Productive demand, sectoral comovement, and total capacity utilization

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Questions

1. How important are demand shocks for explaining business cycle fluctuations, with focus on the Solow residual and sectoral comovement? (Lucas (1981),Smets and Wouters (2007), Christiano and Fitzgerald (1998))
2. What role do goods market frictions play, and what do they imply for capacity utilization?

Key contribution: use capacity utilization jointly with sectoral data to investigate these questions in a setting in which goods market frictions give rise to a productive role for demand

Motivation

Motivated by two strands of the literature

1. Gap between TFP and utilization-adjusted counterpart (Basu, Fernald, and Kimball (2006))
2. Sectoral comovement ⇒ definition of recession from NBER

   A recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy

Demand shocks and effect on measured productivity

- In a standard neoclassical model, prices adjust so that all produced output is sold
  $\Rightarrow$ output is just a function of capital and labor
- Under goods market frictions, output depends on how many customers show up
- Reverses causality between consumption and TFP relative to neoclassical model

Capacity utilization

- Total capacity utilization is the ratio of an output index to a capacity index
- Coverage
  - 89 detailed industries (71 manufacturing, 16 mining, 2 utilities)
  - Primarily correspond to industries at the 3 or 4-digit NAICS
  - Estimates are available for various groups (durables and non-durables, total manufacturing, mining, utilities, and total industry)
- Source data
  - Capacity data reported in physical units from government sources, trade sources
  - Responses to the Bureau of the Census's Quarterly Survey of Plant Capacity (QSPC)
  - Trends through peaks in production for a few mining and petroleum series

Motivation: Utilization measures and output

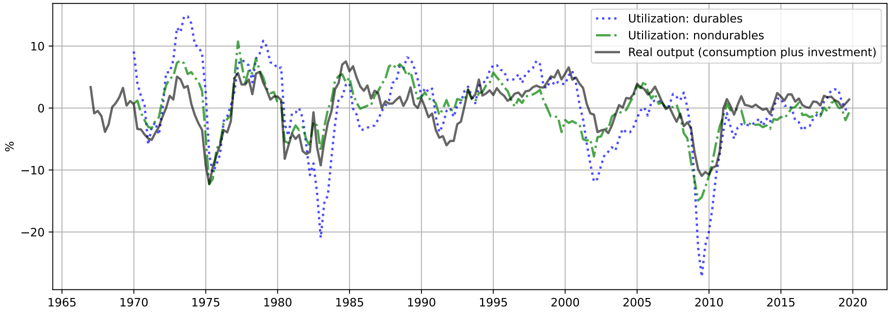

Figure 1: Total capacity utilization in non-durable and durable goods and output, here defined as consumption plus investment. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past $(p = 4, h = 8)$.

- Utilization measures comove positively and are procyclical
- Utilization in durables is significantly more volatile than non-durables

Details on total capacity utilization | Second moments

Motivation: sectoral comovement (hours)

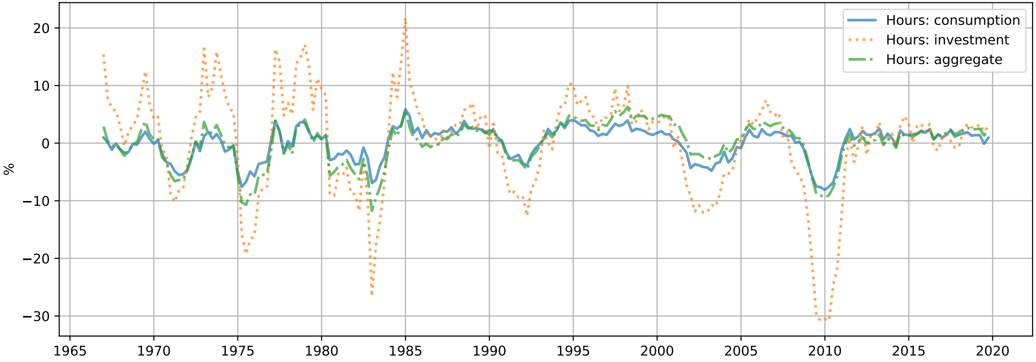

Figure 2: Sectoral and aggregate hours. Hours in consumption is the sum of labor hours in non-durables and services, hours in investment is the sum of labor hours in durables and construction. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past $(p = 4, h = 8)$.

Related literature

1. Purifying Solow residual:
   Basu, Fernald, and Kimball (2006), Fernald (2014)
2. Goods market frictions and firm productivity
   Moen (1997), Michaillat and Saez (2015), Bai, Rios-Rull, and Storesletten (2024), Huo and Ríos-Rull (2018), Qiu and Ríos-Rull (2022), Petrosky-Nadeau and Wasmer (2015), Bethune, Rocheteau, and Rupert (2015)
3. Sectoral comovement and imperfect intersectoral factor mobility
   Long and Plosser (1983), Christiano and Fitzgerald (1998), Horvath (2000), Katayama and Kim (2018)
4. Total capacity utilization
   Christiano, Eichenbaum, and Trabandt (2016), Qiu and Ríos-Rull (2022)
5. News shocks
   Jaimovich and Rebelo (2009), Schmitt-Grohé and Uribe (2012), Katayama and Kim (2018)

Production technology

- 2 consumption sectors (goods mc and services sc) and an investment sector
- Each uses capital k and labor n to produce output
- Stochastic trend to technology X
- Potential output given capital utilization rate h and fixed cost $\nu_j X$.

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\}$$

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k}, \alpha_k + \alpha_n \leq 1$$

- Fixed costs implies that labor productivity rises with sales

Matching technology

- **Competitive search**: households shop in markets indexed by price, market tightness, and quantity
- Each market is subject to Cobb-Douglas matching function

$$M_j(D,T) = A_j D^{\phi} T^{1-\phi}$$

  where D is aggregate shopping effort and T is the measure of firms (normalize $T = 1$)
- Implied matching rates:

$$\Psi_{jd}(D) = M/D = A_j D^{\phi-1}, \quad \Psi_{jT}(D) = M/T = A_j D^{\phi}$$

  so that D describes market tightness
- Once a match is formed, goods are traded at the price $p_j, j \in \{mc, sc, i\}$
- The real quantity of goods purchased given search effort d_j in sector j

$$y_j = d_j \Psi_{jd}(D) F_j \quad j \in \{mc, sc, i\}$$

Preferences

- Households have preferences over search effort, consumption, and a labor composite following BRS

