

Subjective Housing Price Expectations, Falling Natural Rates, and the Optimal Inflation Target*

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Abstract

U.S. households' housing price expectations deviate systematically from full-information rational expectations: (i) expectations are updated on average too sluggishly, (ii) expectations initially underreact but subsequently overreact to housing price changes, and (iii) households are overly optimistic (pessimistic) about housing price growth when the price-to-rent ratio is high (low). We show that weak forms of housing price growth extrapolation allow to simultaneously replicate the behavior of housing prices and these deviations from rational expectations as an equilibrium outcome. Embedding housing price growth extrapolation into a sticky price model with a lower-bound constraint on nominal interest rates, we show that lower natural rates of interest increase the volatility of housing prices and thereby the volatility of the natural rate of interest. This exacerbates the relevance of the lower bound constraint and causes Ramsey optimal inflation to increase strongly with a decline in the natural rate of interest.

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1 Introduction

The large and sustained booms and busts in housing prices in advanced economies are often attributed to households’ excessively optimistic or pessimistic beliefs about future housing prices (Piazzesi and Schneider, 2006; Kaplan, Mitman, and Violante, 2020). This view is supported by a nascent literature that documents puzzling patterns in housing price expectations. Survey measures of expected future housing prices have been found to be influenced by past changes in housing prices, but appear to underreact to these changes. Additionally, they fail to account for the tendency of housing prices to revert to their mean over time (Kuchler and Zafar, 2019; Case, Shiller, and Thompson, 2012; Ma, 2020; Armona, Fuster, and Zafar, 2019).

Documenting deviations of households’ expectations about future housing prices from the full-information rational expectations benchmark is interesting, but does in itself not provide insights into the implications of these deviations for housing markets or the design of monetary policy. Understanding these implications requires a structural equilibrium model that jointly captures the quantitative patterns of households’ deviations from rational expectations and the behavior of housing prices. We construct such an equilibrium model and use it to derive the monetary policy implications of the observed deviations from rational housing price expectations. To the best of our knowledge, this paper is the first to pursue this task. The key monetary policy insight we derive is that the Ramsey optimal inflation target increases much more strongly with a fall in the natural rate of interest than in a setting with rational housing price expectations.¹

We begin our analysis by comprehensively quantifying the dimensions along which households’ housing price expectations deviate from the full-information rational expectations benchmark. We do so using a single data set, so as to establish a coherent set of quantitative targets for our structural equilibrium model with subjective housing price expectations. Our findings reveal three key dimensions in which household expectations deviate from full-information rational expectations. First, expectations about future housing prices exhibit sluggish updating over time. Second, and new to the housing literature, households’ expectations about housing price growth covary positively with market valuation, as measured by the price-to-rent ratio, while actual future housing price growth covaries negatively with market valuation. Third, households initially underreact to observed housing price growth, i.e., are too pessimistic in the first few quarters, but later overreact and exhibit—after approximately twelve quarters—overly optimistic expectations.

We then construct and calibrate a simple housing model with optimizing households and Bayesian belief updating, giving rise to extrapolative expectations about housing price growth. The model reproduces—as an equilibrium outcome—the three deviations of households’ beliefs from full-information rational expectations mentioned above, as well as important patterns of the behavior of U.S. housing prices, in particular, the large and protracted swings in the price-to-rent ratio over time. Despite its simplicity, the model demonstrates a surprisingly good quantitative fit.

The simple model offers two important economic lessons: (1) Deviations from rational expectations (RE) are key for understanding observed housing price dynamics. In particular, subjective beliefs help substantially to explain the observed volatility in the price-to-rent ratio, volatility in housing price growth, and the strong autocorrelation in housing price growth. In contrast, a setting with rational housing price expectations is unable to jointly match these features of housing prices. (2) The effects of subjective housing price beliefs on equilibrium

¹The Ramsey optimal inflation target is the average inflation rate that the Ramsey planner commits to implement.

housing price dynamics are stronger in a low rate environment, i.e., housing prices become more volatile when the natural rate of interest is lower. Consistent with this prediction, several advanced economies, including the United States, have experienced—concurrently with a decline in the natural rate of interest—a considerable rise in housing price volatility, as we document.

We then analyze the monetary policy implications associated with subjective housing price beliefs and the resulting housing price dynamics. To this end, we introduce subjective beliefs that give rise to housing price extrapolation into a New Keynesian model featuring a housing sector and a lower bound constraint on nominal interest rates. Like the simple model, this full general equilibrium model quantitatively replicates the behavior of housing prices and the patterns of deviations from rational housing price beliefs. To allow for a meaningful examination of monetary policy in the presence of subjective beliefs, subjective housing beliefs are introduced in a manner that prevents monetary policy from manipulating household beliefs to its advantage.²

We show analytically that the presence of subjective beliefs gives rise to housing price gaps, i.e., to deviations of housing prices from their efficient level, and that these housing price gaps have two important macroeconomic effects. First, a positive housing price gap, i.e., an inefficiently high housing price, makes it optimal for agents to allocate more resources towards housing investment. For a given level of output, this depresses private consumption and thereby real wages, which manifests itself as a negative cost-push term in the Phillips curve. Second, a positive housing price gap also exerts positive pressure on the equilibrium output gap. When policy seeks to keep the output gap stable, it must then implement higher (lower) real interest rates than in a setting with rational expectations, whenever the housing price gap is expected to increase (decrease). Since housing price gaps become more volatile when the average natural rate of interest is low, this feature implies that the *volatility of the natural rate increases* when the average level of the natural rate falls.

The fact that a lower average level of the natural rate also implies more natural rate volatility dramatically exacerbates the lower-bound problem for monetary policy.³ A more restrictive lower bound forces monetary policy to rely more heavily on promising future inflation in order to lower the real interest rate. Consequently, the Ramsey optimal inflation target increases considerably as the average natural rate falls. In our calibrated model, we find that the optimal inflation target increases by approximately one third of a percentage point in response to a one percentage point decline in the natural rate, with the increase being non-linear and becoming stronger for very low levels of the natural rate. In contrast, under rational expectations, the optimal inflation target is nearly invariant to the average level of the natural rate, because the volatility of the natural rate does not increase as its average level falls.

We also investigate the optimal policy response to housing demand shocks. Under rational expectations, neither inflation nor the output gap respond to housing demand shocks. Yet, in a setting where households extrapolate observed housing price growth, housing shocks move housing prices and these housing price movements get amplified by belief revisions. The belief revisions lead to persistent housing price gaps, i.e., deviations of the housing price from its first-best level, to which monetary policy finds it optimal to react. Interestingly, however, housing price gaps generate conflicting effects. On the one hand, inefficiently high housing prices generate negative

²The model also addresses the critique by Barsky, House, and Kimball (2007) regarding sticky price models with durable goods. Consistent with the data, the model suggests that housing demand responds more strongly to monetary disturbances than non-housing demand, despite housing prices being fully flexible and goods prices being sticky.

³The lower bound problem for monetary policy arises because nominal interest rates cannot fall (significantly) below zero whenever there is free convertibility of deposits into cash.

cost-push pressures, which call for a decrease in the policy rate. On the other hand, inefficiently high housing prices trigger a boom in housing investment, exerting upward pressure on the output gap, which calls for an increase in the policy rate. In our calibrated model, the second effect quantitatively dominates. Optimal monetary policy thus ‘leans against’ housing price movements, with the optimal strength of the reaction depending on the direction of the shock: following a positive housing preference shock, the increase in the interest rate (nominal and real) is more pronounced than the interest rate decrease following a negative housing demand shock. The presence of the lower bound constraint thus strongly attenuates the degree to which monetary policy leans against negative housing demand shocks.

In a final step, we analyze whether macroprudential policies could be employed to address the housing market inefficiencies generated by the presence of subjective price expectations. Specifically, we consider taxes on housing that are dynamically adjusted so as to reduce housing price fluctuations with the goal to insulate monetary policy from the macroeconomic effects of housing price gaps. We find that the required taxes would have to be large, very volatile, and often very negative. It thus seems unlikely that existing macroprudential tools allow addressing the monetary policy trade-offs arising from subjective housing price expectations.

This paper is related to work by [Andrade, Galí, Le Bihan, and Matheron \(2019, 2021\)](#) who also study how the optimal inflation target depends on the natural rate of interest in a setting with a lower bound constraint. In line with our findings, they show that an increase in the inflation target is a promising approach to deal with the lower bound problem. While their work considers optimized Taylor rules in a medium-scale sticky price model without a housing sector and rational expectations, the present paper studies Ramsey optimal policy in a model featuring a housing sector and subjective housing expectations.

A number of papers consider Ramsey optimal policy in the presence of a lower-bound constraint, but also abstract from housing markets and the presence of subjective beliefs ([Eggertsson and Woodford, 2003](#); [Adam and Billi, 2006](#); [Coibion, Gorodnichenko, and Wieland, 2012](#)). This literature finds that lower bound episodes tend to be short and infrequent under Ramsey optimal policy, so that average inflation is very close to zero. The present paper shows that this conclusion is substantially altered in the presence of subjective housing price expectations.

Optimal monetary policy in the presence of subjective beliefs has previously been analyzed in [Caines and Winkler \(2021\)](#) and [Adam and Woodford \(2021\)](#). These papers abstract from the lower bound constraint and consider different belief setups that are not calibrated to replicate patterns of deviations from rational housing price expectations as observed in survey data. We show that taking into account the existence of a lower bound constraint on nominal interest rates is quantitatively important for understanding how the optimal inflation target responds to lower natural rates.

2 Empirical properties of housing price expectations

In this section, we document three key dimensions in which households’ housing price expectations deviate in systematic ways from full-information rational expectations (RE). First, expectations about future housing prices are updated sluggishly. Second, and new to the literature, housing price growth expectations covary positively with the price-to-rent ratio, while actual housing price growth correlates negatively. Third, households’ expectations initially underreact but later overreact to observed housing price growth.

2.1 Data

We measure households' expectations about housing price growth using the Michigan household survey. The survey provides subjective expectations about nominal four-quarter-ahead housing price growth, $E_t^{\mathcal{P}}[q_{t+4}/q_t]$, where q_t is the housing price, for the period 2007-2021. We study both mean and median household expectations.⁴ The survey also provides housing price growth expectations over the next five years, which we analyze in a robustness exercise.⁵

We measure housing prices using the S&P/Case-Shiller U.S. National Home Price Index and measure housing prices q_t at quarterly frequency by the average of the monthly housing price index. We consider both nominal and real housing prices with real housing prices being obtained by deflating nominal housing prices with the CPI.

2.2 Sluggish updating about expected housing prices

We start by documenting that household expectations about the future level of housing prices are updated too sluggishly. This can be tested following the approach of [Coibion and Gorodnichenko \(2015\)](#), which uses regressions of the form

$$q_{t+4} - E_t^{\mathcal{P}}[q_{t+4}] = a^{CG} + b^{CG} \cdot (E_t^{\mathcal{P}}[q_{t+4}] - E_{t-1}^{\mathcal{P}}[q_{t+3}]) + \varepsilon_t. \quad (1)$$

The regression projects forecast errors about the future housing price level on the change in the four-quarter-ahead expected housing price. Under the full-information RE hypothesis, information that is contained in agents' information set, i.e., past forecasts, should not predict future forecast errors ($H_0: b^{CG} = 0$).

We estimate equation (1) for nominal and real housing price growth and using mean and median expectations, respectively. We compute subjective expectations about the future housing price level as $E_t^{\mathcal{P}}[q_{t+4}] = E_t^{\mathcal{P}}[q_{t+4}/q_t] q_t$, where $E_t^{\mathcal{P}}[q_{t+4}/q_t]$ is the housing price growth expectations from the Michigan survey and q_t the S&P/Case-Shiller index. When considering real housing prices, we deflate the nominal housing price growth expectations using the subjective (mean/median) inflation expectations from the Michigan survey.

Table 1 reports the estimated b^{CG} coefficients from regression (1). We find that, inconsistent with the full-information RE hypothesis, the estimated coefficient is positive and statistically significant at the 1% level in all considered specifications. This implies that housing price expectations are updated too sluggishly: when updating their expectations upwards (downwards), households on average underpredict (overpredict) the level of future housing prices. The magnitude of the estimates is also large in economic terms: a coefficient estimate of two suggests that forecast revisions should approximately be three times as strong than they actually are.

2.3 Opposing cyclicity of actual and expected housing price growth

We next document that actual and expected housing price growth covary differently with housing market valuation. In particular, households' expectations about future housing price

⁴Analyzing the dynamics of individual expectations over time is difficult because households in the Michigan survey are sampled at most twice.

⁵The main analysis focuses on the short horizon expectations because these determine housing prices according to our model.

Table 1: Sluggish adjustment of housing price expectations

	Mean expectations	Median expectations
<i>Nominal housing prices</i>		
\widehat{b}^{CG}	2.22*** (0.507)	2.85*** (0.513)
<i>Real housing prices</i>		
\widehat{b}^{CG}	2.00*** (0.332)	2.47*** (0.366)

Notes: This table shows the estimates of regression (1) for nominal and real housings prices and using mean and median expectations, respectively. The standard errors in parentheses are robust with respect to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

growth covaries positively with the price-to-rent ratio PR_t , while actual future housing price growth covaries negatively with PR_t . We estimate a pair of regressions of the form

$$E_t^P \left[\frac{q_{t+4}}{q_t} \right] = a + c \cdot PR_{t-1} + u_t \quad (2)$$

$$\frac{q_{t+4}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t, \quad (3)$$

where c captures the covariation of households' expectations about four-quarter-ahead housing price growth with the (lagged) price-to-rent ratio and \mathbf{c} documents the covariation of realized housing price growth with the price-to-rent ratio. Full information rational expectations imply that $c = \mathbf{c}$, whenever the agents' information set includes the past price-to-rent ratio.

Table 2 reports the regression results. Across all specifications, expected housing price growth covaries positively with the price-to-rent ratio, while realized housing price growth covaries negatively: expected housing price growth is thus pro-cyclical, while realized housing price growth is counter-cyclical. This pattern is akin to the one documented in stock markets (Adam, Marcet, and Beutel, 2017). Since the predictor variable used in these regression equations is highly persistent, we compute a small sample-adjusted estimate for the difference ($\mathbf{c} - c$) (Stambaugh, 1999; Adam et al., 2017). This small sample-adjusted difference turns out to be highly statistically significant in all specifications. Households are therefore overly optimistic (pessimistic) in times of housing booms (busts).

Quantitatively, the results imply that a two standard deviation increase of the price-to-rent ratio by 15.5 units increases the mean household expectations about four-quarter-ahead real housing price growth by around 0.5%. Actual four-quarter ahead housing price growth, however, falls by around 1.5%, so that the forecast error is approximately 2%.

2.4 Initial under- and subsequent over-reaction of housing price growth expectations

Finally, we document that households initially underreact to observed housing price growth but overreact later on. This exercise provides a unified assessment of the previous two findings: While

Table 2: Cyclicalities of expected vs. actual housing price growth

	\hat{c} (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
<i>Nominal housing prices</i>				
Mean expectations	0.033 (0.008)	-0.102 (0.007)	0.006	0.000
Median expectations	0.014 (0.001)	-0.102 (0.007)	0.009	0.000
<i>Real housing prices</i>				
Mean expectations	0.030 (0.017)	-0.113 (0.009)	-0.003	0.000
Median expectations	0.010 (0.004)	-0.113 (0.009)	0.006	0.000

Notes: \hat{c} is the estimate of c in equation (2) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (3). Standard errors in parentheses are based on Newey-West with four lags. The [Stambaugh \(1999\)](#) small-sample bias correction is reported in the second-to-last column and the last column reports the associated p -values for the null hypothesis $c = \mathbf{c}$, using the small sample bias correction.

the results in Table 1 show that households update short-term housing price beliefs on average too sluggishly, the results in Table 2 indicate over-optimism when the current market valuation is high, which points to some form of overreaction to past housing price increases. It turns out that both patterns can be jointly understood by considering the dynamic response of actual and expected housing price growth to housing price changes.

