

Nonequilibrium lattice dynamics in two-dimensional materials

Fabio Caruso

March 1st, 2021

Surface Science 2021

**Frontiers of Electronic-Structure Theory:
Focus on Electron-Phonon Interactions**

C | A | U

Christian-Albrechts-Universität zu Kiel

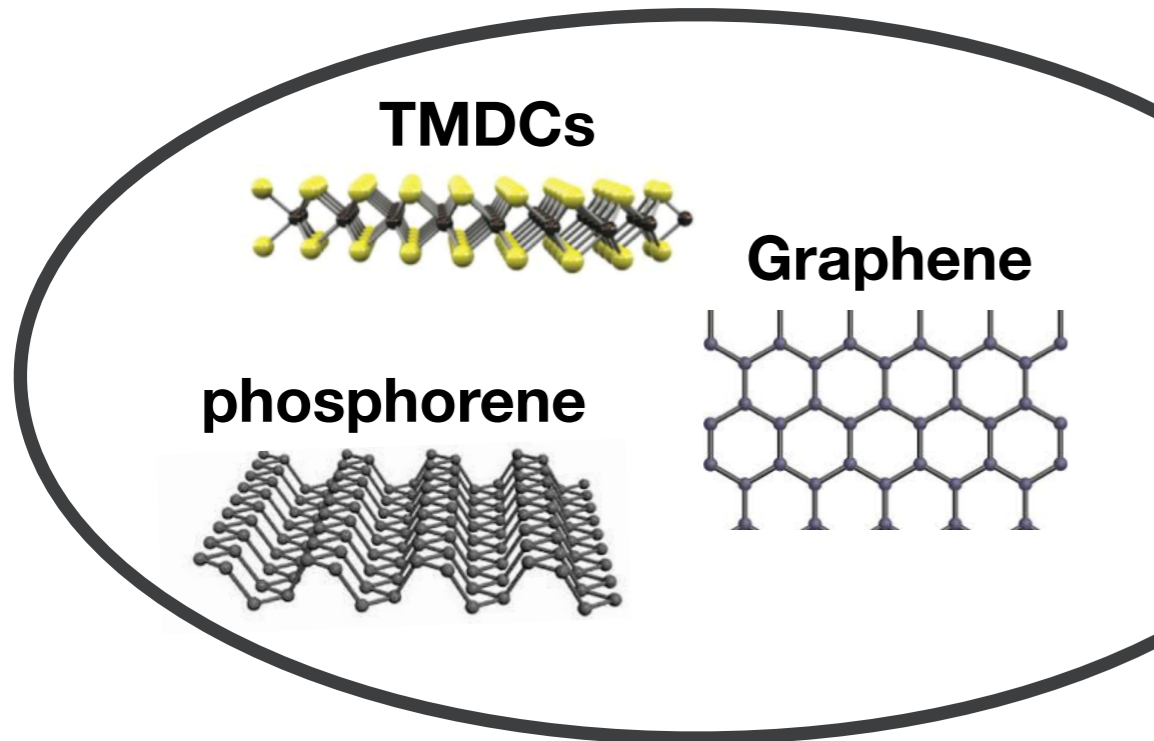
Funded by

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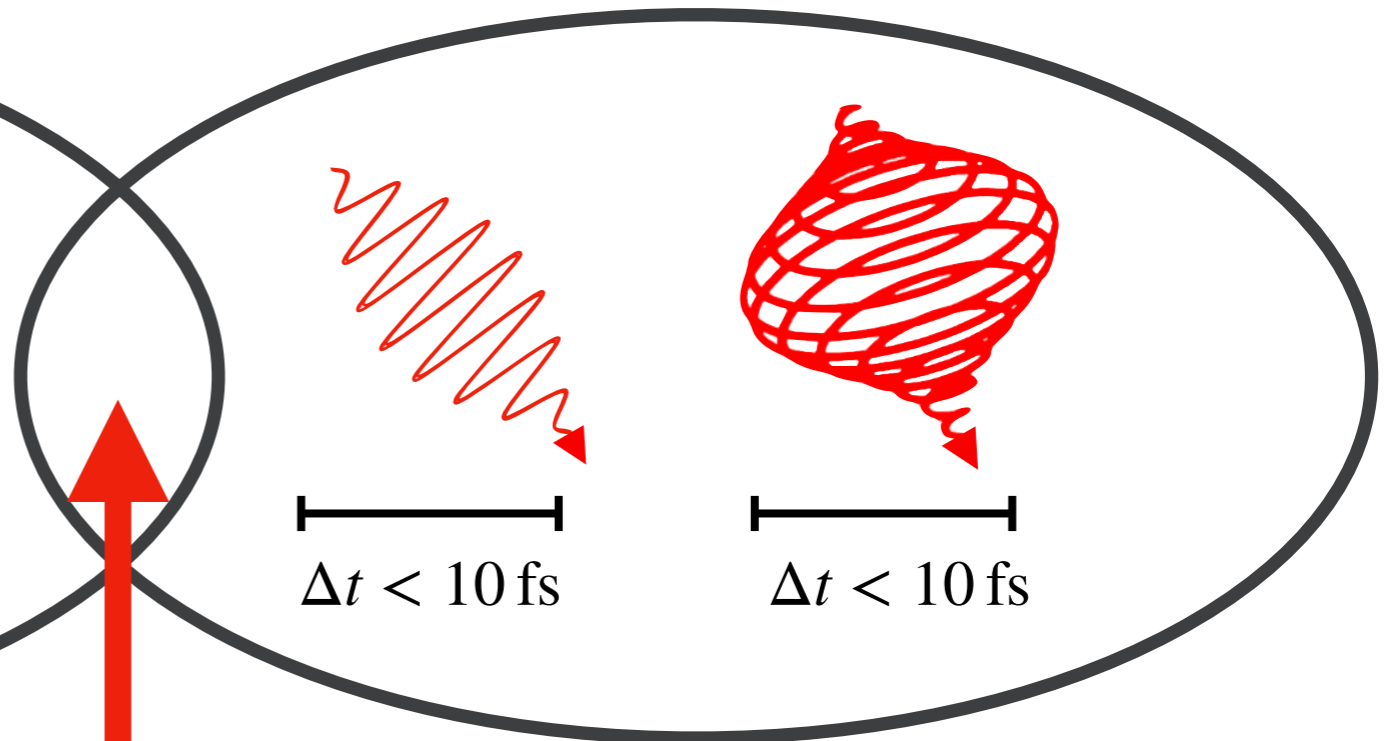
Deutsche
Forschungsgemeinschaft
German Research Foundation

Nonequilibrium dynamics in two dimensions

Materials science in 2D



Advent of femto-second lasers



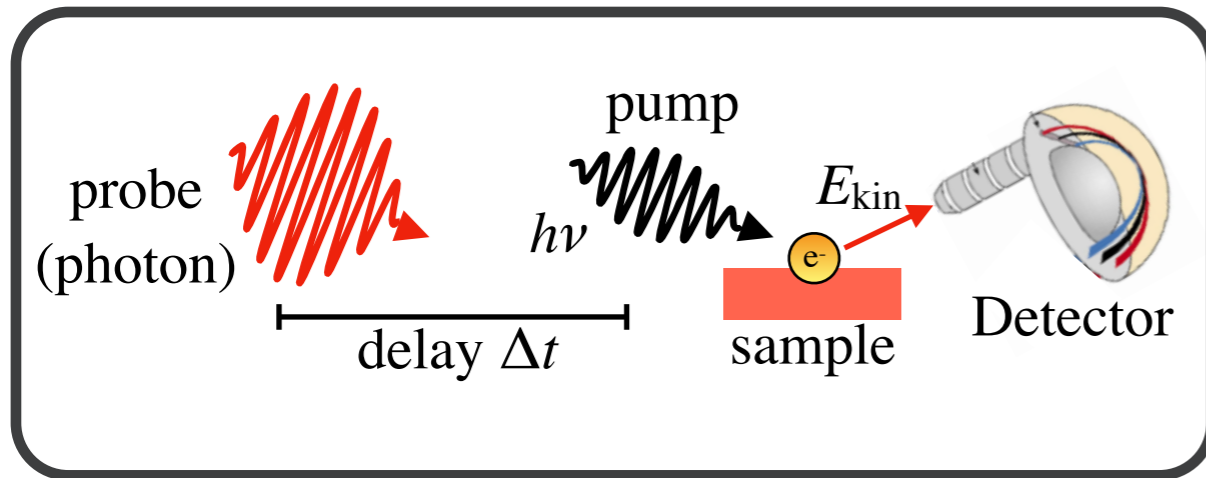
- Reduced dielectric screening
- Quantum-confinement in 2D
- Strong light-matter interactions
- Non-trivial topological properties

- Pump-probe spectroscopies
- Electron and phonon dynamics
- Light-driven phenomena

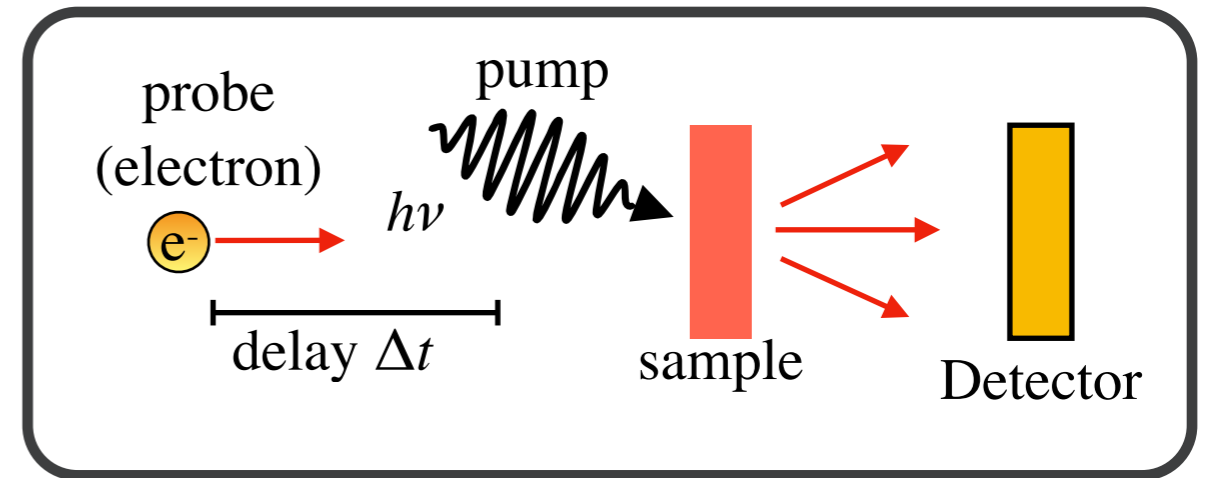
**Properties on-demand
using light pulses**