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta} S$$

and

$$S = \left(c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} \right)^{\gamma} S_{-1}^{1-\gamma}$$

captures role of short-run wealth effects via γ

- Aggregate consumption C and total search effort $d = d_{mc} + d_{sc} + \theta_i d_i$
- Preference shifters $\theta = \{\theta_b, \theta_d, \theta_i, \theta_n\}$

Parameterizing wealth effects on labor supply

Consumption aggregator

- Consumption is bundle of goods y_{mc} and services y_{sc}

$$c = [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + \omega_{sc}^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \quad (1)$$

  such that $\omega_{mc} + \omega_{sc} = 1$
- Elasticity of substitution $\xi = 1/(1-\rho_c)$
- Price index

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}}$$

- Normalize $p_c = 1$

Imperfect labor mobility across sectors

- Assume imperfect substitutability between labor used in consumption and investment sectors (Horvath (2000) and Katayama and Kim (2018))

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{2}$$

- Elasticity of substitution $1/\theta$ measures intersectoral labor mobility
- Induces wage dispersion
- As $\theta \to 0$, $n^a \to n_c + n_i = n$ (perfect mobility benchmark)
- For θ fixed, if $\omega = n_c/n$, then $n^a = n_c + n_i = n$

Differentiated labor and labor unions

- Continuum of monopolistically competitive labor unions in sector j provide services to firms
- Total labor is a CES aggregate of specialized types

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

- Pay workers W^* per unit and rent to firms at rate $W(s)$
- Rebate earnings to workers

Investment

- Households shop for investment goods, accumulate and install capital in each sector, and collect rental income

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}, S_{mc} = S_{sc}, \delta_{mc} = \delta_{sc}$$

  where $i = i_{mc} + i_{sc} + i_i$

- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta_j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2$$

  $\Rightarrow \sigma_{aj} = \delta''_j(1)/\delta'_j(1)$ is the elasticity of marginal utilization cost wrt h at $h = 1$

- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S_j(x) = \frac{\Psi_j}{2}(x-1)^2$$

Role of different ingredients

1. Variable capital utilization/endogenous depreciation ⇒ standard component of utilization; amplification and propagation of technology shocks
2. Limited factor mobility ⇒ sectoral comovement and autocorrelation of labor hours (Horvath (2000), Katayama and Kim (2018))
3. Parametric short-run wealth effects ⇒ sectoral comovement and contribution of news shocks to technology (Jaimovich and Rebelo (2009))
4. External habit formation ⇒ smooth consumption response without implying very high risk aversion
5. Investment adjustment costs ⇒ hump-shaped impulse responses of investment/sectoral comovement (Christiano, Eichenbaum, and Evans (2005))
6. Fixed costs ⇒ Procyclical measured productivity (Christiano, Eichenbaum, and Trabandt (2016))
7. Differentiated labor/labor unions ⇒ wage markups and shocks (Schmitt-Grohé and Uribe (2012))

Timing

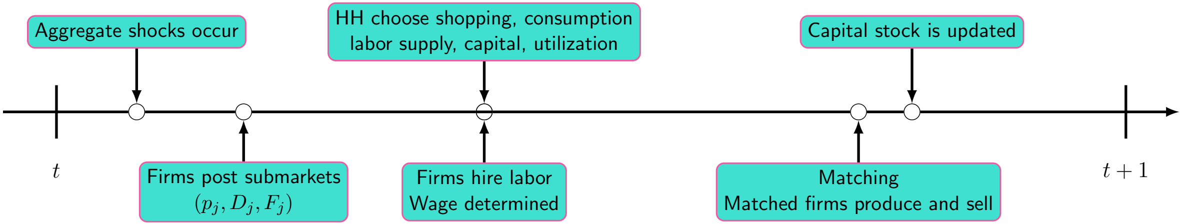

Figure 3: Timing

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates given markets $(p_j, D_j, F_j), j \in \{mc, sc, i\}$ and the aggregate state of the economy Λ

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_{mc}, k'_{sc}, k'_i)|\Lambda\}$$

s.t.

$$y_j = d_j \Psi_{jd}(D_j) F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_{j\in\{mc,sc,i\}} y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

subject to endogenous depreciation δ_j, investment adjustment cost S_j, and consumption and labor aggregators (1) and (2)

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$$

Details of household problem

Demand curve for non-durables and services and shopping wedge

- Combine FOC of non-durables mc and services sc and aggregate

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{mc, sc\}$$

- The elasticity ϕ represents a shopping wedge

$$\frac{u_j}{\lambda p_j} = \frac{1}{1-\phi} \Rightarrow \frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}}, \quad \phi = (u_j - \lambda p_j)/u_j$$

- Demand curve and shopping wedge yield marginal utility of wealth $\lambda = \Gamma^{-\sigma}(1-\phi)$

Optimal shopping effort and demand

- HH equate marginal disutility of shopping effort to marginal utility of consumption in each sector

$$-\frac{u_d}{u_j} = \overbrace{\phi A_j D_j^{\phi-1}}^{\Psi'_{jT}(D)} F_j \quad j \in \{mc, sc\} \tag{3}$$

- Two interpretations of (3)
  1. MRS between consumption and shopping effort $(-u_d/u_j)$ equals MRT (increase firm matching probability $\Psi'_{jT}(D)\times$ output sold)
  2. MRS equals HH matching probability multiplied by quantity of output sold and the shopping wedge
- Express value of investment shopping by converting into consumption units using relative price

$$-\frac{u_d}{u_{mc}}\theta_i = \frac{p_i}{p_{mc}}\phi A_i D_i^{\phi-1} F_i$$

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Firms' problem

- A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, F_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$
$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$
$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p_j, D_j, F_j) \geq V(\Lambda, k_{mc}, k_{sc}, k_i)$$
$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Details of firm problem

Firm factor demands

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\}$$

- Input demand depends positively on shopping effort
- Matching function elasticity ϕ appears as separate factor
- Additional output relaxes participation constraint of households and effectively reduces input cost
- Wage paid by firm is a markup of (variable) wage received by workers

$$W_j = \mu_j W_j^*$$

with difference $W_j - W_j^*$ rebated to HH as fixed wage

Labor share of income

- Labor share of income is key component to constructing Solow residual
- Define fixed cost share $\nu_j^R = \nu_j X/(z_j f - \nu_j X)$
- Write sectoral labor share of income as

$$\frac{W_j n_j}{p_j Y_j} = \frac{\alpha_n (1 + \nu_j^R)}{1 - \phi}$$

- Provided $\nu_j^R = \nu^R$ for all j, overall labor share of income is

$$\frac{W n}{Y} = \frac{\alpha_n (1 + \nu^R)}{1 - \phi}$$

A simple static model

- Consider simple static model with no investment; homogeneous labor as only input, $f = zn^{\alpha_n}$; and GHH preferences between c, d, n

