Productive demand, sectoral comovement, and total capacity utilization

Mario Rafael Silva [1] Marshall Urias [2]

[1]Hong Kong Baptist University

[2]HSBC Business School, Peking University

Introduction

Motivation

1. Which shocks drive business cycle fluctuations? (Lucas (1981) and Smets and Wouters (2007))
2. Why different sectors exhibit positive comovement in terms of input and output? (Christiano and Fitzgerald (1998))
3. What is the contribution of demand shocks to productivity?

Definition of recession from NBER

> *A recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy*

Key contribution: use capacity utilization jointly with sectoral data to investigate these questions in a setting in which goods market frictions give rise to a productive role for demand

Motivation

1. Which shocks drive business cycle fluctuations? (Lucas (1981) and Smets and Wouters (2007))
2. Why different sectors exhibit positive comovement in terms of input and output? (Christiano and Fitzgerald (1998))
3. What is the contribution of demand shocks to productivity?

Definition of recession from NBER

> *A recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy*

Key contribution: use capacity utilization jointly with sectoral data to investigate these questions in a setting in which goods market frictions give rise to a productive role for demand

Motivation

1. Which shocks drive business cycle fluctuations? (Lucas (1981) and Smets and Wouters (2007))
2. Why different sectors exhibit positive comovement in terms of input and output? (Christiano and Fitzgerald (1998))
3. What is the contribution of demand shocks to productivity?

Definition of recession from NBER

> *A recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy*

Key contribution: use capacity utilization jointly with sectoral data to investigate these questions in a setting in which goods market frictions give rise to a productive role for demand

Demand shocks and effect on measured productivity

- In a standard neoclassical model, prices adjust so that all produced output is sold $\Rightarrow$ output is just a function of capital and labor
- Output generally responds weakly to demand shocks through increases in labor hours
- Under goods market frictions, output depends on how many customers show up
- Reverses causality between consumption and TFP relative to neoclassical model

Motivation: Solow-residual vs. utilization-adjusted counterpart

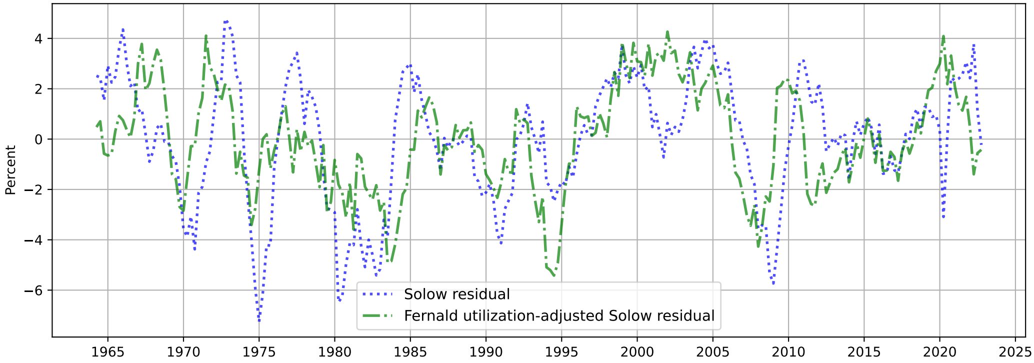

Figure 1: Time series of Solow residual and utilization-adjusted counterpart, following the methodology in Fernald (2014). Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past $(p = 4, h = 8)$

Motivation: measures of utilization

- Two utilization measures
  1. Define Fernald utilization as the difference in cyclical variation between Solow residual and utilization-adjusted counterpart
  2. Capacity utilization: ratio of output index and capacity index in manufacturing, mining, and electric, and gas utilities.
- Capacity index is provided by the Federal Reserve System Board of Governors and characterized as 'the highest level of output a plant can sustain within the confines of its resources.'

Details on total capacity utilization

Motivation: positive comovement between utilization measures and output

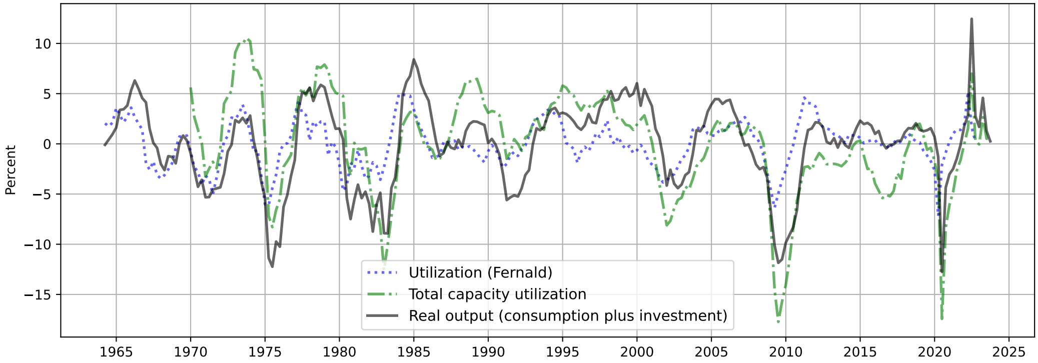

Figure 2: Time series of total capacity utilization; Fernald utilization, following the methodology in Fernald (2014); and output (here defined as consumption plus investment). Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Motivation: sectoral comovement (hours)

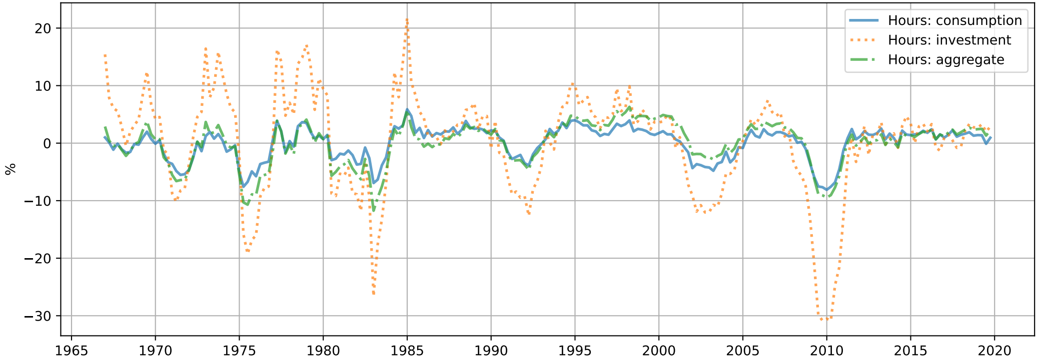

Figure 3: Sectoral and aggregate hours. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Motivation: sectoral comovement (utilization)