Nonequilibrium dynamics in two dimensions

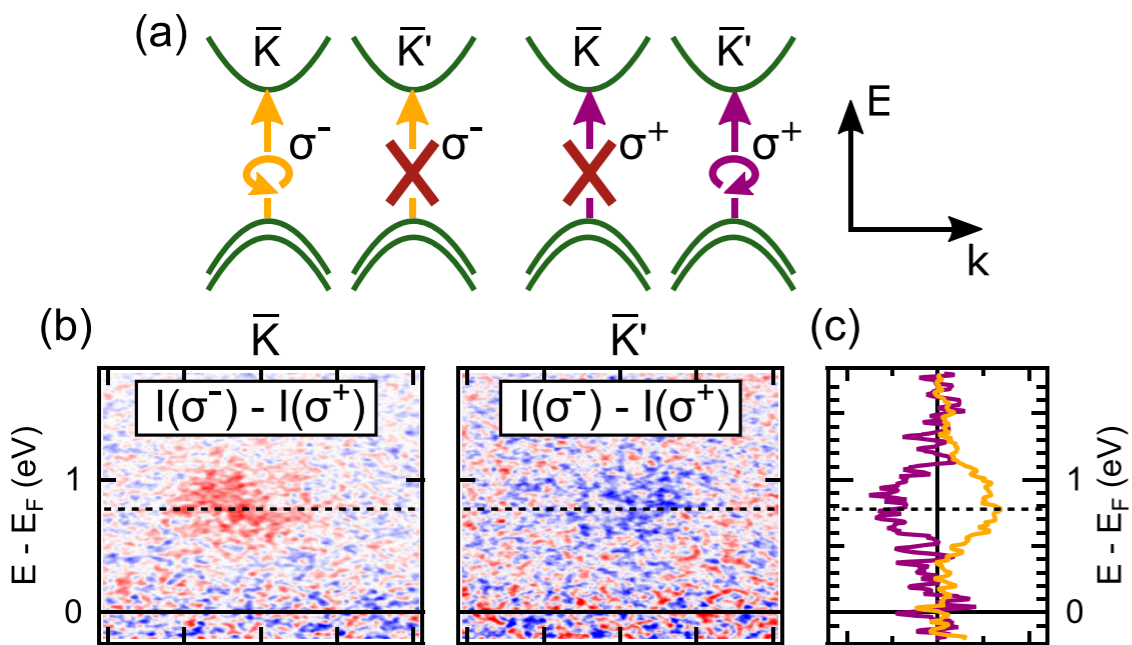
Time- and angle-resolved photoemission spectroscopy (trARPES)



Femto-second electron diffuse scattering (FEDS)

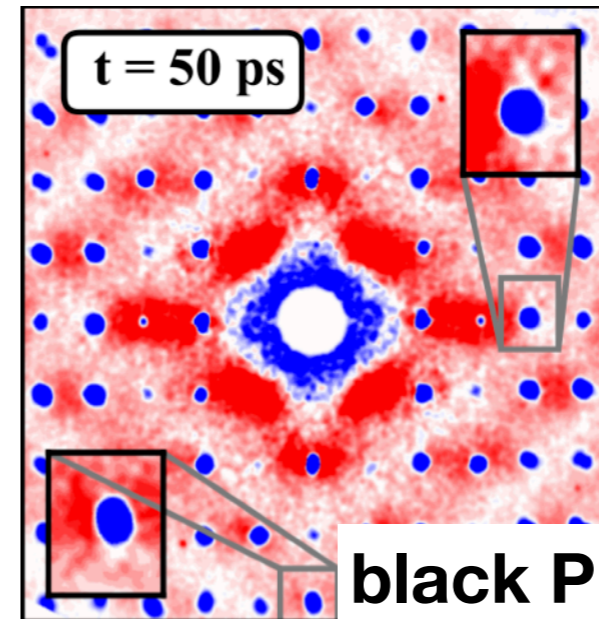


Circular Dichroism (WS_2)

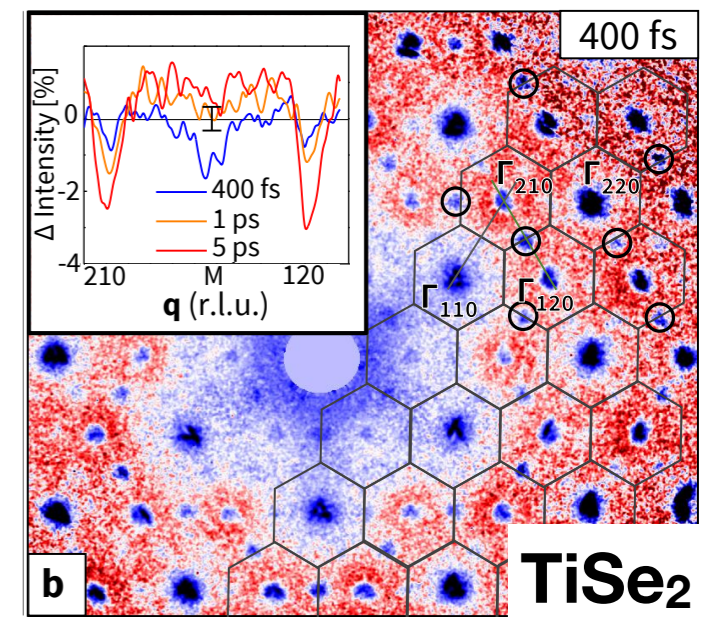


Beyer et al., PRL **123**, 236802 (2019)

Anisotropic lattice dynamic



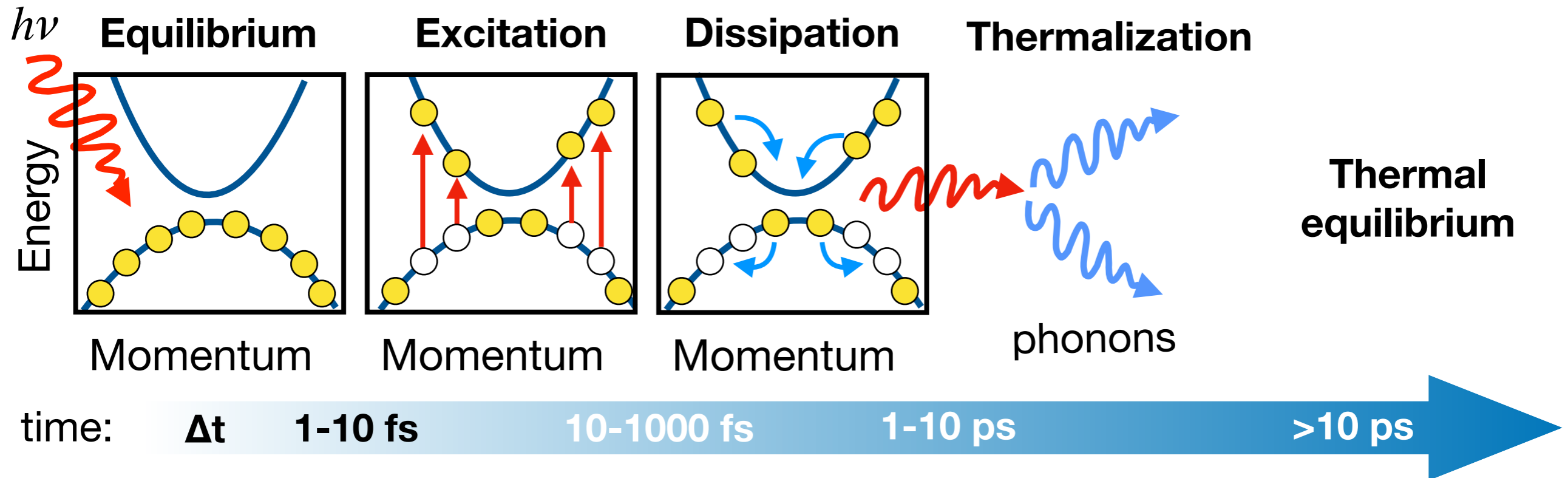
Seiler et al.,
arXiv:2006.12873 (2020)



Otto et al.,
arXiv:1912.03559 (2020)

What is the role of phonons and electron-phonon interactions in the nonequilibrium dynamics?

Ultrafast dynamics from first principles



Which level of theory?

- Nonequilibrium Green's functions (& DMFT)
- Time-dependent density-function theory
- Time-dependent Boltzmann equation
- Quantum Master equation
- Molecular dynamics / Path integrals
- Non-thermal lattice models

Check-list:

- Electron / phonon dynamics
- Electron-phonon coupling
- Full momentum resolution

Equilibrium and nonequilibrium regimes in a quantum system

Distribution function (occupation number):

$f_{n\mathbf{k}}^\sigma$ Number of **electrons** in band n with momentum \mathbf{k}

$n_{q\nu}$ Number of **phonons** in branch ν with momentum q

Equilibrium:

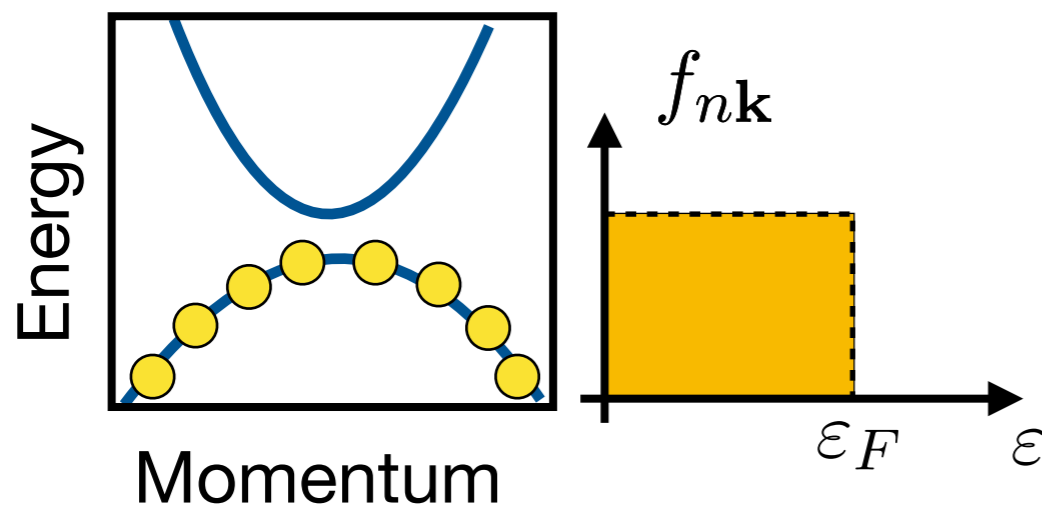
Fermi-Dirac statistics:

Bose-Einstein statistics:

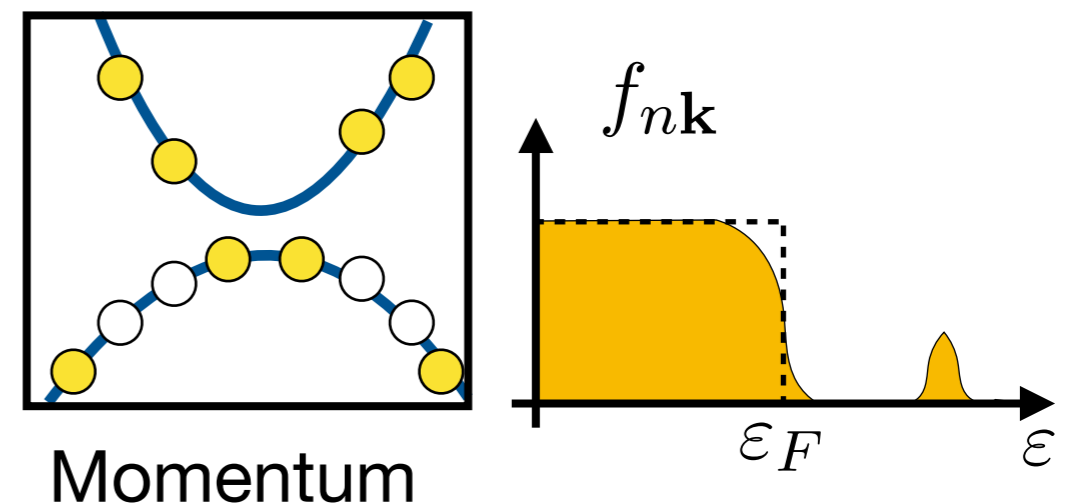
$$f_{n\mathbf{k}}^0(\mu, T) = [e^{(\varepsilon_{n\mathbf{k}} - \mu)/k_B T} + 1]^{-1}$$

$$n_{q\nu}^0(T) = [e^{\hbar\omega_{q\nu}/k_B T} - 1]^{-1}$$

Equilibrium



Excitation



Out of equilibrium:

$$f_{n\mathbf{k}} \neq f_{n\mathbf{k}}^0 \quad \text{and / or} \quad n_{q\nu} \neq n_{q\nu}^0$$

The Boltzmann equation in solid-state physics



distribution
function

collision integral

$$\frac{\partial f}{\partial t} = \Gamma_{\text{collisions}}$$

+ other terms
(fields, etc.)