Shopping	$\theta_d D^{1/\eta} = \phi A D^{\phi-1} z n^{\alpha_n}$
Consumption	$C = A D^{\phi} z n^{\alpha_n}$
Labor demand	$(1-\phi)W = \dfrac{\alpha_n C}{n}$
Labor supply	$\theta_n n^{1/\nu} = (1-\phi)W$

- Labor share $\tau \equiv Wn/C = \alpha_n/(1-\phi)$ used for computing the Solow residual

$$SR \equiv \frac{C}{n^{\tau}} = A D^{\phi} z n^{\alpha_n - \tau}$$

Equilibrium in static setting

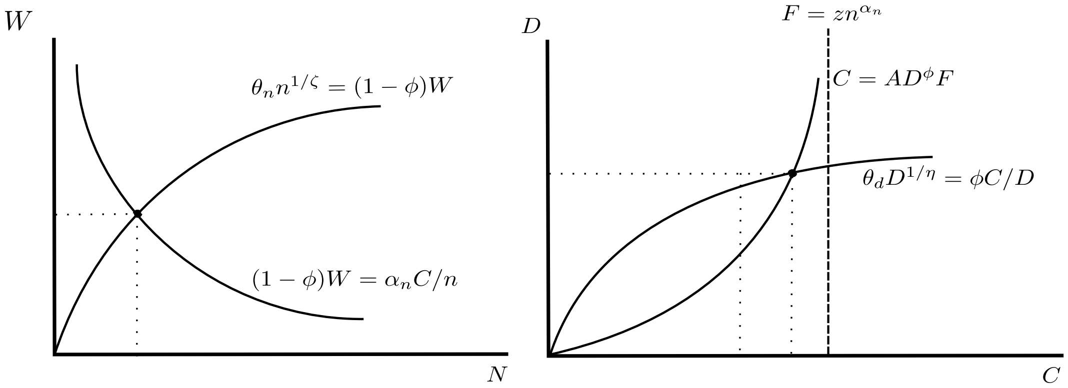

Figure 4: Equilibrium of static model

Demand shock: reduction in θ_d

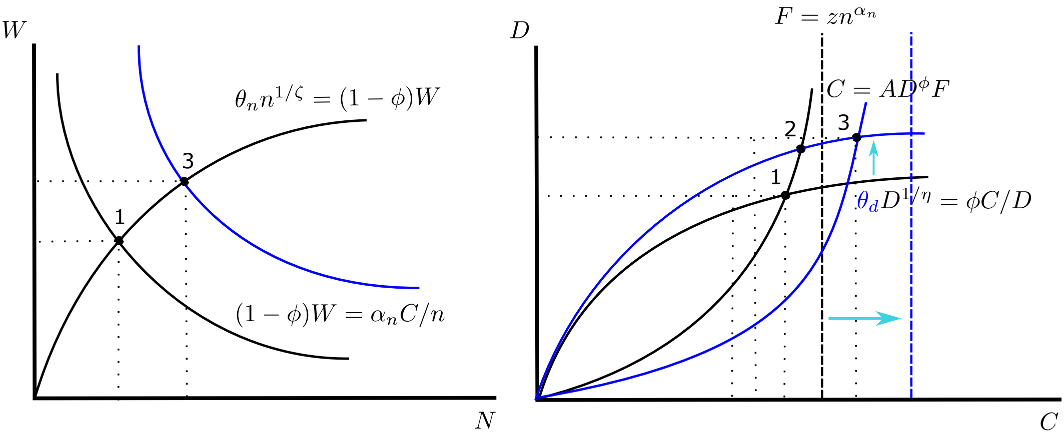

Figure 5: Reduction of shopping disutility in static model

Sectoral Solow residual

- Write sectoral Solow residual as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi} (z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k - 1 + \tau} n_{jt}^{\alpha_n - \tau})}{1 + \nu_{jt}^R}$$

  given

  1. steady-state labor income share τ

- Rewrite using growth rates $dx_t = \Delta \log x_t$

$$dSR_{jt} = \overbrace{\phi dD_{jt}}^{\text{Shopping}} + \underbrace{\alpha_k dh_{jt}}_{\text{Capital utilization}} + \overbrace{dz_{jt} + (1-\alpha_k) dX_t}^{\text{Technology}}$$

$$+ \underbrace{(\alpha_k - 1 + \tau) dk_{jt} + (\alpha_n - \tau) dn_{jt}}_{\text{Input share mismeasurement}} + \overbrace{d(1 + \nu_{jt}^R)}^{\text{Fixed costs}}$$

Capacity utilization and connection to Solow residual

- Define capacity in sector j following Qiu and Ríos-Rull (2022)

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

- Capacity utilization in sector j is the ratio of output to capacity (stationary measure):

$$util_j \equiv \frac{Y_j}{cap_j} = \frac{A_j D_j^{\phi}(z_j h_j^{\alpha_k} X^{1-\alpha_k} k_j^{\alpha_k} n_j^{\alpha_n} - \nu_j X)}{z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X}$$

- Capacity utilization in growth rates

$$dutil_{jt} = \phi dD_{jt} + (1 + \nu_{ss}^R)\alpha_k dh_{jt}$$

- If $\nu_j = 0$, then Solow residual growth rate simplifies to

$$dSR_{jt}|_{\nu_j=0} = \overbrace{dutil_{jt}}^{\text{Utilization}} + \underbrace{dz_{jt} + (1-\alpha_k)dX_t}_{\text{Technology}} + \overbrace{(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}}^{\text{Input share mismeasurement}}$$

Aggregate measures

- Output

$$Y = C + p_i^{ss} I$$

- Using base-year prices makes results independent of numeraire choice Explanation
- Solow residual and capacity utilization

$$SR = \sum_j \frac{Y_j}{Y} SR_j, \quad util = \sum_j \frac{Y_j}{Y} util_j$$

BRS as special case

- Model nests Bai, Rios-Rull, and Storesletten (2024) (BRS) by shutting down additional frictions: Equilibrium
  - $\gamma = 0$
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_j = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- In contrast to BRS, estimate ϕ and η instead of calibrating using shopping time or price dispersion targets
- Also add stationary technology shock; otherwise use same prior distributions

Table 10: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 1: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{z, z_I, \theta_n, \theta_d\}$.

