portfolio, nor to their past performance. We may term this the *buy and hold* strategy.

Strategy 2: Order the assets in the portfolio by their forecast return over the next six months, from lowest return to highest return. Then sell down the position which has the lowest forecast six monthly return. If the value ofthis position is larger than the required amount of cash, we need sell only part of it, otherwise sell all of it and move on to the asset with the next higher forecast return. Continue in this way, selling positions with low forecast returns until we have the required amount of cash. We term this the *momentum* strategy.

Strategy 3: This is the same as Strategy 2 except that we order assets from highest forecast six monthly return to lowest rather than from lowest to highest Thus the first position we reduce will be in the asset with highest forecast six monthly return. We term this the *contrarian* strategy.

Strategy 4: Order the assets in the portfolio by their realized return since the bond issue date (when the portfolio was originally bought) from lowest realized return to highest realized return. Then generate cash by selling down the position whose asset has the lowest realized return. If we need to generate more cash after this position is sold entirely, move on to the position in the asset with the next higher realized return. We term this the *cut losses* strategy.

Strategy 5: This is the same as Strategy 4 except that we order assets from highest realized return to lowest rather than from lowest to highest. This strategy amounts to "taking profit" from the position in the asset which achieved the greatest return to date, which we will therefore term the *take profit* strategy.

We shall compare the performance of these alternative strategies and their implications in Section 5, but here we will construct an example based on the Argus price forecasts (see Figure 9) to illustrate the different selling orders under the alternative strategies listed by name for simplicity as:-

1. *Buy and Hold*, the benchmark strategy in which all sales are portfolio increments with fixed weights
2. *Momentum*, sell lowest 6 month price forecast elements first
3. *Contrarian*, sell highest 6 month price forecast elements first
4. *Cut Losses*, sell lowest realized return to date assets first
5. *Take Profit*, sell highest realized return to date assets first.

For bond valuation in Section 5, rare earth oxides and titanium are priced in in USD in global markets, to value the REO/Ti portfolio we must discount USD cash flows with a US discount yield curve model and convert USD figures to EUR.

Figure 17 shows the EUR-USD rate since the inception of the euro,from which it may be seen that the rate has been declining on average since the crisis.

For forward Monte Carlo scenario generation we will use the standard Garman-Kohlhagen (1983) exchange rate model. We shall also need a EUR discount yield curve model to price the bond in EUR using either or both forms of collateral.

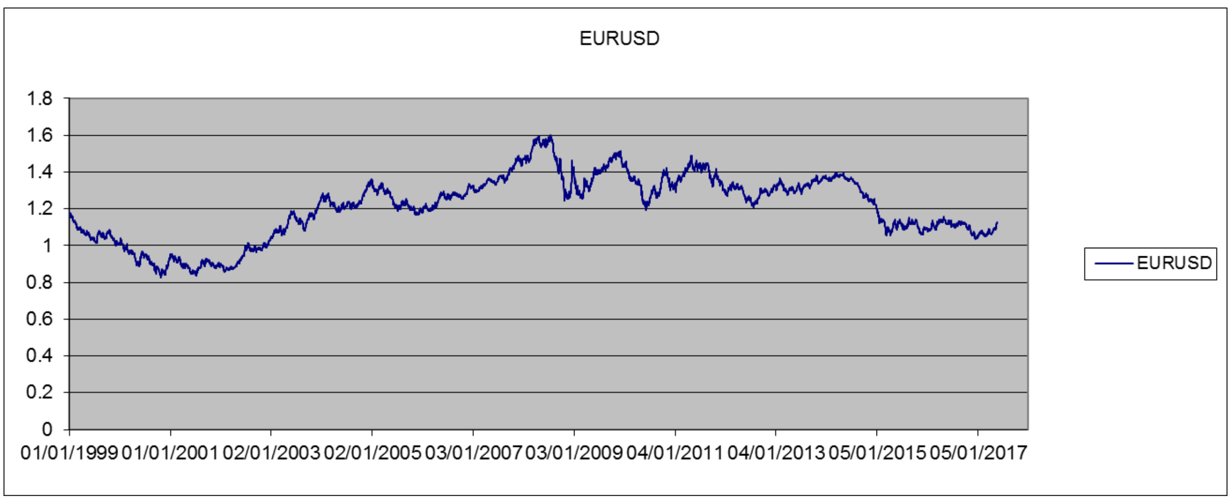


Figure 17. Evolution of the EUR-USD exchange rate from 1st January 1999

For our trading strategy illustrative example, we assume that the EUR-USD exchange rate follows a random path (i.e. an arbitrary choice of one of the 1,000,000 scenarios that we actually simulated from 1st July 2017) and we consider the situation at the first coupon payment date on 1st January 2018. Each 100 EUR bond is backed by (225/3.5 =) 64.285 EUR of REO/Ti collateral. For easy reference we repeat here the table of payments in the Escrow subsection of the Collateral Manager's model above.

Period	Payment Date	Fee Payment	Bondholder payment	Escrow Level
0	2017-07-01	0.714	0	14.142
1	2018-01-01	0.714	3	22.7689
2	2018-07-01	0.714	3	31.2449
3	2019-01-01	0.714	3	42.071
4	2019-07-01	0.714	3	52.672
5	2020-01-01	0.714	11.626	56.922
6	2020-07-01	0.714	11.476	61.022
7	2021-01-01	0.714	13.826	64.972
8	2021-07-01	0.714	13.601	68.772
9	2022-01-01	0.714	15.876	72.422
10	2022-07-01	0.714	15.576	75.922
11	2023-01-01	0.714	17.776	57.4317
12	2023-07-01	0.714	17.401	39.3164
13	2024-01-01	0.714	19.526	19.076
14	2024-07-01	0	19.076	0

Initially (at the bond issue date and before the first fee payment ismade) we require 14.857 EUR/bond in escrow. This is just enough to pay the first 6-monthly fee of 0.714 EUR/bond and leave 14.142 EUR/bond in escrow as required by the above schedule.

The initial collateral portfolio in terms of % weights and the EUR/bond value is:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	9.70	9.70	9.70	9.70	9.70	9.70	41.79
value	6.23	6.23	6.23	6.23	6.23	6.23	26.87.

Between date 0 and date 1 (2018-01-01) all assets increase in value (in USD terms) except for Tb which falls. The forecast price for Tb at the end of 2017 is 425 USD/t while the latest price for Tb (March 2017) is 440USD/t, so that a price drop may be expected.

Overall, the collateral portfolio rises in value from 64.285 EUR/bond to 67.75 EUR/bond. The value in EUR of each holding has also increased (except for Tb), as the FX rate fell only slightly from 0.91133 to 0.89903 EUR/USD:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	9.99	9.90	9.73	9.71	8.86	10.22	41.59
value	6.77	6.71	6.59	6.58	6.00	6.93	28.18.