We investigate the dynamic response of households' forecast errors about housing price growth in response to realized housing price growth following the approach in [Angeletos, Huo, and Sastry \(2020\)](#), who analyze forecast errors about unemployment and inflation. We also consider the dynamic evolution of realized cumulative housing price growth to interpret the behavior of forecast errors in light of actual realizations. We estimate local projections ([Jordà, 2005](#)) of the form

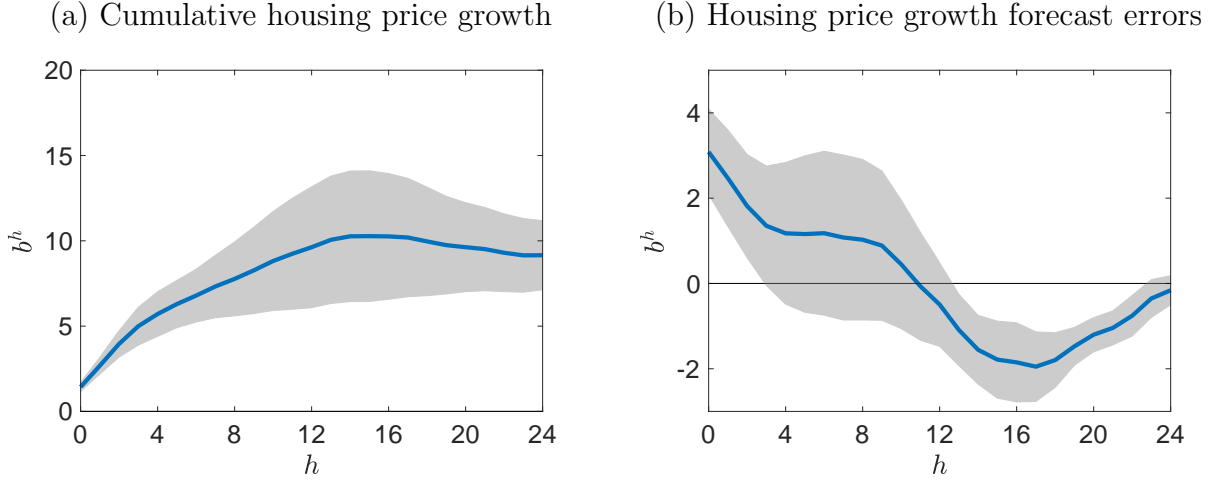
$$X_{t+h} = a^h + b^h \frac{q_{t-1}}{q_{t-2}} + u_t^h, \quad (4)$$

where X_{t+h} is either the cumulative capital gain q_{t+h+4}/q_t , or the forecast error about four-quarter-ahead housing price growth $q_{t+h+4}/q_{t+h} - E_{t+h}^P[q_{t+h+4}/q_{t+h}]$. u_t^h is a potentially autocorrelated and heteroskedastic residual term.

Figure 1 (a) shows the dynamic response of cumulative housing price growth to realized housing price growth, as captured by the estimated coefficients b^h . The initial housing price growth is not only persistent, but increases further over time, reaching a plateau after around twelve quarters. This finding is consistent with the high autocorrelation displayed by housing price growth.

Figure 1 (b) shows the dynamic response of forecast errors to realized housing price growth. Forecast errors are initially positive but later on—once cumulative housing price growth reaches its plateau—become negative before eventually disappearing. The positive values in the initial periods indicate that agents' expectations react too sluggishly: realized housing price growth is persistently larger than expected. This implies that initially expectations underreact to the observed change in housing prices. Subsequently, when housing price growth has fully materialized and housing prices plateau, agents are still optimistic about future housing price growth, thereby

Figure 1: Dynamic responses to a realized housing price growth



Notes: Panel (a) shows the dynamic response of cumulative real housing price growth at horizon h to a one standard deviation innovation in housing price growth. Panel (b) reports the dynamic response of housing price forecast errors at horizon h of one-year ahead expectations to a one standard deviation innovation in housing price growth. Positive (negative) values indicate that realized housing price growth exceeds (falls short of) expected housing price growth. The shaded area shows the 90% confidence intervals, standard errors are robust with respect to autocorrelation and heteroskedasticity (Newey-West with $h + 1$ lags).

over-estimating future housing price growth. This is consistent with the fact that housing price growth expectations display the wrong cyclicity with housing market valuation. It also implies that households entirely miss the mean-reversion in housing price growth: forecast errors turn negative once housing prices stop increasing, with the negative forecast errors disappearing only slowly over time. This pattern is consistent with the experimental evidence provided in [Armona et al. \(2019\)](#).

2.5 Robustness

Our results are robust to modifications of the empirical specifications along a number of dimensions.

Five-year ahead expectations. We investigate the robustness of our results by considering different horizons for housing price growth expectations. Appendix [A.1](#) shows that our results in Sections [2.2](#) and [2.3](#) are robust to this.⁶

Instrumental-variable estimation. Appendix [A.2](#) shows that our findings in Section [2.2](#) that households update their expectations sluggishly are robust to using an instrumental-variable approach for estimating regression (1), in which forecast revisions are instrumented with monetary policy shocks obtained via high-frequency identification.

Sluggish updating of housing price growth expectations. Appendix [A.3](#) shows that similar results emerge when using actual and expected housing price growth in equation (1)

⁶The sample is too short to obtain significant results for the analysis performed in section [2.4](#).

instead of the level and expected level of the housing price.

Cyclicalities of housing price forecast errors. Appendix A.4 shows that similar results as in Section 2.3 are obtained when first subtracting equation (2) from (3) and estimating the resulting equation with forecast errors on the left-hand side, as in Kohlhas and Walther (2021) who do not consider housing related variables.

Forecast error dynamics with median expectations. In Appendix A.5, we show that the nominal forecast error responses look very similar to the ones for real forecast errors presented in Section 2.4. Likewise, using median expectations instead of mean expectations makes no noticeable difference for the results.

Excluding the COVID-19 period. Appendix A.6 shows that all our results are robust to ending the estimation sample in 2019, thereby excluding the COVID-19 period.

Analysis using regional data. As is well known, housing prices often display considerable regional variation across the United States. We thus check whether the three deviations from full-information RE documented above are also present in regional housing prices and housing price beliefs. Appendix A.7 uses regional housing price indices and exploits local information contained in the Michigan survey that allows grouping survey respondents into different U.S. regions. Repeating the above analyses at the regional level shows that one obtains quantitatively similar results.

3 Extrapolative housing price expectations, equilibrium housing prices, and the natural rate of interest

This section presents a stylized housing model in which Bayesian learning causes households to extrapolate past housing price growth into the future. The model makes equilibrium predictions about the joint dynamics of housing prices and housing price beliefs, with housing prices depending on housing price beliefs and housing price beliefs being influenced by past housing price behavior. Despite the simplicity of the model, the equilibrium dynamics quantitatively replicate key features of U.S. housing price behavior and the patterns of deviations from rational expectations documented in Section 2.

The model predicts that large parts of observed housing price volatility are due to the presence of subjective housing price beliefs. In addition, it predicts that low levels of the natural rate of interest give rise to increased housing price volatility. As we show, this prediction is consistent with the evolution of natural rates and housing prices in advanced economies over the past decades.

3.1 A simple model with extrapolative housing price beliefs

Consider a unit mass of identical households.⁷ Each household chooses consumption $C_t \geq 0$, bonds B_t , housing units to own $D_t \in [0, D^{max}]$, and housing units to rent $D_t^R \geq 0$ to maximize lifetime

⁷The fact that households are identical is not common knowledge among households.

utility⁸

$$E_t^{\mathcal{P}} \left[\sum_{t=0}^{\infty} \beta^t [C_t + \xi_t^d (D_t + D_t^R)] \right]$$

subject to the budget constraint

$$C_t + B_t + (D_t - (1 - \delta)D_{t-1})q_t + R_t D_t^R = Y_t + (1 + r_t)B_{t-1}$$

for all $t \geq 0$, where Y_t is the total endowment (assumed to be sufficiently large), q_t the real price of housing, r_t the real interest rate, R_t the real rental price, and δ the housing depreciation rate. ξ_t^d denotes the preference for housing and is an exogenous determinant of housing prices. Household expectations are based on the subjective probability measure \mathcal{P} , as specified below.

The households' optimality condition for owned housing implies that equilibrium housing prices are determined by the asset pricing equation⁹

$$q_t = \xi_t^d + \beta(1 - \delta)E_t^{\mathcal{P}}[q_{t+1}] \quad (5)$$

and the optimality condition for rental units gives

$$R_t = \xi_t^d. \quad (6)$$

The optimal consumption-savings decision

$$1 = \beta(1 + r_t) \quad (7)$$

implies that the real interest rate r_t is constant and equal to the natural rate of interest $r^* = 1/\beta - 1$ at all times.

We now introduce subjective beliefs that give rise to housing price growth extrapolation, using the setup in [Adam, Marcet, and Nicolini \(2016\)](#). Households perceive housing price growth to evolve according to

$$\frac{q_t}{q_{t-1}} = b_t + \varepsilon_t, \quad (8)$$

where $\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$ is a transitory component of housing price growth and b_t a persistent component, which itself evolves according to $b_t = b_{t-1} + \nu_t$, with $\nu_t \sim iiN(0, \sigma_\nu^2)$. Households observe the realized housing price growth (q_t/q_{t-1}) and use Bayesian belief updating to estimate from observed housing price growth the persistent and transitory components. With conjugate prior beliefs, the subjective conditional one-step-ahead housing price growth expectation,

$$\gamma_t \equiv E_t^{\mathcal{P}} \left[\frac{q_{t+1}}{q_t} \right] \quad (9)$$

evolves according to the learning equation

$$\gamma_t = \min \left\{ \gamma_{t-1} + \frac{1}{\alpha} \left(\frac{q_{t-1}}{q_{t-2}} - \gamma_{t-1} \right), \bar{\gamma} \right\}, \quad (10)$$

⁸The constraint $D_t \leq D^{max}$ ensures existence of optimal plans in the presence of subjective housing beliefs. It is chosen such that it will never bind in equilibrium: housing supply D is fixed and satisfies $0 < D < D^{max}$.

⁹For the household, the first-order conditions may hold for some contingencies only with inequality under the subjectively optimal plans, due to the presence of short and long constraints. This explains why rational households can hold price expectations that differ from the discounted sum of future rents, see [Adam and Marcet \(2011\)](#) for details and [Adam and Nagel \(2022\)](#) for related arguments.

Table 3: Calibration of the simple model

Parameter	Value	Source/target
<i>Preferences and technology</i>		
β	0.9982	Average U.S. natural rate 2007-2021
δ	$\frac{3\%}{4}$	Adam and Woodford (2021)
<i>Exogenous shock processes</i>		
ρ_ξ	0.99	Quarterly autocorrelation of the price-to-rent ratio of 0.99
σ_ξ	0.67%	Standard deviation of the price-to-rent ratio pre-1990
<i>Subjective belief parameters</i>		
$1/\alpha$	0.7%	Adam et al. (2016)
$\bar{\gamma}$	1.0031	Max. percentage deviation of price-to-rent ratio from mean

where $1/\alpha$ is the Kalman gain and $\bar{\gamma}$ an upper bound on housing price growth beliefs, which ensures that housing price growth optimism is bounded from above, so as to keep subjectively expected utility finite. The gain parameter $1/\alpha$ captures the degree of extrapolation, i.e., it determines how strongly past housing price growth surprises feed into households' housing price growth beliefs for the future.

From the asset pricing equation (5) and the definition of subjective beliefs γ_t it follows that the equilibrium housing price is given by

$$q_t = \frac{1}{1 - \beta(1 - \delta)\gamma_t} \xi_t^d. \quad (11)$$

From the rental price (6) it follows that the equilibrium price-to-rent ratio is given by

$$PR_t \equiv \frac{q_t}{R_t} = \frac{1}{1 - \beta(1 - \delta)\gamma_t}. \quad (12)$$

Equations (10)-(12) jointly characterize the equilibrium dynamics of housing prices, subjective beliefs, and the price-to-rent ratio.

3.2 Model Calibration

We now calibrate the six parameters describing our simple model. Two parameter values are chosen from the literature and four are set to target data moments, see Table 3 for a summary.

We tie our hands with regard to the important belief parameter $1/\alpha = 0.7\%$ and set it equal to the value estimated in [Adam et al. \(2016\)](#) for stock price growth expectations. The low value for the Kalman gain implies that agents extrapolate observed capital gains only weakly, because households believe most of the realized capital gains to be due to transitory components. The annual housing depreciation rate is set equal to 3% following [Adam and Woodford \(2021\)](#).

The quarterly discount factor β is set such that the natural interest rate is equal to 0.75%, which is the average value of the U.S. natural rate over the period 2007-2021 estimated by [Holston, Laubach, and Williams \(2017\)](#). The value for the upper belief bound $\bar{\gamma}$ is set to match the maximum observed deviation of the price-to-rent ratio from its mean over the same period. This yields $\bar{\gamma} = 1.0031$.

Table 4: Housing price moments: data vs. model

	Data (2007-2021)	Subj. belief model	RE version of subj. beliefs	Recalibrated RE model
Std(PR_t)	8.76	8.76	2.67	8.76
Corr(PR_t, PR_{t-1})	0.99	0.99	0.99	0.99
Std(q_t/q_{t-1})	0.018	0.011	0.003	0.011
Corr($q_t/q_{t-1}, q_{t-1}/q_{t-2}$)	0.79	0.76	-0.01	-0.02

Notes: The table reports the standard deviation and first-order autocorrelation of price-to-rent ratios and housing price growth in the data (2007-2021), for the baseline model with subjective housing price beliefs, the rational expectations version of the subjective beliefs model (same model parameters) and the recalibrated version of the rational expectations model.

It only remains to specify the process for housing preference shocks. We consider an AR(1) process

$$\log \xi_t^d = (1 - \rho_\xi) \log \underline{\xi}^d + \rho_\xi \log \xi_{t-1}^d + \varepsilon_t^d, \quad (13)$$

where $\varepsilon_t^d \sim iiN(0, \sigma_\xi^2)$. To give the rational expectations version of the model a chance to replicate the observed high quarterly persistence of the price-to-rent ratio, we set $\rho_\xi = 0.99$.¹⁰ The standard deviation of ε_t^d is then chosen such that the model replicates the empirical standard deviation of the price-to-rent ratio over the period 2007-2021, expressed in percent deviation from its mean. This yields $\sigma_\xi = 0.67\%$ for the subjective belief model and $\sigma_\xi = 2.24\%$ for the rational expectations version of the model.¹¹ The latter is more than three times as large because housing prices in the subjective belief model also fluctuate due to changes in the subjective beliefs.

3.3 Match of empirical patterns in housing prices and beliefs

We now show that the simple model with extrapolative housing price beliefs replicates surprisingly well a number of *untargeted* data moments, including the behavior of the price-to-rent ratio, the behavior of housing price growth, and the previously documented deviations of households' expectations from full-information rational expectations (Section 2).

Table 4 shows that the subjective belief model replicates the empirical volatility and autocorrelation of the price-to-rent ratio as well as of housing price growth. While the standard deviation of the price-to-rent ratio is a targeted moment, all other moments are untargeted. The model matches very well the high quarterly autocorrelation of the price-to-rent ratio and the fairly high quarterly autocorrelation of housing price growth. It undershoots somewhat the standard deviation of quarterly housing price growth, illustrating that it features perhaps too little high-frequency variation in prices.¹²

Table 5 and Figure 2 show that the simple model also quantitatively replicates the three key deviations of households' housing price expectations from rational expectations as documented in Section 2. Table 5 illustrates that it matches the sluggish updating about expected housing prices ($b^{CG} > 0$) and the opposing cyclicity of actual and expected housing price growth ($c > 0$ and $\mathbf{c} < 0$). The magnitudes of the coefficients generated by the model closely match the ones obtained

¹⁰The subjective belief model can generate housing price persistence solely via the belief dynamics.

¹¹We normalize the mean of the housing preference shock process ξ^d to 1, but this is irrelevant for the cyclical properties of housing price beliefs and housing prices in which we are interested in here.

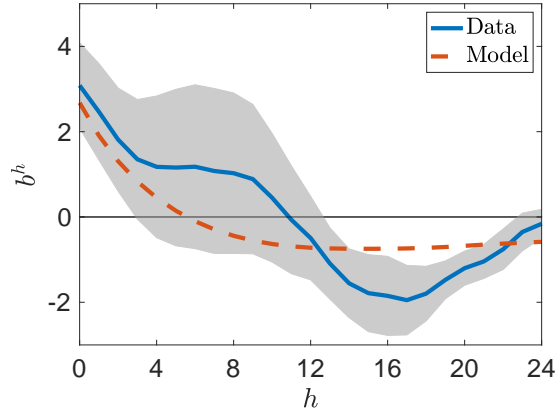
¹²This could easily be remedied by adding some i.i.d. shocks, for example to the discount factor β .