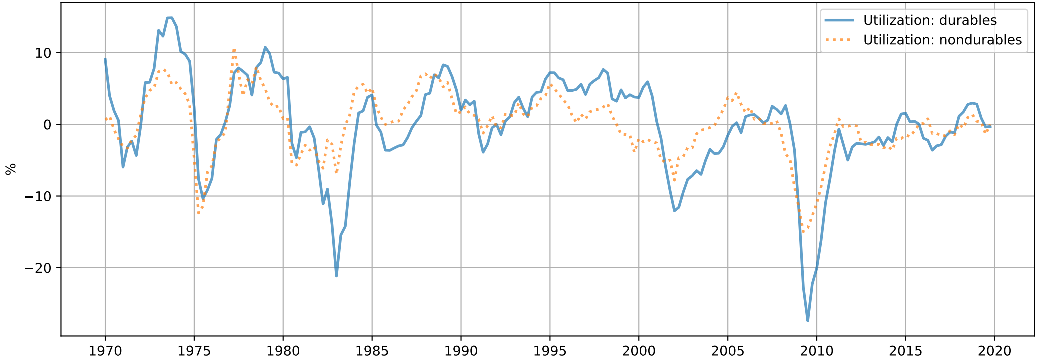

Figure 4: Total capacity utilization in non-durable and durable goods. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Second moments (growth rates)

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, N_I)	Cor(x,x$_{-1}$)
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
N_c	0.57	0.66	0.66	0.87	0.67
N_i	1.94	2.23	0.73	1.00	0.64
Y/N	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_D$	2.27	2.61	0.69	0.84	0.55
$util_{ND}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment.

Related literature

1. Purifying Solow residual:
   Basu, Fernald, and Kimball (2006), Fernald (2014)
2. Goods market frictions and firm productivity
   Moen (1997), Bai, Ríos-Rull, and Storesletten (2023), Huo and Ríos-Rull (2018), Qiu and Ríos-Rull (2022), Petrosky-Nadeau and Wasmer (2015), Bethune, Rocheteau, and Rupert (2015)
3. Sectoral comovement and imperfect intersectoral factor mobility
   Long and Plosser (1983), Christiano and Fitzgerald (1998), Horvath (2000), Jaimovich and Rebelo (2009), Katayama and Kim (2018)
4. Total capacity utilization Qiu and Ríos-Rull (2022)
5. News shocks
   Schmitt-Grohé and Uribe (2012), Katayama and Kim (2018)

Production model with shocks and dynamics

Production technology

- 2 consumption sectors (goods M_c and services S_c) and an investment sector
- Each uses capital k and labor n to produce output
- Aggregate state $\nabla = (\theta, K)$ where $K = (K_{mc}, K_{sc}, K_i)$
- Potential output given utilization rate h and fixed cost ν_j.

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{m_c, s_c, i\}$$

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k}$$

- Households eat consumption goods, invest, and rent capital to firms
- Fixed costs implies that labor productivity rises with sales

Matching technology

- **Competitive search**: households shop in markets indexed by price, market tightness, and quantity
- Each market is subject to Cobb-Douglas matching function

$$M = AD^{\phi}T^{1-\phi}$$

  where D is aggregate shopping effort and T is the measure of firms.
- Implied matching rates:

$$\Psi_d(D) = M/D = AD^{\phi-1}$$
$$\Psi_T(D) = M/T = AD^{\phi}$$

  so that D describes market tightness ($T = 1$)

Matching technology

- Once a match is formed, goods are traded at the price $p_j, j \in \{m_c, s_c, i\}$
- The real quantity of goods purchased is

$$y_j = d_j A_j D_j^{\phi-1} F_j$$

where d_j is a household's search effort in sector j

Consumption aggregator

- Consumption is bundle of goods M_c and services S_c

$$c = [\omega_c^{1-\rho_c} m_c^{\rho_c} + (1-\omega_c)^{1-\rho_c} s_c^{\rho_c}]^{1/\rho_c} \tag{1}$$

- Elasticity of substitution $= 1/(1-\rho_c)$
- Price index

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}}$$

  such that $\omega_{mc} + \omega_{sc} = 1$
- Normalize $p_c = 1$

Households

- Households have preferences over search effort, consumption, and a labor composite following Bai, Ríos-Rull, and Storesletten (2023)

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1-\sigma}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta} S$$

and

$$S = \left(c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta}\right)^{\gamma} S_{-1}^{1-\gamma}$$

given

- aggregate consumption C
- total search effort $d = d_{mc} + d_{sc} + \theta_i d_i$
- preference shifters $\theta = \{\theta_b, \theta_d, \theta_i, \theta_n\}$

Parameterizing wealth effects on labor supply

- Parameter γ regulates strength of wealth effects while preserving balanced growth in labor supply
  - $\gamma \to 0$: GHH, Greenwood, Hercowitz, and Huffman (1988) (BRS with $ha = 0$)

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta}$$

  - $\gamma \to 1$: KPR, King, Plosser, and Rebelo (1988)

$$\Gamma = \left(c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta}\right)(1 - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta})$$

- Standard additively separable preferences arise with $\gamma = \sigma = 1$
- Parameter ζ is Frisch elasticity in special case $\gamma = ha = 0$

Imperfect mobility across sectors

- Assume imperfect substitutability between labor used in consumption and investment sectors (Horvath (2000) and Katayama and Kim (2018))

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

- Elasticity of substitution $1/\theta$ measures intersectoral labor mobility
- Induces wage dispersion
- As $\theta \to 0$, $n^a \to n_c + n_i = n$ (MRS $\to 1$)
- For θ fixed, if $\omega = n_c/n$, then $n^a = n_c + n_i = n$

Households

- Households shop for investment goods, accumulate and install capital in each sector, and collect rental income

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

where $i = i_{mc} + i_{sc} + i_i$

- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta_j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2$$

- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S_j(x) = \frac{\Psi_j}{2}(x-1)^2$$

$\Rightarrow$ generates hump-shaped output and investment irf's (autocorrelated growth rates)

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, F_j), j \in \{mc, sc, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_c, k_i, p, D, F) = \max_{d_j, n_c, n_i, m_c, s_c, i_c, i_i, k'_j, h'_j} u(m_c, s_c, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad \text{s.}t.$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and consumption and labor aggregators

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, k_c, k_i, p, D, F)$$

Details of household problem

Optimal shopping effort

- Households equate marginal disutility of shopping effort to marginal utility of output in new matches

$$-u_d = u_j \overbrace{\phi A_j D_j^{\phi-1}}^{\text{new matches}} F_j \quad j \in \{m_c, s_c\} \tag{2}$$

$$-u_d \theta_i = \frac{u_{mc} p_i}{p_{mc}} \phi A_i D_i^{\phi-1} F_i \tag{3}$$

- Relative price p_i/p_{mc} converts investment goods into units of consumption goods
- Shopping wedge given marginal utility of wealth λ

$$\frac{u_j}{\lambda p_j} = \frac{1}{1-\phi} \Rightarrow \frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}}$$