At this point we have 14.142 EUR/bond in escrow. We need to pay the bond holders 3 EUR/bond, the collateral manager 0.714 EUR/bond and leave 22.7689 EUR/bond in escrow for future payments. This requires 3 + 0.714 + 22.7689 = 26.4829 EUR/bond of which we currently have 14.142 in escrow already. Thus we need to raise 26.482 - 14.14 = 12.340 EUR/bond by selling collateral.

How we do this depends on the chosen trading strategy, so we consider each trading strategy in turn and show for each how the cash is generated.

Buy and Hold

We need to generate 12.340 EUR/bond from the collateral portfoliowhich is currently worth 67.75 EUR/bond, so we simply sell a fraction12.34/67.75 = 18.2% of each holding. This doesn't change any of the portfolio asset weights but decreases each holding's value by 18.2%.

Momentum

We look at the forecast return of each asset over the coming six months and order them from low return to high return. For the purpose of this ordering it does not matter which currency we value them in so we may simply use the USD forecast figures.

Price forecasts for date 1 and date 2 and return:

	Ce	**Dy**	**Nd**	**Pr**	**Tb**	**Y**	**Ti**
date 1	2000.645	208.978	43842.047	55315.343	425.040	4301.173	20.056
date 2	2120.978	214.096	45963.920	57202.730	432.414	4518.786	21.236
return%	6.015	2.449	4.84	3.41	1.73	5.06	5.88.

Ordering: Tb Dy Pr Nd Y Ti Ce.

We raise cash by selling as much as is needed, starting with Tb. Since the value of our Tb holding is only 6.00EUR/bond, we sell all the Tb and need to raise 6.340 EUR/bond from the rest of the portfolio, starting with Dy. Our holding in DY is 6.71 EUR/bond so we sell 6.340 EUR/bond of it, leaving a holding in Dy of 0.369 EUR/bond.

After these sales the collateral portfolio is:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	12.21	0.67	11.90	11.87	0.00	12.50	50.85
value	6.77	0.37	6.59	6.58	0.00	6.93	28.18.

Contrarian

This is the same as the Momentum strategy but reverses the order of assets:

Ordering: Ce Ti Y Nd Pr Dy Tb.

Since the holding in Ce is 6.77 EUR/bond we sell all of it and then sell 5.57 EUR/bond of the Ti holding.

After these sales the collateral portfolio is:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	12.21	0.67	11.90	11.87	0.00	12.50	50.85
value	6.77	0.37	6.59	6.58	0.00	6.93	28.18.

Cut Losses

We order the assets in the portfolio by their realized return since the bond issue date (when the portfolio was originally bought) from low realized return to high realized return.

Realized prices and returns:

	Ce	**Dy**	**Nd**	**Pr**	**Tb**	**Y**	**Ti**
date 0	1819.272	191.693	40921.231	51722.396	435.696	3819.924	18.862
date 1	2000.645	208.978	43842.047	55315.343	425.040	4301.173	20.056
return%	9.97	9.02	9.92	7.14	6.95	-2.45	12.60.

Order: Tb Ti Pr Nd Dy Ce Y.

Thus the first asset sold is Tb (we hold 6 EUR/bond and sell all of it), then Ti of which we hold 28.18 EUR/bond and sell 6.340 EUR/bond of it.

After these sales the portfolio is:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	12.21	12.10	11.90	11.87	0.00	12.50	39.42
value	6.77	6.71	6.59	6.58	0.00	6.93	21.84.

Take Profit

This is the same as Cut Losses but reverses the order of assets:

Order: Y Ce Dy Nd Pr Ti Tb.

Since the holding in Y is 6.93 EUR/bond we sell all of it and then sell 5.41 EUR/bond of the Ce holding.

After these sales the collateral portfolio is:

	Ce	Dy	Nd	Pr	Tb	Y	Ti
%	2.44	12.10	11.90	11.87	10.83	0.00	50.85
value	1.35	6.71	6.59	6.58	6.00	0.00	28.18.

Discount yield curve

To compute fair values of structured products under Q and perform risk evaluation simulations under P (and Q), we use a Gaussian affine 3-factor yield curve model for discounting which captures the simultaneous movement of short, medium and long rates. This model, termed the *economic factor model*(EFM) (Campbell, 2000; Medova *et al*., 2005; Dempster*et al*., 2010), has many parameters which need to be calibrated using historical market data. Its evolution under the *risk-adjusted* (*risk neutral*) probabilities corresponding to the pricing measure Q is determined by *stochastic differential equations* (SDEs) whose Wiener process increment terms are *correlated*. Its three *unobservable* Gaussian factors **R**, **X** and **Y** represent respectively a *short rate*, a *long rate* and (minus) the *slope* (difference) between an un- observable continuously compounded *instantaneous* short rate $\mathbf{R}^* := \mathbf{X} + \mathbf{Y}$ and the long rate.

Since the data to which we are fitting our model has evolved according to the actual *market* probabilities (historical frequencies), i.e. under the market measure P rather than the risk adjusted probabilities under the pricing measure Q, we must include parameters for the *market prices of risk* in the SDEs describing its factor evolution. Thus we include three parameters γ to allow for *constant* market prices of risk of the factors. The model used admits a closed form solution for zero-coupon bond prices (Medova *et al*., 2005) from which zero coupon bond yields may be derived at t in affine form. The bond prices corresponding to these yields, after replacing negative yields less than -33 bps (which occur with low probability) by -33 bps, provide appropriate discount factors for security forward payments whose present value under P or Q is sought at any time t (see the Appendix for more details).

Combined collateral

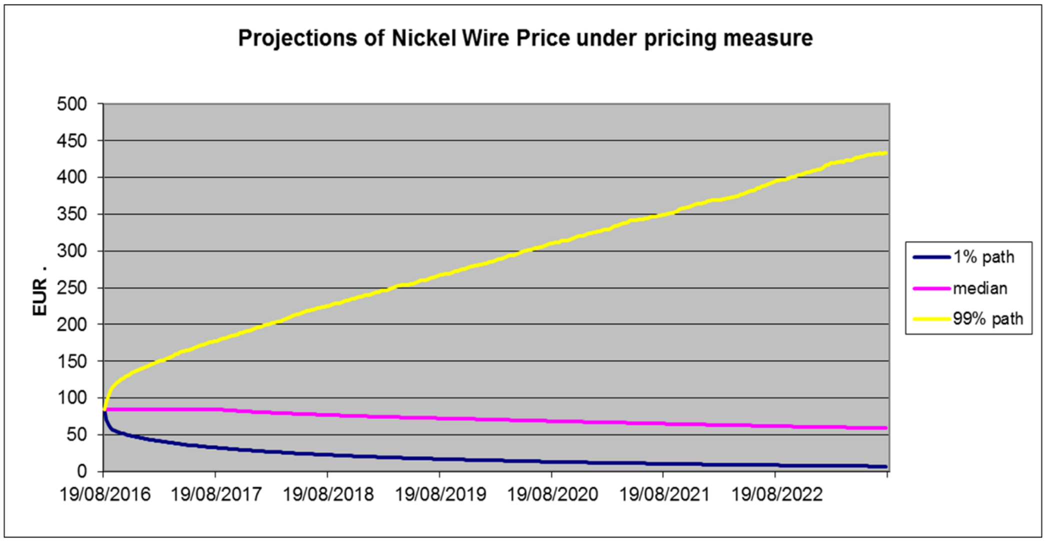
This is the full collateral for the bond for which results will be reported and sensitivity and robustness results given in in Section 5.