Role of capacity utilization on parameter estimates

Table 11: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
e_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 2: Estimation of baseline BRS model with to sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 12: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Std. dev.		
D	1.54	1.69
$util$	0.15	1.49
FEVD of demand shocks θ_d		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 3: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The BRS dataset includes growth rates of output, investment, labor productivity, and the relative price of investment. The second column adds variable total capacity utilization. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_D. See Table 11.

Highlights of adding capacity utilization

- Shopping-related parameters are more precisely estimated, and demand channel is stronger
- Capacity utilization volatility rises by 10 times, much closer to empirical value
- Forecast error variance contribution of θ_d rises dramatically
- Why not just use shopping time data?
  - Shocks to goods market frictions can also rise from fluctuations in matching efficiency, which cannot be separately identified
  - Shopping time can be contaminated with leisure

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Bayesian estimation

- Time period: $1964Q1 - 2019Q4$, quarterly frequency
- Use seven observables in growth rates:

$$(C, I, n_c, n_i, util_{ND}, util_D, p_i)$$

- Use sectoral data on output and labor following Katayama and Kim (2018)
- Construct output from sum of private consumption and private investment (as BRS)
- Note that sectoral dataset implicitly targets labor productivity in each sector

Estimation procedure

Calibration

Targets	Value	Parameter	Calibrated value/posterior mode
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average per capita growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.6
Labor supply	–	ζ	1.97
Elasticity of matching function	–	ϕ	0.84
Elasticity of shopping effort cost	–	η	0.65
Fixed cost share of capacity	–	ν_R	0.42
Habit persistence	–	ha	0.40
Third group: normalizations			
SS output	1	z_{mc}	0.45
Relative price of services	1	z_{sc}	0.69
Relative price of investment	1	z_i	0.36
Fraction time spent working	0.30	θ_n	3.85
Capacity utilization of nondurables	0.81	A_{mc}	2.51
Capacity utilization of services	0.81	A_{sc}	1.49
Capacity utilization of investment sector	0.81	A_i	3.33
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	0.014
Physical capital to output ratio	2.75	α_k	0.242
Labor share of income	0.67	α_n	0.074

Calibration details

Stochastic processes

- The growth rate of the stochastic trend $g_t = X_t / X_{t-1}$ follows an AR(1) process in logs as BRS

$$\log g_t = (1-\rho_g)\log \bar{g} + \rho_g \log g_{t-1} + e^0_{g,t} + \overbrace{e^4_{g,t-4}}^{\text{anticipated shock}}$$

  where $e^0_{g,t} \sim N(0, \sigma^0_g)$ and $e^4_{g,t} \sim N(0, \sigma^4_g)$.
- Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e^0_{v,t} + e^4_{v,t-4}$$

  where $e^0_{v,t} \sim N(0, \sigma^0_v)$ and $e^4_{v,t} \sim N(0, \sigma^4_v)$.
- Set $z_i = z_c z_I$, where z_I is independent of z_c
- Also impose $e^4_{\theta_n, t-4} = 0$ for all t
- Stationarize trending variable by dividing by X_t (X_{t-1} in case of predetermined capital stock K_{jt})

Posterior estimates: structural parameters

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	beta	1.50	0.25	1.81	0.18	1.58	2.09
ha	beta	0.50	0.20	0.42	0.05	0.35	0.50
ζ	gamm	0.72	0.25	1.85	0.13	1.64	2.00
γ	beta	0.50	0.20	0.32	0.04	0.25	0.38
ϕ	beta	0.32	0.20	0.86	0.04	0.79	0.93
η	gamm	0.20	0.15	0.56	0.12	0.38	0.73
ξ	gamm	0.85	0.10	0.92	0.06	0.82	1.02
ν_R	beta	0.20	0.10	0.33	0.09	0.17	0.44
σ_{ac}	invg	1.00	1.00	1.37	0.34	0.71	1.88
σ_{ai}	invg	1.00	1.00	0.54	0.15	0.33	0.73
Ψ_c	gamm	4.00	1.00	4.82	0.35	4.26	5.40
Ψ_i	gamm	4.00	1.00	4.18	0.74	3.12	5.31
θ	gamm	1.00	0.50	1.55	0.50	0.93	2.32

Parameters well identified in exercise using artificial data generated from model evaluated at posterior mean

Unconditional forecast error variance decomposition: grouped shocks

Table 5: Forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	35.1	0.01	64.1	0.73	0.08
SR	41.3	0.73	52.9	3.12	2.00
I	38.1	0.01	54.9	6.90	0.03
p_i	54.5	0.00	45.2	0.12	0.14
n_c	14.5	14.3	31.2	23.6	16.5
n_i	18.6	1.28	26.6	13.4	40.1
$util$	13.0	0.01	86.1	0.84	0.03
D	2.36	0.00	97.6	0.06	0.00
h	31.2	0.01	68.0	0.78	0.02

Table 5: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Breakdown of demand and technology shocks

Model comparison

Table 6: Comparison of model specification

			Remove			
	Data	Baseline	Fixed cost	VCU	SDS	SDS and utilization data
Log marginal likelihood (LML)	–	4531.0	4516.9	4470.9	4202.2	–
Δ LML	–	0	-14.1	-60.1	-328.8	–
90% HPDI band ϕ	–	(0.8, 0.94)	(0.84, 0.96)	(0.2467, 0.3452)	(0.69, 0.72)	(0.56, 0.70)
FEVD(Y, SDS)	–	64.1	58.7	54.01	–	–
FEVD(SR, SDS)	–	52.9	36.3	54.2	–	–
Var(util)/Var(SR)	–	0.87	0.65	0.77	1.49	0.11
std(Y)	0.87	1.62	1.63	2.00	60.5	0.6
std($util_{ND}$)	1.26	1.15	1.1	1.27	47.9	0.27
std($util_D$)	2.27	2.98	3.25	2.44	85.6	1.18
std(n_c)	0.57	0.53	0.63	0.53	17.3	0.48
std(n_i)	1.94	1.83	1.92	1.76	39.6	1.66
Cor(C, I)	0.54	0.63	0.55	0.58	0.99	0.26
Cor($util_{ND}, util_D$)	0.75	0.57	0.53	0.62	1.00	-0.71
Cor(n_c, n_i)	0.87	0.77	0.81	0.84	1.00	0.82
Cor($util_{ND}, util_{ND,-1}$)	0.51	0.36	0.40	-0.040	0.999	0.17
Cor($util_D, util_{D,-1}$)	0.55	0.55	0.69	0.043	0.999	0.42

Impulse responses under baseline: negative 1 sd shock e_D (shopping disutility shock)