Demand curve for non-durables and services

- First order condition with respect to non-durables m_c and services s_c together with aggregation implies

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

given elasticity of substitution $\xi = 1/(1-\rho_c)$

- Demand curve and shopping wedge yield $\lambda = \Gamma^{-\sigma}(1-\phi)$

Optimal labor supply

- Households optimally divide their labor hours between sectors

$$\frac{n_c}{n_i} = \frac{\omega}{1-\omega}\left(\frac{W_c^*}{W_i^*}\right)^{1/\theta}$$

- Absent wage dispersion, a share ω of hours is in consumption sector

$$-u_n\left(\frac{n_c}{n_a}\right)^{\theta}\omega^{-\theta} = u_c(1-\phi)W_c^*$$
$$-u_n\left(\frac{n_i}{n_a}\right)^{\theta}(1-\omega)^{-\theta} = u_c(1-\phi)W_i^*$$

Simplified shopping effort and relative price of investment

- Take ratio of (2) and (3) and rearrange for the relative price of investment

$$\frac{p_i}{p_j} = \theta_i \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi-1} \frac{z_c}{z_i} \frac{f(h_j k_j, n_j) - \nu_j}{f(h_i k_i, n_i) - \nu_i}$$

- If p_i rises relative to p_j, then investment goods are more valuable in terms of consumption, so D_i/D_j increases

Firms' problem

- A representative firm in sector $j \in \{m_c, s_c, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, F_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Details of firm problem

Firm factor demands

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{m_c, s_c, i\} \quad W_{mc} = W_{sc}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{m_c, s_c, i\}$$

- Matching function elasticity ϕ appears as separate factor
- Additional output relaxes participation constraint of households

Relative price of investment

- Take ratio of labor factor demand for i and j and rearrange for p_i/p_j

$$\frac{p_i}{p_j} = \frac{n_i W_i}{n_j W_j} \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi} \frac{z_j f(h_j k_j, n_j)}{z_i f(h_i k_i, n_i)}$$

- As D_j/D_i rises, non-durables or services are easier to sell to customers, so p_i/p_j increases
- Comparing to relative price of investment from HH, we find in the special case without fixed costs

$$\frac{D_i}{D_j} = \frac{1}{\theta_i} \frac{n_i W_i}{n_j W_j}$$

- Ratios of wage bill (or labor supply) are informative about relative shopping effort

Sectoral Solow residual

- Write sectoral Solow residual using fixed cost share $\nu_j^R = \nu_j X/(z_j f - \nu_j X)$

$$SR_j \equiv \frac{Y_j}{k_j^{1-\omega} n_j^{\omega}} = \frac{A_j D_j^{\phi}(z_j h_j^{\alpha_k} X^{1-\alpha_k} k_j^{\alpha_k - 1 + \omega} N_j^{\alpha_n - \omega})}{1 + \nu_j^R}$$

given steady-state labor income share ω

- Log linearized (cyclical deviations)

$$\widetilde{SR}_j = \overbrace{\phi \widetilde{D}_j}^{\text{Shopping}} + \underbrace{\alpha_k \widetilde{h}_j}_{\text{Capital utilization}} + \overbrace{\widetilde{z}_j + (1-\alpha_k)\widetilde{X}}^{\text{Technology}} + \underbrace{(\alpha_k - 1 + \omega)\widetilde{k} + (\alpha_n - \omega)\widetilde{n_j}}_{\text{Input share mismeasurement}} - \overbrace{\frac{\nu_{ss}^R}{1+\nu_{ss}^R}\widetilde{\nu}_j^R}^{\text{Fixed cost}}$$

Capacity utilization

- Define capacity in j following Qiu and Ríos-Rull (2022)

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

- Capacity utilization in sector j is the ratio of output to capacity:

$$\begin{aligned} util_j &\equiv \frac{Y_j}{cap_j} \\ &= \frac{A_j D_j^{\phi}(z_j h_j^{\alpha_k} X^{1-\alpha_k} k_j^{\alpha_k} n_j^{\alpha_n} - \nu_j X)}{z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X} \end{aligned}$$

- Stationary measure
- Integrates goods market frictions and variable capital utilization

Capacity utilization and relationship to Solow residual

- Log linearized capacity utilization

$$\widetilde{util}_j = \phi \tilde{D}_j + (1 + \nu_{ss}^R)\alpha_k \tilde{h}_j$$

using

$$\nu_{ss}^R \equiv \frac{\nu_j X}{z_j f - \nu_j X}\Big|_{\text{steady state}}$$

- Both shopping effort and variable capital utilization contribute with weights ϕ and $\alpha_k(1 + \nu^R)$
- If $\nu_j = 0$, then cyclical deviations of Solow residual comprise

$$\widetilde{SR}_j|_{\nu_j=0} = \overbrace{\widetilde{util}_j}^{\text{capacity utilization}} + \underbrace{\widetilde{z}_j + (1-\alpha_k)\widetilde{X}}_{\text{technology}} + \overbrace{(\alpha_k - 1 + \omega)\widetilde{k} + (\alpha_n - \omega)\widetilde{n_j}}^{\text{input share mismeasurement}}$$

Mapping model to data

Aggregate measures

- Output

$$Y = C + p_i^{ss} I$$

- Using base-year prices makes results independent of numeraire choice Explanation
- Solow residual and capacity utilization

$$SR = \sum_j \frac{Y_j}{Y} SR_j, \quad util = \sum_j \frac{Y_j}{Y} util_j$$

BRS as special case

- Model nests BRS by shutting down additional frictions: Equilibrium
  - $\gamma = 0$
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_k = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise

- Fix $\sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- In contrast to BRS, estimate ϕ and η instead of calibrating using shopping time or price dispersion targets

Table 2: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 2: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{Z, Z_I, N, D\}$.

Role of capacity utilization on parameter estimates

Table 3: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_D	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
e_D	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 3: Estimation of baseline BRS model with to sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 4: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Volatility		
θ_D	9.84	2.00
D	1.54	1.69
$util$	0.15	1.49
FEVD		
Y	7.73	63.6
Y/N	2.49	27.0
SR	4.68	39.1

Table 4: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_D. See Table 3.