4. 100% Risk Free Securities with 3rd Party Derivatives

For completeness this section briefly describes an elegant solution to guaranteeing bond holder payments and fees by entering into an OTC derivative exchange with a credit worthy third party institution such as an AA rated bank or reinsurance company. It is based on CSA (2016) and was the first approach with solely nickel wire collateral evaluated for the bond issuer as proof of concept before consideration of principal amortization or fees to advisors and managers.

A put option string is purchased from the third party issuer to guarantee the payments required at each reset date. However the fair value premium for this put option string is too expensive to be feasible so that at the expense of sharing the collateral price upside with the derivative issuer a call option string is sold to this third party by the bond issuer and the call premium is set against the put string premium to make the bond payment guarantee affordable. In fact, a AA rated reinsurance company was willing to undertake this deal in 2016 for the 475M EUR flotation identified below before the broker-dealer became involved. Only recently we learned of a similar deal with a reinsurance broker issuing weather derivatives to protect the revenues of a national electricity generator in dry seasons and sharing upside revenues in wet seasons (Edge 2019).

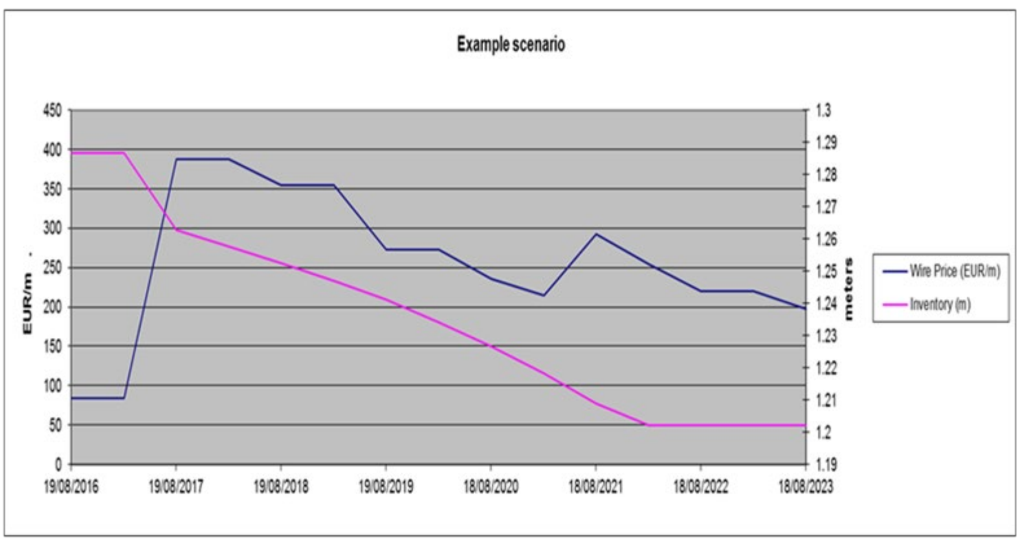
Pricing



Source: CSA (2016)

Figure 18. Q projections of the nickel wire price at 8.9.16

Using the same techniques as described above involving the economic factor yield curve model for discounting under the Q pricing measure we obtain projections of the nickel wire price and inventory evolution. Figure 19 shows the evolution of a typical scenario of wire price from a starting price of €86.90 per metre and the corresponding nickel wire inventory reduction for a bond with a 6% coupon. All values in the figure are per 100 EUR bond principal.



Source: CSA (2016)

Figure 19. Representative scenario of semi-annual wire price and inventory level under Q

Initial security values and risk with EFM discounting

The initial security values – bond and derivative strings -- with the 3 factor affine model under the pricing measure (Q) at 8 September 2016 from wire price €86.90 per metre are as follows.

Bond value without collateral a B^+ credit rating and 0 recovery (under P) is **92.58** with a **22.98**% 5-year default probability.

Bond value with nickel wire collateral is **79.43**% of par with 99.7% confidence interval (79.13,79.74) and **80.64**% *5- year default probability*.

European put string value is **48.81** with 99.7% confidence interval (48.13,49.50) and is a *perfect hedge* as with it in place the bond pays 128.00% with **zero default** probability.

European call string value with a *floor* of 1% of the bond principal at each coupon date and a *cap* of 45% (denoted 1/45) is **22.03** to pay **26.8%** of the net put premium in *cash*.

Determination of Net Derivative Premia

Table 20 shows the net premia for a range of bond issue principals.

Issue Amount M EUR	Put Premium All Payments percent	Put Premium Principal Only	Call Premium 1% Floor 45% Cap	Net Premium All Payments	Net Premium Principal
400	30.8295	30.8270	33.2555	-2.4260	-2.4285
450	35.7809	35.7764	29.5252	6.2557	6.2512
500	40.4396	40.4323	26.4697	13.9699	13.9626
600	48.8103	48.7939	22.0336	26.7767	26.7603
700	55.9709	55.9390	19.3191	36.6518	36.6199
800	62.0527	61.9969	17.6495	44.4032	44.3474

Table 20. Net derivative premia by issue size

Optimal bond issue size determination at 19.9.16

Repeating the calculations in Table 20 at finer range of issue sizes leads initially to a 475M EUR issue.

Issue Amount M EUR	Put Premium All Payments per cent	Call Premium 0.3% Floor 15% Cap	No. Calls Sold	Total Call Value	Net Premium	Call Total Min/Max Claim*
400	30.8295	10.8254	2	21.6508	9.1787	8.4 / 30
425	33.3382	10.1567	2	20.3134	13.0248	8.4 / 30
450	35.7809	9.53126	2	19.0625	16.7184	8.4 / 30
475	**38.1497**	**8.9663**	**3**	**26.8988**	**11.2509**	**12.6 / 45**
500	40.4396	8.4606	3	25.3819	15.0577	12.6 / 45
525	42.6526	8.0088	3	24.0265	18.6261	12.6 / 45
550	44.7853	7.6115	3	22.8335	21.9518	12.6 / 45
575	46.8361	7.2590	4	29.0362	17.7999	16.8 / 60
600	48.8103	6.9490	4	27.7960	21.0143	16.8 / 60

* Per cent of remaining bond payments at exercise of the multiple call strings

5. Bond Price and Default Risk

This section reports the price and default probability of the presently proposed bond under respectively the risk-neutral measure Q and the market measure P. Real world models under the market measure P often incorporate estimated *market prices of risk* (MPR) which in many cases, such as for the EFM 3 factor affine discount model used here, are set to zero to derive the corresponding risk-neutral pricing model under the pricing measure Q.