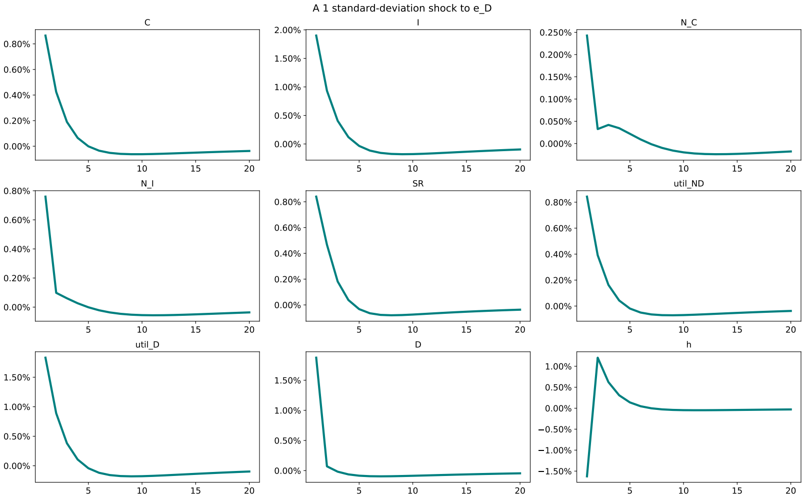

Figure 6: The vertical axis measures response in growth rates.

Impulse responses under baseline: positive 1 sd shock e_{zc} (neutral technology shock)

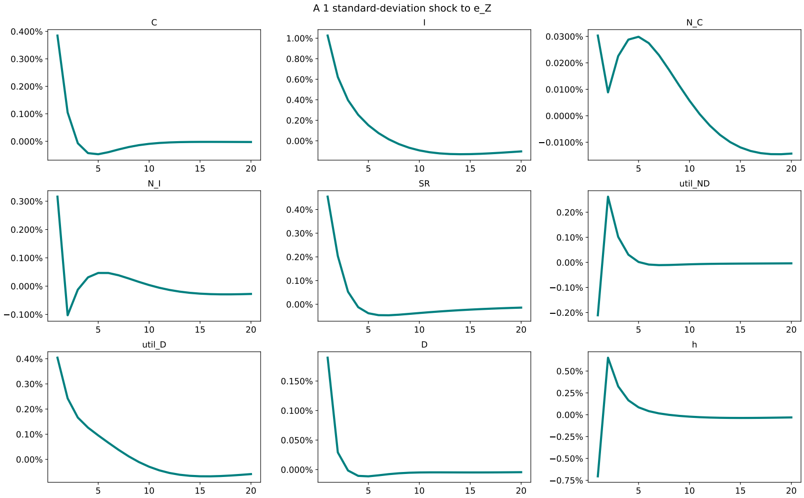

Figure 7: The vertical axis measures response in growth rates.

Impulse responses under baseline: positive 1 sd shock e_b (discount-factor shock)

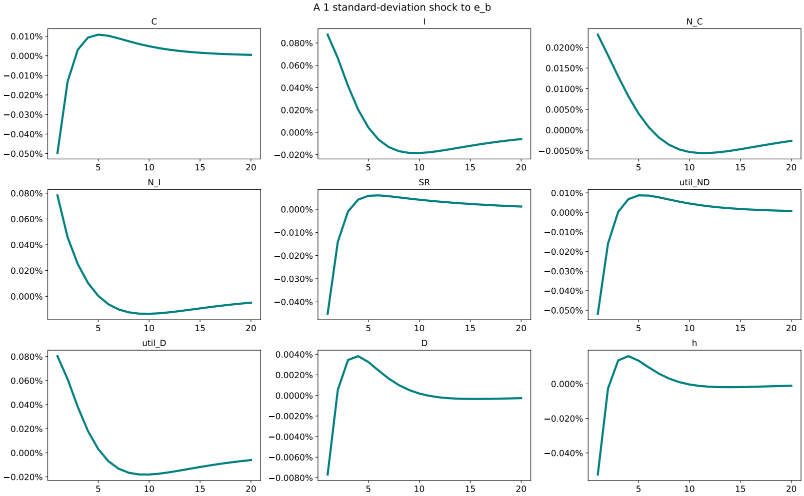

Figure 8: The vertical axis measures response in growth rates.

Conclusion

- Estimate precise, high value of key parameter ϕ and shopping-effort shocks without relying on shopping time data
- Shocks to shopping effort and its news component explain a major part of the forecast error variance of standard variables and utilization
- Explains sectoral comovement and utilization volatility well
- Removing fixed costs and variable capital utilization reduces model fit but does not change main findings
- Model is incapable of fitting data without search demand shocks
  1. Search effort (e_d) shocks are unique in generating positive comovement between sectoral output, input, and utilization
  2. Both technology shocks (e_{zc} and e_g) induce negatively correlated movements in utilization growth$\Rightarrow$ utilization of nondurables falls

Second moments (growth rates)

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_I)	Cor(x,x$_{-1}$)
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
n_c	0.57	0.66	0.66	0.87	0.67
n_i	1.94	2.23	0.73	1.00	0.64
Y/N	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_d$	2.27	2.61	0.69	0.84	0.55
$util_{nd}$	1.26	1.45	0.61	0.65	0.51

Table 7: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment.

Back to utilization comovement

Data series

ID	Description	Source
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPIC1	Real gross private domestic investment	BEA
COMPRNFB	Wages (real compensation per hour)	BLS
CNP160V	Civilian non-institutional population	BLS
GDPDEF	GDP Deflator	BEA
SR	Solow residual	Fernald (2014), FRB of San Francisco
Util	Total capacity utilization	Federal Reserve Board of Governors
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Back to second moments

Construction of variables

Symbol	Description	Construction
C	Nominal consumption	$PCEND + PCESV$
I	Nominal gross private domestic investment	GPDI
Deflator	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
c	Real per capita consumption	$\frac{C}{Pop*P_c}$
i	Investment	$\frac{I}{Pop*P_i}$
y	Real per capita output	$c + i$
N_c	Labor in consumption sector	Labor in nondurables and services, BLS
N_i	Labor in investment sector	Labor in construction and durables, BLS
N	Aggregate labor	$N_c + N_i$
P_i	Price index: investment goods	$A006RD3Q086SBEA$
P_c	Price index: consumption goods	$DPCERD3Q086SBEA$
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: non-durables	Federal Reserve Board
$util_D$	Total capacity utilization: durables	Federal Reserve Board
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