Highlights of adding capacity utilization

- Shopping-related parameters are more precisely estimated, and demand channel is stronger
- Capacity utilization volatility rises by 10 times, much closer to empirical value
- FEVD of output, labor productivity, and the Solow residual rises dramatically

Calibration

- Several parameters are estimated
- Remaining parameters are calibrated using long-run targets, normalizations, and subset of estimated parameters θ_R
- Long-run targets are physical capital to output ratio, investment share, and labor share permit identification of depreciation rate, capital share, and labor shares

Calibration details

Calibration

Targets	Value	Parameter	Calibrated value/posterior mode
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.6
Labor supply	–	ζ	1.97
Elasticity of matching function	–	ϕ	0.84
Elasticity of shopping effort cost	–	η	0.65
Fixed cost share	–	ν_R	0.42
Habit persistence	–	ha	0.40
Third group: normalizations			
SS output	1	z_{mc}	0.45
Relative price of services	1	z_{sc}	0.69
Relative price of investment	1	z_i	0.36
Fraction time spent working	0.30	θ_n	3.85
Capacity utilization of non-durables	0.81	A_{mc}	2.51
Capacity utilization of services	0.81	A_{sc}	1.49
Capacity utilization of investment sector	0.81	A_i	3.33
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	0.014
Physical capital to output ratio	2.75	α_k	0.242
Labor share of income	0.67	α_n	0.074

Estimation

Bayesian estimation

1. Sample from posterior distribution

$$P(\Theta|Y) = \frac{L(Y|\Theta)P(\Theta)}{P(Y)}$$

given marginal likelihood

$$P(Y) = \int L(Y|\Theta)P(\Theta)d\theta$$

2. Impute shock processes and compute forecast error variance decomposition
3. Incorporate prior information (e.g. microeconomic) and parameter restrictions
4. Evaluate model fit using marginal likelihood $\Rightarrow$ implicitly penalizes parameter complexity

Estimation procedure

Sets of observable variables

- Time period: $1964Q1 - 2019Q4$
- Use seven observables in growth rates: Smets and Wouters (2007), Bai, Ríos-Rull, and Storesletten (2023)

$$(C, I, N_c, N_i, util_{ND}, util_D, p_i)$$

- Use sectoral data on output and labor use following Katayama and Kim (2018)
- Construct output from sum of private consumption and private investment (as BRS)
- Note that sectoral dataset implicitly targets labor productivity in in each sector

Nonstatinary technology

- Incorporate stochastic trend X
- Shock $g_t = X_t / X_{t-1}$ follows AR(1) process in logs as Bai, Ríos-Rull, and Storesletten (2023):

$$\log g_t = (1 - \rho_g) \log \overline{g} + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{gt}, \varepsilon_{gt} \sim N(0, 1)$$

- In special case $\rho_g = 0$, neutral technology is random walk with drift

$$\log X_t = \log X_{t-1} + \log \overline{g} + \sigma_g \varepsilon_{gt}$$

- We stationarize trending variable by dividing them by X_t (X_{t-1} in case of predetermined capital stock K_t)

Shock processes

- Additional persistence compare to preference shocks aids identification
- Also consider stationary neutral shock z_c and investment-specific shock z_i
- Indexing
  - Let stationary technology shock on investment firms be $z_I z_c$, where z_I is independent of z_c
  - Let $z_i \equiv z_c z_I$
- Estimate shock processes $\{\theta_b, \theta_d, \theta_i, \theta_n, g, z_c, z_i, \mu_c, \mu_i\}$, each AR(1) with
  - persistence $\{\rho_b, \rho_d, \rho_i, \rho_n, \rho_g, \rho_{zc}, \rho_{zi}, \rho_{\mu c}\rho_{\mu i}\}$
  - conditional sd $\{\sigma_b, \sigma_d, \sigma_i, \sigma_n, \sigma_g, \sigma_{zc}, \sigma_{zi}, \sigma_{\mu c}, \sigma_{\mu i}\}$ of unanticipated component
  - conditional sd $\{\sigma_{b,-4}, \sigma_d, \sigma_i, \sigma_{g,-4}, \sigma_{zc,-4}, \sigma_{zi,-4}, \sigma_{\mu c,-4}, \sigma_{\mu i,-4}\}$ of anticipated component

Key exercises

1. Posterior estimates
2. FEVD of baseline model
3. Model comparison: marginal likelihood, FEVD of demand shocks, contribution of utilization, second moments

Posterior estimates: structural parameters

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	beta	1.50	0.25	1.81	0.18	1.58	2.09
ha	beta	0.50	0.20	0.42	0.05	0.35	0.50
ν	gamm	0.72	0.25	1.85	0.13	1.64	2.00
γ	beta	0.50	0.20	0.32	0.04	0.25	0.38
ϕ	beta	0.32	0.20	0.86	0.04	0.79	0.93
η	gamm	0.20	0.15	0.56	0.12	0.38	0.73
ξ	gamm	0.85	0.10	0.92	0.06	0.82	1.02
ν_R	beta	0.20	0.10	0.33	0.09	0.17	0.44
σ_{ac}	invg	1.00	1.00	1.37	0.34	0.71	1.88
σ_{ai}	invg	1.00	1.00	0.54	0.15	0.33	0.73
Ψ_C	gamm	4.00	1.00	4.82	0.35	4.26	5.40
Ψ_I	gamm	4.00	1.00	4.18	0.74	3.12	5.31
θ	gamm	1.00	0.50	1.55	0.50	0.93	2.32

Posterior estimates: shock processes on shopping effort

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ_D	beta	0.600	0.2000	0.936	0.0164	0.9108	0.9638
ρ_{DI}	beta	0.600	0.2000	0.995	0.0052	0.9887	0.9999
e_D	gamm	0.010	0.0100	0.040	0.0081	0.0276	0.0523
$e_{D,-4}$	gamm	0.010	0.0100	0.007	0.0061	0.0001	0.0165
e_{DI}	gamm	0.010	0.0100	0.002	0.0014	0.0001	0.0040
$e_{DI,-4}$	gamm	0.010	0.0100	0.020	0.0011	0.0177	0.0212

$\rho_D, \rho_{DI}, e_D, e_{DI,-4}$ are estimated reasonably precisely

Unconditional forecast error variance decomposition

Table 7: Forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	35.42	0.01	63.76	0.72	0.09
SR	46.62	0.66	48.16	2.75	1.81
I	38.2	0	55.11	6.66	0.03
p_i	53.82	0	45.93	0.1	0.15
n_c	15.36	14.55	30.5	22.21	17.38
n_i	18.34	1.33	25.7	13.06	41.58
$util$	13.13	0.01	85.97	0.86	0.03
D	2.36	0	97.58	0.06	0.01
h	32.14	0.02	66.99	0.83	0.03