Table 5: Patterns of deviations from rational expectations: data vs. model

	Data		Model
	Mean expectations	Median expectations	
b^{CG} from (1)	1.68 (0.355)	2.12 (0.394)	2.09
c (in %) from (2)	0.030 (0.172)	0.010 (0.043)	0.030
c (in %) from (3)	-0.113 (0.009)	-0.113 (0.009)	-0.063

Notes: This table shows the model-implied regression coefficients of regressions (1), (2) and (3) for a natural rate of 0.75% (annualized) in the first column and the empirical results (for real housing prices) in the second and third column.

Figure 2: Dynamic forecast error response: data vs. model

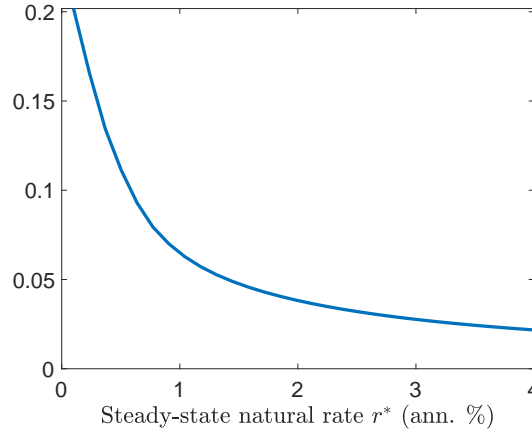


Notes: The figure shows impulse-response functions of housing price forecast errors of one-year ahead expectations to a one standard deviation innovation in housing price growth from the model and the data. The shaded area shows the 90%-confidence intervals of the empirical estimates, standard errors are robust with respect to autocorrelation and heteroskedasticity (Newey-West with $h + 1$ lags).

using survey data, except that the model underpredicts somewhat the counter-cyclicality of actual housing price growth. Figure 2 shows that the simple model also matches the dynamic response of forecast errors, where model-implied forecast errors are computed as $FE_{t+h}^{model} = \frac{q_{t+4+h}}{q_{t+h}} - (\gamma_{t+h})^4$. These results are a unique success of extrapolative beliefs: these model-based statistics would be identically equal to zero for the rational expectations model.

We now use the subjective belief model to understand the quantitative importance of subjective housing beliefs for housing market outcomes. To this end, we consider the calibrated subjective believe model but impose rational housing expectations. The resulting outcomes are reported in the second to last column in Table 4: the standard deviation of the price-to-rent ratio and of housing price growth both decrease by about 70% and the autocorrelation of housing price growth essentially drops to zero. This shows that the majority of the observed fluctuations in the PR ratio in housing price growth is due to the presence of subjective beliefs. In addition, the autocorrelation

Figure 3: Volatility of the price-to-rent ratio in the subjective belief model



Notes: The figure reports the standard deviation of the price-to-rent ratio, relative to its mean value, in the subjective belief model as a function of the steady state natural rate of interests.

of housing price growth is almost exclusively due to the presence of subjective beliefs.

The rational expectations version of the model not only fails to match the observed patterns of deviations from rational expectations, but also has difficulties in matching actual housing price behavior. This is illustrated in the last column in Table 4, which calibrates the rational expectations version of the model in a way that it also matches the standard deviation of the price-to-rent ratio. This requires $\sigma_\xi = 2.24\%$, which is more than three times higher than in the model under subjective beliefs. Perhaps not surprisingly, the rational expectations model has difficulties in generating persistent housing returns: it cannot account for the high autocorrelation in observed housing price growth.

3.4 Implications of falling natural rates for housing price dynamics

This section shows that lower natural rates of interest are associated with higher housing price volatility: this holds true in the data and in the subjective belief model and will be key for understanding the monetary policy outcomes in the next section.

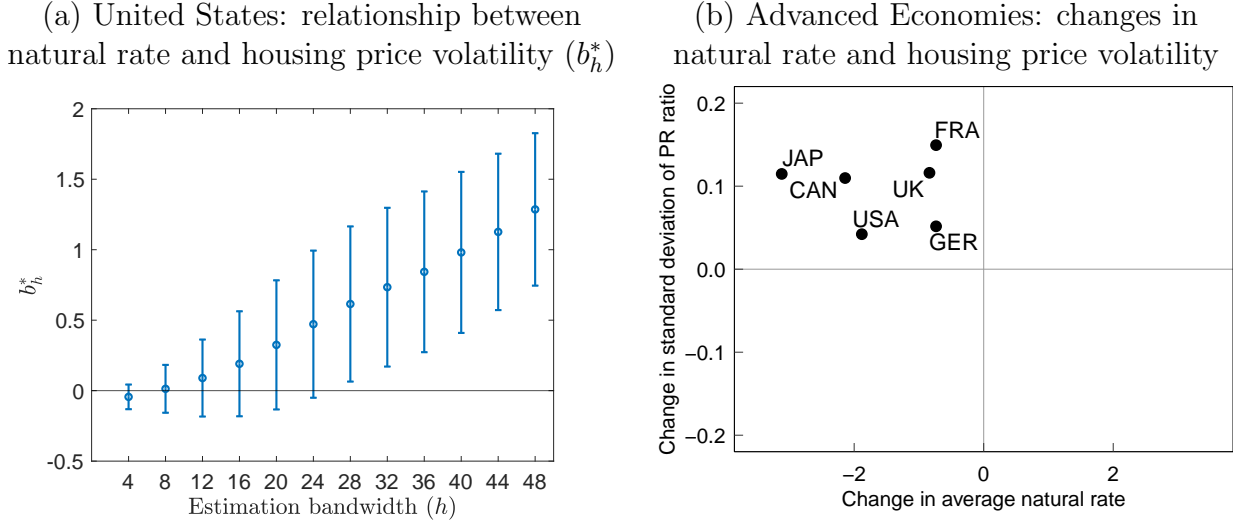
In the subjective belief model, a higher discount factor $\beta < 1$ gives rise to a lower natural rate of interest $r^* = 1/\beta - 1$ (equation (7)). A high discount factor also implies that any given change in housing price growth expectations leads to a larger change in equilibrium housing prices (equation (11)). Larger realized housing price growth in turn produces stronger revisions in beliefs in the future (equation (10)) and thus feeds stronger price growth in the subsequent period. Through this feedback loop, lower natural rates generate stronger momentum and more volatility in beliefs. This leads to more volatility in the price-to-rent ratio (equation (12)).

Figure 3 depicts the model-implied relationship between the natural rate and the volatility of the price-to-rent ratio. The figure measures volatility by the standard deviation of the price-to-rent ratio divided by its mean, to control for the fact that housing prices increase when the natural rate falls.¹³ It shows that lower natural rates are associated with larger housing price volatility, with the effect becoming quite non-linear for low levels of the natural rate.

Next, we document the relationship between natural rates and housing volatility in the data.

¹³The volatility measure is thus equal to the standard deviation of the percent deviation of the price-to-rent ratio from its mean.

Figure 4: The natural rate and housing price fluctuations in the data



Notes: Panel (a) reports the regression coefficient b_h^* from equation (14) together with 68% Newey-West error bands using h quarterly lags. Panel (b) plots the pre-/post-1990 changes in the average natural rate, using estimates from Holston et al. (2017), against the changes in the volatility of the price-to-rent ratio for different advanced economies. Volatility is again defined as the standard deviation (pre-/post-1990 periods) relative to the period-specific mean value.

Let r_t^* denote the long-run level of the natural rate at time t . Under standard assumptions, the long-run level is a function of exogenous fundamentals only. In the simple model introduced above, the discount rate is the only fundamental. In the full model presented in the next section, the long-run level (or the steady state) also depends on long-run productivity growth and preferences. In any case, the exogeneity of r_t^* allows considering regressions of the form

$$\frac{\text{Std}_h(PR_t)}{\text{Mean}_h(PR_t)} = a_h^* - b_h^* \cdot r_t^* + u_{t,h}, \quad (14)$$

where $\text{Std}_h(PR_t) \equiv \text{Std}(PR_{t-\frac{h}{2}}, \dots, PR_{t+\frac{h}{2}})$ denotes the standard deviation of the price-to-rent ratio using a window of $h + 1$ quarters centered around period t and $\text{Mean}_h(PR_t)$ the average price-to-rent ratio over the same window. The coefficients b_h then capture the causal effect of the natural rate on housing price volatility.

Panel (a) in Figure 4 reports the coefficients b_h^* for the United States for different estimation bandwidths h using the long-run natural rate estimates from Holston et al. (2017). For narrow bandwidths, we obtain insignificant results, which is likely due to the difficulties associated with reliably estimating the standard deviation and the mean of the price-to-rent ratio. For larger bandwidths, the coefficients become positive, statistically significant and quantitatively quite large: for $h=48$ quarters, the estimate implies that a 1 percentage point drop in the natural rate gives rise approximately to a 1-2 percentage point increase in housing volatility.¹⁴ This shows that the volatility of U.S. housing prices is rising as the long-run natural rate falls.

Panel (b) in Figure 4 shows that a similar relationship is present in other advanced economies: it plots the change in the average level of the natural rate from the period before 1990 to the period after 1990 for the U.S., Canada, France, Germany, and the United Kingdom, against the

¹⁴Due to measurement error in the regressor r_t^* , this is likely an underestimation of the true effect.

change in the volatility of the price-to-rent ratio. In all six advanced economies, the price-to-rent ratio has become more volatile as the average level of the natural rate declined.

Taken together, the structural mechanism in the subjective belief model and the empirical evidence suggest that the volatility of the price-to-rent ratio increases when the average level of the natural rate of interest falls.

4 Full model with extrapolative housing price beliefs

To analyze the monetary policy implications of falling natural interest rates and rising housing price volatility, we embed subjective beliefs into a general equilibrium sticky price model with a housing sector. All agents are internally rational, see [Adam and Marcet \(2011\)](#), and maximize utility/profits given their subjective beliefs about housing prices. The policy model is related to the one studied in [Adam and Woodford \(2021\)](#) but features belief distortions that are outside the class of absolutely continuous distortions permitted within their setup. This allows deriving monetary policy implications in a setup featuring quantitatively credible forms of belief distortions. In addition, the present model takes into account the lower-bound constraint on nominal interest rates, which we show to be quantitatively important for understanding how the optimal inflation target responds to lower average levels of the natural rate of interest.

4.1 Model setup: households, goods producers, house producers, and the government

We outline the most important model features in this section. A detailed description of the model can be found in [Appendix B](#).

Households. The economy is made up of identical infinitely-lived households that own the good-producing firms as well as the firms in the housing construction sector.¹⁵ Households obtain utility from consumption and from housing and disutility from working, with lifetime utility

$$E_0^P \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{\lambda}{1+\nu} \int_0^1 H_t(j)^{1+\nu} dj + \xi_t^d (D_t + D_t^R) \right], \quad (15)$$

where C_t is consumption of the final good, $H_t(j)$ is labor supply of type j and $w_t(j)$ the associated real wage. ξ_t^d is the time-varying preference for housing services in the form of owned houses D_t or rented houses D_t^R .

Households optimize subject to the budget constraints

$$C_t + B_t + (D_t - (1 - \delta)D_{t-1})q_t + R_t D_t^R = \int_0^1 w_t(j)H_t(j)dj + \frac{B_{t-1}}{\Pi_t}(1 + i_{t-1}) + \frac{\Sigma_t}{P_t} + \frac{\Sigma_t^d}{P_t} + \frac{T_t}{P_t}. \quad (16)$$

B_t is the real value of government bonds, Π_t is the gross inflation rate, and i_t the nominal interest rate. R_t is the real rental rate for housing. T_t denotes lump-sum taxes and transfers. Profits of firms accrue to households: Σ_t from intermediate goods producers and Σ_t^d from housing constructors.

¹⁵As before, the fact that households are identical is not common knowledge.

Households can buy and sell houses at price q_t . Owned houses D_t depreciate at rate δ . Households have subjective beliefs about housing prices but rational expectations about other variables. For tractability, we assume here that agents form beliefs about the housing price in terms of marginal utility units,

$$q_t^u \equiv \frac{q_t}{C_t}, \quad (17)$$

which provides a measure of whether housing is currently expensive or inexpensive in units that are particularly relevant for households.¹⁶ As introduced in Section 3, we assume that households extrapolate housing price growth (now in marginal utility units), $\gamma_t^u \equiv E_t^{\mathcal{P}} \left[\frac{q_{t+1}^u}{q_t^u} \right]$, according to the learning equation

$$\gamma_t^u = \min \left\{ \gamma_{t-1}^u + \frac{1}{\alpha} \left(\frac{q_{t-1}^u}{q_{t-2}^u} - \gamma_{t-1}^u \right), \bar{\gamma}^u \right\}. \quad (18)$$

Households discount future utility at the rate $\beta \in (0, 1)$. Since our model is formulated in terms of growth-detrended variables, the discount rate β jointly captures the time preference rate $\tilde{\beta}$ and the steady-state growth rate of marginal utility. Letting g_c denote the steady-state trend growth rate of consumption, we have

$$\beta = \tilde{\beta} \frac{1}{(1 + g_c)}. \quad (19)$$

When the growth rate g_c of the economy falls, the discount rate β increases. A decline in the trend growth rate of the economy can thus be captured by an increase in the detrended discount factor $\tilde{\beta}$. Declining trend growth causes the steady-state natural interest rate to fall, which is in line with the estimates provided in [Holston et al. \(2017\)](#) (see Appendix A.8). Appendix B.1 provides further details on the household optimization problem and the optimality conditions.

Final and intermediate goods producers. A representative final good producer provides an aggregate consumption good

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \quad (20)$$

Each intermediate good i is supplied by a monopolistically competitive producer with a common technology in which (industry-specific) labor $h_t(i)$ is the only variable input:

$$y_{i,t} = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \quad (21)$$

and where A_t denotes exogenous total factor productivity. Intermediate goods producers are subject to a [Calvo \(1983\)](#) price adjustment friction and maximize the value of the firm to the households, using households' subjectively optimal consumption plans to discount profits. The government levies a constant and negative sales tax τ to induce marginal cost pricing in the steady state. Appendix B.2 provides details on the intermediate and final goods producers' optimization problem.

¹⁶Specifying subjective beliefs in units of marginal utility leaves the ability of the learning rule to replicate the survey evidence unchanged. This is because log consumption preferences imply that contributions from fluctuations in marginal utility are orders of magnitude smaller than those generated by subjective beliefs.

Housing construction firm. A representative housing construction firm operates an isoelastic housing production function to build new houses

$$\tilde{d}_t = \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}, \quad (22)$$

where k_t denotes investment into new houses and with $\tilde{\alpha} \in (0, 1)$. Appendix B.3 provides further details and optimality conditions.

Government. The government imposes a constant sales tax τ on intermediate goods revenues, issues nominal bonds, and pays lump-sum taxes and transfers T_t to households. The real government budget constraint is given by

$$B_t = B_{t-1} \frac{1 + i_{t-1}}{P_t/P_{t-1}} + \frac{T_t}{P_t} - \tau Y_t.$$

Lump-sum taxes and transfers are set such that they keep real government debt constant at some initial level B_{-1}/P_{-1} . Appendix B.4 provides further details as well as the market clearing conditions.

Equilibrium. The optimality conditions of households, intermediate and final good producers, and housing constructors, the government budget constraints, market clearing conditions, and the nominal interest rate i_t set by the monetary policy authority define an *Internally Rational Expectations Equilibrium* (IREE, see Adam and Marcet, 2011), as spelled out in Appendix B.5. To pin down the nominal rate i_t , we will consider Ramsey optimal monetary policy below in Section 5.

4.2 Equilibrium characterization

This section characterizes the equilibrium of the model. To gain analytic insights, we derive a linear-quadratic approximation to the optimal policy problem around the efficient steady state.¹⁷

Asset prices and housing market equilibrium. From the household optimality conditions, the equilibrium housing price is given by

$$q_t^u = \frac{1}{1 - \beta(1 - \delta)\gamma_t^u} \xi_t^d \quad (23)$$

and the price-to-rent ratio by

$$PR_t = \frac{q_t^u}{\xi_t^d}. \quad (24)$$

Variations in subjective beliefs γ_t^u introduce inefficient housing price fluctuations. The economic and welfare implications of these fluctuations can be quantified by the *housing price gap* $\hat{q}_t^u - \hat{q}_t^{u*}$. The housing price gap measures the log deviation of the actual housing price from its welfare-maximizing level, which in percentage deviations from the steady state is given by

$$\hat{q}_t^{u*} \equiv \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\rho_\xi} \hat{\xi}_t^d. \quad (25)$$

¹⁷In this steady state, the government levies a negative output tax $\tau = 1/(1 - \eta)$ on goods producing firms to eliminate steady-state distortions arising from monopolistic competition.