More details on construction of sectoral data

- Closely follows Katayama and Kim (2018)
- Construct consumption and investment as follows

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

- Use HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates
- P_c: combine price indices of nondurable goods (DNDGRG3Q086SBEA) and services (DSERRG3Q086SBEA)
- P_i: use quality-adjusted investment deflator (INVDEV)

More details on construction of sectoral data

- BLS Current Employment Statistics (https://www.bls.gov/ces/data)
- BLS Table B6 contains the number of production and non-supervisory employees by industry
- BLS Table B7 contains average weekly hours of each sector
- We compute total hours for non-durables, services, construction, and durables by multiplying the relevant components of each table
- Construct labor in consumption as sum of non-durables and services
- Construct labor in investment as sum of construction and durables

Parameterizing wealth effects on labor supply

- Parameter γ regulates strength of wealth effects while preserving balanced growth in labor supply
  - $\gamma \to 0$: GHH, Greenwood, Hercowitz, and Huffman (1988) (BRS with $ha = 0$)

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta}$$

  - $\gamma \to 1$: KPR, King, Plosser, and Rebelo (1988)

$$\Gamma = \left(c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta}\right)(1 - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta})$$

- Standard additively separable preferences arise with $\gamma = \sigma = 1$
- Parameter ζ is Frisch elasticity in special case $\gamma = ha = 0$

Back to household

Investment

- Households shop for investment goods, accumulate and install capital in each sector, and collect rental income

$$k_j' = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}, S_{mc} = S_{sc}$$

  where $i = i_{mc} + i_{sc} + i_i$

- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta_j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2$$

  $\Rightarrow \sigma_a = \delta''(1)/\delta'(1)$ is the elasticity of marginal utilization cost wrt h at $h = 1$

- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S_j(x) = \frac{\Psi_j}{2}(x-1)^2$$

  $\Rightarrow$ generates hump-shaped output and investment irf's (autocorrelated growth rates)

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, y_j), j \in \{c, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad \text{s.}t.$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

  and the consumption and labor aggregators

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,y\}\in\Omega} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, y)$$

First order conditions

- Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_c, \mu_i$ be the respective Lagrangian multipliers on the constraints
- FOC

$$
\begin{aligned}
&[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc} \\
&[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc} \\
&[i_c]: \quad -\gamma_i - \lambda p_i + \mu_c \left(1 - S_c'(x_c)x - S_c(x_c)\right) + \beta\theta_b \mathbb{E}\mu_c' S_c'(x_c')(x_c')^2 = 0 \\
&[i_i]: \quad -\gamma_i - \lambda p_i + \mu_i \left(1 - S_i'(x_i)x_i - S_i(x_i)\right) + \beta\theta_b \mathbb{E}\mu_i' S_i'(x_i')(x_i')^2 = 0 \\
&[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \\
&[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \\
&[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \\
&[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \\
&[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \\
&[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Envelope conditions

- Consumption

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{3}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j (u_j - \lambda p_j) \quad j \in \{mc, sc\} \tag{4}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} (u_j - \lambda p_j) \quad j \in \{mc, sc\}$$

- Investment

$$\frac{\partial V^i}{\partial p_i} = -\lambda i = -\lambda (d_i A_i D_i^{\phi-1} F_i) \tag{5}$$

$$\frac{\partial V^i}{\partial D_i} = -(\phi - 1) d_i A_i D_i^{\phi-2} F_i \gamma_i \tag{6}$$

$$\frac{\partial V^i}{\partial F_i} = d_i A_i D_i^{\phi-1} \gamma_i$$

Price-tightness tradeoff

- Take ratio of (3) and (4):

$$\frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)} \tag{7}$$

- Take ratio of (5) and (6)

$$\frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i} \tag{8}$$

Back to household problem

Firms' problem

- A representative firm in sector $j \in \{mc, s, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, y_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Conditional labor demand and wage index

- Consider labor cost minimization problem

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$
$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n}$$

- Take FOC and recognize W_j as Lagrangian multiplier on constraint

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{9}$$

- Wage index for composite labor input in sector j

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

Optimal wage choice of labor union and aggregation

- Problem of labor union

$$\max_{W_j(s)} (W_j(s) - W_j^*) n_j(s) \quad \text{s.t.} \quad (9) \Leftrightarrow$$

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Labor union in each sector choose

$$W_j(s) = \mu_j W_j^*$$

- Labor unions pay same wage and firms choose identical quantities of labor within j

$$W_j(s) = W_j, n_j(s) = n_j$$

- Labor unions rebate earnings to HH in lump-sum fashion (regard as fixed component to wage)

Firm first order conditions

- Let ι_j and ∇_j be the multipliers on participation constraint and production technology

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{10}$$

$$[D_j] \quad \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{11}$$

Firm problem: finding λ and γ_j

- Take ratio of first order conditions for (10) and (11)

$$\frac{D_j}{\phi p_j} = \frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}}$$

- Plug in (7)

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)}$$

- Simplify

$$\lambda \phi p_j = (1 - \phi)(u_j - \lambda p_j) \Rightarrow$$

$$\lambda = u_j(1 - \phi)/p_j$$

so that

$$\gamma_j = \phi u_j$$

Firm problem: finding γ_i

- Take ratio of first order conditions for (10) and (11) for $j = i$:

$$\frac{D_i}{\phi p_i} = \frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}}$$

- Plug in (8)

$$\frac{D_i}{\phi p_i} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i}$$

- Simplify

$$\gamma_i = \frac{\phi}{1-\phi}\lambda p_i$$
$$= \phi \frac{u_j}{p_j} p_i$$

Simplifying shopping conditions

- Plug in values of γ_j to find

$$
\begin{aligned}
-u_d &= \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{m_c, s_c\} \\
-u_d \theta_i &= \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]
\end{aligned}
$$

- Plug in $\lambda = u_{mc}(1-\phi)/p_{mc}$ to simplify labor-leisure tradeoff

$$
u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}
$$

Demand for non-durables and services

- From the expression for λ we have

$$\frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \Rightarrow \phi = (u_j - \lambda p_j)/u_j$$

- Combine with consumption aggregation and price index to find demand curves

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

where $\xi = 1/(1-\rho_c)$ is the elasticity of substitution.