Table 7: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Model comparison

Table 8: comparison of model specification

	Data	M1	M2	M3	M4	M5
LML	–	4526.7	4514.10	4463.10	4192.8	–
Δ LML	–	0	-12.60	-63.60	-333.9	–
90% HPDI band ϕ	–	(0.8, 0.94)	(0.84, 0.96)	(0.2467, 0.3452)	(0.69, 0.72)	(0.56, 0.70)
FEVD(Y, dem)	–	63.76	58.68	54.01	–	–
FEVD(SR, dem)	–	48.16	42.41	48.1	–	–
Var(util)/Var(SR)	–	0.79	0.76	0.69	1.49	0.09
std(Y)	0.87	1.63	1.63	2	59.63	0.6
std($util_{ND}$)	1.26	1.14	1.1	1.27	47.17	0.27
std($util_D$)	2.27	3	3.25	2.44	84.16	1.18
std(N_c)	0.57	0.53	0.63	0.53	17.05	0.48
std(N_i)	1.94	1.8	1.92	1.76	39.08	1.66
Cor(C, I)	0.54	0.62	0.55	0.58	0.99	0.26
Cor($util_{ND}, util_D$)	0.75	0.57	0.53	0.62	1	-0.71
Cor(N_C, N_I)	0.87	0.78	0.81	0.84	1	0.82

Table 8

Impulse responses under baseline: 1 sd shock e_D (shopping disutility shock)

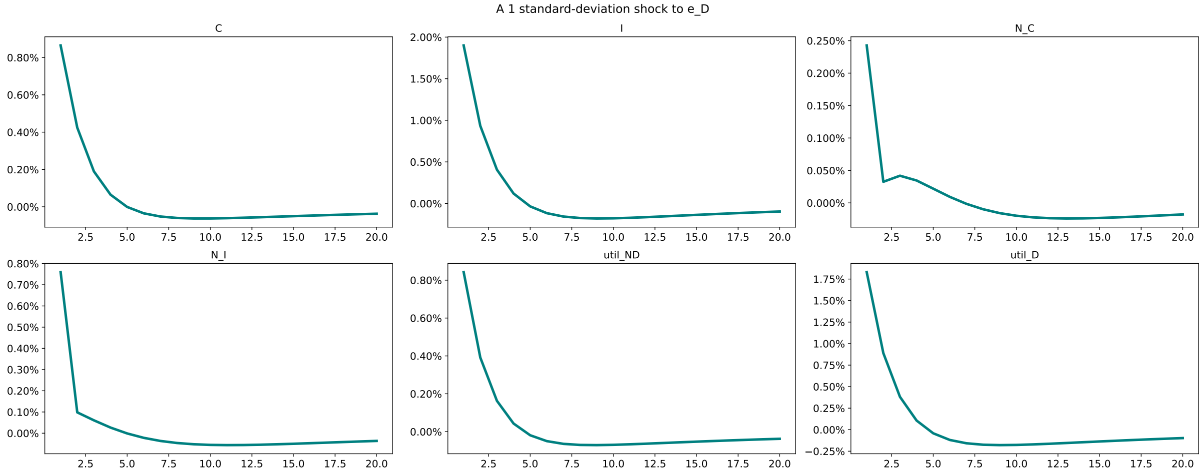

Figure 5

Impulse responses under baseline: 1 sd shock e_b (discount-factor shock)

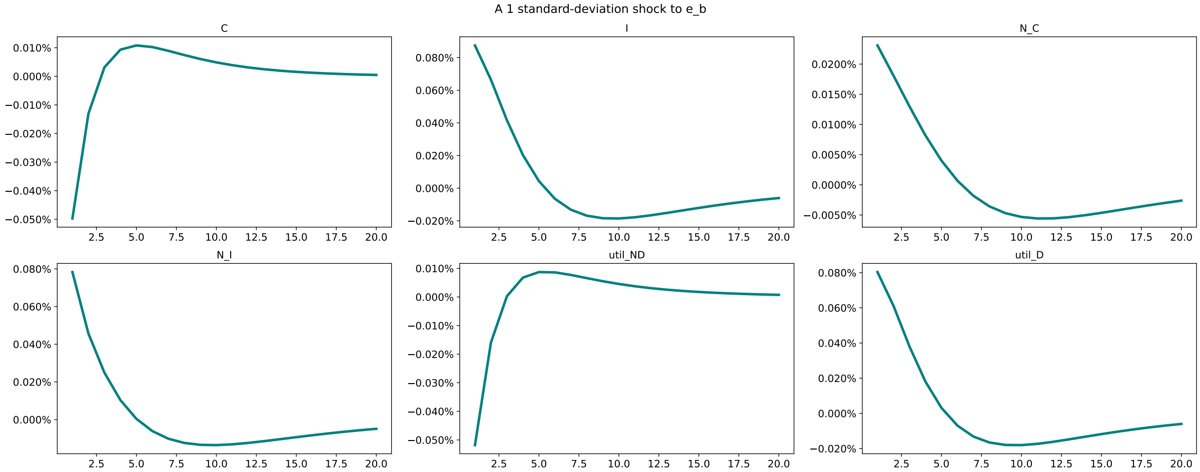

Figure 6

Interpretation

- Search effort (e_D) shocks are unique in generating positive comovement between sectoral output, input, and utilization
- Discount-factor (e_b) shocks generate opposing movements in consumption and investment, and in utilization
- Both technology shocks (e_z and e_g) induce negatively correlated movements in utilization growth

Tentative conclusion

- Precise, high estimate of key parameter ϕ and shopping-effort shocks (no reliance on shopping time data)
- Shocks to shopping effort and its news component explain a major part of the forecast error variance decomposition of output, the Solow residual, the relative price of investment, hours, and utilization
- Explains sectoral comovement and utilization volatility well
- Removing fixed costs and variable capital utilization reduces model fit but does not change main findings
- Model is incapable of fitting data without search demand shocks
  - Search demand shocks are unique in matching all comovement properties
  - Can fit data other than utilization
  - But implied utilization is far less volatile and has negative comovement

Data series

ID	Description	Source
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPIC1	Real gross private domestic investment	BEA
COMPRNFB	Wages (real compensation per hour)	BLS
CNP160V	Civilian non-institutional population	BLS
GDPDEF	GDP Deflator	BEA
SR	Solow residual	Fernald (2014), FRB of San Francisco
Util	Total capacity utilization	Federal Reserve Board of Governors
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Back to second moments

Construction of variables

Symbol	Description	Construction
C	Nominal consumption	$PCEND + PCESV$
I	Nominal gross private domestic investment	GPDI
Deflator	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
c	Real per capita consumption	$\frac{C}{Pop*Deflator}$
i	Investment	$\frac{I}{Pop*Deflator}$
y	Real per capita output	$c+i$
N_c	Labor in consumption sector	Labor in nondurables and services
N_i	Labor in investment sector	Labor in construction and durables
N	Aggregate labor	$N_c + N_i$
P_i	Price index: investment goods	$A006RD3Q086SBEA$
P_c	Price index: consumption goods	$DPCERD3Q086SBEA$
p_i	Relative price of investment	P_i/P_c
util	Total capacity utilization	TCU
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