Under rational expectations, the housing price gap is zero at all times, but under subjective beliefs, the housing price gap is given by¹⁸

$$\widehat{q}_t^u - \widehat{q}_t^{u*} = \left(\frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\gamma_t^u} - \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\rho_\xi} \right) \widehat{\xi}_t^d + \frac{\beta(1 - \delta)(\gamma_t^u - 1)}{1 - \beta(1 - \delta)\gamma_t^u} \quad (26)$$

and depends on the subjective housing beliefs γ_t^u and on the housing preference shocks $\widehat{\xi}_t^d$. The housing price gap is of economic interest because it captures misallocations of output between consumption and housing investment. This follows from equation

$$\left((1 - \tilde{\alpha})\widehat{k}_t - \widehat{c}_t \right) - \left((1 - \tilde{\alpha})\widehat{k}_t^* - \widehat{c}_t^* \right) = \widehat{q}_t^u - \widehat{q}_t^{u*}, \quad (27)$$

in which \widehat{k}_t denotes housing investment, \widehat{c}_t consumption and starred variables indicate the values with efficient housing prices.¹⁹ The previous equation shows that a positive housing price gap affects the ratio between investment and consumption. Intuitively, this occurs because housing prices affect investment incentives. In particular, high housing prices make housing investment more attractive and lead to a higher investment to consumption ratio. This feature explains why the housing price gap is a welfare-relevant object.

Importantly, equation (27) implies that the housing price gap affects consumption even for a *given* level of the output. As a result, the housing price gap will show up in the consumption Euler equation for output, as derived below, because different levels of consumption can be associated with a given path of output, depending on the values assumed by the housing price gap. For the same reasons, the housing gap will affect real wages. The gap will thus also show up in the Phillips curve in addition to output, as we show below.

Finally, we observe that the housing price gap will be more volatile when natural rates of interest are low, because housing prices are more volatile, as discussed in Section 3.4. Detailed derivations underlying the results in this section can be found in Appendix B.8.

Aggregate IS equation and the natural rate of interest. The equilibrium in the goods and housing markets give rise to an aggregate IS equation. It is given by²⁰

$$y_t^{gap} = E_t[y_\infty^{gap}] - E_t \left[\sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+1+k} - \tilde{r}_{t+k}) \right] + \zeta_q (\widehat{q}_t^u - \widehat{q}_t^{u*}), \quad (28)$$

and depends on long-run expectations of the output gap, $E_t[y_\infty^{gap}] \equiv \lim_{T \rightarrow \infty} E_t y_T^{gap}$, the path of future interest rates, and the housing price gap. The variable \tilde{r}_t denotes the exogenous component of the natural interest rate and depends on the productivity shocks in the economy. The parameter $\zeta_q > 0$ summarizes the aggregate demand effects of housing price gaps. It is the only channel through which subjective housing beliefs influence aggregate demand.

In the special case with efficient housing prices (rational housing price expectations), the IS equation (28) implies that setting $i_t - E_t \pi_{t+1} = \tilde{r}_t$ for all $t \geq 0$ is consistent with a constant output gap. In the presence of subjective beliefs, however, such a policy fails to keep the output gap stable due to inefficient housing price fluctuations. A policy that sets real interest rates equal to \tilde{r}_t then delivers

$$y_t^{gap} = E_t[y_\infty^{gap}] + \zeta_q (\widehat{q}_t^u - \widehat{q}_t^{u*}). \quad (29)$$

¹⁸See Appendix B.8 for the derivation.

¹⁹All variables are expressed in log deviations from the steady state.

²⁰See Appendix B.9 for the derivation.

Since $\zeta_q > 0$, a positive housing price gap is then associated with a positive output gap. As discussed before, high housing prices induce higher housing investment and thereby increase aggregate demand, which leads—for unchanged real interest rates—to an increase in aggregate output. Since the output expansion is inefficient, the policymaker might find it optimal to *lean against housing prices*. The extent to which this is optimal will be explored quantitatively in Section 5 below.

With subjective beliefs, the natural rate of interest, i.e., the real interest rate that is consistent with a constant output gap ($y_t^{gap} = E_t[y_\infty^{gap}]$ for all t) is given by²¹

$$r_t^* \equiv \check{r}_t - \zeta_q \left((\hat{q}_t^u - \hat{q}_t^{u*}) - E_t(\hat{q}_{t+1}^u - \hat{q}_{t+1}^{u*}) \right) \text{ for all } t. \quad (30)$$

The expression shows that the natural rate under subjective beliefs differs from its value under efficient housing prices if and only if the housing price gap is expected to change in the subsequent period. The natural rate will exceed its level under efficient housing prices, when the expected housing price gap in the next period is higher than the current housing price gap (and vice versa). Since the housing price gap, as well as its first difference, become more volatile as the steady state natural rate falls, equation (30) shows how a lower steady state level of the natural rate increases the volatility of the natural rate of interest in the presence of subjective beliefs. This effects is absent with rational housing price expectations.

New Keynesian Phillips Curve. The New Keynesian Phillips Curve also depends on housing price gaps. It is given by²²

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa_y y_t^{gap} + \kappa_q (\hat{q}_t^u - \hat{q}_t^{u*}). \quad (31)$$

The coefficients $\kappa_y > 0$ and $\kappa_q < 0$ imply that positive output gaps exert positive inflation pressure and positive housing price gaps exert *negative* cost-push effects. This is because inefficiently high housing prices increase housing investment and, for a given output gap, decrease (non-housing) consumption. The latter raises the marginal utility of consumption and thereby depresses wages and the marginal costs for producing the consumption good. In principle, this allows the model to produce a non-inflationary boom in housing prices and housing investment.

4.3 Model calibration and evaluation

We now calibrate the model to explore the quantitative implications for monetary policy of housing price growth extrapolation. The calibration strategy consists of choosing a set of standard parameter values previously considered in the literature and of matching salient features of the behavior of natural interest rates and housing prices in the United States in the pre-1990 period. We then test the model along two dimensions. First, we consider the model predictions for the period 1991-2021 where the average natural rate was significantly lower.²³ Second, we show in section 4.4 that the model produces reasonable impulse responses to monetary policy disturbances.

²¹See Appendix B.10 for the derivation.

²²See Appendix B.11 for the derivation.

²³We compare across long time spans of 30 years each to obtain more reliable estimates of housing price volatility.

Table 6: Calibration of the full model

Parameter	Value	Source/Target
<i>Preferences and technology</i>		
β	0.9917	Average U.S. natural rate pre 1990
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	-0.0023	Jointly chosen to match (i) steady state $\frac{k}{c}$ ratio of 6.5%
ζ_q	0.29633	and (ii) long-run housing supply elasticity equal to 5
δ	$\frac{3\%}{4}$	Adam and Woodford (2021)
<i>Exogenous shock processes</i>		
$\rho_{\tilde{r}}$	0.8	Adam and Billi (2006)
$\sigma_{\tilde{r}}$	0.1394% [RE: 0.2940%]	Adam and Billi (2006)
ρ_ξ	0.99	Quarterly autocorr. of the price-to-rent ratio of 0.99
σ_ξ	1.65% [RE: 2.33%]	Standard deviation of price-to-rent ratio pre-1990
<i>Subjective belief parameters</i>		
$1/\alpha$	0.7%	Adam et al. (2016)
$\bar{\gamma}^u$	1.0031	Max. percentage deviation of price-to-rent ratio from mean

Calibration to the pre-1990 period. Table 6 summarizes the model parameterization. The quarterly discount factor β is chosen such that the steady-state natural rate equals the pre-1990 average of the U.S. natural rate of 3.34%, as estimated by Holston et al. (2017). The slope of the Phillips curve κ_y , and the welfare weight Λ_y/Λ_π are taken from Adam and Billi (2006, Table 2). The Phillips curve coefficient κ_q and ζ_q are chosen to match the average U.S. housing investment to consumption ratio of 6.5% and the long-run housing supply elasticity of 5, in line with the estimated value in Adam, Marcet, and Kuang (2012) and in the range of estimates in Topel and Rosen (1988).²⁴

As before, we use the AR(1) process (13) for housing demand shocks with a quarterly shock persistence $\rho_\xi = 0.99$. The standard deviation of the innovations to the housing preferences σ_ξ are set such that the model replicates the pre-1990 standard deviation of the price-to-rent ratio. This is achieved by simulating equations (10) and (12), which requires specifying the belief updating parameters α and $\bar{\gamma}^u$. We set $1/\alpha = 0.7\%$ following Adam et al. (2016) as in Section 3 and determine σ_ξ and $\bar{\gamma}^u$ jointly such that (i) we match the volatility of the price-to-rent ratio and (ii) the simulated data matches the maximum deviation of the price-to-rent ratio in the data from its sample mean. This pins down $\bar{\gamma}^u = 1.0031$ and $\sigma_\xi = 1.65\%$.

We also consider an AR(1) process for the exogenous part of the natural rate

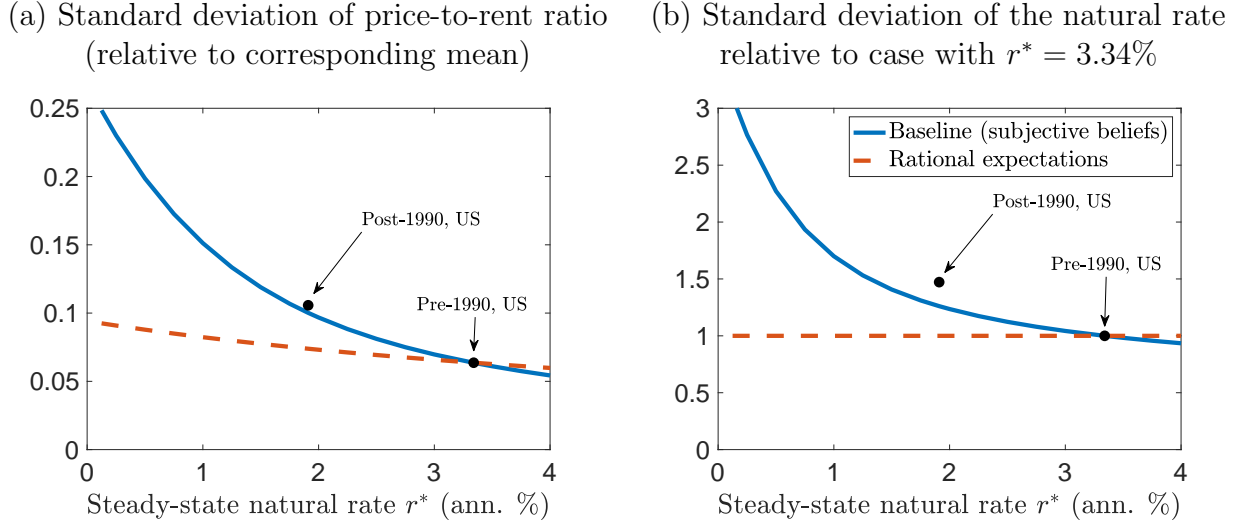
$$\tilde{r}_t = \rho_{\tilde{r}}\tilde{r}_{t-1} + \varepsilon_t^{\tilde{r}}, \quad (32)$$

where $\varepsilon_t^{\tilde{r}} \sim iiN(0, \sigma_{\tilde{r}}^2)$. We set $\rho_{\tilde{r}}$ as in Adam and Billi (2006) and choose $\sigma_{\tilde{r}}$ such that the generalized natural rate for the subjective belief model, defined in equation (30), also matches the natural rate volatility in Adam and Billi (2006). This yields $\sigma_{\tilde{r}} = 0.1394\%$.

We also calibrate a rational expectations version of our model. To this end, we recalibrate the volatilities of the innovations to the housing preference and natural rate shock processes to match the volatility of the price-to-rent ratio and the natural rate in the pre-1990 period. This results

²⁴See Appendix B.12 for details.

Figure 5: Standard deviation of price-to-rent ratio and natural rate



Notes: This figure plots, for different steady-state levels of the natural rate, the standard deviation of the price-to-rent ratio (relative to its mean) and the standard deviation of the natural rate.

in $\sigma_\xi = 2.33\%$ and $\sigma_{\tilde{r}} = 0.2940\%$. Not surprisingly, the RE model requires higher exogenous volatilities to match the empirically observed fluctuations in price-to-rent ratios and natural rates. All other parameters are unchanged relative to the subjective belief model.

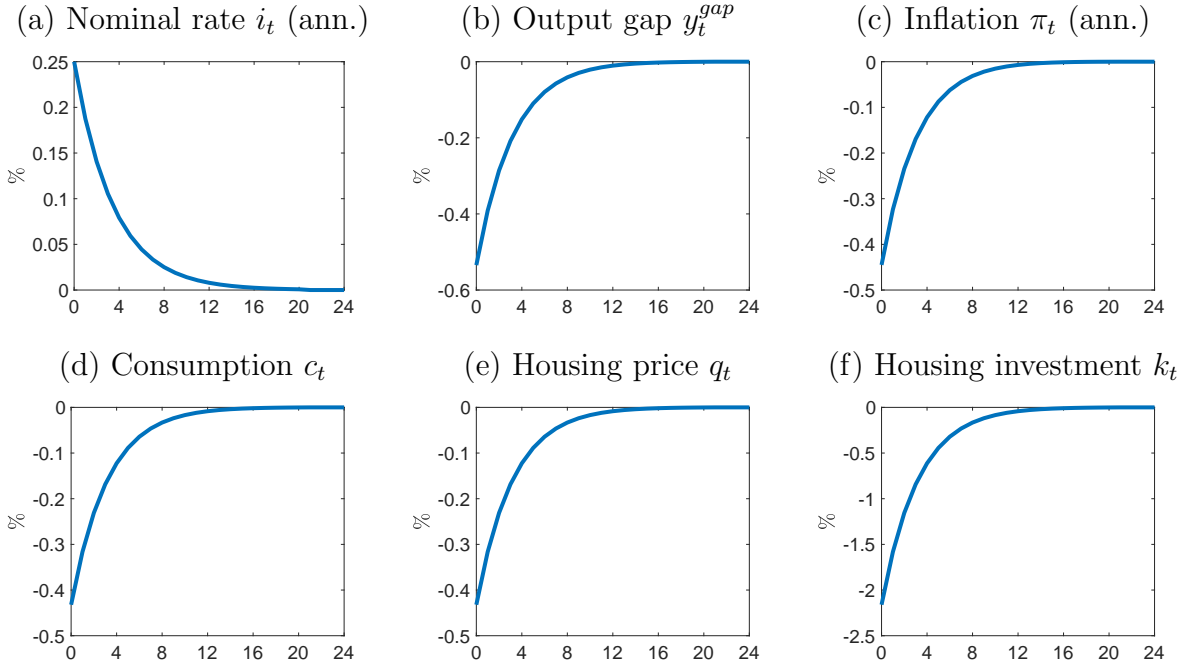
Evaluation of the model in the post-1990 period. Figure 5 illustrates the predictions of the baseline subjective belief model (solid line) for the standard deviation of the price-to-rent ratio (panel (a)) and the standard deviation of the natural rate of interest (panel (b)), conditional on various levels of the steady-state natural rate. The predictions of the RE model are also shown (dashed line). Variations in the steady-state level of the natural rate are achieved via appropriate variations in the discount factor β .²⁵ The dots in Figure 5 report the average values for the pre- and post-1990 U.S. sample periods, where the average natural rate was equal to 3.34% and 1.91%, respectively.²⁶

Since the model has been calibrated to the pre-1990 period, the model matches the pre-1990 data points in Figure 5 with subjective beliefs and under rational expectations. The subjective belief model also performs quite well in matching the post-1990 outcomes, despite the fact that these outcomes are untargeted. In particular, the standard deviation of the price-to-rent ratio and the standard deviation of the natural rate endogenously increase as the natural rate falls, with the magnitudes roughly matching the increase observed in the data. In contrast, the RE model produces no increase in the volatility of the natural rate and only a weak increase in the volatility of the price-to-rent ratio. Matching the increase in the natural rate volatility under rational expectations would require increasing $\sigma_{\tilde{r}}$. We will consider such increases when discussing our quantitative results.

²⁵As discussed before, variations in the discount factor may be driven by variations in the long-term growth rate and/or by variations in time-preferences.

²⁶The reported increase in the standard deviation of the natural rate is again based on the estimates in [Holston et al. \(2017\)](#).