(In a gas:)

**Equilibrium is
re-established
via collisions**

Books:

Ziman, Electrons and phonons, Oxford University Press (1960)

Hang, Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, Springer (1996)

Bonitz, Quantum Kinetic Theory (1998)

Charge and thermal Transport:

Poncé, Li, Reichard, Giustino, Rep. Prog. Phys. 83, 036501 (2019)

Li, Carrete, Katcho, Mingo, Comp. Phys. Comm. 185, 1747 (2014)

Mizokami, Togo, Tanaka Phys. Rev. B 97, 224306, (2018)

Chaput, Phys. Rev. Lett 110, 265506 (2013)

Togo, Chaput, Tanaka, Phys. Rev. B 91, 094306 (2015)

Ultrafast dynamics:

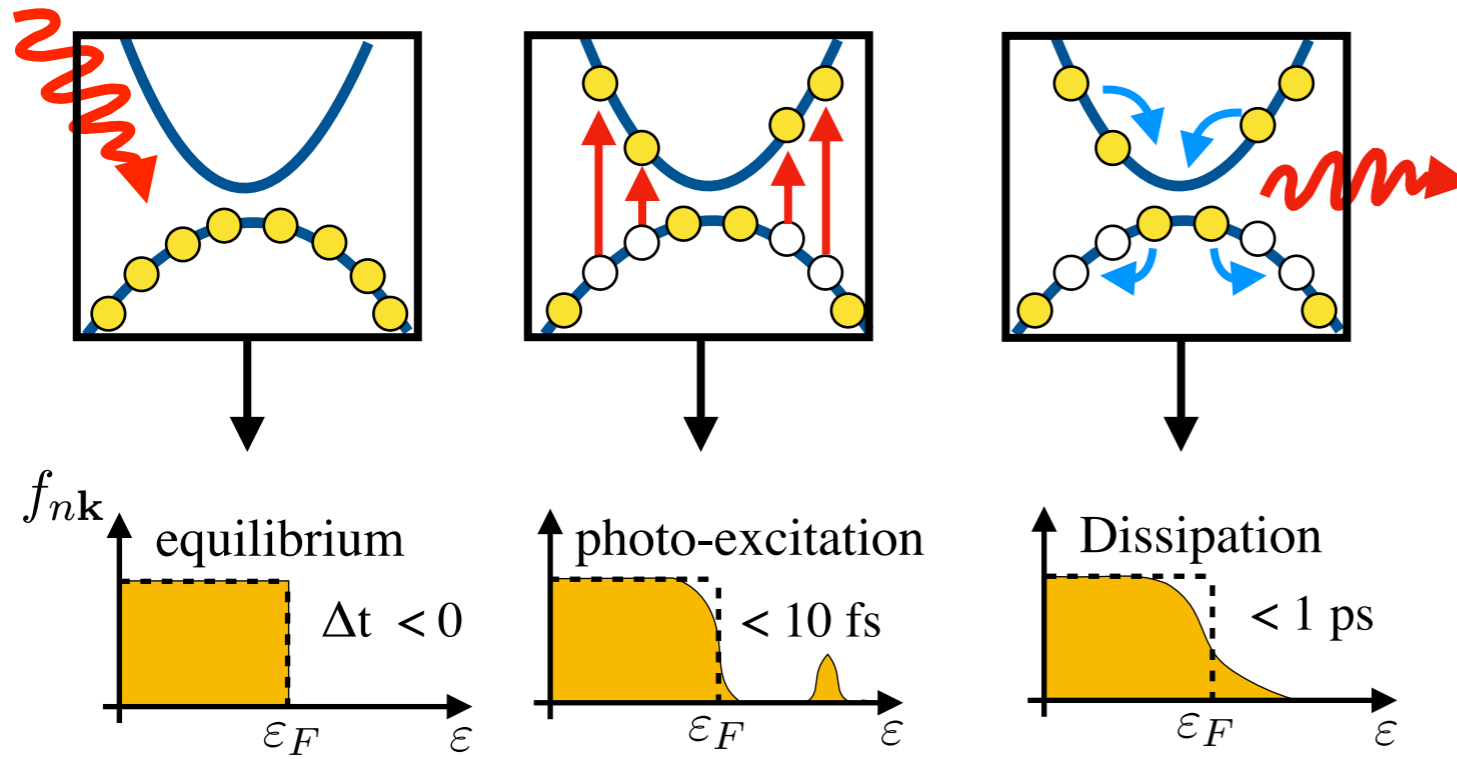
Sadasivam, Chan, Darancet, Phys. Rev. Lett. 119, 136602 (2017)

Bernardi, Eur. Phys. J. B 89, 239 (2016)

Jhalani, Zhou, Bernardi, Nano Letters 17, 5012 (2017)

Caruso, J. Phys. Chem. Lett. 12, 1274 (2021)

Time-dependent Boltzmann equation



Nonequilibrium states are described by

$$f_{n\mathbf{k}}^\sigma \quad n_{\mathbf{q}\nu}$$

time-dependence of $f_{n\mathbf{k}}^\sigma$ $n_{\mathbf{q}\nu}$
 time-dep. Boltzmann eq.:

electron- phonon	electron-photon (dipole int.)	electron- electron
\swarrow	\swarrow	\swarrow
$\frac{\partial f_{n\mathbf{k}}}{\partial t} = I_{n\mathbf{k}}^{\text{e-ph}}[f, n] + \cancel{I_{n\mathbf{k}}^{\text{light}}[f]} + \cancel{I_{n\mathbf{k}}^{\text{e-e}}[f]}$		
$\frac{\partial n_{\mathbf{q}\nu}}{\partial t} = I_{\mathbf{q}\nu}^{\text{e-ph}}[f, n] + I_{\mathbf{q}\nu}^{\text{ph-ph}}[n]$		
\searrow	\searrow	
phonon- electron	phonon- phonon	

First-principles approach to electron and lattice dynamics

$$\frac{\partial f_{n\mathbf{k}}}{\partial t} = I_{n\mathbf{k}}^{\text{e-ph}}[f, n]$$

$$\frac{\partial n_{\mathbf{q}\nu}}{\partial t} = I_{\mathbf{q}\nu}^{\text{e-ph}}[f, n] + I_{\mathbf{q}\nu}^{\text{ph-ph}}[n]$$

Boltzmann equation for the electron and phonon distribution function

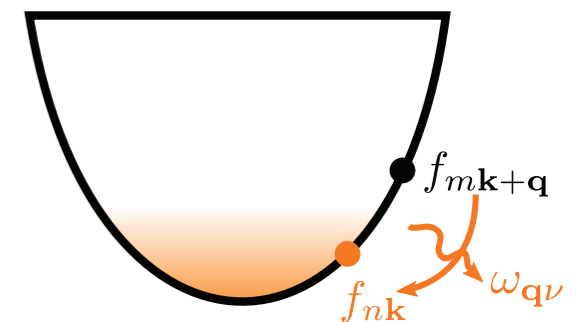
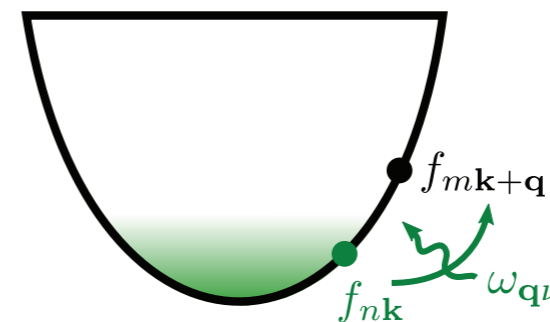
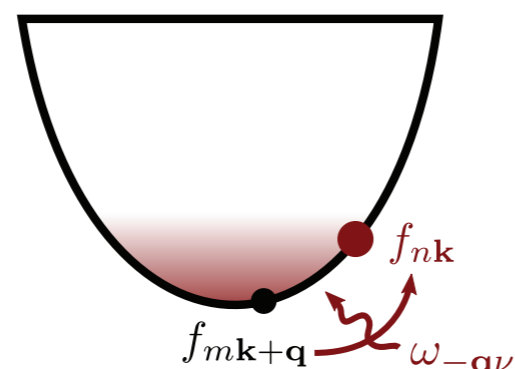
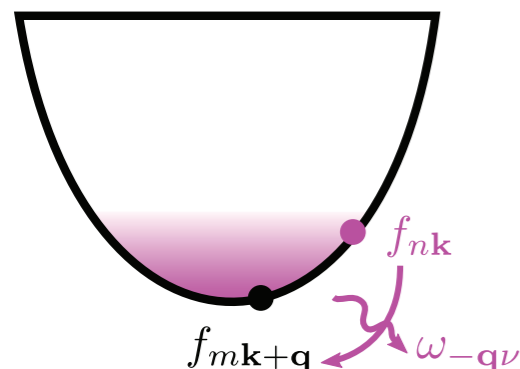
Poncé, et. al, Rep. Prog. Phys. (2019)

Example: electron-phonon scattering rate

$$I_{n\mathbf{k}}^{\text{e-ph}}[f, n] = \frac{2\pi}{\hbar} \sum_{m\nu} \int \frac{d\mathbf{q}}{\Omega_{\text{BZ}}} |g_{m\nu}(\mathbf{k}, \mathbf{q})|^2$$

electron-phonon coupling matrix element

$$\times \left\{ \begin{aligned} &(1 - f_{n\mathbf{k}}) f_{m\mathbf{k}+\mathbf{q}} \delta(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}+\mathbf{q}} + \hbar\omega_{\mathbf{q}\nu})(1 + n_{\mathbf{q}\nu}) \\ &+ (1 - f_{n\mathbf{k}}) f_{m\mathbf{k}+\mathbf{q}} \delta(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}+\mathbf{q}} - \hbar\omega_{\mathbf{q}\nu}) n_{\mathbf{q}\nu} \\ &- f_{n\mathbf{k}} (1 - f_{m\mathbf{k}+\mathbf{q}}) \delta(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}+\mathbf{q}} - \hbar\omega_{\mathbf{q}\nu})(1 + n_{\mathbf{q}\nu}) \\ &- f_{n\mathbf{k}} (1 - f_{m\mathbf{k}+\mathbf{q}}) \delta(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}+\mathbf{q}} + \hbar\omega_{\mathbf{q}\nu}) n_{\mathbf{q}\nu} \end{aligned} \right\}$$