More details on construction of sectoral data

- Closely follows Katayama and Kim (2018)
- Construct consumption and investment as follows

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

- Use HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates
- P_c: combine price indices of nondurable goods (DNDGRG3Q086SBEA) and services (DSERRG3Q086SBEA)
- P_i: use quality-adjusted investment deflator (INVDEV)

More details on construction of sectoral data

- BLS Current Employment Statistics (https://www.bls.gov/ces/data)
- BLS Table B6 contains the number of production and non-supervisory employees by industry
- BLS Table B7 contains average weekly hours of each sector
- We compute total hours for non-durables, services, construction, and durables by multiplying the relevant components of each table
- Construct labor in consumption as sum of non-durables and services
- Construct labor in investment as sum of construction and durables

Capacity utilization

- Coverage
  - 89 detailed industries (71 manufacturing, 16 mining, 2 utilities)
  - Primarily correspond to industries at the 3 or 4-digit NAICS
  - Estimates are available for various groups (durables and non-durables, total manufacturing, mining, utilities, and total industry)
- Source data
  - Capacity data reported in physical units from government sources, trade sources
  - Responses to the Bureau of the Census's Quarterly Survey of Plant Capacity (QSPC)
  - Trends through peaks in production for a few mining and petroleum series

Data download

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, y_j), j \in \{c, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_c, k_i, p, D, y) = \max_{d_{mc}, d_{sc}, d_i, n_c, n_i, c, i_c, i_i, k'_c, k'_i, h_c, h_i} u(m_c, s_c, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad \text{s.}t.$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j / i_{j,-1})]i_j$$

and (1)

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p, D, y\} \in \Omega} \widehat{V}(\Lambda, k_c, k_i, p, D, y)$$

First order conditions

- Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_c, \mu_i$ be the respective Lagrangian multipliers on the constraints
- FOC

$$[m_c]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc}$$

$$[s_c]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc}$$

$$[i_c]: \quad -\gamma_i - \lambda p_i + \mu_c \left(1 - S_c'(x_c)x - S_c(x_c)\right) + \beta\theta_b \mathbb{E}\mu_c' S_c'(x_c')(x_c')^2 = 0$$

$$[i_i]: \quad -\gamma_i - \lambda p_i + \mu_i \left(1 - S_i'(x_i)x_i - S_i(x_i)\right) + \beta\theta_b \mathbb{E}\mu_i' S_i'(x_i')(x_i')^2 = 0$$

$$[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\}$$

$$[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i$$

$$[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^*$$

$$[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^*$$

$$[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\}$$

$$[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}$$

Envelope conditions

- Consumption

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{4}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j (u_j - \lambda p_j) \quad j \in \{mc, sc\} \tag{5}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} (u_j - \lambda p_j) \quad j \in \{mc, sc\}$$

- Investment

$$\frac{\partial V^i}{\partial p_i} = -\lambda i = -\lambda (d_i A_i D_i^{\phi-1} F_i) \tag{6}$$

$$\frac{\partial V^i}{\partial D_i} = -(\phi - 1) d_i A_i D_i^{\phi-2} F_i \gamma_i \tag{7}$$

$$\frac{\partial V^i}{\partial F_i} = d_i A_i D_i^{\phi-1} \gamma_i$$

Price-tightness tradeoff

- Take ratio of (4) and (5):

$$\frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)} \tag{8}$$

- Take ratio of (6) and (7)

$$\frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i} \tag{9}$$

Back to household problem

Firms' problem

- A representative firm in sector $j \in \{m_c, s_c, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, y_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Conditional labor demand and wage index

- Consider labor cost minimization problem

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$
$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n}$$

- Take FOC and recognize W_j as Lagrangian multiplier on constraint

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Wage index for composite labor input in sector j

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

Optimal wage choice of labor union and aggregation

- Problem of labor union

$$\max_{W_j(s)} (W_j(s) - W_j^*) n_j(s) \quad \text{s.t.} \quad (\textbf{??}) \Leftrightarrow$$

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Labor union in each sector choose

$$W_j(s) = \mu_j W_j^*$$

- Labor unions pay same wage and firms choose identical quantities of labor within j

$$W_j(s) = W_j, n_j(s) = n_j$$

- Labor unions rebate earnings to HH in lump-sum fashion (regard as fixed component to wage)

Firm first order conditions

- Let ι_j and ∇_j be the multipliers on participation constraint and production technology

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{10}$$

$$[D_j] \quad \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{11}$$

Firm problem: finding λ and γ_j

- Take ratio of first order conditions for (10) and (11)

$$\frac{D_j}{\phi p_j} = \frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}}$$

- Plug in (8)

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)}$$

- Simplify

$$\lambda \phi p_j = (1 - \phi)(u_j - \lambda p_j) \Rightarrow$$

$$\lambda = u_j(1 - \phi)/p_j$$

so that

$$\gamma_j = \phi u_j$$

Firm problem: finding γ_i

- Take ratio of first order conditions for (10) and (11) for $j = i$:

$$\frac{D_i}{\phi p_i} = \frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}}$$

- Plug in (9)

$$\frac{D_i}{\phi p_i} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i}$$

- Simplify

$$\begin{aligned} \gamma_i &= \frac{\phi}{1-\phi}\lambda p_i \\ &= \phi \frac{u_j}{p_j} p_i \end{aligned}$$

Simplifying shopping conditions

- Plug in values of γ_j to find

$$-u_d = \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{m_c, s_c\}$$

$$-u_d \theta_i = \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]$$

- Plug in $\lambda = u_{mc}(1-\phi)/p_{mc}$ to simplify labor-leisure tradeoff

$$u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}$$

Demand for non-durables and services

- From the expression for λ we have

$$\frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \Rightarrow$$

- Combine with consumption aggregation and price index to find demand curves

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

where $\xi = 1/(1 - \rho_c)$ is the elasticity of substitution.