Figure 6: Impulse responses to a monetary policy shock under a Taylor rule



Notes: The figure shows the impulse response functions of the subjective belief model to a contractionary monetary policy shock when monetary policy follows a Taylor rule.

4.4 The effects of monetary policy on housing prices and beliefs

The dynamics of housing prices in marginal utility units, q_t^u , are unaffected by monetary policy, even if housing prices in units of consumption, q_t , do depend on policy. As a result, the object about which agents learn does not depend on policy and the policymaker cannot ‘manipulate’ households’ subjective housing price beliefs in a way to achieve outcomes that are potentially better than under rational expectations.²⁷ This allows side-stepping the otherwise thorny issue of how the learning rule should respond to the conduct of monetary policy.

Importantly, the model replicates the fact that housing demand and housing investment respond more strongly to monetary policy disturbances than non-housing demand, even though housing prices are flexible and goods price sticky. The model thus avoids the pitfalls of sticky price models with durable goods described in Barsky et al. (2007). Concretely, in response to an exogenous shift in the path of nominal interest rates, i , the change in housing investment and consumption satisfies at all times

$$\frac{d \log k_t}{d i} = \frac{1}{1 - \tilde{\alpha}} \cdot \frac{d \log C_t}{d i}, \quad (33)$$

where $1/(1 - \tilde{\alpha}) > 1$ is the price elasticity of housing supply, see Appendix B.7 for the proof.²⁸

Figure 6 depicts the impulse-response functions of the subjective belief model to a contractionary monetary policy shock, when monetary policy sets the nominal interest rate according to a Taylor rule: $i_t = 1.5\pi_t + \varepsilon_t^{MP}$, with ε_t^{MP} denoting an exogenous monetary policy shock. We see that the output gap, inflation, consumption, and housing investment all decrease

²⁷This is a key distinction to the setups analyzed in Molnar and Santoro (2014), Mele, Molnar, and Santoro (2020), and Caines and Winkler (2021).

²⁸Our calibration uses a supply elasticity of $1/(1 - \tilde{\alpha}) = 5$, see Appendix B.12.

following a contractionary shock. Importantly, housing investment decreases substantially more strongly than non-durable consumption. Also, housing prices are affected by monetary policy, even though housing prices in marginal utility terms, \widehat{q}_t^u , are independent of monetary policy. In the remainder of this paper, we move away from Taylor rules, and instead characterize Ramsey optimal monetary policy.

5 Optimal monetary policy in the presence of extrapolative housing price expectations

We now examine how the conduct of optimal monetary policy is affected by subjective housing price beliefs and the average level of the natural rate of interest. To this end, we derive the policymaker's Ramsey problem and show how housing price growth extrapolation generates new monetary policy trade-offs. We focus in the main text on the quantitative implications of housing price beliefs for the optimal inflation target and the policy response to housing demand shocks, but Appendix C.4 characterizes the optimal targeting rule analytically.

5.1 The Ramsey optimal policy problem

We consider optimal monetary policy under commitment with a policymaker that maximizes household utility subject to the constraint that prices and allocations constitute an internally rational expectations equilibrium (IRRE). The policymaker holds rational expectations, i.e., understands that the private sectors' housing price beliefs are distorted and thus acts under a probability measure different from the one entertained by households.²⁹ To obtain analytic insights, we derive the quadratic approximation to the non-linear policy problem:³⁰

$$\max_{\{\pi_t, y_t^{gap}, \widehat{q}_t^u, i_t \geq i\}} -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_{\pi} \pi_t^2 + \Lambda_y (y_t^{gap})^2 + \Lambda_q (\widehat{q}_t^u - \widehat{q}_t^{u*})^2 \right) \quad (34)$$

subject to:

$$y_t^{gap} = E_t [y_{\infty}^{gap}] - E_t \left[\sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}) \right] + \zeta_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \quad (35)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + \kappa_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \quad (36)$$

$$\widehat{q}_t^u - \widehat{q}_t^{u*} = \left(\frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\gamma_t^u} - \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\rho_{\xi}} \right) \widehat{\xi}_t^d + \frac{\beta(1 - \delta)(\gamma_t^u - 1)}{1 - \beta(1 - \delta)\gamma_t^u} \quad (37)$$

$$\text{Belief updating: } \begin{cases} \gamma_t^u = \min \left\{ \gamma_{t-1}^u + \frac{1}{\alpha} \left(\frac{q_{t-1}^u}{q_{t-2}^u} - \gamma_{t-1}^u \right), \overline{\gamma}^u \right\} \\ q_t^u = \frac{1}{1 - \beta(1 - \delta)\gamma_t^u} \xi_t^d \end{cases} \quad (38)$$

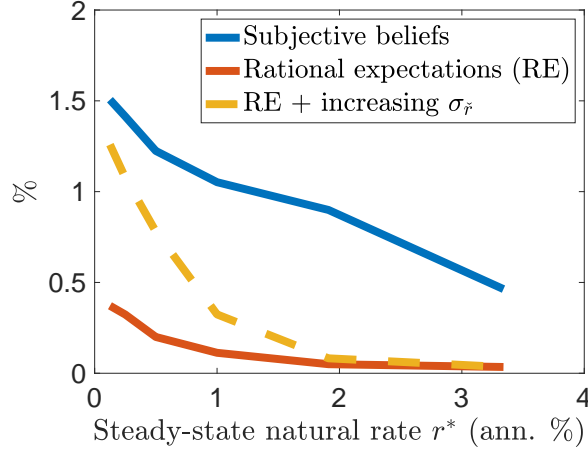
for $t \geq 0$, and initial pre-commitments.

The policymaker's objective (34) is derived in Appendix C.2 and involves the standard terms of squared inflation and the squared output gap, but also depends on the squared housing price gap. The latter arises because deviations of housing prices from their efficient level distort—for a given level of the output gap—housing investment, as explained in Section 4.2. The IS equation

²⁹Benigno and Paciello (2014) refer to such a policymaker as a “paternalistic” policymaker.

³⁰Appendix C.1 presents the non-linear optimal policy problem.

Figure 7: Average inflation under optimal monetary policy



Notes: The figure reports the optimal inflation target for different levels of the steady-state natural rate in the presence of a zero lower bound constraint. The red line shows the optimal target for the case with rational housing price beliefs and the blue line the one with subjective housing price beliefs. The yellow line shows the optimal average inflation under RE where the exogenous volatility of the natural rate is adjusted such that it matches the endogenous volatility increase under subjective beliefs.

(35) and the New Keynesian Phillips Curve (36) are constraints associated with optimal private sector behavior. They both depend on the housing price gap, as discussed in Section 4.2. The housing price gap is determined by equation (37) and depends on subjectively expected housing price growth γ_t^u . The dynamics of γ_t^u are jointly determined by the equations in (38).

Since monetary policy cannot directly influence the housing price gap, as explained in Section 4.4, the loss from housing price gaps in the welfare function can be ignored for the computation of optimal policy; housing price gaps thus matter for policy only through their effect on the IS equation and the Phillips curve.

The policymaker's choice of the nominal interest rate i_t is subject to an effective lower bound $i_t \geq \underline{i}$, where the bound $\underline{i} < 0$ is expressed in terms of deviations from the interest rate in a zero-inflation steady state. For the special case with a zero lower bound, we have $\underline{i} = -(1 - \beta)/\beta$. In the absence of a lower bound constraint or when economic shocks never cause the bound to become binding, the IS equation (35) can be dropped from the policy problem.

Interestingly, the expectations showing up in the monetary policy problem (34)-(38) are all rational. The way subjective housing price expectations affect the monetary policy problem are thus fully captured through their effects on the housing price gap.

Finally, in the special case with rational housing price expectations, the housing gap is equal to zero at all times, so that the monetary policy problem reduces to the textbook case with a lower bound constraint.

We recursify the optimal policy problem following Marcet and Marimon (2019) and numerically solve for the value functions and optimal policy functions, see Appendix C.3 for details.

5.2 The optimal inflation target

We define the optimal inflation target as the average inflation rate emerging under Ramsey optimal monetary policy. The inflation target thus captures the average inflation rate that the Ramsey planner commits to implement in the stochastic equilibrium. Depending on the realization of

shocks, actual inflation will fluctuate around this average value, as discussed below.

Figure 7 depicts the optimal inflation target for different steady-state levels of the natural rate of interest.³¹ The figure graphs the optimal target for the setup with subjective housing beliefs (upper solid line), for the case with rational expectations (RE) about housing prices (lower solid line), and for a third case discussed below (dashed line).

Under RE, the optimal target is close to zero for any steady-state level of the natural rate. This confirms earlier findings in Adam and Billi (2006), who considered a high value for the steady-state natural rate and found that the presence of a lower bound constraint cannot justify targeting significantly positive inflation rates. It may be surprising that this holds true also for low steady-state levels of the natural rate: under RE, optimal policy seeks to track the natural rate of interest, but a low steady-state level implies that this is not always feasible. Policy must then use promises of future inflation to lower real interest rates and this more often, the lower is the steady-state natural rate. Yet, lower-bound episodes are relatively infrequent and short-lived, independently of the steady-state level of the natural rate. As a result, the inflation target does not react strongly to the steady-state (or average) level of the natural rate.

This invariance to the average level of the natural rate differs substantially from the findings in Andrade et al. (2019). They show that the optimal target should move up approximately one-to-one with a fall in the average natural rate under rational expectations. Besides considering a medium-scale sticky price model without housing, a main difference to our approach is that Andrade et al. (2019) study Taylor rules with optimized intercepts rather than optimal monetary policy. Coibion et al. (2012) show that this makes a big difference for how the optimal inflation target responds to lower average values of the natural rate compared to the case with Ramsey optimal policy.

The upper line in Figure 7 shows that the situation is fundamentally different with subjective housing beliefs. The optimal inflation target is overall substantially higher and also depends more strongly on the average natural rate of interest. In particular, a fall in the steady-state natural rate from its pre-1990 average of 3.34% to its post-1990 average of 1.9% already causes the optimal inflation target to increase by approximately 0.5 percentage points. The difference relative to the case with rational expectations arises because the volatility of the natural rate endogenously increases once the natural rate drops, due to the higher volatility of the housing price gaps. This reinforces the stringency of the zero lower bound constraint associated with a lower average value of the natural rate and causes the central bank to engage more often in inflation promises.

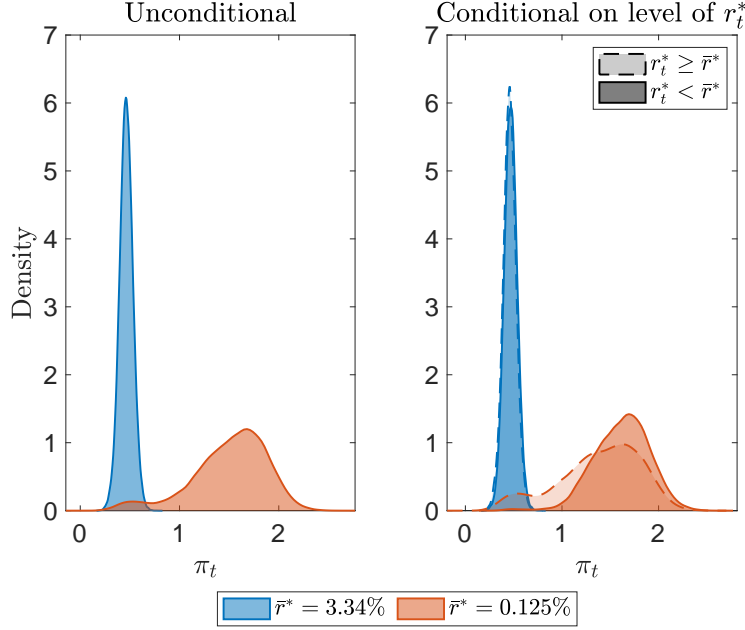
The optimal inflation target with subjective housing beliefs is also substantially higher than the optimal target under rational expectations. This holds true even for the pre-1990 average level of the natural rate (3.34%) for which the calibration implies that the natural rate is equally volatile for both belief specifications.

This outcome is due to two reasons: First, fluctuations in the housing price gap also generate cost-push terms in the Phillips curve. Second, belief fluctuations induce more persistent variations in the natural rate than the exogenous natural rate shocks. This puts further upward pressure on the optimal inflation rate, as it requires larger and more persistent inflation promises by the central bank.

To illustrate the role of the *volatility* of the natural rate, the dashed line in Figure 7 depicts the optimal inflation rate under rational expectations, when we set the volatility of the (exogenous) natural rate in the RE model such that it matches the volatility of the natural rate in the subjective belief model, for each considered level of the natural rate. While the optimal inflation rate increases somewhat relative to the benchmark RE setting, the level of the optimal inflation target still falls

³¹The average level is computed by simulating the model under the optimal monetary policy for 100,000 periods.

Figure 8: Distribution of inflation rates under optimal monetary policy



Notes: The figure reports the distribution of inflation rates (in annualized %) for two levels of the steady-state natural rate (also annualized) in the presence of a zero lower bound constraint. The left panel graphs the unconditional distributions of inflation. The right panel conditions distributions on whether the natural rate, r_t^* , is above or below its steady-state level.

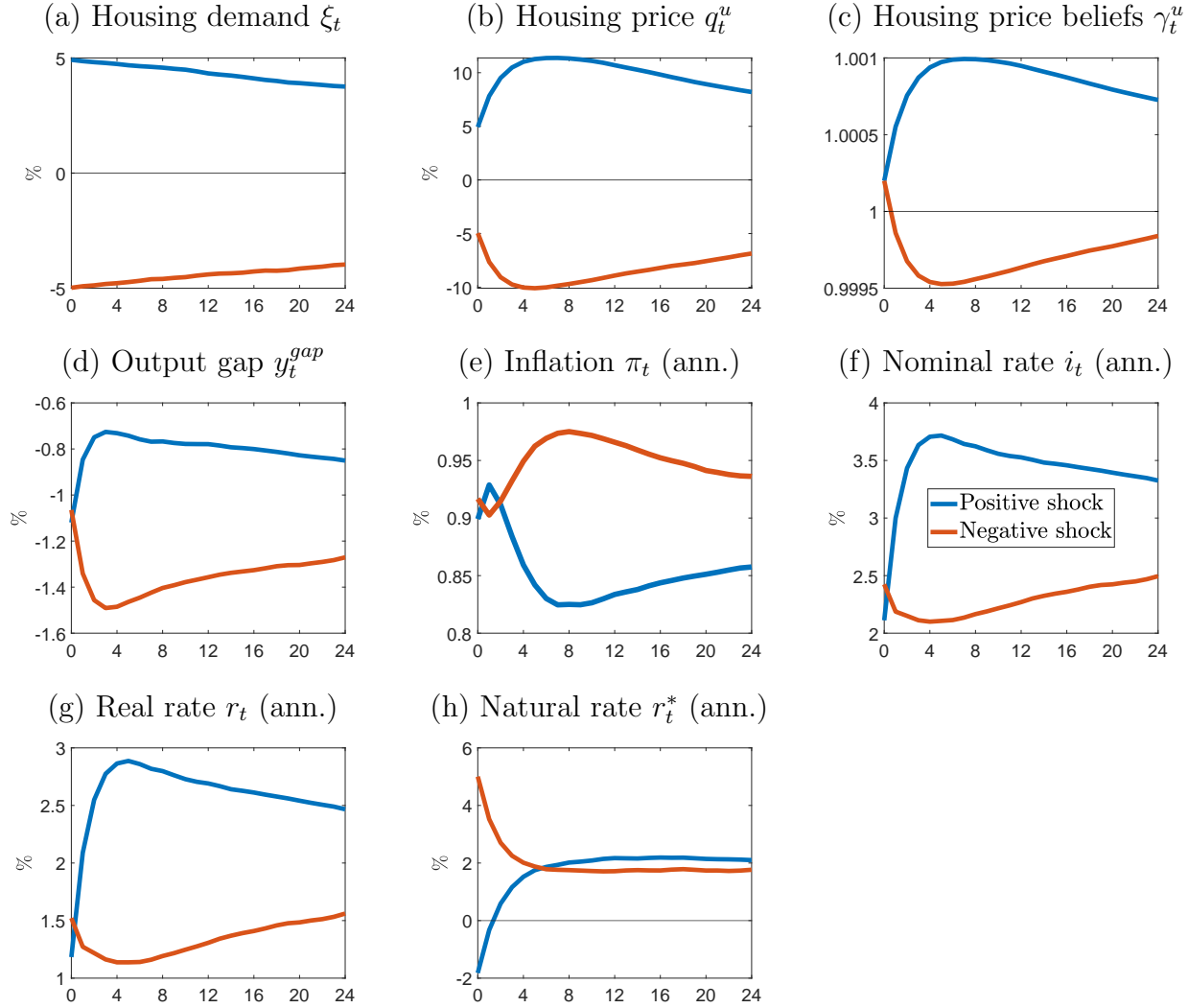
short of the one implied by subjective beliefs. However, the optimal inflation target now depends more strongly on the steady-state level of the natural rate and approaches the optimal inflation targets under subjective beliefs when the steady-state natural rate becomes very low.