Ultrafast dynamics from first principles

WANNIER90

Wannier-function interpolation

G. Pizzi et al.,
J. Phys. Cond. Matt. 32, 165902 (2020)



Electron-phonon coupling

S. Poncé et al.,
Comp. Phys. Comm. (2017)

Coupled-dynamics of electrons and phonons

$$\frac{\partial f_{n\mathbf{k}}}{\partial t} = I_{n\mathbf{k}}^{\text{e-ph}}[f, n]$$
$$\frac{\partial n_{\mathbf{q}\nu}}{\partial t} = I_{\mathbf{q}\nu}^{\text{e-ph}}[f, n] + I_{\mathbf{q}\nu}^{\text{ph-ph}}[n]$$



Density functional theory

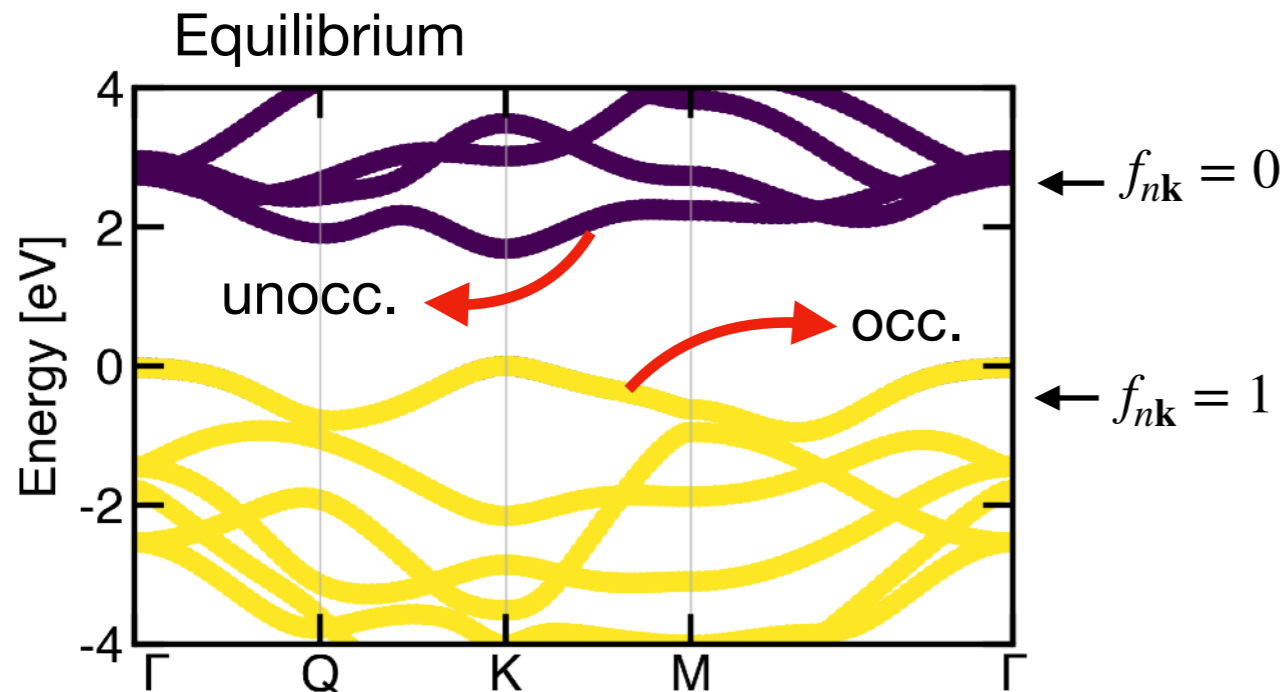
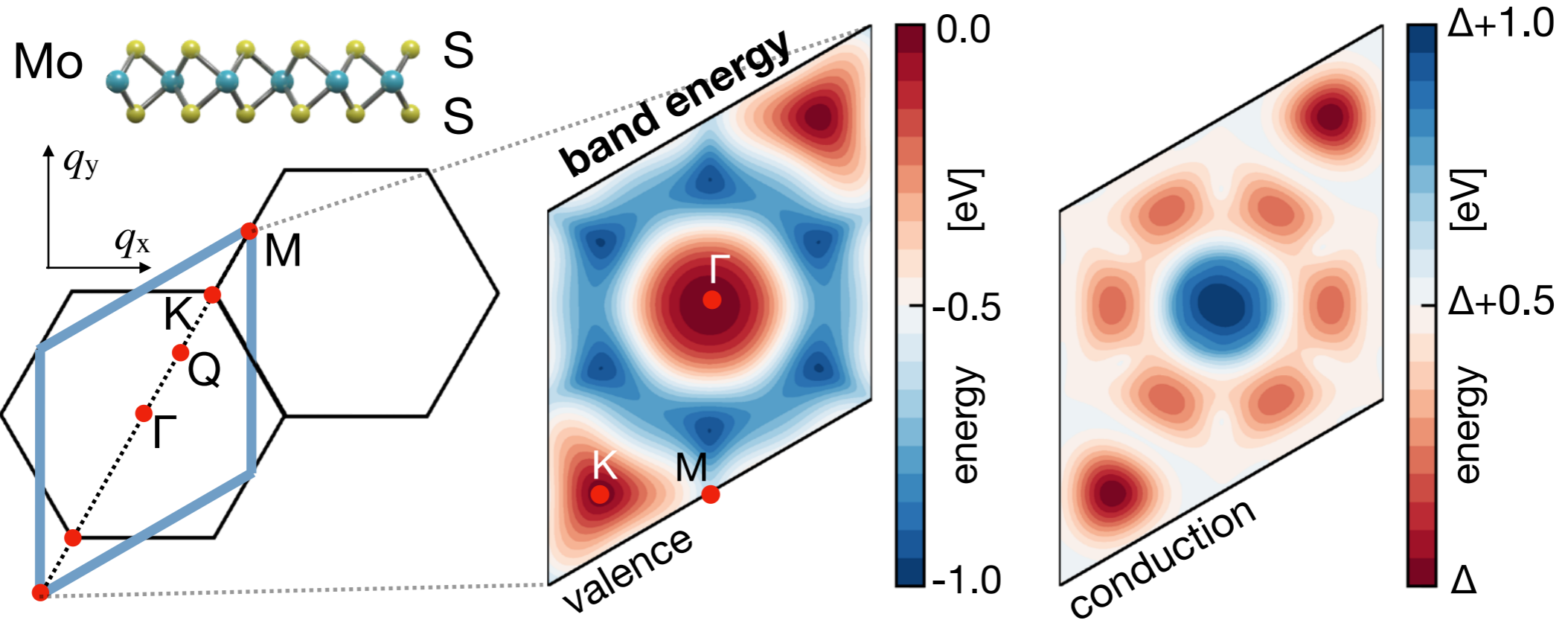
P. Giannozzi et al.,
J. Phys.: Condens. Matter 29, 465901 (2017)