After describing the data used to price the bond with full collateral we will give the resulting bond price and default probability under P of interest to investors, together with the default probability under Q which is interesting to analysts for comparison of the impacts of collateral structures. In particular, in order to understand the impact of trading the REO/Ti portfolio, we shall also give the bond valuation with only this collateral.

In this section, we will also value the bond with separately only REO/Ti collateral and only nickel wire collateral in order to compare the latter results with those of the non-amortized 6% coupon bond treated in CSA (2016). Taken all together this allows assessment of the positive impact on bond value and default probability of the currently higher nickel wire price, the amortization structure of the bond and the addition of trading the second collateral portfolio of rare earth oxides and titanium.

Data description

The computational results reported in this section are based on the following data. The data range and observation counts for the various models are as listed (everything uses as much data as possible, which is every trading day in the case of daily observations).

1. **Nickel wire price model**
   2169 daily observations, 2008-11-25 to 2017-04-28

2. **EUR-USD exchange rate**
   1279 daily observations, 2012-04-30 to 2017-04-28

3. **EFM discount model**

   EURIBOR/EUR swap data
   4677daily observations, 1999-01-06 to 2017-04-28

   USD LIBOR/USD swap data
   4055 daily observations, 2001-01-02 to 2017-04-28

4. **REO/Ti price models**
   86 monthly observations, November 2009 to April 2017

5. **Scandium volatility**
   27 annual price observations, 1990 to 2017

The data sources used were:

Nickel Wire Catalogue price data:	Alfa Aesar
EURIBOR data:	euribor-ebf.eu
EUR constant maturity swap data:	Bloomberg.
EUR/USD FX swap data	Bloomberg
REO/Ti price data	Argus Media
Scandium price data	Argus Media.

Collateralized bond with the Take Profit REO/Ti trading strategy

Figure 21 shows the projected evolution to maturity of the collateral portfolio value distribution with initial weights given in Table 9 using the optimal trading strategy from its initial 934 (= 709+225) M EUR value.

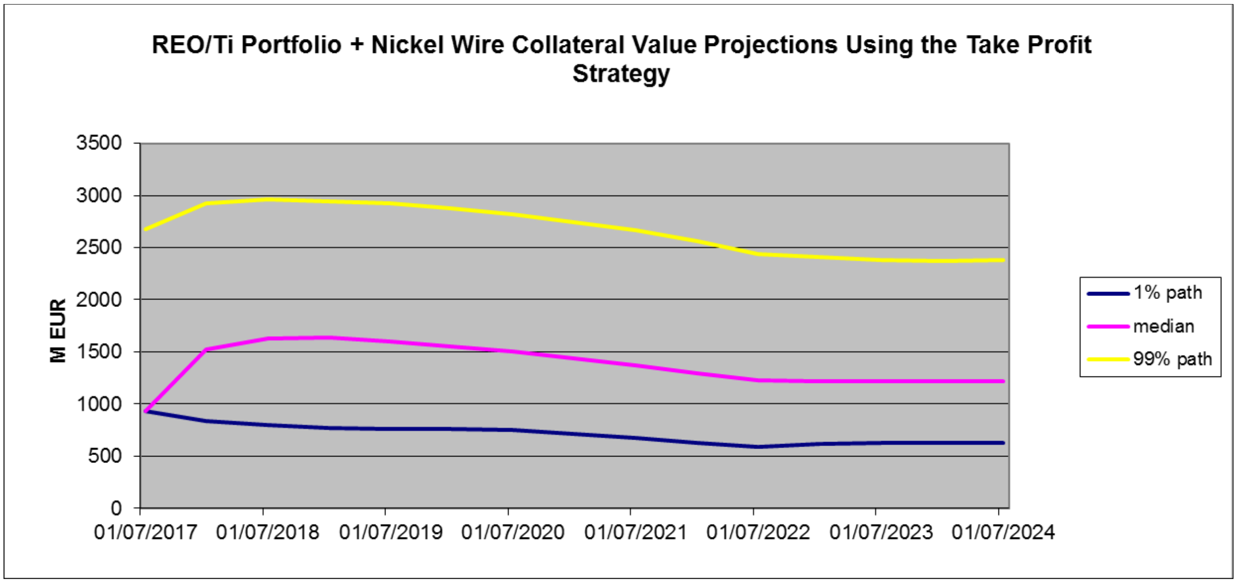


Figure 21. Monte Carlo collateral value projections from inception to maturity

The figure shows a 7 year median forecast from 1 July 2017 for the state distribution of the collateral value based on 1,000,000 simulated scenarios with a monthly time step and initial nickel wire price 116 EUR. The 1% and 99% confidence levels around the median path projection are also shown.

The median path of collateral value demonstrates that after taking account of all necessary payments, including the 6% coupon and those to achieve a 12% IRR, the median value of the collateral increases significantly from its initial value of € 934 M to € 1,218 M, or 30%, by bond maturity.

We shall see that this is due to the possibility of initially large trading gains from selling the REO/Ti portfolio purchased from the bond proceeds.Trading gains are sufficiently high that there is at least a 50% chance of retiring the bond issue after 2 years with more than enough extra to cover all costs and any premium that the bond holders would naturally require for early repayment. In the sequel we shall assume *no early repayment*, as this would require further modelling and computation to incorporate such costs.

Market price of the bond at inception:	**€ 554.197 M**
Corresponding market price of the 100 EUR bond:	€ 158.342
Six standard deviation pricing uncertainty: (99.7% confidence interval)	€ 158.29 to € 158.394

Bond 5 year default probability (under the market measure P)[4]: **0**

Median residual wire value: **€ 1,218 M**

For future reference we note that the default probability of the bond under the risk-neutral pricing measure Q is 25.349%, assuming no recovery after default. Based on a million paths, the default probabilities (in per cent terms) are accurate to the first decimal place.

To understand the contribution of the REO/Ti collateral portfolio with initial weights given in Table 9 to this high yielding bond with 0 default probability, Figure. 20 shows the projected evolution from its initial € 225 M value to maturityof the REO/Ti trading portfolio value distribution using the optimal Take Profit trading strategy.

The figure shows trading gains sufficiently high that there is at least a 1% chance of retiring the bond issue after 2 years with enough extra to cover costs.