We can gain further insights into the inflation process under Ramsey optimal policy by considering the stochastic distribution of inflation rates. The left panel in Figure 8 depicts the unconditional distribution of inflation for the lowest and highest steady-state levels of the natural rate considered in Figure 7. It shows that inflation is tightly centered around the optimal inflation target when the steady-state natural rate is high. For a low steady-state natural rate, the inflation distribution shifts to the right, but also displays a long left tail of low inflation rates. This shows that it is optimal for monetary policy to sometimes implement inflation rates that lie significantly below the target.

The right panel in Figure 8 explains why periods of low inflation are optimal: it depicts the inflation distribution conditional on the natural rate being above or below its steady-state value. The left tail with low inflation rates emerges in situations where the natural rate is temporarily high. In such situations, the policymaker does not need to promise future inflation to lower real interest rates, simply because the lower bound constraint is sufficiently far from being binding.³² This shifts the inflation distribution closer to the one emerging with a high steady-state level for the natural rate. Temporarily high natural rates thus make below-target inflation optimal in a setting where the steady-state natural rate is low. In contrast, it is optimal to induce higher inflation rates—sometimes substantially above the average inflation rate—when the natural rate is below its steady state level. These higher inflation rates capture the promises the monetary

³²Due to other state variables shifting around, the lower bound still binds with some probability, but this probability is lower when the natural rate is above its steady-state level.

Figure 9: Impulse responses to a housing preference shock under optimal monetary policy



Notes: The figure reports the average impulse responses of the economy under subjective beliefs (at $r^* = 1.91\%$) after a three-standard-deviation housing demand shock. The blue lines show the responses after a positive shock and the red lines after a negative shock.

authority makes when currently constrained by the lower bound. This differs from a setting with a high steady-state natural rate: conditional distributions are then almost independent of whether the natural rate is above or below its steady-state value, see the right panel in Figure 8.

5.3 Asymmetric leaning against housing demand shocks

We now examine the optimal monetary policy response to housing demand shocks. Under RE, housing demand shocks do not affect the housing price gap and thus also do not generate policy trade-offs. This is different with subjective housing price beliefs. Housing demand shocks then move housing prices and thereby housing price expectations. The latter then affect the housing price gap, which generates movements in the natural rate and leads to cost-push terms.

We show that this makes it optimal to ‘lean against’ housing demand shocks in the presence of subjective beliefs. Yet, due to the lower bound constraint, the response to positive and

negative housing demand shocks displays considerable asymmetry: it is optimal that real rates increase considerably following positive housing demand shocks, while they fall only weakly following negative shocks. Since positive (negative) housing demand shocks generate a persistent housing price increase (decrease), monetary policy leans strongly against shocks that give rise to housing price booms but only weakly against shocks producing a housing price bust.

The top row in Figure 9 shows the response of housing-related variables to a persistent positive/negative housing demand shock of 5%.³³ On impact, the shock triggers housing price growth of an equal amount, which then triggers belief revisions that fuel further movements of the housing price in the same direction. The positive shock, for instance, pushes housing prices up by about 5% on impact, with belief momentum generating approximately another 5% in the first six quarters after the shock. This causes the housing price gap to become significantly positive (not shown in the figure). Once actual housing price increases start to fall short of the expected housing price increases, the housing boom reverts direction.

Higher housing prices push up housing investment, which causes upward pressure on the output gap. Optimal monetary policy leans strongly against the housing price and increases nominal and real interest rates. It does so despite the fact that the natural rate of interest falls in response to the shock. The policy response causes a fall in inflation, which is amplified by the fact that the increase in housing prices and investment increases the marginal utility of consumption, hence, dampens wages and marginal costs. A positive housing demand shock thus results—in the presence of subjective housing beliefs—in a disinflationary housing boom episode under optimal monetary policy.

The policy response to a positive housing demand shock is much stronger than that to a negative housing demand shock. In particular, nominal and real interest rates fall considerably less following a negative shock realization. This is so because a negative housing price gap is inflationary and inflation is already high to start with. Negative housing demand shocks thus move inflation further away from its optimal level of zero.³⁴ Yet, policy still ‘leans against’ the housing price decrease: real interest rates fall despite the fact that the natural rate increases.

The fact that leaning against housing prices can be optimal in the presence of housing price growth extrapolation is in line with results in Caines and Winkler (2021), who consider a setting with “conditionally model consistent beliefs” in which expectations about many variables differ from rational expectations, and with results in Adam and Woodford (2021), who consider a setting where the policymaker fears “worst-case” belief distortions about inflation and housing price expectations. As none of these papers consider a lower-bound constraint, the policy response to positive and negative shocks is symmetric in their settings.

6 The role of macroprudential policy

It is often argued that macroprudential policies can be used to stabilize financial markets and that this would allow monetary policy to ignore disturbances coming from the housing sector, see Svensson (2018) for a prominent exposition of this view. In this section, we evaluate the quantitative plausibility of this view within our setup with subjective housing beliefs.

³³We initialize the economy at its ergodic mean and then hit the economy with a one-time shock of three standard deviations. We then average the subsequent response over the possible future shock realizations. We assume a steady-state natural rate equal to its post-1990 mean (1.91%).

³⁴While the output gap is moved closer to its optimal level, the weight on the output gap in the welfare function is two orders of magnitude smaller than that on inflation, see Table 6.

We show below that fully eliminating fluctuations in the housing price gap requires imposing large and very volatile macroprudential taxes. None of the macroprudential instruments currently available in advanced economies appear suited to achieve effects anywhere near the required size. In addition, we show that it is often necessary to implement substantial subsidies, which, to the best of our knowledge, none of the available macroprudential instruments can achieve. Less aggressive policies that aim at only partly eliminating the housing price gap still require considerable tax volatility, because fluctuations in subjective beliefs turn out not to be independent of the tax policy pursued.

We consider a setup in which the policymaker can tax or subsidize the ownership of housing. While actual macroprudential policies often operate via constraints imposed on the banking sector, their ultimate effect is to make housing more or less expensive to households. For this reason, we consider taxes and subsidies at the household level. Specifically, we analyze a proportional and time-varying tax τ_t^D that is applied to the rental value of housing in every period t . A household owning D_t units of houses, then has to pay taxes of

$$\tau_t^D D_t R_t \quad (39)$$

units of consumption.³⁵ We find this specification more plausible than a policy that taxes the market value of housing, as it is difficult to determine market values in real time. A setup that taxes the physical housing units, i.e., where taxes are equal to $\tau_t^D D_t$, delivers very similar results, but is analytically more cumbersome. Furthermore, the tax setup in equation (39) is equivalent to a setup where taxes directly affect household utility, i.e., where the utility contribution from owning houses would instead be given by $\xi_t^d (1 - \tau_t^D) D_t$ and no monetary taxes would have to be paid. We prefer the formulation in equation (39) because it allows expressing taxes in monetary units.

In the presence of these taxes, housing prices under subjective beliefs are given by

$$q_t^u = \frac{(1 - \tau_t^D) \xi_t^d}{1 - \beta(1 - \delta)\gamma_t^u}, \quad (40)$$

and the housing price *gap* in percentage deviations from the steady state (where $\tau^D = 0$) is

$$\hat{q}_t^u - \hat{q}_t^{u*} = \frac{(1 - \beta(1 - \delta)) (1 - \tau_t^D) \hat{\xi}_t^d}{1 - \beta(1 - \delta)\gamma_t^u} + \frac{\beta(1 - \delta)(\gamma_t^u - 1)}{1 - \beta(1 - \delta)\gamma_t^u} - \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\gamma_t^u} \tau_t^D - \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\rho_\xi} \hat{\xi}_t^d. \quad (41)$$

This equation shows that the macroprudential policy must eliminate housing price gap fluctuations that are due to housing demand shocks $\hat{\xi}_t^d$ and due to fluctuations in subjective housing price growth expectations γ_t^u . Doing so requires setting the tax according to

$$\tau_t^{D*} = \frac{\beta(1 - \delta)}{1 + \hat{\xi}_t^d} \left(\frac{(\gamma_t^u - \rho_\xi)}{1 - \beta(1 - \delta)\rho_\xi} \hat{\xi}_t^d + \frac{1}{1 - \beta(1 - \delta)} (\gamma_t^u - 1) \right). \quad (42)$$

To understand what the preceding equation implies for the behavior of taxes, one has to take into account that the fluctuations in subjective beliefs γ_t^u depend themselves on the tax: the tax influences housing prices, see equation (40), and thus—via housing price growth extrapolation—the evolution of subjective beliefs.

³⁵To keep the rest of the model unchanged, the household also needs to expect lump sum tax rebates that are equal to the amount of subjectively expected tax payments.

Table 7: Taxes and housing price gap fluctuations for alternative tax sensitivities λ^D

Tax sensitivity λ^D	Housing price gap $\hat{q}_t^u - \hat{q}_t^{u*}$	Housing taxes τ_t^D		
		Std. dev.	Maximum	Minimum
0.0	14.2%	0.0%	0.0%	0.0%
0.2	9.8%	2.4%	7.0%	-12.1%
0.4	6.4%	4.2%	13.8%	-21.7%
0.6	3.7%	5.7%	18.0%	-30.0%
0.8	1.7%	7.0%	21.3%	-36.2%
1.0	0.0%	8.0%	23.9%	-41.8%

Notes: The table reports the standard deviation of the housing gap, $\hat{q}_t^u - \hat{q}_t^{u*}$, as well as the standard deviation, minimum value and maximum value of the macroprudential tax τ_t^D , for different tax sensitivities λ^D .

To analyze the behavior of taxes, we consider the calibrated subjective belief model from Section 4.3 for the case where the average natural rate is equal to its post-1990 average (1.91%). We consider also intermediate forms of taxation that do not aim at fully eliminating the housing gap, by specifying taxes as

$$\tau_t^D = \lambda^D \tau_t^{D*}, \quad (43)$$

where $\lambda^D \in [0, 1]$ is a sensitivity parameter. Our prior setup assumed $\lambda^D = 0$, while fully eliminating the housing price gap using macroprudential policy requires setting $\lambda^D = 1$. We then simulate the dynamics of housing prices, beliefs and taxes for alternative values of λ^D .

Table 7 reports the main outcomes. It shows that a higher tax sensitivity λ^D steadily reduces the standard deviation of the housing price gap (second column). However, the standard deviation of taxes has to steadily increase. For a policy that fully eliminates the housing price gap ($\lambda^D = 1$), the standard deviation of taxes is a staggering 8% of the rental value of housing. Taxes reach maximum values up to 24% and minimum values deeply in negative territory, with subsidies above 40% of the rental value. These taxes fully stabilize the housing price gap but still induce substantial variation in subjective beliefs. The latter explains why taxes have to remain rather volatile. Intermediate policies, say ones that set $\lambda^D = 0.4$, substantially reduce the volatility of the housing gap, but still require rather volatile taxes and often very large subsidies.

Given the outcomes in Table 7, we conclude that the currently available macroprudential instruments will likely not be able to insulate the monetary authority from disturbances in the housing sector arising from housing price growth extrapolation.

7 Conclusion

This paper empirically documents three key deviations of households' housing price expectations from full-information rational housing price expectations and constructs a structural equilibrium model that jointly replicates the behavior of housing prices and the patterns of subjective beliefs. The model shows that subjective housing price beliefs significantly contribute to housing price fluctuations and that lower natural rates of interest generate increased volatility for housing prices and the natural rate.

Optimal monetary policy responds to lower and more volatile natural rates by implementing

higher average inflation rates. Monetary policy should also lean against housing price fluctuations induced by housing demand shocks, with reactions to housing price increases being more forceful than the reaction to housing price downturns. None of these features is optimal if households hold rational housing price expectations. This highlights the importance of basing policy advice on economic models featuring empirically plausible specifications for household beliefs.

References

- ADAM, K. AND R. M. BILLI (2006): “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates,” *Journal of Money, Credit and Banking*, 38(7), 1877–1905.
- ADAM, K. AND A. MARCET (2011): “Internal Rationality, Imperfect Market Knowledge and Asset Prices,” *Journal of Economic Theory*, 146, 1224–1252.
- ADAM, K., A. MARCET, AND J. BEUTEL (2017): “Stock Price Booms and Expected Capital Gains,” *American Economic Review*, Vol. 107 No. 8, 2352–2408.
- ADAM, K., A. MARCET, AND P. KUANG (2012): “House Price Booms and the Current Account,” in *NBER Macroeconomics Annual 2011*, ed. by D. Acemoglu and M. Woodford, 77 – 122.
- ADAM, K., A. MARCET, AND J. P. NICOLINI (2016): “Stock Market Volatility and Learning,” *Journal of Finance*, 71(1), 33–82.
- ADAM, K. AND S. NAGEL (2022): “Expectations Data in Asset Pricing,” in *forthcoming in: Handbook of Economic Expectations*, Elsevier.
- ADAM, K. AND M. WOODFORD (2021): “Robustly Optimal Monetary Policy in a New Keynesian Model with Housing,” *Journal of Economic Theory*, 198, Article 105352.
- ANDRADE, P., J. GALÍ, H. LE BIHAN, AND J. MATHERON (2019): “The Optimal Inflation Target and the Natural Rate of Interest,” *Brookings Papers on Economic Activity*, 2019(2), 173–255.
- (2021): “Should the ECB Adjust its Strategy in the Face of Lower r^* ?” *Journal of Economic Dynamics and Control (forthcoming)*.
- ANGELETOS, G.-M., Z. HUO, AND K. A. SASTRY (2020): “Imperfect Macroeconomic Expectations: Evidence and Theory,” in *NBER Macroeconomics Annual 2020*, Vol. 35.
- ARMONA, L., A. FUSTER, AND B. ZAFAR (2019): “Home Price Expectations and Behaviour: Evidence from a Randomized Information Experiment,” *The Review of Economic Studies*, 86, 1371–1410.
- BARSKY, R. B., C. L. HOUSE, AND M. S. KIMBALL (2007): “Sticky-Price Models and Durable Goods,” *American Economic Review*, 97, 984–998.
- BENIGNO, P. AND L. PACIELLO (2014): “Monetary Policy, Doubts and Asset Prices,” *Journal of Monetary Economics*, 64, 85–98.
- CAINES, C. AND F. WINKLER (2021): “Asset Price Beliefs and Optimal Monetary Policy,” *Journal of Monetary Economics*, 123, 53–67, federal Reserve Board mimeo.

- CALVO, G. A. (1983): “Staggered Contracts in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12, 383–398.
- CASE, K. E., R. J. SHILLER, AND A. K. THOMPSON (2012): “What Have They Been Thinking? Homebuyer Behavior in Hot and Cold Markets,” *Brookings Papers on Economic Activity*, 265–315.
- CLARIDA, R., J. GALÍ, AND M. GERTLER (1999): “The Science of Monetary Policy: Evidence and Some Theory,” *Journal of Economic Literature*, 37, 1661–1707.
- COIBION, O. AND Y. GORODNICHENKO (2015): “Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts,” *American Economic Review*, 105, 2644–78.
- COIBION, O., Y. GORODNICHENKO, AND J. WIELAND (2012): “The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?” *Review of Economic Studies*, 79, 1371–1406.
- EGGERTSSON, G. AND M. WOODFORD (2003): “The Zero Interest-Rate Bound and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, (1), 139–211.
- FUJIWARA, S., Y. IWASAKI, I. MUTO, K. NISHIZAKI, AND N. SUDO (2016): “Developments in the Natural Rate of Interest in Japan,” *Bank of Japan Review*.
- GORODNICHENKO, Y. AND M. WEBER (2016): “Are sticky prices costly? Evidence from the stock market,” *American Economic Review*, 106, 165–99.
- GÜRKAYNAK, R. S., B. SACK, AND E. SWANSON (2005): “The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models,” *American economic review*, 95, 425–436.
- HOLSTON, K., T. LAUBACH, AND J. C. WILLIAMS (2017): “Measuring the natural rate of interest: International trends and determinants,” *Journal of International Economics*, 108, S59 – S75, 39th Annual NBER International Seminar on Macroeconomics.
- JORDÀ, (2005): “Estimation and Inference of Impulse Responses by Local Projections,” *American Economic Review*, 95, 161–182.
- KAPLAN, G., K. MITMAN, AND G. L. VIOLANTE (2020): “The Housing Boom and Bust: Model Meets Evidence,” *Journal of Political Economy*, 128, 3285–3345.
- KOHLHAS, A. N. AND A. WALTHER (2021): “Asymmetric Attention,” *American Economic Review*, 119, 2879–2925.
- KUCHLER, T. AND B. ZAFAR (2019): “Personal Experiences and Expectations about Aggregate Outcomes,” *Journal of Finance*, VOL. LXXIV.
- MA, C. (2020): “Momentum and Reversion to Fundamentals: Are They Captured by Subjective Expectations of House Prices?” *Journal of Housing Economics*, 49, 101687.
- MARCET, A. AND R. MARIMON (2019): “Recursive contracts,” *Econometrica*, 87, 1589–1631.