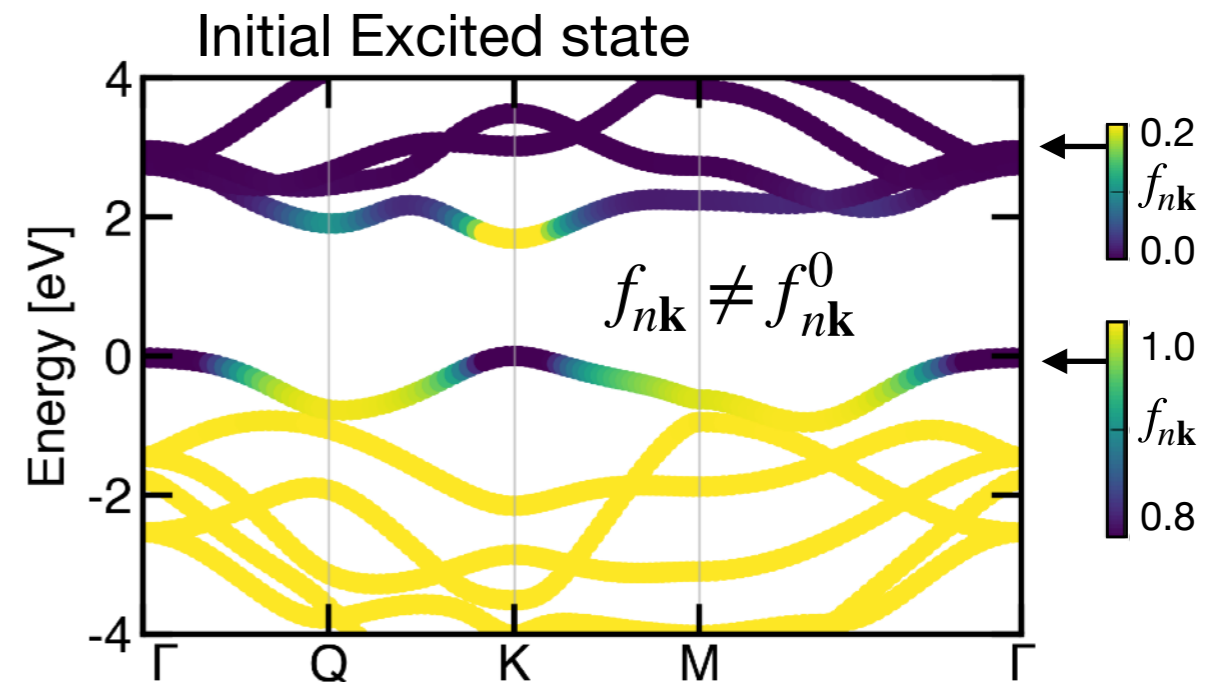
Third-order force constant

W. Li et al., Comp. Phys. Comm. 185,
1747 (2014)

Nonequilibrium electron dynamics in monolayer MoS₂

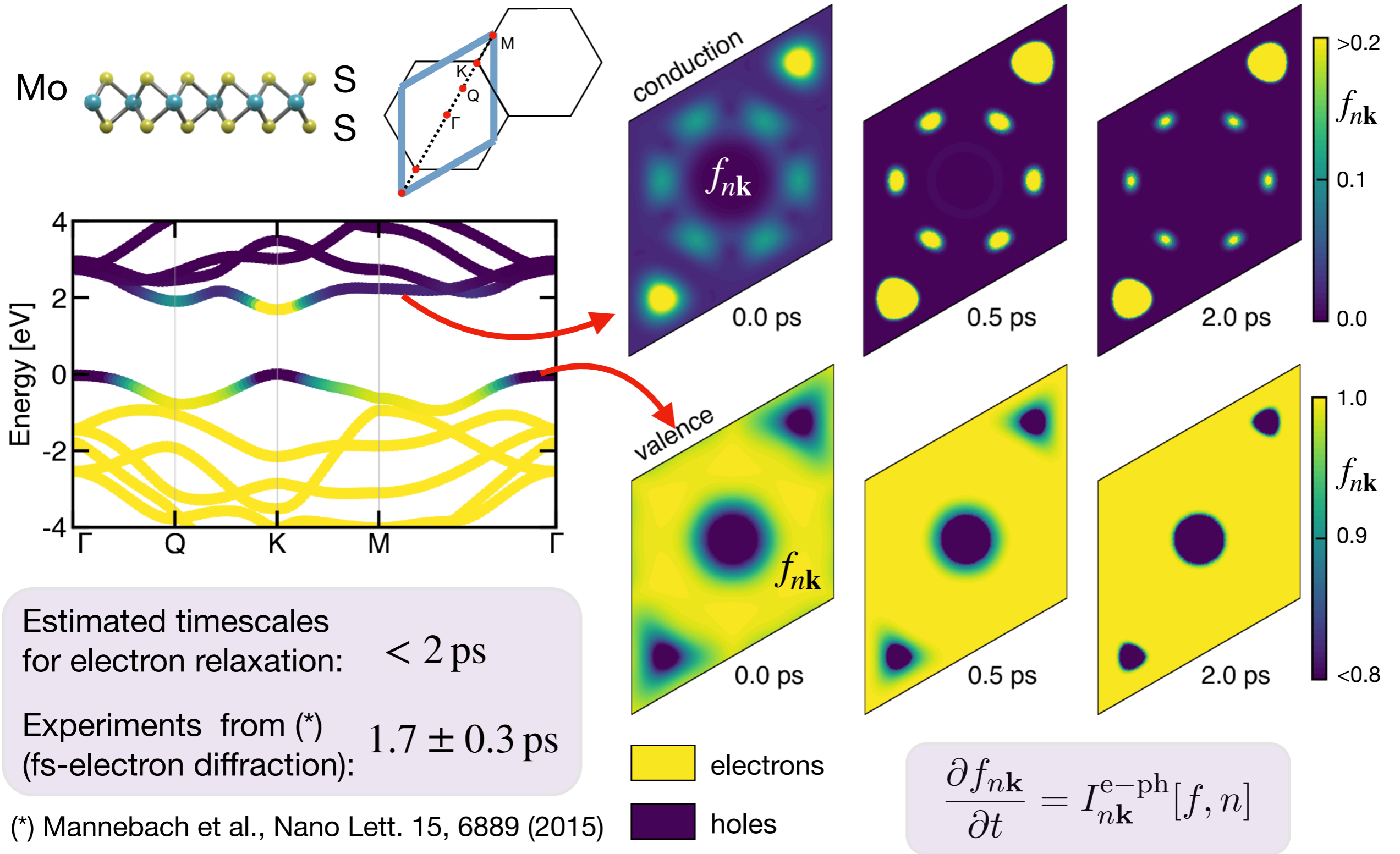


$$f_{n\mathbf{k}}^0(\mu, T) = [e^{(\varepsilon_{n\mathbf{k}} - \mu)/k_B T} + 1]^{-1}$$



Excited carrier density: $n=10^{14} \text{ cm}^{-2}$

Nonequilibrium electron dynamics



Nonequilibrium phonon dynamics

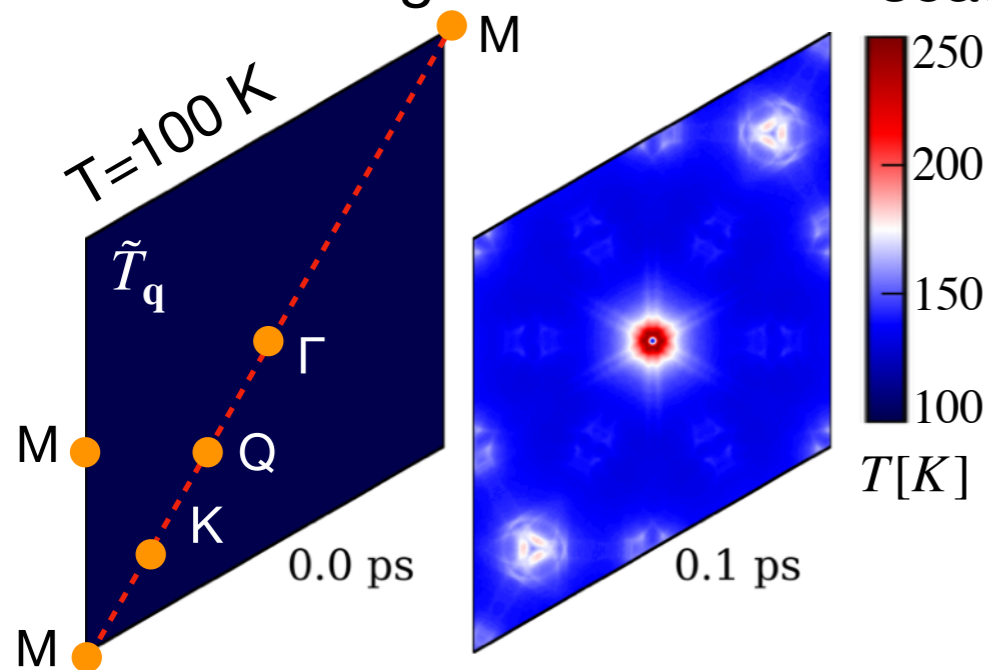
$$\frac{\partial n_{\mathbf{q}\nu}}{\partial t} = I_{\mathbf{q}\nu}^{\text{e-ph}}[f, n] + I_{\mathbf{q}\nu}^{\text{ph-ph}}[n]$$

effective vibrational temperature

$$T_{\mathbf{q}\nu} = \hbar\omega_{\mathbf{q}\nu} [k_B \ln(1 + n_{\mathbf{q}\nu})]^{-1}$$

electron-phonon scattering

phonon-phonon scattering

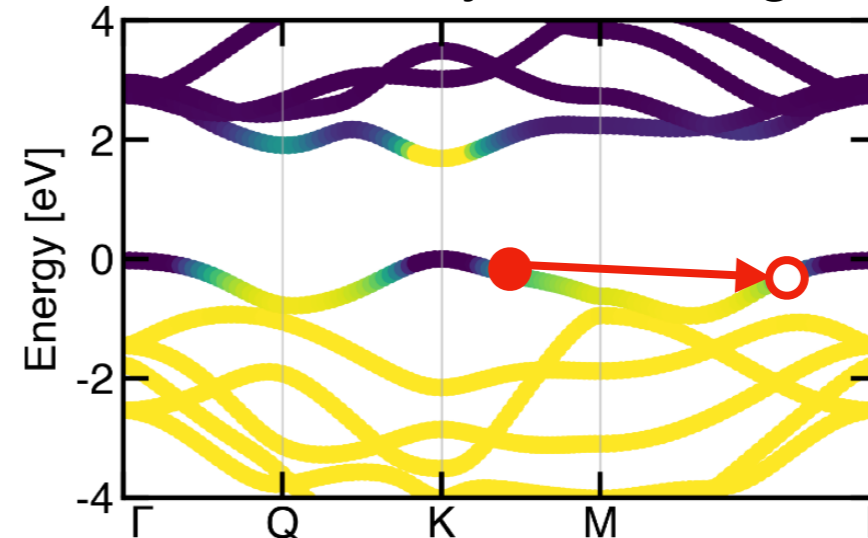


vibrational temperature

Energy cons.: $\hbar\omega_{\mathbf{q}\nu} = \varepsilon_{n\mathbf{k}} - \varepsilon_{n\mathbf{k}'}$

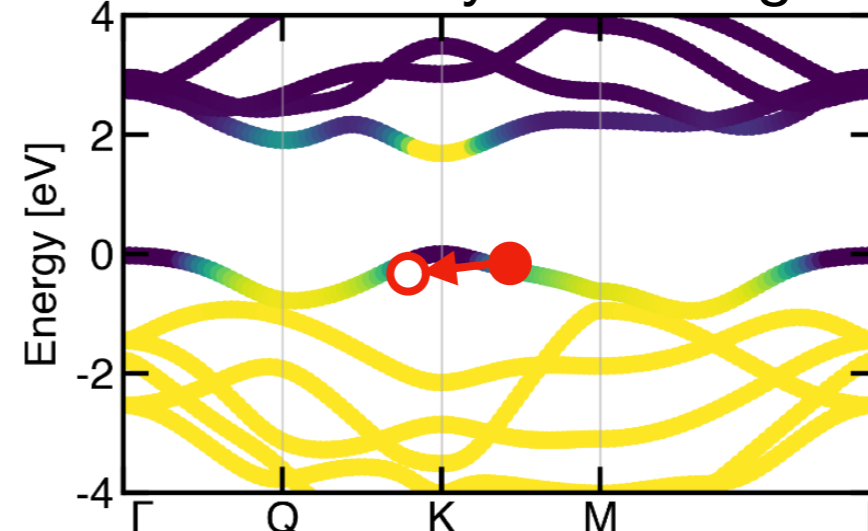
Momentum cons.: $\mathbf{q} = \mathbf{k}' - \mathbf{k}$

Intervalley scattering



allowed phonon momenta:
 $\mathbf{q} \simeq \mathbf{K}$

Intravalley scattering



allowed phonon momenta:
 $\mathbf{q} \simeq \Gamma$

Nonequilibrium phonon dynamics

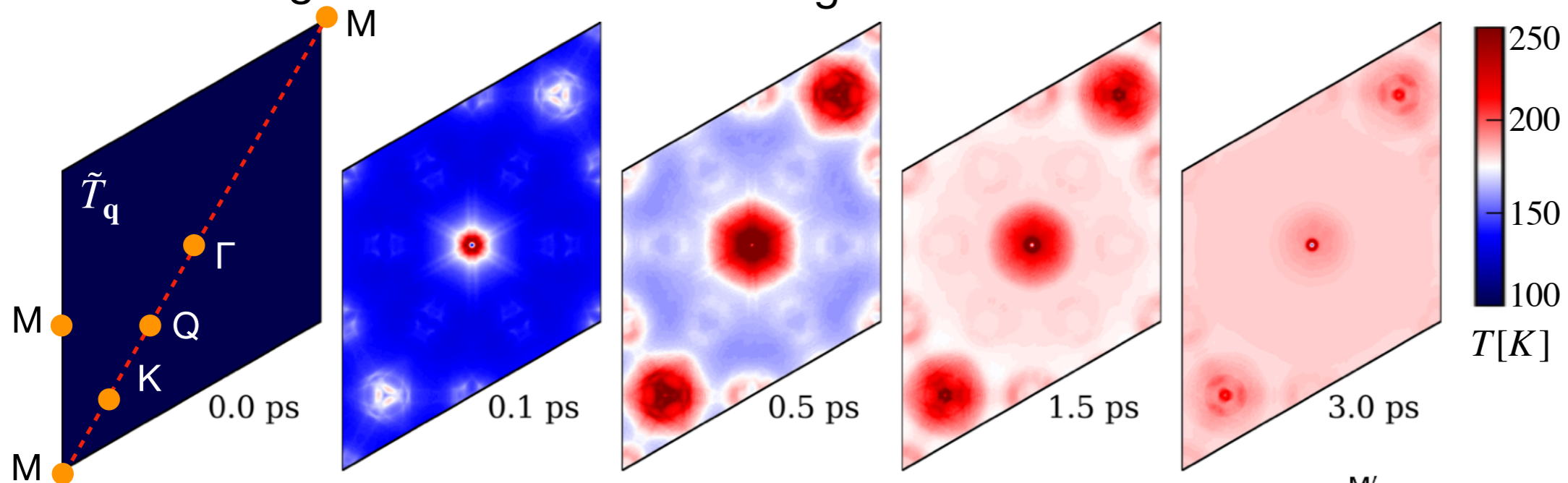
$$\frac{\partial n_{\mathbf{q}\nu}}{\partial t} = I_{\mathbf{q}\nu}^{\text{e-ph}}[f, n] + I_{\mathbf{q}\nu}^{\text{ph-ph}}[n]$$

effective vibrational temperature

$$T_{\mathbf{q}\nu} = \hbar\omega_{\mathbf{q}\nu} [k_B \ln(1 + n_{\mathbf{q}\nu})]^{-1}$$