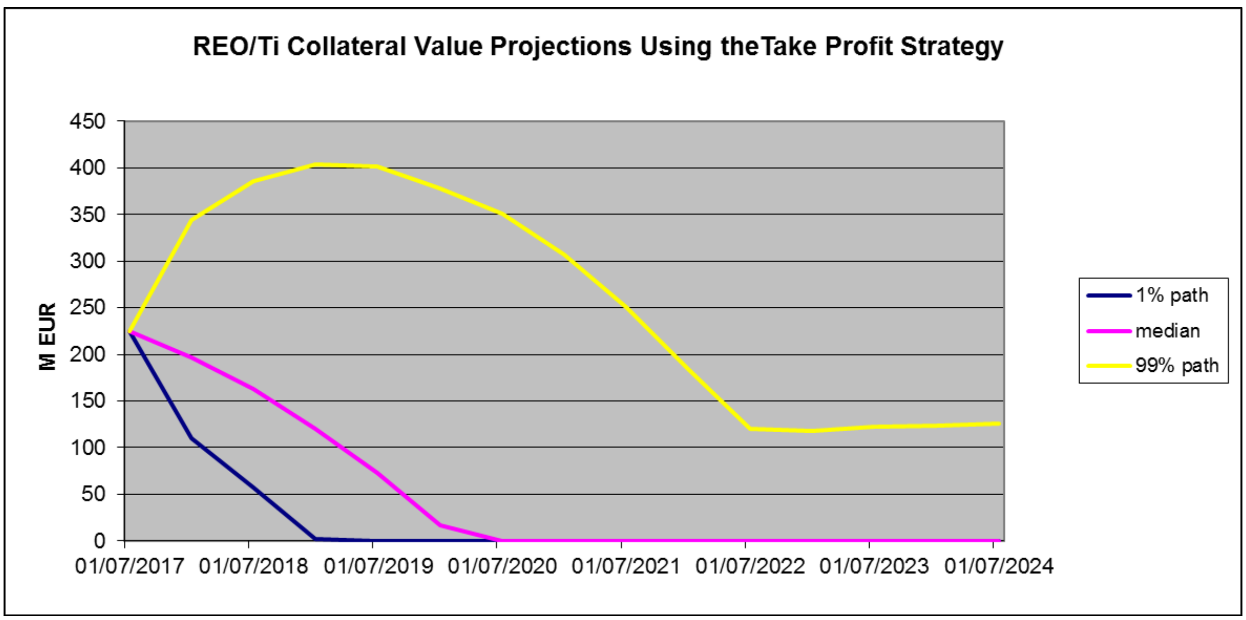


Figure 20. Monte Carlo REO/Ticollateral value projections from inception to maturity

For robustness evaluation we now present computational results which allow comparison of the actual bond with more basic bond structures and earlier findings, as well as an evaluation of the surprising relatively insignificant impacts on bond price of alternative REO/Ti portfolio trading strategies.

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[4] Note that because initially 4 senior coupons are escrowed and no junior coupons are paid until the third year, this seven year bond can only default from year three to maturity.

First we value the bond with only the REO/Ti and nickel wire collateral separately, and we compare results for the latter with earlier results for a simpler bond structure (CSA 2016).

Bond with REO/Ti collateral

We consider the bond with € 350 M principal but with unrealistic collateral only the initially purchased REO/Ti trading portfolio of value € 225 M.

Market price of the bond at inception: **€ 252.921 M**

Corresponding market price of the 100 EUR bond: € 72.263

Six standard deviation pricing uncertainty: € 72.207 to € 158.320
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: **97.715 %**

Not surprisingly, with only € 225 M of collateral for a € 350 M principal, the default probability in this case is high and the market value of the bond is well below par. In fact, default under the risk-neutral pricing measure is virtually certain, with probability 99.687% and survival probability only 31 bps.

However, since we *assumed* for the purpose of modelling the bond with the full collateral of the previous section that the purchased elements of the initial REO/Ti portfolio will all be sold *before* any nickel wire, Figure 20 in the previous section which was developed from these calculations gives an accurate picture of the potential favourable consequences of REO/Ti trading for the combined collateral.

Bond with nickel wire collateral

We first compare a standard €100 bond analysed in CSA (2016) with the present bond with only nickel wire collateral. All payments are met by nickel wire sales at reset dates from an initial nickel wire collateral of 6,112,126 m.

Standard bond at 19.8.16 with nickel wire collateral (wire price: 83.9 EUR /m)

Collateral value: € 512.8 M Principal: € 100 Maturity: 7 years Coupon: 4% p.a.

Semi-annual reset Rolling 2 year (4) coupons escrowed

P (mean reverting) market measure *default probability*: **0%**

Q (risk neutral) pricing measure default probability 80.6%

Bond value: **€ 79.4**

Expected residual wire value (based on 600 M EUR principal): **€ 14.8**

Amortized bond at 28.4.17 with nickel wire collateral (wire price: 116 EUR / m)

Collateral value: € 709.0 M Principal: € 100 Maturity: 7 years Coupon: 6% p.a.

Semi-annual reset IRR: 12% p.a. Semi-annual junior coupons last 5 years: € 3.62

Total return last 5 years: 13.24% (€13.24) p.a.

Rolling 4 (2 years) coupons and IRR payments escrowed

Amortization schedule last 5 years: 10, 15, 20, 25, 30

Initial costs: 27.71 EUR Running costs (7 years up front): 1.43 EUR p.a.

Rolling 2 year capital repayments and 3 year running cost payments escrowed

P (mean reverting) market measure *default probability*: **0%**

Q (risk neutral) pricing measure default probability: 49.7%

Bond *value*: **€ 144.5**

Expected *residual wire value* (based on € 350 M principal): **€ 61.0**

Although both bonds have 0 probability of default, it is interesting to note how much the amortization, reduced principal, higher coupon and nickel wire collateral have reduced the risk-neutral pricing measure Q probabilty of default from 80% to 50%.

Figure 21 shows the evolution of the nickel wire collateral value to bond maturity. It should be noted that the initial rise in median collateral value is due to the mean reversion of the wire price after starting the simulation in the lower tail of the value distribution. In the full collateral case this rise is augmented by the trading profits of the REO/Ti collateral portfolio.