- MELE, A., K. MOLNAR, AND S. SANTORO (2020): “On the perils of stabilizing prices when agents are learning,” *Journal of Monetary Economics*, 115, 339–353.
- MOLNAR, K. AND S. SANTORO (2014): “Optimal monetary policy when agents are learning,” *European Economic Review*, 66, 39–62.
- PIAZZESI, M. AND M. SCHNEIDER (2006): *Equilibrium Yield Curves*, University of Chicago Press, vol. 21 of *NBER Macroeconomics Annual*, chap. 6, 389–442.
- STAMBAUGH, R. F. (1999): “Predictive Regressions,” *Journal of Financial Economics*, 54, 375–421.
- SVENSSON, L. E. O. (2018): *Festschrift in Honour of Vitor Constancio*, European Central Bank, chap. The Future of Monetary Policy and Macroprudential Policy, 69–123.
- TOPEL, R. AND S. ROSEN (1988): “Housing Investment in the United States,” *Journal of Political Economy*, 96, 718–740.

Appendix A Robustness of empirical results

A.1 Five-year-ahead housing price growth expectations

While for our baseline results in Section 2 we focus on short-term housing price expectations, our findings equally hold for medium-term five-year-ahead expectations. We estimate the five-year analogue of regression (1) as follows:

$$q_{t+20} - E_t^{\mathcal{P}} [q_{t+20}] = a^{CG} + b^{CG} \cdot (E_t^{\mathcal{P}} [q_{t+20}] - E_{t-1}^{\mathcal{P}} [q_{t+19}]) + \varepsilon_t. \quad (\text{A.1})$$

Table A.1 reports the estimates of b^{CG} showing that five-year expectations are updated sluggishly.

Table A.1: Sluggish adjustment of five-year-ahead housing price expectations

	Mean expectations	Median expectations
\widehat{b}^{CG}	6.95*** (1.703)	6.89*** (1.680)

Notes: This table reports the empirical estimates of regression (A.1) using nominal housing price expectations. The reported standard errors are robust with respect to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

We also run five-year-ahead versions of the regressions (2) and (3):

$$E_t^{\mathcal{P}} \left[\frac{q_{t+20}}{q_t} \right] = a + c \cdot PR_{t-1} + u_t \quad (\text{A.2})$$

$$\frac{q_{t+20}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t. \quad (\text{A.3})$$

Table A.2 shows that five-year-ahead housing price growth expectations correlate positively with the price-to-rent ratio, whereas actual housing price growth correlates negatively.

Table A.2: Expected vs. actual housing price growth using five-year-ahead housing price expectations

	\hat{c} (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
Mean expectations	0.045 (0.0001)	-1.889 (0.01997)	0.0159	0.000
Median expectations	0.044 (0.00024)	-1.889 (0.01997)	0.0155	0.000

Notes: \hat{c} is the estimate of c in equation (A.2) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (A.3). The [Stambaugh \(1999\)](#) small-sample bias correction is reported in the second-to-last column and the last column reports the p -values for the null hypothesis $c = \mathbf{c}$. Newey-West standard errors using four lags in parentheses.

A.2 IV estimation of sluggish belief updating

To ensure that the results obtained from regression (1) in Section 2 are not driven by forecast revisions being correlated with the error term, we follow Coibion and Gorodnichenko (2015) by adopting an IV approach. Specifically, we consider monetary policy shocks as an instrument for forecast revisions. We identify daily monetary policy shocks as changes of the current-month federal funds future in a 30-minute window around scheduled FOMC announcements (following the approach in Gürkaynak, Sack, and Swanson, 2005; Gorodnichenko and Weber, 2016). We then aggregate shocks to quarterly frequency by assigning daily shocks partly to the current quarter and partly to the consecutive quarter, based on the number of remaining days in the current quarter. Table A.3 reports the results of the IV regression. The coefficients are positive and statistically significant, with point estimates that are even larger than the ones reported in Section 2.

Table A.3: Instrumental variable regression

	Mean expectations	Median expectations
<i>Nominal housing prices</i>		
\hat{b}^{CG}	2.85** (1.259)	3.84*** (1.497)
First-stage F -statistic	21.88	17.78
<i>Real housing prices</i>		
\hat{b}^{CG}	2.62*** (0.745)	3.45*** (0.649)
First-stage F -statistic	44.49	34.13

Notes: \hat{b}^{CG} report the results from regression (1), instrumenting forecast revisions using monetary policy shocks, obtained via high-frequency identification. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.3 Sluggish adjustment of housing price growth expectations

Regression (1) in Section 2 studies sluggish adjustment of expectations about the housing price level. Similar results can be obtained when considering expectations about housing price growth. *Specification 1* in Table A.4 reports the regression coefficient when one replaces actual and expected housing price levels on the left-hand side of equation (1) with actual and expected housing price growth. The coefficient estimates remain positive and highly statistically significant. *Specification 2* in Table A.4 reports results when replacing expectations about housing price levels with expectations about housing price growth on the right-hand side of equation (1) and *Specification 3* reports results when replacing levels by (actual and expected) housing price growth on both sides of equation (1). The coefficient estimates remain positive, but the significance levels are lower for Specifications 2 and 3.

Table A.4: Sluggish adjustment of housing price growth expectations

	Mean expectations	Median expectations
<i>Specification 1</i>		
<i>Nominal housing prices</i>		
\hat{b}^{CG}	0.023*** (0.005)	0.030*** (0.005)
<i>Real housing prices</i>		
\hat{b}^{CG}	0.024*** (0.004)	0.031*** (0.004)
<i>Specification 2</i>		
<i>Nominal housing prices</i>		
\hat{b}^{CG}	492* (279)	182 (210)
<i>Real housing prices</i>		
\hat{b}^{CG}	302* (164)	158 (168)
<i>Specification 3</i>		
<i>Nominal housing prices</i>		
\hat{b}^{CG}	5.20* (2.896)	2.16 (2.06)
<i>Real housing prices</i>		
\hat{b}^{CG}	3.23* (1.678)	2.06 (1.835)

Notes: This table shows the results of regression (1) in terms of house-price growth rates instead of house-price levels. See the text above for more details on the specifications. Standard errors (in parentheses) are robust to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels:

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.4 Cyclicalities of housing price forecast errors

A similar version of the test from [Adam et al. \(2017\)](#) presented in Section 2, is proposed by [Kohlhas and Walther \(2021\)](#). In this case, we regress forecast errors about housing prices on the price-to-rent ratio. Formally, we estimate

$$\frac{q_{t+4}}{q_t} - E_t^{\mathcal{P}} \left[\frac{q_{t+4}}{q_t} \right] = \alpha + \gamma \cdot PR_{t-1} + \varepsilon_t. \quad (\text{A.4})$$

Table A.5 shows the results. We find a negative and statistically significant coefficient in all cases. Thus, consumers tend to become too optimistic (pessimistic) when they observe high (low) housing valuations, inconsistent with rational expectations.

Table A.5: Forecast errors and price-to-rent ratios

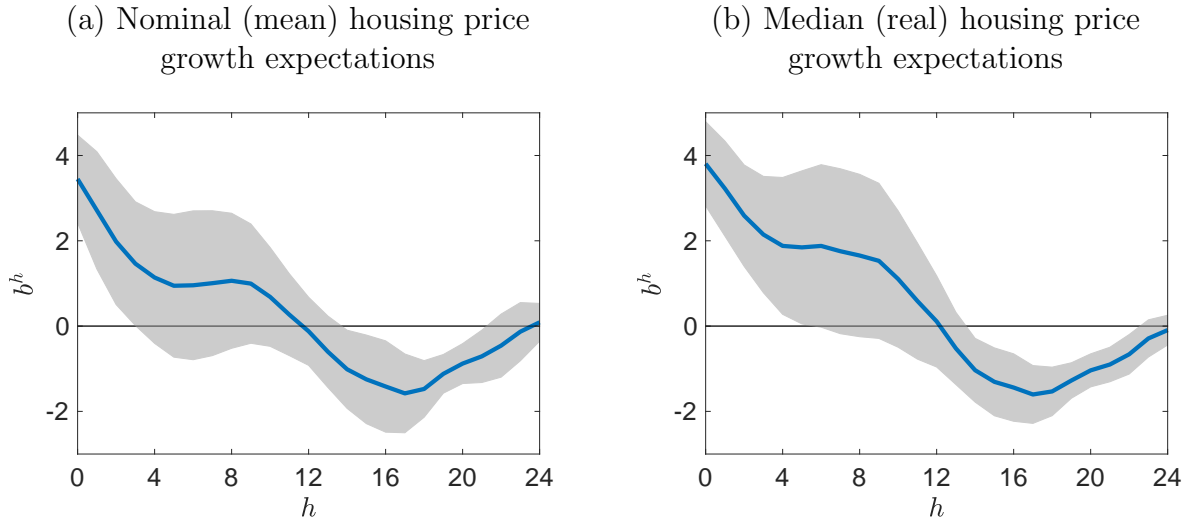
	Mean expectations	Median expectations
<i>Nominal housing prices</i>		
$\hat{\gamma}$	-0.5^{***} (0.09)	-0.5^{***} (0.10)
<i>Real housing prices</i>		
$\hat{\gamma}$	-0.5^{***} (0.08)	-0.5^{***} (0.10)

Notes: This table shows the results of regression (A.4), whereas the estimated regression coefficients (and standard errors) are multiplied by one hundred for better readability. The reported standard errors are robust with respect to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.5 Dynamics of forecast errors with median and nominal housing price expectations

Figure A.1 shows alternative specifications of the dynamic forecast error responses presented in Section 2. Panel (a) presents the response of forecast errors for nominal housing prices. Panel (b) shows the response of forecast errors for real housing prices (as in Section 2) but considering median expectations. The figure shows that these responses are very close to the baseline specification shown in Section 2.

Figure A.1: Dynamic Forecast error response to realized housing price growth



Notes: Panel (a) shows impulse-response functions of nominal housing price growth forecast errors to a one standard deviation innovation in housing price growth. Panel (b) shows the impulse-response functions of median (real) housing price growth forecast errors of one-year ahead expectations to a one standard deviation innovation in housing price growth. The shaded area shows the 90% confidence intervals, standard errors are robust with respect to heteroskedasticity and autocorrelation (Newey-West with $h + 1$ lags).

A.6 Results when excluding the COVID-19 period

The empirical results reported in Section 2 are based on the entire period for which household-survey expectations are available, i.e., 2007-2021. This section reports results obtained when ending the sample in 2019, thereby excluding the recent COVID-19 period. Tables A.6 and A.7 show that our results are qualitatively and quantitatively robust to excluding observations from the years 2020 and 2021.

Table A.6: Sluggish adjustment of housing price expectations: excluding coronavirus crisis

	Mean expectations	Median expectations
<i>Nominal housing prices</i>		
\hat{b}^{CG}	2.18*** (0.503)	2.80*** (0.502)
<i>Real housing prices</i>		
\hat{b}^{CG}	1.97*** (0.332)	2.43*** (0.360)

Notes: This table shows the results of regression (1) excluding the coronavirus crisis, i.e., we exclude the years 2020 and 2021. The reported standard errors are robust with respect to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.7: Expected vs. actual housing price growth: excluding coronavirus crisis

	\hat{c} (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
<i>Nominal housing prices</i>				
Mean expectations	0.058 (0.0066)	-0.065 (0.0126)	0.0036	0.000
Median expectations	0.018 (0.0010)	-0.065 (0.0126)	0.0118	0.042
<i>Real housing prices</i>				
Mean expectations	0.0614 (0.0136)	-0.0483 (0.0090)	-0.0009	0.000
Median expectations	0.196 (0.0034)	-0.483 (0.0090)	0.076	0.017

Notes: This table shows the results of regressions (2) and (3) excluding the coronavirus period, i.e., we exclude the years 2020 and 2021. \hat{c} is the estimate of c in equation (2) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (3). The small-sample bias correction is reported in the second to last column and the last column reports the p -values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Newey-West standard errors using four lags in parentheses.

A.7 Regional housing prices and expectations

This appendix considers regional variation in housing prices and housing price expectations. This is possible because the Michigan Survey reports the location of respondents using four different regions: West, North East, North Central (or Midwest) and South. As the Case-Shiller Price Index is not available at the regional level, we construct regional housing price index using the Case-Shiller Index that is available for twenty large U.S. cities. Following the definition of the regions in the Michigan Survey, we assign the twenty cities to the four regions and aggregate city price indices to a regional index using two alternative approaches. A first approach weighs cities by population (as of 2019) within each region, while a second approach uses equal weights for all cities within a region.

Table A.8 lists the cities, the assigned region, and the population weights, calculated as the ratio of the population in the considered city, divided by the total population across all cities in the respective region. We deflate nominal housing price indices by the aggregate CPI and obtain real housing price expectations by deflating nominal (mean) expectations with region-specific (mean) inflation expectations.

Table A.8: Regions, cities and their weights

City	Region	Weight	City	Region	Weight
Denver	West	$\frac{0.705}{10.595}$	Chicago	North Central	$\frac{2.71}{4.189}$
Las Vegas	West	$\frac{0.634}{10.595}$	Cleveland	North Central	$\frac{0.385}{4.189}$
Los Angeles	West	$\frac{3.97}{10.595}$	Detroit	North Central	$\frac{0.674}{4.189}$
Phoenix	West	$\frac{1.633}{10.595}$	Minneapolis	North Central	$\frac{0.42}{4.189}$
Portland	West	$\frac{0.645}{10.595}$	Atlanta	South	$\frac{0.488}{4.209}$
San Diego	West	$\frac{1.41}{10.595}$	Charlotte	South	$\frac{0.857}{4.209}$
San Francisco	West	$\frac{0.874}{10.595}$	Dallas	South	$\frac{1.331}{4.209}$
Seattle	West	$\frac{0.724}{10.595}$	Miami	South	$\frac{0.454}{4.209}$
Boston	North East	$\frac{0.68}{9.1}$	Tampa	South	$\frac{0.387}{4.209}$
New York	North East	$\frac{8.42}{9.1}$	Washington DC	South	$\frac{0.692}{4.209}$

Notes: This table lists the twenty cities for which the Case-Shiller Home Price Index is available, the assigned region, and the population weights.

Table A.9 reports the region-specific estimates of b^{CG} from regression equation (1). All point estimates are significantly positive with magnitudes broadly in line with the estimates at the national level. This shows that households also sluggishly update expectations about regional houseprices, consistent with the findings reported for the national level reported in the main text.

Table A.10 reports the region-specific estimates of c and \mathbf{c} from regressions (2) and (3). Since regional price-to-rent ratios are not available, the regression uses real housing prices on the right-hand side. In line with our findings at the aggregate level, we find $c > 0$ and $\mathbf{c} < \mathbf{0}$ in all the regions with the differences being largely highly statistically significant.

Figure A.2 shows the dynamic forecast errors responses to a one standard deviation innovation in the real housing capital gain in each of the four regions, for population-weighted and equal-weighted averages of city price indices. In line with the baseline findings, households' housing price growth expectations initially underreact but later overshoot.