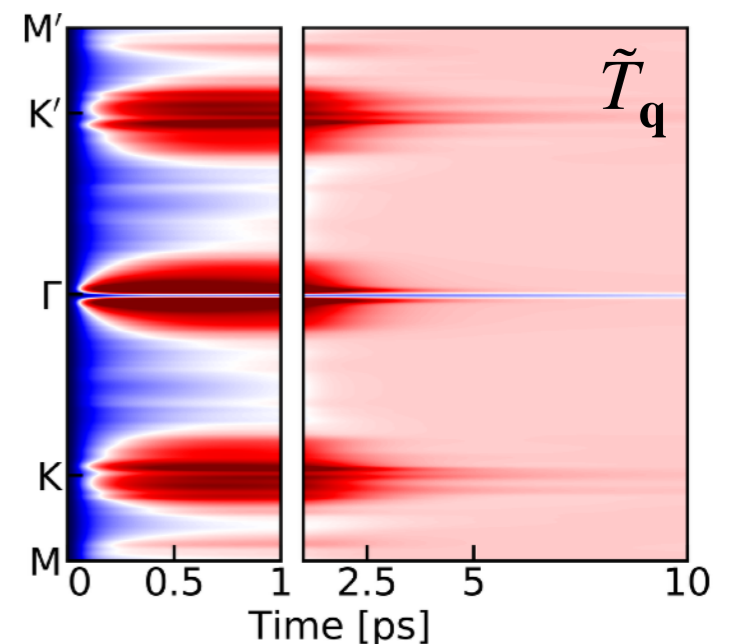
electron-phonon scattering

phonon-phonon scattering

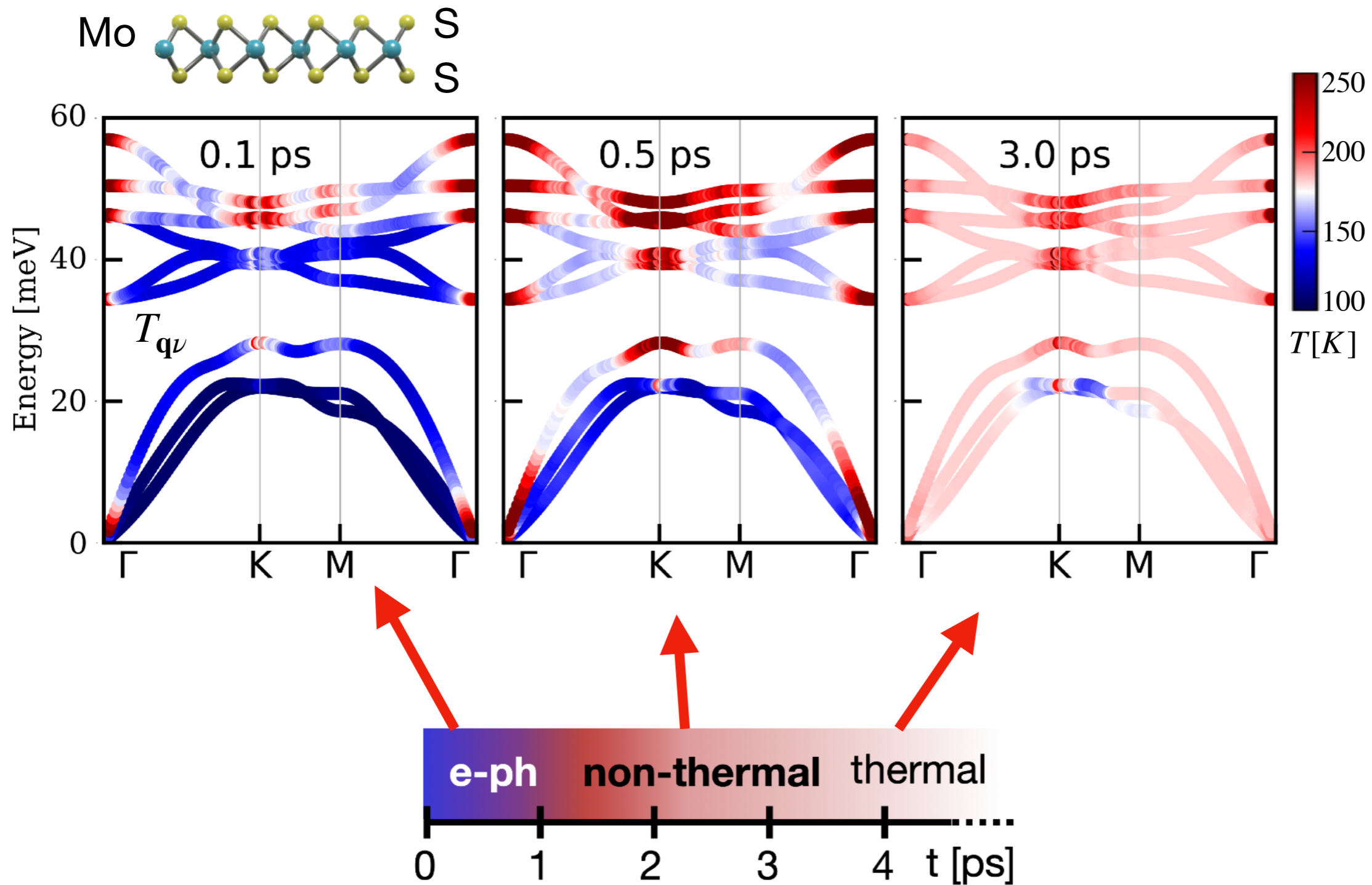


Energy cons.: $\hbar\omega_{\mathbf{q}\nu} = \varepsilon_{n\mathbf{k}} - \varepsilon_{n\mathbf{k}'}$

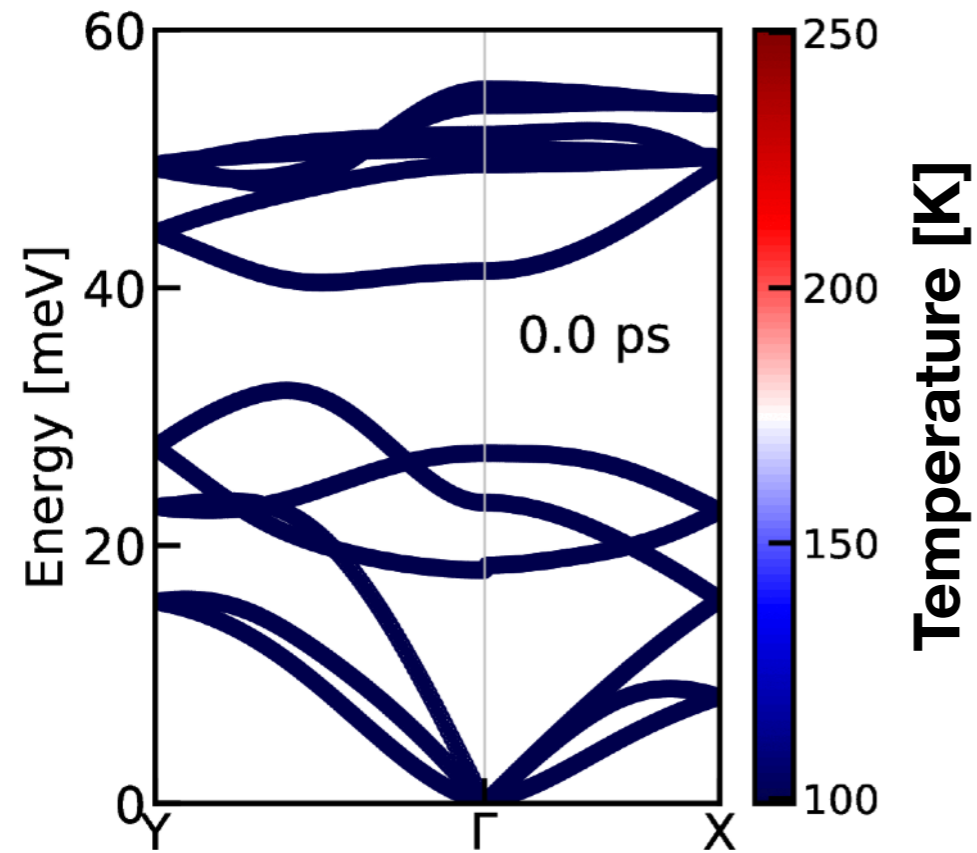
Momentum cons.: $\mathbf{q} = \mathbf{k}' - \mathbf{k}$



Regimes of nonequilibrium phonon dynamics



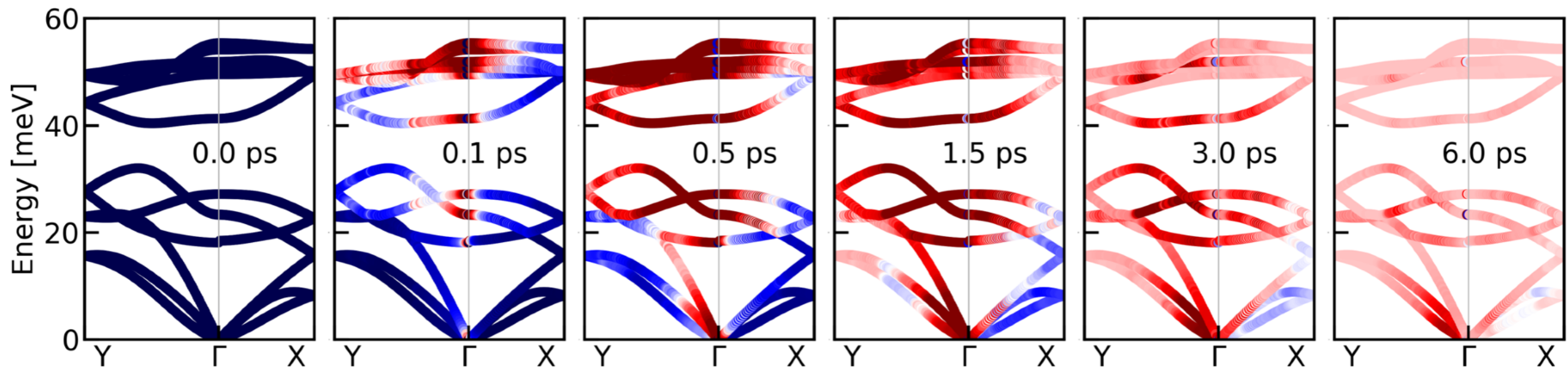
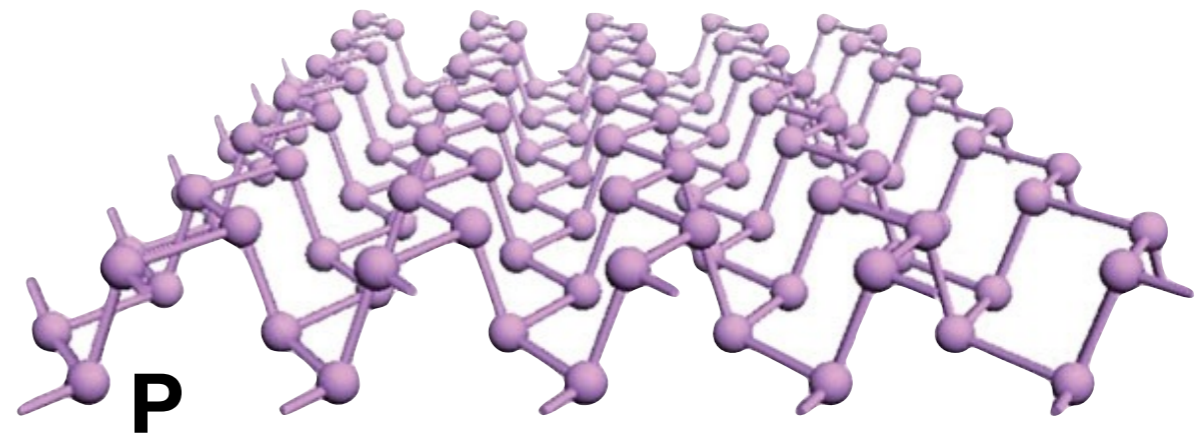
Nonequilibrium lattice dynamics: phosphorene