Tobin's Q

- Solve for value of investment: $j \in \{c, i\}$

$$\lambda p_i + \gamma_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$
$$\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$
$$\frac{\lambda p_i}{1-\phi} = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

- Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption
- We can rearrange as

$$\frac{p_i}{1-\phi} = Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}Q_j' S_j'(x_j')(x_j')^2$$

Tobin's Q

- Rewrite optimal choice of utilization: $j \in \{mc, sc, i\}$

$$\delta_h(h_j)Q_j = R_j$$

- Euler equation

$$Q_j = \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j'h_j'\right] \quad j \in \{mc, sc, i\}$$

Solving for firm multipliers

$$
\begin{aligned}
\iota_j &= \frac{A_j q_j^{\phi} F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda} \\
\nabla_j &= p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j} \\
&= p_j A_j D_j^{\phi} + \frac{A_j D_j^{\phi} \gamma_j}{\lambda} \\
&= p_j A_j D_j^{\phi} + A_j D_j^{\phi} \frac{\phi}{1-\phi} p_j \\
&= A_j D_j^{\phi} \left(p_j + \frac{\phi}{1-\phi} p_j \right) \\
&= \frac{p_j A_j D_j^{\phi}}{1-\phi}
\end{aligned}
$$

Back to firm problem

Simplified optimality conditions for firm

$$
\begin{aligned}
(1-\phi)\frac{W_c}{p_j} &= A_j(D_j)^{\phi} z_c f_{N_j} \quad j \in \{m_c, s_c\} \\
\frac{W_c}{R_j} &= \frac{f_{N_c}}{f_{K_c}} \\
(1-\phi)\frac{W_i}{p_i} &= A_i(D_i)^{\phi} z_i f_{N_i} \\
\frac{W_i}{R_i} &= \frac{f_{N_i}}{f_{K_i}}
\end{aligned}
$$

Firm factor demands

$$(1-\phi)\frac{W_c}{p_j} = \alpha_n \frac{Y_j + A_j D_j^{\phi} \nu_j}{N_j} \quad j \in \{m_c, s_c, i\}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j + A_j D_j^{\phi} \nu_j}{h_j K_j} \quad j \in \{m_c, s_c, i\}$$

Summary of equilibrium conditions

$$\theta_n (N^a)^{1/\nu} \left(\frac{N_c}{N^a}\right)^{\theta} \omega^{-\theta} = (1-\phi)\frac{W_c}{\mu_c \zeta}$$

$$\theta_n (N^a)^{1/\nu} \left(\frac{N_i}{N^a}\right)^{\theta} (1-\omega)^{-\theta} = (1-\phi)\frac{W_i}{\mu_i \zeta}$$

$$N^a = \left[\omega^{-\theta} N_c^{1+\theta} + (1-\omega)^{-\theta} N_i^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

$$\theta_d D^{1/\eta} = \phi p_j \frac{Y_j}{D_j} \quad j \in \{m_c, s_c\}$$

$$\theta_i \theta_d D^{1/\eta} = \phi p_i \frac{I}{D_i}$$

$$\frac{p_i}{1-\phi} = Q_j [1 - S_j'(x_j) x_j - S_j(x_j)] + \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} Q_j' S_j'(x_j')(x_j')^2$$

$$Q_j = \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} \left[(1 - \delta_j(h_j')) Q_j' + R_j' h_j'\right] \quad j \in \{c, i\}$$

Summary of equilibrium conditions

$$C = [\omega_c^{1-\rho_c} Y_{mc}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc}^{\rho_c}]^{1/\rho_c}$$

$$Y_j = p_j^{-1/(1-\rho_c)} \omega_j C \quad j \in \{mc, sc\}$$

$$C = p_{mc} Y_{mc} + p_{sc} Y_{sc}$$

$$\lambda = \Gamma^{-\sigma}(1-\phi)$$

Summary of equilibrium conditions

$$\delta_h(h_j)Q_j = R_j, \quad j \in \{mc, sc, i\}$$

$$Y_j = A_j(D_j)^{\phi}(z_j(h_jK_j)^{\alpha_k}(N_j)^{\alpha_n} - \nu_j) \quad j \in \{m_c, s_c, i\}$$

$$I = I_c + I_i$$

$$K'_{mc} + K'_{sc} = (1-\delta_c(h_{mc}))K_{mc} + (1-\delta_c(h_{sc}))K_{sc}) + [1 - S_c(x_c)]I_c$$

$$K'_i = (1-\delta_i(h_i))k_i + [1 - S_i(x_i)]I_i$$

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{Y_j + A_jD_j^{\phi}\nu_j}{N_j} \quad j \in \{m_c, s_c, i\}$$

$$\frac{W_j}{R_j} = \frac{\alpha_n}{\alpha_k}\frac{h_jK_j}{N_j} \quad j \in \{m_c, s_c, i\}$$

Explanation of numeraire dependence

- Quantity movements may depend on the numeraire in a multisector model
- Consider positive shock to Z^C: relative price of consumption goods falls
- In terms of the investment good, consumption may fall even though actual units purchased rises
- However, if the consumption good were the numeraire, the investment good instead rises in price, so output rises by more
- Reasoning is symmetric with a positive Z^I shock
- Using base-year prices eliminates dependence as by Bai, Ríos-Rull, and Storesletten (2023)
- Fisher index also eliminates dependence on base year, but it is equivalent in the case of a first-order approximation.
- See Duernecker, Herrendorf, Valentinyi et al. (2017) for a detailed discussion

Back to mapping

Calibration

Details: depreciation

- Over sample, the average annual growth rate of output is 1.8%
- Set $\overline{g} = 0.45\%$ (1.8% annual growth)
- Capital accumulation (ignoring adjustment costs)

$$g\widehat{K}' = (1-\delta)\widehat{K} + g\widehat{I}$$

so that in steady state

$$\delta = 1 - \overline{g} + \frac{I}{K}$$

- Let investment share $\kappa = p_i I/Y = 0.2$ and $p_i K/Y = 2.75(4) = 11$
- Hence, $\delta = 0.2/11 - 0.0045 = 1.37\%$

Details: labor share α_n

- Rearrange FOC for labor demand

$$p_j = (1-\phi)\frac{W_j N_j}{\alpha_n A_j (D_j)^{\phi} F_j}$$

Hence,

$$W_j N_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu^R)$$

where $\nu^R = \nu_j/(F_j)$ and thus labor share is

$$\frac{\sum W_j N_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C+p_i I}{Y}(1+\nu^R) = \frac{\alpha_n}{1-\phi}(1+\nu^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1+\nu^R)$

Details: capital share α_k and deprecation parameter σ_b

- $R_j = R$ in steady state
- Note $\beta(\overline{g})^{-\sigma} = 1/(1+r) \Rightarrow \overline{g} - 1 \approx (r-\rho)/\gamma$
- Implies $\rho \approx r - \gamma\overline{g}$ (so we must have $r \geq \gamma\overline{g}$)
- Steady-state Euler

$$\begin{aligned} Q &= \beta\overline{g}^{-\gamma}[(1-\delta)Q + R] \Rightarrow \\ (1+r)Q &= (1-\delta)Q + R \\ (r+\delta)Q &= R \end{aligned}$$

- Steady-state optimal utilization

$$\sigma_b = \frac{R}{Q} = r + \delta$$

- Combine with steady state Tobin's Q: $p_i/(1-\phi) = Q$ and we find

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Details: capital share α_k and deprecation parameter σ_b

- Firm optimization yields

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{K_j}(1+\nu^R)$$

- Note

$$\frac{Y_j}{K_j} = \frac{Y}{K} \quad \forall K$$

and hence

$$r + \delta = \alpha_k \frac{Y}{K}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{K}{Y}$$

Using $r, \delta, K/Y, \nu^R$, we recover $\alpha_k = 0.216$

Details: weight of services ω_{sc}

- We pin down the weight of services ω_{sc} as the empirical measure $S_c = Y_{sc}/C$ and set $S_c = 0.65$.
- The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

and plug in S_c, using $\omega_{sc} = S_c$:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

so that $p_{mc} = p_{sc}$

- Given normalization $p_{sc} = 1$, all consumption goods prices equal unity.