Tobin's Q

- Solve for value of investment: $j \in \{c, i\}$

$$\lambda p_i + \gamma_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

$$\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

$$\frac{\lambda p_i}{1-\phi} = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

- Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption
- We can rearrange as

$$\frac{p_i}{1-\phi} = Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}Q_j' S_j'(x_j')(x_j')^2$$

Tobin's Q

- Rewrite optimal choice of utilization: $j \in \{mc, sc, i\}$

$$\delta_h(h_j)Q_j = R_j$$

- Euler equation

$$Q_j = \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h'_j))Q'_j + R'_j h'_j\right] \quad j \in \{mc, sc, i\}$$

Solving for firm multipliers

$$\iota_j = \frac{A_j q_j^{\phi} F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

$$\begin{aligned}
\nabla_j &= p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j} \\
&= p_j A_j D_j^{\phi} + \frac{A_j D_j^{\phi} \gamma_j}{\lambda} \\
&= p_j A_j D_j^{\phi} + A_j D_j^{\phi} \frac{\phi}{1-\phi} p_j \\
&= A_j D_j^{\phi} \left(p_j + \frac{\phi}{1-\phi} p_j \right) \\
&= \frac{p_j A_j D_j^{\phi}}{1-\phi}
\end{aligned}$$

Back to firm problem

Simplified optimality conditions for firm

$$(1-\phi)\frac{W_c}{p_j} = A_j(D_j)^{\phi} z_c f_{N_j} \quad j \in \{m_c, s_c\}$$

$$\frac{W_c}{R_j} = \frac{f_{N_c}}{f_{K_c}}$$

$$(1-\phi)\frac{W_i}{p_i} = A_i(D_i)^{\phi} z_i f_{N_i}$$

$$\frac{W_i}{R_i} = \frac{f_{N_i}}{f_{K_i}}$$

Firm factor demands

$$(1-\phi)\frac{W_c}{p_j} = \alpha_n \frac{Y_j + A_j D_j^{\phi} \nu_j}{N_j} \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j + A_j D_j^{\phi} \nu_j}{h_j K_j} \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$\theta_n (n^a)^{1/\nu} \left(\frac{n_c}{n^a}\right)^{\theta} \omega^{-\theta} = (1-\phi)\frac{W_c}{\mu_c \zeta}$$

$$\theta_n (n^a)^{1/\nu} \left(\frac{n_i}{n^a}\right)^{\theta} (1-\omega)^{-\theta} = (1-\phi)\frac{W_i}{\mu_i \zeta}$$

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

$$\theta_d D^{1/\eta} = \phi p_j \frac{Y_j}{D_j} \quad j \in \{mc, sc\}$$

$$\theta_i \theta_d D^{1/\eta} = \phi p_i \frac{I}{D_i}$$

$$\frac{p_i}{1-\phi} = Q_j [1 - S_j'(x_j) x_j - S_j(x_j)] + \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} Q_j' S_j'(x_j')(x_j')^2$$

$$Q_j = \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} \left[(1 - \delta_j(h_j')) Q_j' + R_j' h_j'\right] \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$C = [\omega_c^{1-\rho_c} Y_{mc}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc}^{\rho_c}]^{1/\rho_c}$$

$$Y_j = p_j^{-1/(1-\rho_c)} \omega_j C \quad j \in \{mc, sc\}$$

$$C = p_{mc} Y_{mc} + p_{sc} Y_{sc}$$

$$\lambda = \Gamma^{-\sigma}(1-\phi)$$

Summary of equilibrium conditions

$$
\begin{aligned}
\delta_h(h_j)Q_j &= R_j, \quad j \in \{mc, sc, i\} \\
Y_j &= A_j(D_j)^{\phi}(z_j(h_jK_j)^{\alpha_k}(n_j)^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\} \\
I &= I_c + I_i \\
K'_j &= (1-\delta_j(h_j))k_j + [1 - S_j(x_j)]I_j \quad j \in \{mc, sc, i\} \\
(1-\phi)\frac{W_j}{p_j} &= \alpha_n \frac{Y_j + A_jD_j^{\phi}\nu_j}{N_j} \quad j \in \{mc, sc, i\} \\
\frac{W_j}{R_j} &= \frac{\alpha_n}{\alpha_k}\frac{h_jK_j}{n_j} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Explanation of numeraire dependence

- Quantity movements may depend on the numeraire in a multisector model
- Consider positive shock to Z^C: relative price of consumption goods falls
- In terms of the investment good, consumption may fall even though actual units purchased rises
- However, if the consumption good were the numeraire, the investment good instead rises in price, so output rises by more
- Reasoning is symmetric with a positive Z^I shock
- Using base-year prices eliminates dependence as by Bai, Rios-Rull, and Storesletten (2024)
- Fisher index also eliminates dependence on base year, but it is equivalent in the case of a first-order approximation.
- See Duernecker, Herrendorf, Valentinyi et al. (2017) for a detailed discussion

Back to mapping

Details: depreciation

- Over sample, the average annual growth rate of output is 1.8%
- Set $\overline{g} = 0.45\%$ (1.8% annual growth)
- Capital accumulation (ignoring adjustment costs)

$$g\widehat{K}' = (1-\delta)\widehat{K} + g\widehat{I}$$

so that in steady state

$$\delta = 1 - \overline{g} + \frac{I}{K}$$

- Let investment share $\kappa = p_i I/Y = 0.2$ and $p_i K/Y = 2.75(4) = 11$
- Hence, $\delta = 0.2/11 - 0.0045 = 1.37\%$

Details: labor share α_n

- Rearrange FOC for labor demand

$$p_j = (1-\phi)\frac{W_j n_j}{\alpha_n A_j (D_j)^{\phi} F_j}$$

Hence,

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu^R)$$

where $\nu^R = \nu_j/(F_j)$ and thus labor share is

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C+p_i I}{Y}(1+\nu^R) = \frac{\alpha_n}{1-\phi}(1+\nu^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1+\nu^R)$

Details: capital share α_k and deprecation parameter σ_b

- $R_j = R$ in steady state
- Note $\beta(\overline{g})^{-\sigma} = 1/(1+r) \Rightarrow \overline{g} - 1 \approx (r - \rho)/\gamma$
- Implies $\rho \approx r - \gamma\overline{g}$ (so we must have $r \geq \gamma\overline{g}$)
- Steady-state Euler

$$\begin{aligned} Q &= \beta\overline{g}^{-\gamma}[(1-\delta)Q + R] \Rightarrow \\ (1+r)Q &= (1-\delta)Q + R \\ (r+\delta)Q &= R \end{aligned}$$

- Steady-state optimal utilization

$$\sigma_b = \frac{R}{Q} = r + \delta$$

- Combine with steady state Tobin's Q: $p_i/(1-\phi) = Q$ and we find

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Details: capital share α_k and deprecation parameter σ_b

- Firm optimization yields

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{K_j}(1+\nu^R)$$

- Note

$$\frac{Y_j}{K_j} = \frac{Y}{K} \quad \forall K$$

and hence

$$r + \delta = \alpha_k \frac{Y}{K}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{K}{Y}$$

Using $r, \delta, K/Y, \nu^R$, we recover $\alpha_k = 0.216$

Details: weight of services ω_{sc}

- We pin down the weight of services ω_{sc} as the empirical measure $S_c = Y_{sc}/C$ and set $S_c = 0.65$.
- The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  and plug in S_c, using $\omega_{sc} = S_c$:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

  so that $p_{mc} = p_{sc}$
- Given normalization $p_{sc} = 1$, all consumption goods prices equal unity.