nonequilibrium dynamics of the lattice

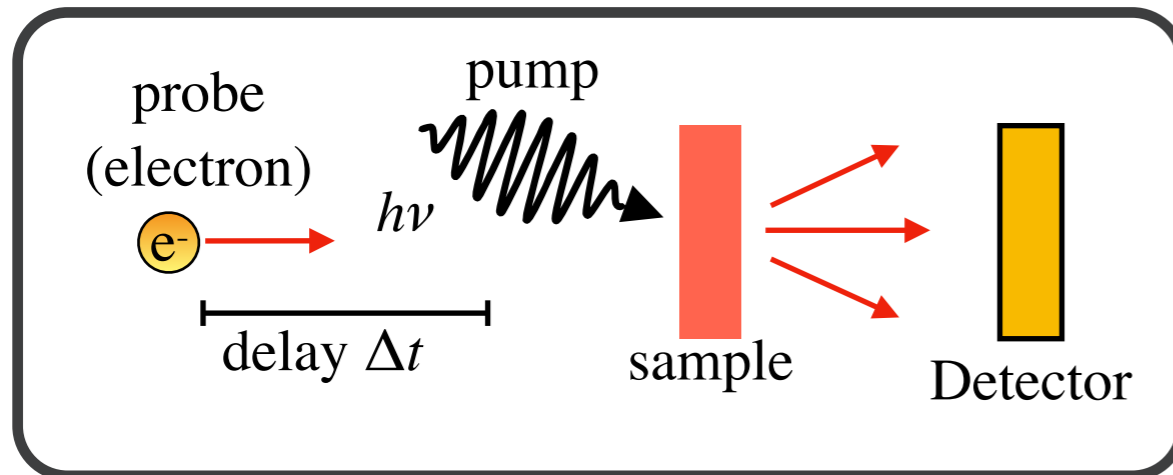


phase space for electronic transitions

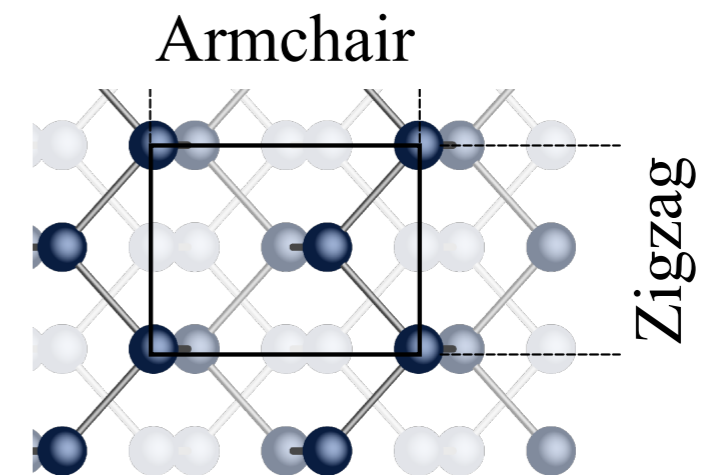


Electron diffuse scattering: black Phosphorus

Femto-second electron diffuse scattering (FEDS)



Black phosphorus



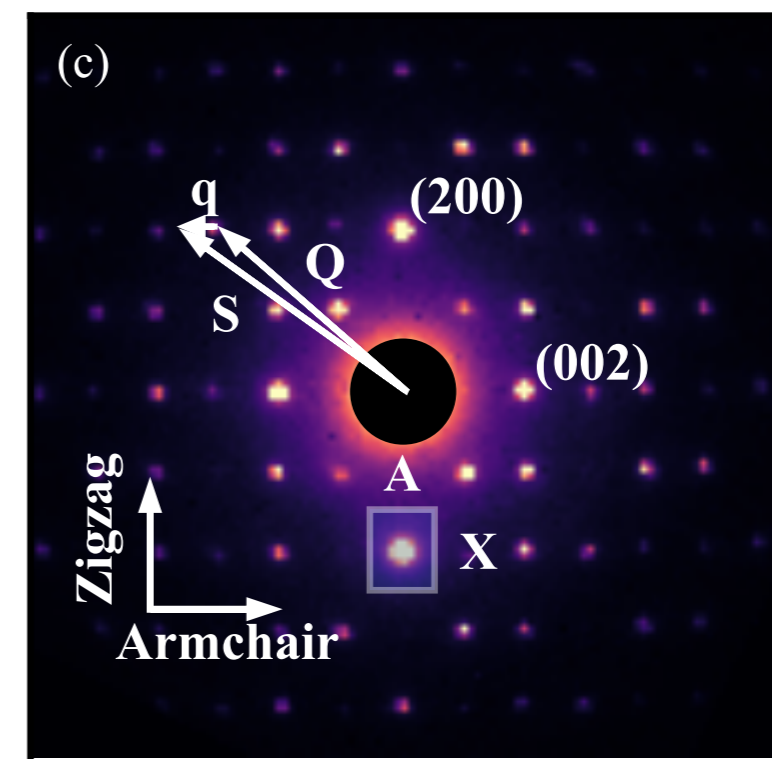
Experiments: FHI Berlin



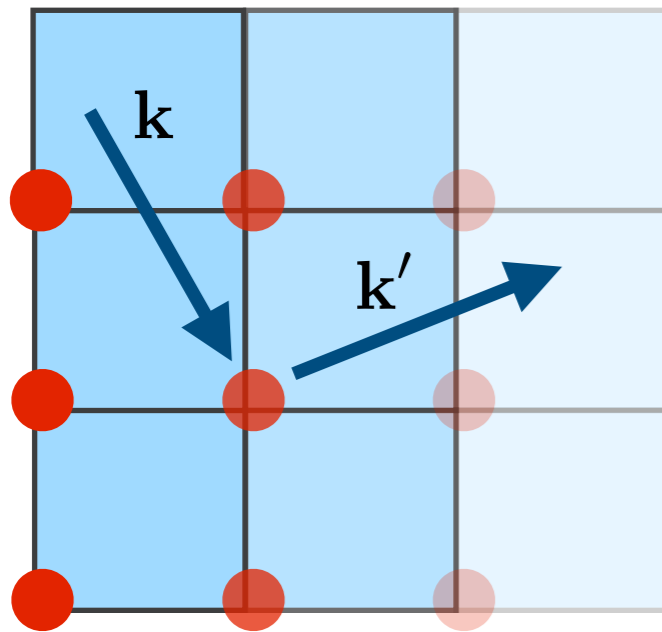
Helene Seiler



Ralph Ernstorfer



Diffraction from a lattice



Bragg's law for interference condition:

$$A(\mathbf{q}) = \sum_p^{N_p} f_0 \exp [i\mathbf{q} \cdot \mathbf{R}_p]$$

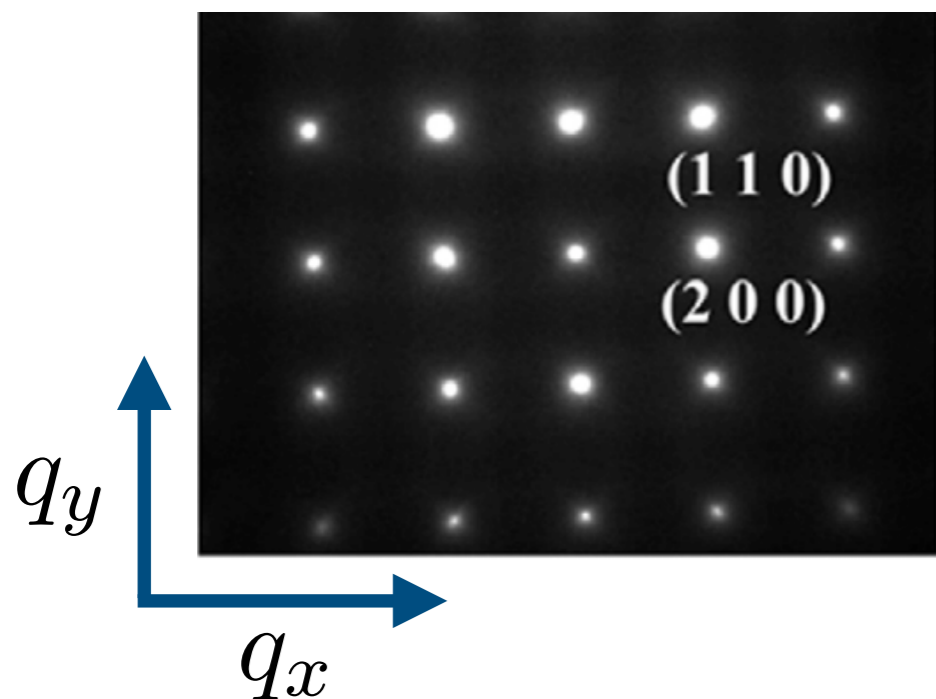
amplitude

Scattering
cross section

$\mathbf{q} = \mathbf{k}' - \mathbf{k}$
Transferred
momentum

Lattice
vector

example: Cubic lattice

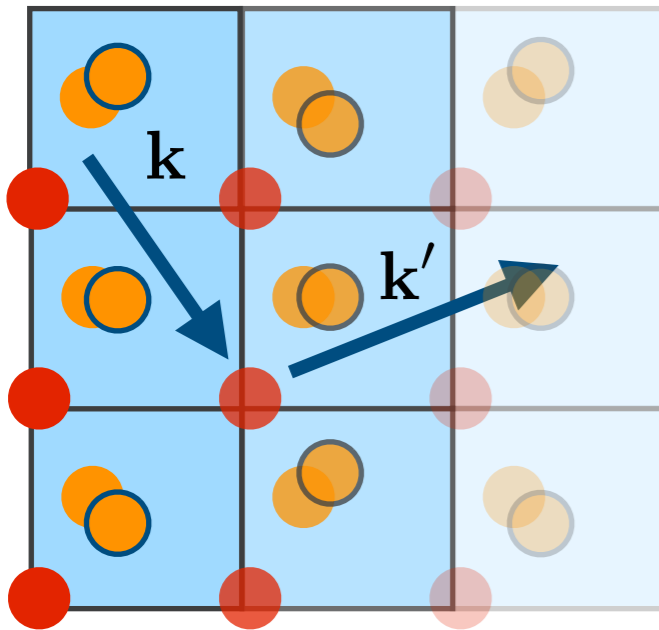


Scattering intensity for a perfect lattice:

$$I(\mathbf{q}) = |A(\mathbf{q})|^2 \simeq \delta_{\mathbf{q},\mathbf{G}}$$

Reciprocal
lattice vector

Diffraction from a vibrating lattice



Zero-phonon term:

$$\langle I_0(\mathbf{S}) \rangle_T = N_p^2 |f_0|^2 \exp(-2W_T) \delta_{\mathbf{S}, \mathbf{G}}$$