Details: matching technology coefficient A_j

- Given $\Psi_j = A_j D_j^{\phi}$, the matching technology coefficient satisfies

$$A_j = \frac{\Psi_j}{D_j^{\phi}}$$

- Need to find D_j for each j

Details: matching technology coefficient A_j

- We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j \rightarrow$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

- Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Details: matching technology coefficient A_j

- Consider ratio in shopping optimality conditions between m_c and i:

$$\frac{D_{mc}}{D_i} = \frac{p_{mc}}{p_i}\frac{Y_{mc}}{Y_i}$$
$$= (1-\omega_{sc})\frac{1-I/Y}{I/Y}$$

- Hence,

$$D_{mc} = (1-S_c)(1-I/Y)D$$
$$D_{sc} = S_c(1-I/Y)D$$
$$D_i = (I/Y)D$$

Estimation

Balanced growth and transformation of variables

- Output, consumption, investment, wages, and capital grow at common rate g_t
- Transform each trending variable y_t determined at time t

$$\widehat{y}_t = \frac{y_t}{X_t}$$

so that $\log \widehat{y}_t$ represents log deviation from stochastic trend

- Capital stock K_t is determined at $t-1$, so we deflate by X_{t-1}

$$\widehat{K}_t = \frac{K_t}{X_{t-1}}$$

- Transform preferences to make shopping stationary

$$\Gamma_t = c_t - haC_{t,-1} - X_t \theta_{dt} \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt} \frac{(n_t^a)^{1+1/\nu}}{1+1/\nu} \zeta_t$$

Equations modified by growth

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$
\begin{aligned}
C_t^{obs} &= \log C_t - \log C_{t-1} + g_t - \overline{g} \\
I_t^{obs} &= \log I_t - \log I_{t-1} + g_t - \overline{g} \\
w_t^{obs} &= \log w_t - \log w_{t-1} + g_t - \overline{g}
\end{aligned}
$$

- Stationary series

$$
\begin{aligned}
N_{jt}^{obs} &= \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\} \\
p_{i,t}^{obs} &= \log p_{i,t} - \log p_{i,t-1} \\
util_{j,t}^{obs} &= \log util_{j,t} - \log util_{j,t-1}
\end{aligned}
$$

Vector of observable variables

Vector of observables

$$
= \begin{bmatrix} \Delta \log(C_t) \\ \Delta \log(I_t) \\ \Delta \log(N_{ct}) \\ \Delta \log(N_{it}) \\ \Delta \log(util_{ND,t}) \\ \Delta \log(util_{D,t}) \\ \Delta \log(p_{it}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Estimation procedure

- Estimate mode of posterior distribution by maximizing log posterior function (combines priors and likelihood)
- Use Metropolis-Hastings algorithm to sample posterior distribution and to evaluate marginal likelihood of the model
  - Sample of 300,000 draws (neglect first 20%)
  - Hessian defines transition probability that generates new proposed draw
- Check convergence and identification (trace plots)

Back to estimation

On the use of growth rates for estimation

- Major macroeconomic series are difference-stationary
- For such data, growth rates preserves all dynamics of a series
- Other filters (such as HP filter/Hamilton filter) extract specific frequencies of time series
- Latter may be reasonable for *description* depending on the notion of business cycle

Bai, Y., J.-V. Ríos-Rull, and K. Storesletten (2023): "Demand Shocks as Technology Shocks," .

Basu, S., J. G. Fernald, and M. S. Kimball (2006): "Are technology improvements contractionary?," *American Economic Review*, 96(5), 1418–1448.

Bethune, Z., G. Rocheteau, and P. Rupert (2015): "Aggregate unemployment and household unsecured debt," *Review of Economic Dynamics*, 18(1), 77–100.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, 113(1), 1–45.

Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2016): "Unemployment and business cycles," *Econometrica*, 84(4), 1523–1569.

Christiano, L. J., and T. J. Fitzgerald (1998): "The Business Cycle: It's still a Puzzle," *Federal-Reserve-Bank-of-Chicago-Economic-Perspectives*, 4, 56–83.

Fernald, J. (2014): "A quarterly, utilization-adjusted series on total factor productivity," Citeseer.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, pp. 402–417.

Horvath, M. (2000): "Sectoral shocks and aggregate fluctuations," *Journal of Monetary Economics*, 45(1), 69–106.

Huo, Z., and J.-V. Ríos-Rull (2018): "Financial frictions, asset prices, and the great recession," *CEPR Discussion Paper No. DP11544*.

Jaimovich, N., and S. Rebelo (2009): "Can news about the future drive the business cycle?," *American Economic Review*, 99(4), 1097–1118.

Katayama, M., and K. H. Kim (2018): "Intersectoral labor immobility, sectoral comovement, and news shocks," *Journal of Money, Credit and Banking*, 50(1), 77–114.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988): "Production, growth and business cycles," *Journal of Monetary Economics*, 21(2/3), 196–232.

Long, J. B., and C. I. Plosser (1983): "Real business cycles," *Journal of political Economy*, 91(1), 39–69.

Lucas, R. E. (1981): "Studies in business-cycle theory," .

Moen, E. R. (1997): "Competitive search equilibrium," *Journal of Political Economy*, 105(2), 385–411.

Petrosky-Nadeau, N., and E. Wasmer (2015): "Macroeconomic dynamics in a model of goods, labor, and credit market frictions," *Journal of Monetary Economics*, 72, 97–113.

Qiu, Z., and J.-V. Ríos-Rull (2022): "Procyclical productivity in new keynesian models," Discussion paper, National Bureau of Economic Research.

Schmitt-Grohé, S., and M. Uribe (2012): "What's news in business cycles," *Econometrica*, 80(6), 2733–2764.

Smets, F., and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American economic review*, 97(3), 586–606.