Details: matching technology coefficient A_j

- Given $\Psi_j = A_j D_j^{\phi}$, the matching technology coefficient satisfies

$$A_j = \frac{\Psi_j}{D_j^{\phi}}$$

- Need to find D_j for each j

Details: matching technology coefficient A_j

- We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j \rightarrow$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

- Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Details: matching technology coefficient A_j

- Consider ratio in shopping optimality conditions between m_c and i:

$$\frac{D_{mc}}{D_i} = \frac{p_{mc}}{p_i}\frac{Y_{mc}}{Y_i}$$
$$= (1-\omega_{sc})\frac{1-I/Y}{I/Y}$$

- Hence,

$$D_{mc} = (1-S_c)(1-I/Y)D$$
$$D_{sc} = S_c(1-I/Y)D$$
$$D_i = (I/Y)D$$

Balanced growth and transformation of variables

- Output, consumption, investment, wages, and capital grow at common rate g_t
- Transform each trending variable y_t determined at time t

$$\widehat{y}_t = \frac{y_t}{X_t}$$

so that $\log \widehat{y}_t$ represents log deviation from stochastic trend

- Capital stock K_t is determined at $t-1$, so we deflate by X_{t-1}

$$\widehat{K}_t = \frac{K_t}{X_{t-1}}$$

- Transform preferences to make shopping stationary

$$\Gamma_t = c_t - haC_{t,-1} - X_t \theta_{dt} \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt} \frac{(n_t^a)^{1+1/\nu}}{1+1/\nu} \zeta_t$$

Equations modified by growth

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$
\begin{aligned}
C_t^{obs} &= \log C_t - \log C_{t-1} + g_t - \overline{g} \\
I_t^{obs} &= \log I_t - \log I_{t-1} + g_t - \overline{g} \\
w_t^{obs} &= \log w_t - \log w_{t-1} + g_t - \overline{g}
\end{aligned}
$$

- Stationary series

$$
\begin{aligned}
N_{jt}^{obs} &= \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\} \\
p_{i,t}^{obs} &= \log p_{i,t} - \log p_{i,t-1} \\
util_{j,t}^{obs} &= \log util_{j,t} - \log util_{j,t-1}
\end{aligned}
$$

Vector of observable variables

Vector of observables

$$
= \begin{bmatrix} \Delta \log(C_t) \\ \Delta \log(I_t) \\ \Delta \log(N_{ct}) \\ \Delta \log(N_{it}) \\ \Delta \log(util_{ND,t}) \\ \Delta \log(util_{D,t}) \\ \Delta \log(p_{it}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Estimation procedure

- Estimate mode of posterior distribution by maximizing log posterior function (combines priors and likelihood)
- Use Metropolis-Hastings algorithm to sample posterior distribution and to evaluate marginal likelihood of the model
    - Sample of 300,000 draws (neglect first 20%)
    - Hessian defines transition probability that generates new proposed draw
- Check convergence and identification (trace plots)

Back to estimation

On the use of growth rates for estimation

- Major macroeconomic series are difference-stationary
- For such data, growth rates preserves all dynamics of a series
- Other filters (such as HP filter/Hamilton filter) extract specific frequencies of time series
- Latter may be reasonable for *description* depending on the notion of business cycle

FEVD: breakdown of search demand shocks

Table 8: Forecast error variance decomposition

	e_D	$e_{D,\text{news}}$	e_{DI}	$e_{DI_{\text{news}}}$
Y	93.61	1.14	0.08	5.16
SR	92.91	1.06	0.11	5.92
I	77.04	0.85	0.35	21.76
p_i	6.12	0.12	0.98	92.77
N_c	80.37	1.76	0.21	17.66
N_i	70.78	1.08	0.23	27.91
$util$	93.91	1.14	0.08	4.88
D	98.20	1.49	0.00	0.30
h	90.95	1.72	0.05	7.28

Table 8: Contribution of components to forecast error variance decomposition of search shocks.

FEVD: breakdown of technology shocks

Table 9: Forecast error variance decomposition

	e_g	$e_{g_{\text{news}}}$	e_Z	$e_{Z_{\text{news}}}$	e_{ZI}	$e_{ZI_{\text{news}}}$
Y	4.30	33.78	35.27	19.91	6.50	0.24
SR	6.05	48.75	24.99	13.40	6.61	0.22
I	0.89	6.83	42.13	20.60	28.54	1.01
p_i	0.01	0.07	23.26	15.94	57.85	2.86
N_c	2.59	23.97	18.96	19.74	33.09	1.64
N_i	1.75	16.13	20.72	19.43	39.37	2.60
$util$	0.22	4.27	39.98	33.81	20.19	1.53
D	1.94	23.11	42.21	26.15	6.17	0.42
h	0.51	3.03	46.53	41.13	8.16	0.64
$tech$	6.84	63.47	14.73	11.20	3.57	0.19

Table 9: Contribution of components to forecast error variance decomposition of technology shocks.

Back to FEVD

BRS as special case

- Model nests Bai, Ríos-Rull, and Storesletten (2023) (BRS) by shutting down additional frictions: Equilibrium
  - $\gamma = 0$
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_j = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- In contrast to BRS, estimate ϕ and η instead of calibrating using shopping time or price dispersion targets
- Also add stationary technology shock; otherwise use same prior distributions

Table 10: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 10: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{z, z_I, \theta_n, \theta_d\}$.

Role of capacity utilization on parameter estimates

Table 11: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_D	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
e_D	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 11: Estimation of baseline BRS model with to sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 12: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Std. dev.		
D	1.54	1.69
$util$	0.15	1.49
FEVD of demand shocks		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 12: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The BRS dataset includes growth rates of output, investment, labor productivity, and the relative price of investment. The second column adds variable total capacity utilization. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_D. See Table 11.

Back to estimation

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