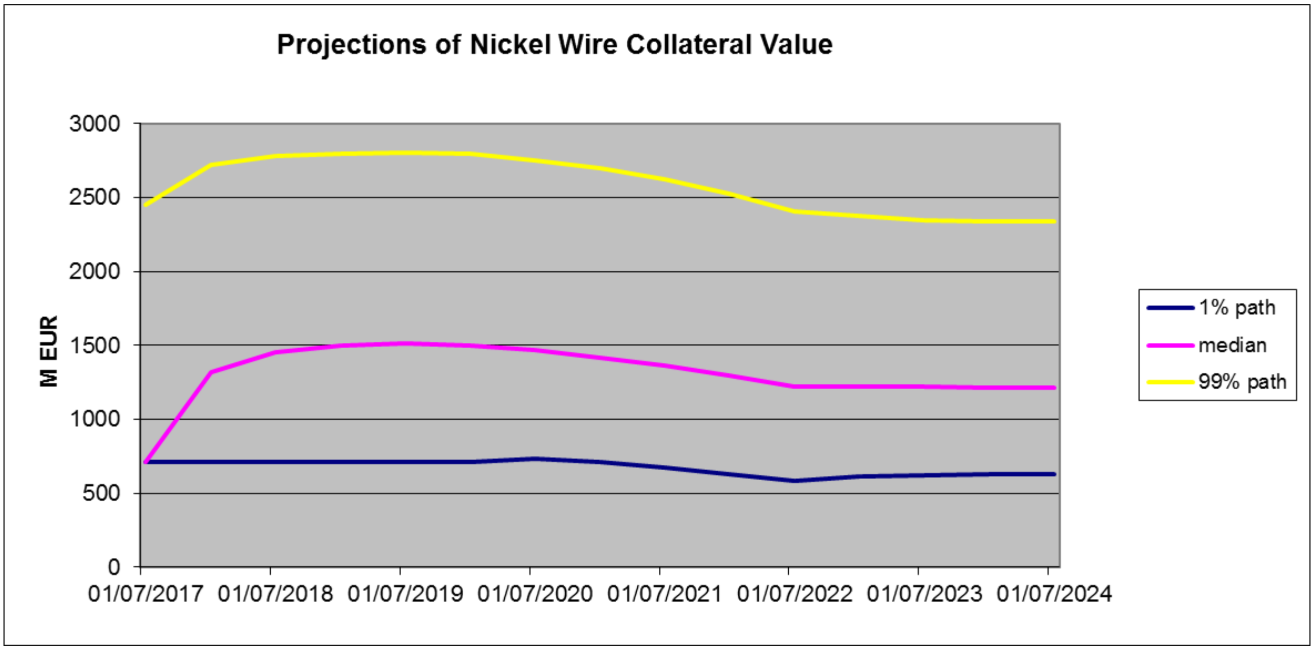

Figure 21. Monte Carlo nickel wire collateral value projections from inception to maturity using the take profit strategy

Trading strategy comparison

Finally, we study impacts on bond price and cost of five alternative trading strategies for the REO/Ti portfolio introduced in Section 3. For convenience we list them here again by name:-

1. *Buy and Hold*, the benchmark strategy in which all sales are portfolio increments with fixed asset weights
2. *Momentum*, sell lowest 6 month price forecast elements first
3. *Contrarian*, sell highest 6 month price forecast elements first
4. *Cut Losses*, sell lowest realized return to date assets first
5. *Take Profit*, sell highest realized return to date assets first.

We now list the corresponding results for the 100 EUR bond.

Buy and hold market price of the 100 EUR bond: **€ 158.310**

Six standard deviation pricing uncertainty: € 158.218 to € 158.362
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: 17.167%

Bond 5 year default probability under the pricing measure Q: 25.393%

Momentum market price of the 100 EUR bond: **€ 158.271**

Six standard deviation pricing uncertainty: € 158.218 to € 158.323
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: 17.187%

Bond 5 year default probability under the pricing measure Q: 25.479%

Contrarian market price of the 100 EUR bond: **€ 158.302**

Six standard deviation pricing uncertainty: € 158.249 to € 158.354
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: 17.214%

Bond 5 year default probability under the pricing measure Q: 25.380%

Cut losses market price of the 100 EUR bond: **€ 158.244**

Six standard deviation pricing uncertainty: € 158.192 to € 158.296
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: 17.230%

Bond 5 year default probability under the pricing measure Q: 25.533%

Take profit market price of the 100 EUR bond: **€ 158.342**

Six standard deviation pricing uncertainty: € 158.29 to € 158.394
(99.7% confidence interval)

Bond 5 year default probability under the market measure P: 17.157%

Bond 5 year default probability under the pricing measure Q: 25.349%

The *take profit strategy,*which ignores the 6 month forecasts, gives the highest bond price and lowest Q default probability and hence has been employed for the reported results. It is followed closely by the benchmark buy and hold strategy which also ignores the forecasts.

However, although the differences in impact on bond price and Q default probabilities of the alternative trading strategies are statistically significant due to the 1 M scenarios employed in generating the results, they are strikingly small. This is reassuring for the stability of the predicted results relative to the actual trading strategies employed in practice by the Collateral Manager of the issue during the life of the bond.

6. Conclusion

This paper develops the technical pricing and default risk analysis for a 350 M EUR 7 year amortized corporate bond issue backed by a nickel wire inventory as collateral worth 709 M EUR at the catalogue wire price of 116 EUR per metre. This research has been commissioned through Hanover Square Capital (UK) Limited, who are regulated by the UK Financial Conduct Authority, for a client, and Hanover will act as Collateral Manager for the issue. The client was advised by a number of other groups including Schloer Consulting Group, Argus Media, STE SpA and Storm Harbour Securities LLP.

Based on the general view that high tech metals, and metals more generally, are currently underpriced, the issuer wished to utilize their physical inventory in order to invest in more high tech metals before their prices return to more usual levels around 300 EUR per metre under the generally rising demand for high tech metals depicted in Figure 10. It is therefore intended to immediately use up to 225 M EUR of the bond issue proceeds to buy a portfolio of rare earth oxides and titanium to act a second form of physical collateral whose subsequent profitable selling over the lifetime of the bond will help to return a 12% internal rate of return to investors in the senior secured notes of the issue, provide working capital to the issuer and preserve or enhance the value of the nickel wire inventory.

The extensive development work outlined in this paper was required for the actual bond issue, which upon completion necessitates an ongoing commitment by the Collateral Manager to report current pricing and risk management for it to maturity.

The complex stochastic modelling and Monte Carlo simulation analysis reported here, based in part on specially developed modelling of the nickel wire catalogue price and the price projections of Argus Media for the rare earth oxides and titanium, supports an optimnistic view. Indeed, after accounting for all ongoing costs, we find a zero default probability for the bond issue – a situation seldom seen to accompany the assumed 12% IRR. Noting the short term negative trend reversal for rising long term high tech metal demand, together with the financial collateral structure of the bond, the likelihood of actually achieving the necessary sales by the collateral manager in time to meet all obligations to the bond holders is high.

Acknowledgements

This paper was scheduled to be presented as the first keynote address at the 7th International Conference on Futures and Other Derivatives on 19th October 2018 at Fudan University as part of the Chinese Futures Association annual meeting. Unfortunately the author was unable to attend the conference due to unforeseen circumstances. The research reported here has been undertaken in collaboration with M H A Davis, G W P Thompson and A Sood and further work will be reported in Davis *et al.* (2019).

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Appendix

Bond structure description

The holder receives a payment on each of the dates t_i, i=1,…,14 where

t_0 = 2017-07-01
t_1 = 2018-01-01
t_2 = 2018-07-01
t_3 = 2019-01-01
t_4 = 2019-07-01
t_5 = 2020-01-01
t_6 = 2020-07-01
t_7 = 2021-01-01
t_8 = 2021-07-01
t_9 = 2022-01-01
t_10 = 2022-07-01
t_11 = 2023-01-01
t_12 = 2023-07-01
t_13 = 2024-01-01
t_14 = 2024-07-01 .