Debye-Waller factor:

$$2W_T = \frac{1}{M_\kappa N_p} \sum_{\mathbf{q}\nu} \left[\sum_{\alpha\alpha'} S_\alpha S_{\alpha'} e_{\kappa\alpha}^\nu(\mathbf{q}) e_{\kappa\alpha'}^{\nu*}(\mathbf{q}) \right] E_{\mathbf{q}\nu, T}$$

One-phonon (dynamical) structure factor

$$\begin{aligned} \langle I_1(\mathbf{S}, E) \rangle_T &= |f_0|^2 \exp(-2W_T) \frac{\hbar^2 N_p}{2M_\kappa} \sum_{\mathbf{q}\nu} \left[\sum_{\alpha\alpha'} S_\alpha S_{\alpha'} e_{\kappa\alpha}^\nu(\mathbf{q}) e_{\kappa\alpha'}^{\nu*}(\mathbf{q}) \right] \frac{1}{\hbar\omega_{\mathbf{q}\nu}} \quad (16) \\ &\times \left[\delta(\mathbf{S} + \mathbf{q}) n_{\mathbf{q}\nu, T} \delta(E + \hbar\omega_{\mathbf{q}\nu}) + \delta(\mathbf{S} - \mathbf{q}) (n_{\mathbf{q}\nu, T} + 1) \delta(E - \hbar\omega_{\mathbf{q}\nu}) \right]. \end{aligned}$$

from the time-dependent Boltzmann equation

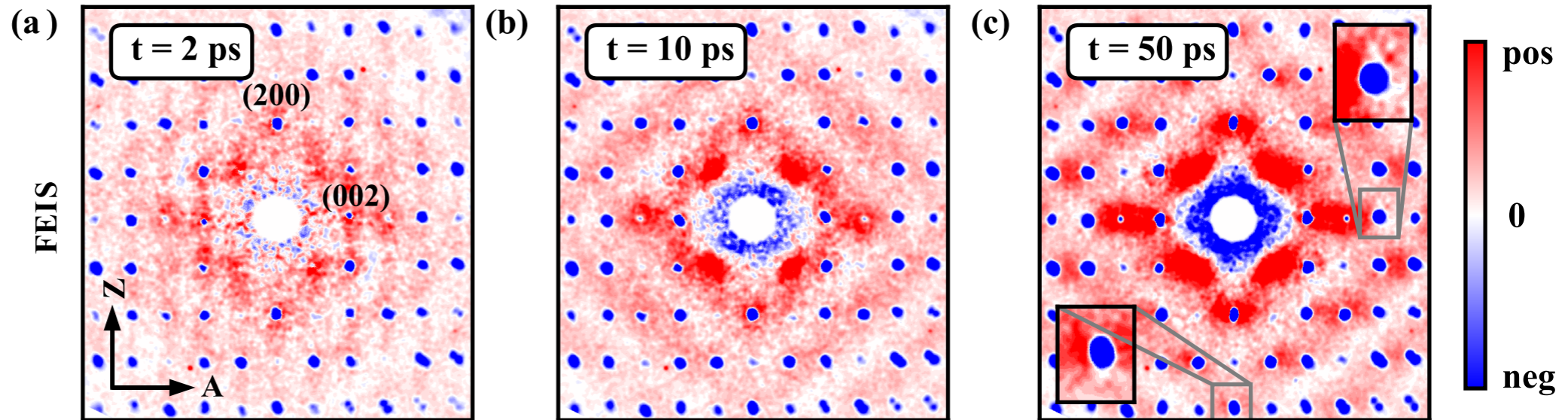
First-principles approach to Femto-second electron diffuse scattering (FEDS)

Microscopic origin of non-equilibrium lattice dynamics in bP

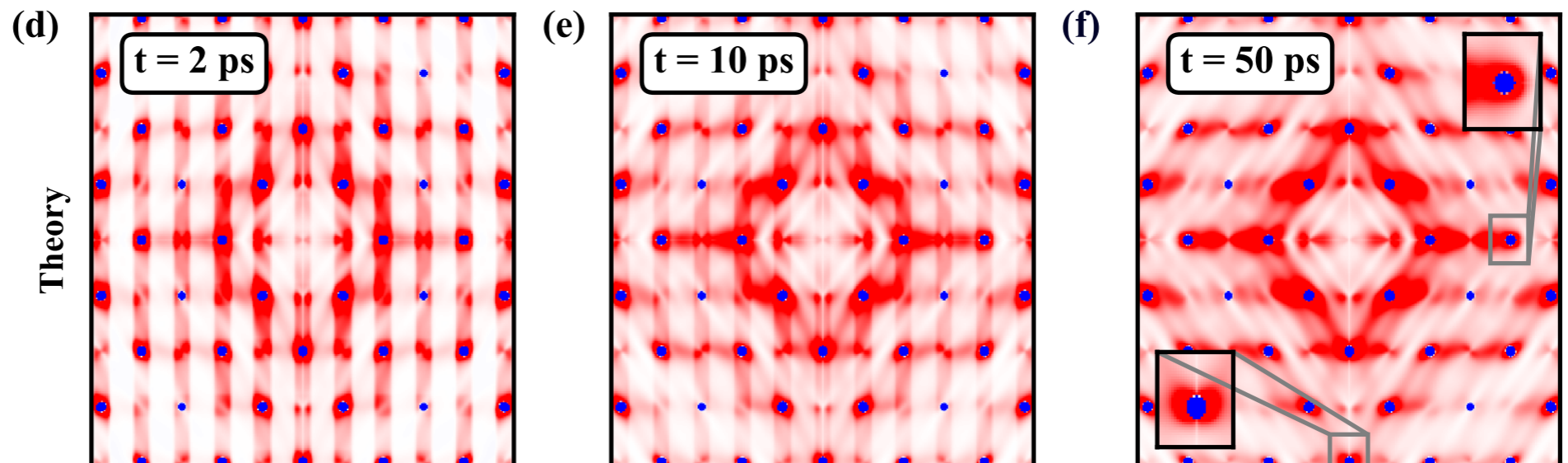
Intensity difference:

$$I(\mathbf{S}, t) - I(\mathbf{S}, t < t_0)$$

Experiment:

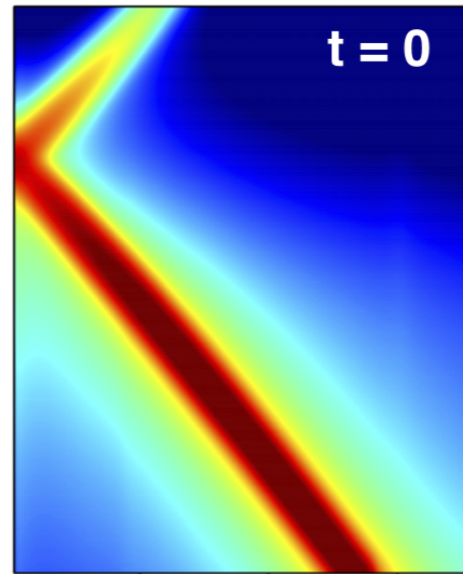


Theory:



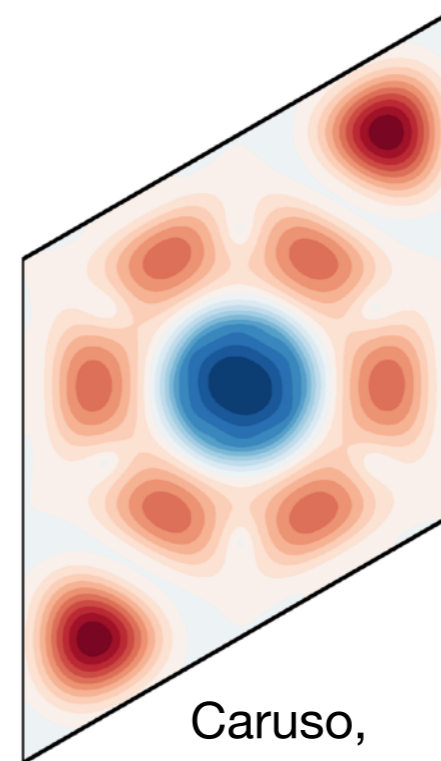
Marios Zacharias

Ultrafast dynamics of electrons and phonons



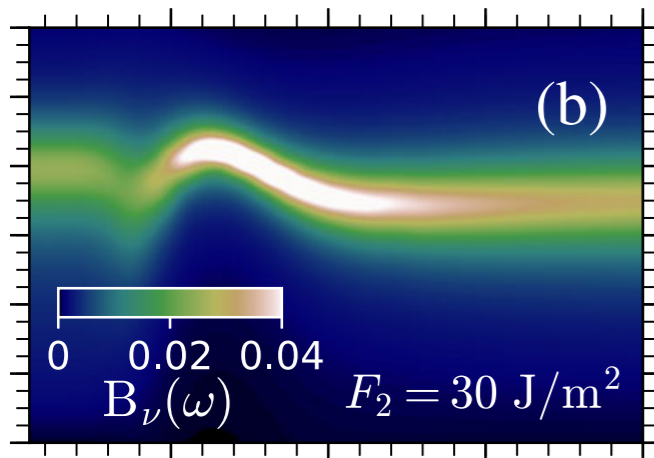
t = 0 Nonequilibrium carrier dynamics in graphene

Caruso, Novko, Draxl, Phys. Rev. B **101** 035128 (2020)



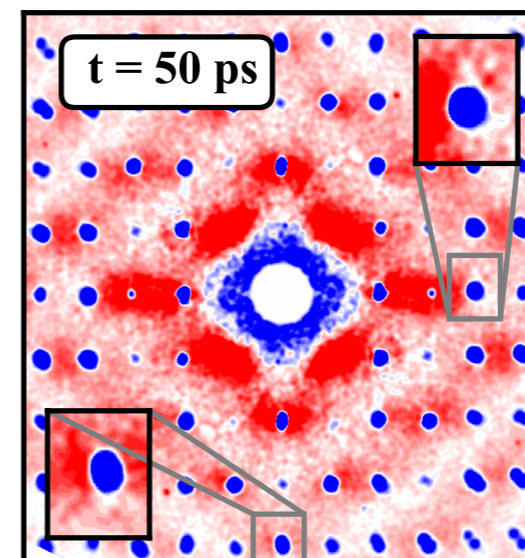
Nonequilibrium lattice dynamics in MoS₂

Caruso, J. Phys. Chem. Lett. **12**, 1274 (2021)



(b) Transient phonon renormalization in MgB₂

Novko, Caruso, Draxl, Cappelluti, Phys. Rev. Lett. **124** 077001 (2020)



Signatures of nonequilibrium lattice dynamics in phosphorene

Seiler, Zahn, Zacharias, et al. arXiv (2020)

Acknowledgements

THEORY

Humboldt-Universität zu Berlin

- Claudia Draxl
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- Marios Zacharias

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- Dino Novko

EXPERIMENTS

Fritz Haber Institute, Berlin

- Helene Seiler
- Daniela Zahn
- Ralph Ernstorfer



EPW:

S. Poncé et al.,
Comp. Phys. Comm. (2017)



Quantum Espresso:

P. Giannozzi et al., J. Phys.:
Condens. Matter 29, 465901 (2017)



Wannier90: G. Pizzi et al., J. Phys.
Cond. Matt. 32, 165902 (2020)



ShengBTE:

W. Li et al., Comp. Phys.
Comm. 185, 1747 (2014)



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