We assume that the bond has a notional of 100 Euros. Let principal_i be the amount of principal outstanding at t_i. Then

principal_0 = 100
principal_i = principal_{i-1} - A_i   i=1,…,14,

where A_i is the amount of *principal repaid* at date t_i given by the amortization schedule (see below).

The payment received by the holder at t_i, i=1,…,4 is

payment_i = 0.5*principal_{i-1}*coupon_rate + A_i

and for i=5,…,14,

payment_i = 0.5*principal_{i-1}*coupon_rate + A_i + IRR_payment

where coupon_rate is 6% and IRR_payment is a fixed amount chosen to make the IRR of the bond equal to 12%.

The *amortization schedule* is given by:

$A_1 = 0$
$A_2 = 0$
$A_3 = 0$
$A_4 = 0$
$A_5 = 5$
$A_6 = 5$
$A_7 = 7.5$
$A_8 = 7.5$
$A_9 = 10$
$A_{10} = 10$
$A_{11} = 12.5$
$A_{12} = 12.5$
$A_{13} = 15$
$A_{14} = 15$

With these parameters, the bond payments excluding the IRR_payment are:

date	interest payment	principal repaid	total payment
1	3	0	3
2	3	0	3
3	3	0	3
4	3	0	3
5	3	5	8
6	2.85	5	7.85
7	2.7	7.5	10.2
8	2.475	7.5	9.975
9	2.25	10	12.25
10	1.95	10	11.95
11	1.65	12.5	14.15
12	1.275	12.5	13.775
13	0.9	15	15.9
14	0.45	15	15.45.

The IRR_payment made at t_i for $i=5,\dots,14$ is 3.62604 for a total of 36.26 EUR.

Nickel wire price evolution (EUR)

Market price (market measure P)

Under the *market* (real-world) *measure* P assume that the *nickel wire price* ***N*** follows a pure jump process with *jump intensity* λ.[5] Given that a jump occurs at time t, the distribution of ***N***(t) is assumed to be independent of {***N***(s): s<t} and lognormal with time-invariant parameters, and we define

$$\mu := E[\log \boldsymbol{N}(t)] \qquad \sigma^2 := \text{var}[\log \boldsymbol{N}(t)].$$

To calibrate the model we choose $\hat{\lambda}$ to be the historical jump rate (approximately 1.7 jumps per year) and estimate μ and σ^2 by matching the first two moments of the historical distribution of {***N***(t): t is a jump time}. To simulate paths of ***N*** we assume a daily time step and that at most once jump occurs in each day. Scenarios from such a simulation under the market measure P incorporate a *risk premium*.

Risk neutral price (pricing measure Q)

Under the equivalent martingale measure Q, we assume the nickel wire *price process* ***N*** is still a pure jump process with *intensity* λ and that, given a jump occurs at time t, the distribution of ***N***(t) conditional on {***N***(s): s<t} is lognormal with mean

$$E^Q[\boldsymbol{N}(t)] = N(t-)[r(t)/\lambda + 1],$$

where r(t) is the *instantaneous short rate* at time t (so that the discounted wire price is a Q-martingale) and that the conditional variance of $\log \boldsymbol{N}$ is σ^2, the same as under the market (real-world) measure.

For the pure jump model under Q, the conditional expected value of the wire price ***N***(t) at a jump time t is derived as follows.

In a small time interval dt, the probability of a jump is λdt, so the expected value of ***N***(t+dt) is

$$(\lambda dt)N(t)[r(t)/\lambda + 1] + (1 - \lambda dt)N(t),$$

which simplifies to $N(t)[1 + r(t)dt]$, so that ***N***(t) grows at instantaneous short rate r(t) in expectation and hence under Q the discounted wire price is a *martingale* with expectation 0.

[5] Throughout this report we use boldface to denote stochastic entities, sometimes conditionally stochastic.

REO/Ti price evolution (USD)

Argus Media (Argus, 2017a,b) have produced annual forecasts from 2016 over the 7 year life of the bond for the rare earth oxides and titanium whose normalized forecast paths are depicted in Figure 9. We use these un-normalizedand monthly historical data for each REO and Ti from 2010 to April 2017 to analyse and model the ten price processes of these materials for forward simulation to bond maturity *T*.

Market prices (P)

For *i*=1,…,10 and *t*=0,…,T let f_{it} denote the Argus forecast value for high tech materiel *i* at time *t.* Using linear interpolation this may be converted to the continuous time price function $f_i(t)$. Then the *market price* $\boldsymbol{R}_i$ of materiel *i* evolves under the market measure *P* according to the *geometric Brownian motion* (GBM) given by the stochastic differential equation (SDE)

$$\frac{d\boldsymbol{R}_{it}}{R_{it}} = f_i(t)dt + \sigma_i d\boldsymbol{W}_{it} \ ,$$

where dW_{it} is a Wiener process increment correlated with those of the price processes of the other materiels. Note that $f_i(t)$ incorporates the market view of risk, i.e. the *market price of risk*, at time t.

The discrete time approximation of the GBM SDE (here with daily time steps) is given by

$$(\boldsymbol{R}_{i,t+1} - R_{i,t})/R_{i,t} = (f_i(t+1) - f_i(t))\Delta t + \sigma_i \Delta\boldsymbol{W}_{it} \ , \qquad ,$$

where $\Delta\boldsymbol{W}_t$ is a standard normal variable independent at each time step.

Risk neutral prices (Q)

Following Harrison-Pliska using the Girsanov theorem, for *i*=1,…,10 the *risk neutral price* processes for the high tech materiels under the pricing measure Q satisfy the SDEs given by

$$\frac{d\boldsymbol{R}_{it}}{R_{it}} = rdt + \sigma_i d\boldsymbol{W}_{it} \ ,$$

where *r* is the *instantaneous short rate*, currently *r* = - 0.0033 for the Eurozone, with discretization

$$(\boldsymbol{R}_{i,t+1} - R_{i,t})/R_{i,t} = r\Delta t + \sigma_i \Delta\boldsymbol{W}_{it} \ ,$$

With negative exponential discounting of these processes at rate *r,* the resulting zero expectation processes are *martingales*.

The historical high tech material series is used to estimate the volatility parameters σ_i.

Discount yield curve

Initial experiments with option structuring and pricing in CSA (2016) were carried out using negative exponential discounting at the *fixed* EUR spot rate r = - 0.0039.

To find the continuously compounded *risk free rate r* more generally, a discount curve $\{D(s), s \geq 0\}$ is calibrated to an observable market forward curve from inception at time *t* using EURIBOR rates at the short end and bootstrapping EUR constant maturity swap rates thereafter. Inverting the standard fixed floating swap formula recursively at each time point converts the resulting linearly interpolated market curve to the discount curve representing the yield on zero coupon bonds of each maturity, see Yong (2007).

A *par interest rate swap* is a standard contract between two counterparties to exchange cash flows. At set time intervals termed *reset dates* one pays a predetermined *fixed* rate of interest on the *nominal* value, the other a *floating* rate, until the *maturity* date of the contract. The floating leg of the swap fixes the interest rates for each payment at the rate of a published interest rate. The fixed rate, known as the *swap rate,* is that interest rate which makes the fair value of the par swap 0 at inception. Thus the cash flows of the two legs of a par swap are those of a pair of bonds with face value the swap nominal, one fixed rate, and the other floating rate.

Since the swap market is highly liquid with many par swaps traded every day, it is possible to obtain rates for swaps with a set of *constant* maturities from 1 to 30 years from the market each day.[6] From the market swap rates a swap curve which gives the rates for constant maturity swaps (CMS) of *all* maturities may be constructed each day. This market-determined curve may be used to price *over-the-counter* (OTC) structured products between an issuer and a specific client counterparty.

To compute fair values of structured products under Q and perform risk evaluation simulations under P, we use a Gaussian affine 3-factor yield curve model for discounting which captures the simultaneous movement of short, medium and long rates. This model, termed the *economic factor model* (EFM) (Campbell, 2000; Medova *et al*., 2005; Dempster *et al*., 2010), has many parameters which need to be calibrated using historical market data.

Its evolution under the *risk-adjusted* (*risk neutral*) probabilities corresponding to the pricing measure Q is determined by *stochastic differential equations* (SDEs) whose Wiener process

[6] This is by contrast with the market yields for Treasury bonds whose actual maturities each day depend on a discrete number of previous auction dates and must be adjusted to approximate constant maturity.

increment terms are *correlated*. Its three *unobservable* Gaussian factors **R**, **X** and **Y** represent respectively a *short rate*, a *long rate* and (minus) the *slope* (difference) between an unobservable continuously compounded *instantaneous* short rate $\mathbf{R}^* := \mathbf{X} + \mathbf{Y}$ and the long rate.

Since the data to which we are fitting our model has evolved according to the actual *market* probabilities (historical frequencies), i.e. under the market measure P rather than the risk adjusted probabilities under the pricing measure Q, we must include parameters for the *market prices of risk* in the SDEs describing its factor evolution. Thus we include three parameters γ to allow for *constant* market prices of risk of the factors.

The model used admits a closed form solution for zero-coupon bond prices (Medova *et al*., 2005) from which zero coupon bond yields may be derived at t in affine form involving the vector of *parameters* θ as

$$\mathbf{y}(t, T) = \tau^{-1}[A(\tau, \theta)\mathbf{R}_t + B(\tau, \theta)\mathbf{X}_t + C(\tau, \theta)\mathbf{Y}_t + D(\tau, \theta)]$$

for given maturity $\tau := T - t$. The bond prices corresponding to these yields, after replacing negative yields less than -39 bps (which occur with low probability) by -39 bps, provide appropriate discount factors for security forward payments whose present value under P or Q is sought at any time t.

Interest rate data and calibration

Since the **R**, **X** and **Y** Gaussian factors are not observable directly, we use the *Kalman filter* to estimate the model parameters θ after correcting the annually compounded observed swap rates for continuous compounding in the model. The parameters θ include the constant volatilities of the three factors which produce a *conservative* forward view of zero coupon bond yield volatilities based on historical observations to valuation date. The full estimation procedure is described in Dempster *et al.* (2010), see also Yong (2007).

The data used to calibrate the model to various valuation dates consists of appropriate publically available *daily* rates: swap and EURIBOR data from 6 January 1999 to 28 April 2017, a total of 4,677 observations.

We interpolate the swap curve linearly to obtain swap rates at all maturities, then use the 1, 3 and 6 month EURIBOR rates and the EUR swap curve to recursively back out a zero-coupon bond yield curve for each day from the basic swap pricing equation. This latter is the input data used by the model calibration code which is run for securities at inception and subsequent valuation dates as required.

When the parameters have been estimated, we can compute the yield curve based on the posterior means for the three factors **R**, **X**, and **Y** at historical dates and compare this to the actual yield curve deduced from the (linearly interpolated) historical swap curve on that day. These fits on representative days may not accurately capture the very long end of the yield curve, which might require a fourth factor. They are however acceptably accurate up to 10 year maturity and in any event generally err on the conservative side. Moreover, the out-of-sample forecasting ability of this model, which is relevant to accurate pricing, is high.[7]

Currency rate evolution

In considering European *foreign exchange* (FX) options, Garman & Kolhagen (1983) observed that Merton's (1973) equity option valuation formulae for call and put options paying in the foreign currency, with a *continuous* dividend process, could be applied to the valuation of such an option with the dividend process replaced by the foreign (deliverable) *risk free rate* corresponding to the continuously compounded return on a foreign zero coupon bond with the option's maturity. The Garman-Kolhagen *call option prices* (premia) in the foreign (e.g. the US dollar) or domestic (e.g. the euro) currencies are widely used as a basis for pricing currency options by traders in the FX markets, where they are usually (incorrectly) referred to as simply *Black-Scholes prices* in honour of the seminal paper on equity option pricing by Black & Scholes (1973).

When paying in the domestic currency these theoretical option prices are based on the *assumption* that the underlying *exchange rate* stochastic process ***S*** evolves in *continuous time* according to *geometric Brownian motion* (GBM) in terms of an expected value or *drift,* which is the difference between the foreign and domestic risk free rates, and the *volatility* (standard deviation) σ of the proportional change in the exchange rate (*return*) per unit time (usually expressed per annum). This process under the *risk adjusted probabilities* satisfies the stochastic differential equation given by

$$\frac{d\boldsymbol{S}_t}{S_t} = (r_f - r_d)dt + \sigma d\boldsymbol{W}_t \quad ,$$

with discrete time approximation (here with daily time steps)

$$(\boldsymbol{S}_{t+1} - S_t)/S_t = (r_f - r_d)\Delta t + \sigma\Delta\boldsymbol{W}_t \quad ,$$

where $\Delta\boldsymbol{W}_t$ is a standard normal variable independent at each time step.

[7] Although assessment of the calibration estimates and overall in-sample goodness-of-fit require detailed technical knowledge this information is available on request. A recent (confidential unpublished) independent assessment of our model's out-of-sample forecasting ability shows that it is high and superior to both naïve forecasts and those based on an alternative sophisticated forecasting model (Christensen *et al*., 2007) currently used by many central banks for policy making.

FX data and calibration

Daily EUR-USD exchange rate data was obtained from the European Central Bank and daily one month EUR and USD LIBOR rates from the British Banking Association. The foreign and domestic rates in the above discrete time equation are fixed at the continuously compounded versions of say 1 month LIBOR rates at inception. The evolution of these rates for the US dollar (USD) and the euro (EUR) have caused the spread between them to have to have a positive drift since the Eurobond Crisis leading to a positive drift in the EUR-USD exchange rate return under the risk adjusted probabilities.