Table A.9: Sluggish adjustment of housing price expectations across regions crisis

	Weighted	Unweighted
$\widehat{b}^{CG,W}$	2.00*** (0.411)	1.95*** (0.374)
$\widehat{b}^{CG,NE}$	1.24*** (0.385)	1.15*** (0.441)
$\widehat{b}^{CG,NC}$	1.97*** (0.461)	1.95*** (0.459)
$\widehat{b}^{CG,S}$	1.74*** (0.385)	1.94*** (0.393)

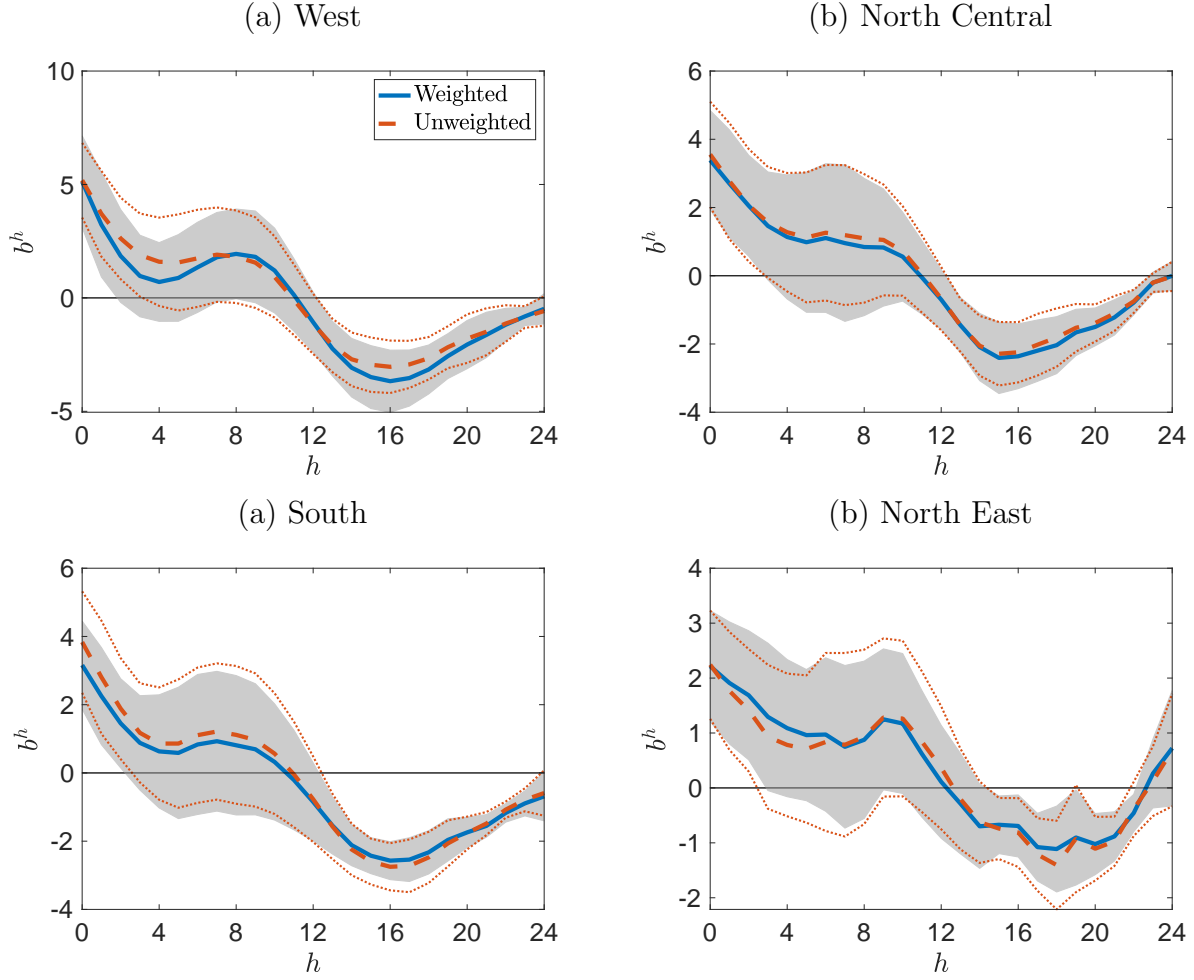
Notes: This table shows the results of regression (1) using regional housing prices and expectations. The superscripts *W*, *NE*, *NC* and *S* denote the regions West, North East, North Central (or Midwest) and South, respectively. The reported standard errors are robust with respect to heteroskedasticity and autocorrelation (Newey-West with four lags). Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.10: Expected vs. actual housing price growth across regions

	\hat{c} (in %)	$\hat{\mathbf{c}}$ (in %)	bias (in %) $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
<i>West</i>				
Population-weighted	0.109 (0.0036)	-0.216 (0.1360)	0.090	0.083
Equally weighted	0.132 (0.0034)	-0.132 (0.1197)	0.137	0.183
<i>North Central</i>				
Population-weighted	0.045 (0.0089)	-0.544 (0.0256)	0.008	0.000
Equally weighted	0.088 (0.0118)	-0.458 (0.0769)	0.0191	0.000
<i>North East</i>				
Population-weighted	0.013 (0.0089)	-0.474 (0.0072)	0.001	0.000
Equally weighted	0.126 (0.0187)	-0.315 (0.0838)	0.023	0.000
<i>South</i>				
Population-weighted	0.210 (0.0023)	-0.008 (0.1067)	0.137	0.144
Equally weighted	0.163 (0.0044)	-0.238 (0.1250)	0.055	0.014

Notes: This table shows the results of regressions (2) and (3) for different regions. \hat{c} is the estimate of c in equation (2) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (3). The small-sample bias correction is reported in the second to last column and the last column reports the p -values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Newey-West standard errors using four lags in parentheses.

Figure A.2: Regional dynamic forecast error responses to realized housing price growth

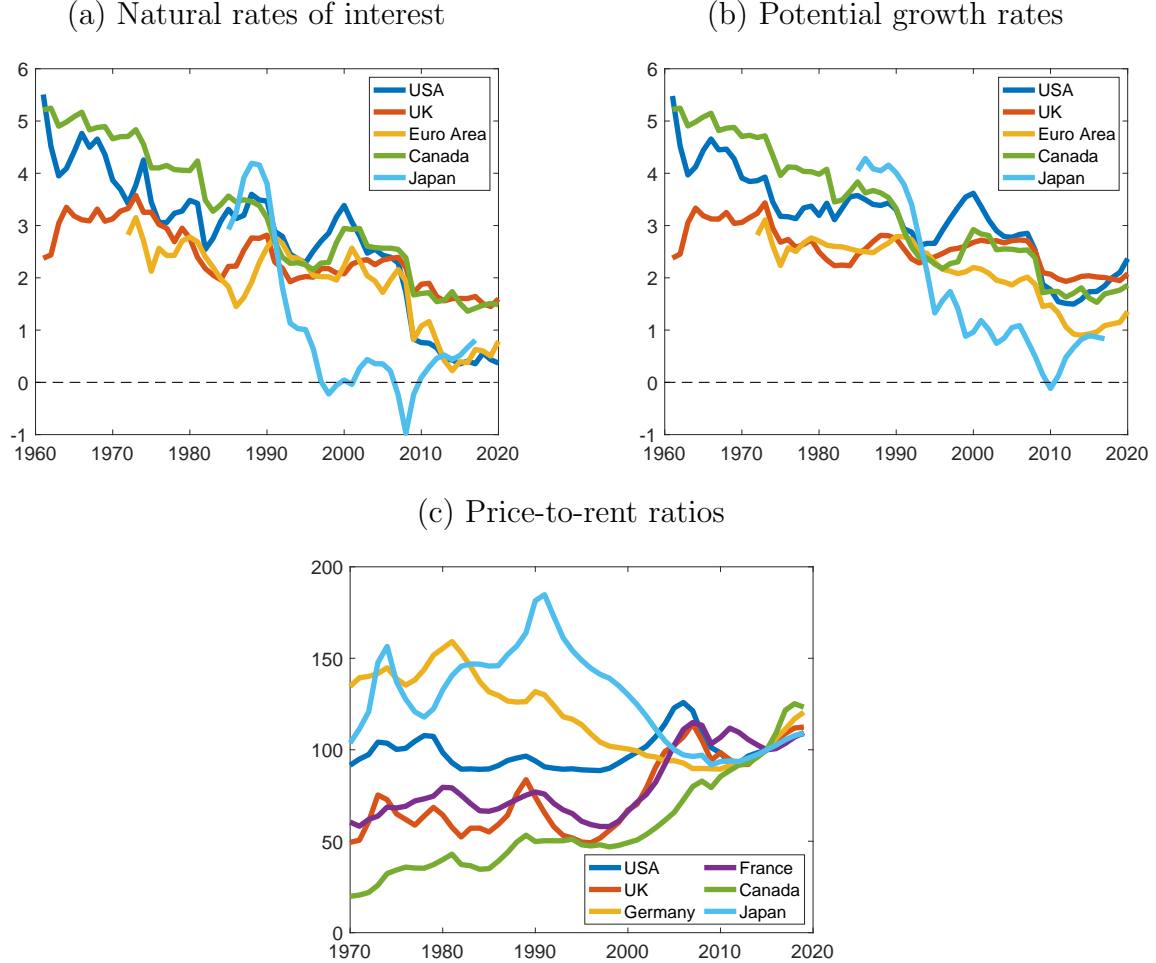


Notes: The figure shows the dynamic response of real housing price growth forecast errors across the four different regions (in which cities' housing indices are weighted by their population share) to a one standard deviation innovation in housing price growth. Solid lines indicate results for population-weighted city housing price indices. Dashed lines show results for equal-weighted indices. The shaded area shows the 90%-confidence intervals, standard errors are robust with respect to autocorrelation and heteroskedasticity (Newey-West with $h + 1$ lags).

A.8 The volatility of price-to-rent ratio and of the natural rate

Figure A.3 shows the evolution of natural rates of interest and price-to-rent ratios for the U.S., Canada, France, Germany, and the United Kingdom, which we use in Section 2. The natural rates are estimated by [Holston et al. \(2017\)](#) and [Fujiwara, Iwasaki, Muto, Nishizaki, and Sudo \(2016\)](#). The price-to-rent ratios are taken from the OECD. We convert the quarterly series of natural rates to annual series by taking arithmetic averages and the quarterly series of price-to-rent ratios to annual series by taking harmonic averages.

Figure A.3: Natural rates, potential growth rates, and price-to-rent ratios

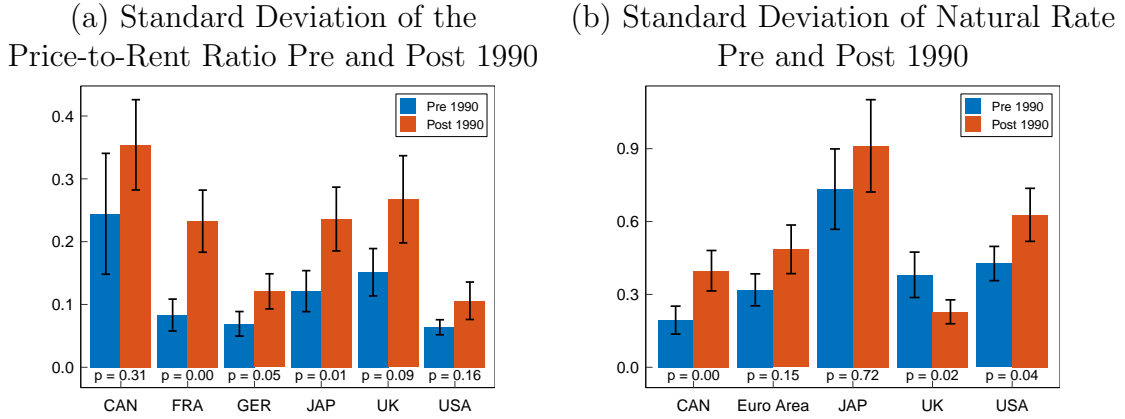


Notes: This figure shows the evolution of the natural rate of interest and potential growth rates taken from [Holston et al. \(2017\)](#) and [Fujiwara et al. \(2016\)](#) and price-to-rent ratios (right panel) for advanced economies

Figure A.4 plots the volatility of the price-to-rent ratio (left panel) and the standard deviation of the natural rate (right panel), respectively before 1990 (blue bars) and after 1990 (red bars), along with 90% confidence bands. The reported volatilities of the price-to-rent ratios are the standard deviations relative to the period-specific mean values, in line with the model. The reported volatilities of the natural rates of interest are the standard deviations of the fluctuations around a linear time trend, in order to isolate high-frequency volatility that can be related to natural rate fluctuations in the model around a fixed steady state value of the natural rate.

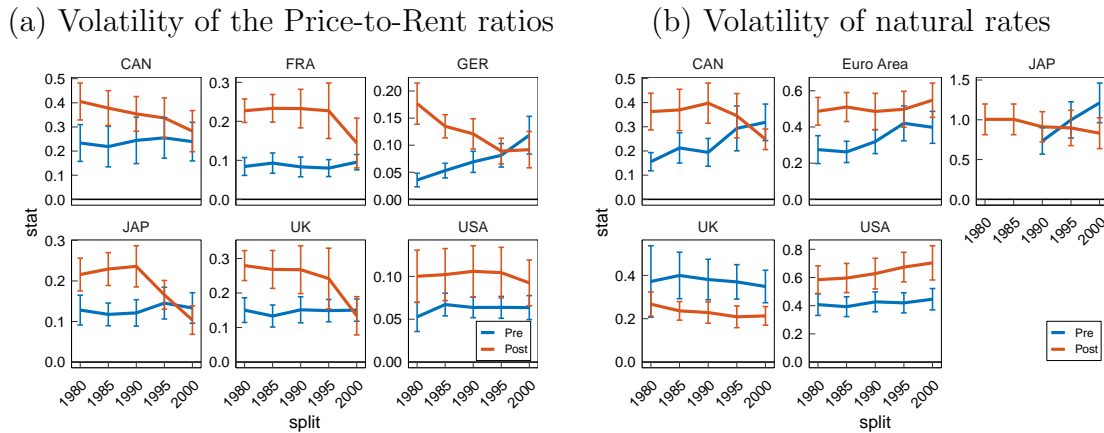
Figure A.6 shows the volatility price-to-rent ratio using the same linear detrending approach. The p -values below the respective bars are for the null hypothesis of no change in the volatility. The increase in the volatility of the PR ratio and the natural rate were statistically significant in most of the advanced economies. The evidence is not always statistically significant due to the high autocorrelation of the price-to-rent ratio and the natural rate, which makes it difficult to estimate standard deviations precisely. Figure A.5 shows that the reported volatility increases are not driven by the exact point where we split the data, instead looks often similar for other split points.

Figure A.4: Volatility of the PR ratio and natural rates pre and post 1990.



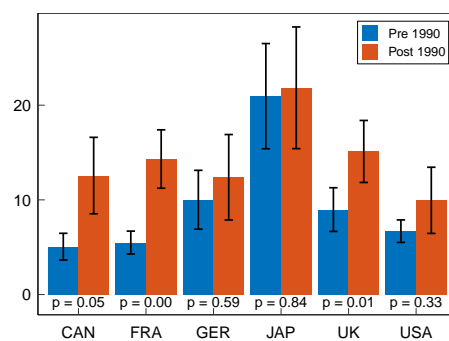
Notes: The black lines denote the 90%-confidence bands. The p -value corresponds to the test whether or not the values changed from pre to post 1990. The reported volatilities of the price-to-rent ratios are the standard deviations relative to the period-specific mean values. The reported volatilities of the natural rates of interest are the standard deviations of the fluctuations around a linear time trend.

Figure A.5: Robustness of housing and natural rate volatility increases with different sample splits



Notes: Panel (a) shows the standard deviation of the price-to-rent ratio, and panel (b) shows the standard deviation of the natural rate for different advanced economies, computed for varied subsamples. The blue lines show the estimates for the pre-period, and the red lines for the post-period, when the sample is split at the year marked on the horizontal axis. The whiskers denote 90%-confidence bands.

Figure A.6: Standard deviation of the detrended PR ratio pre and post 1990



Note: The black lines denote the 90%-confidence bands. The p -value corresponds to the test whether or not the values changed from pre to post 1990.

Appendix B Details of the full model

B.1 Household optimality conditions

Internally rational households choose state-contingent sequences for the choice variables $\{C_t, H_t(j), D_t, D_t^R, B_t\}$ so as to maximize (15), subject to the budget constraints (16), taking as given their beliefs about the processes $\{P_t, w_t(j), q_t, R_t, i_t, \Sigma_t/P_t, \Sigma_t^d/P_t, T_t/P_t\}$, as determined by the (subjective) measure \mathcal{P} .

The first order conditions give rise to an optimal labor supply relation

$$w_t(j) = \lambda (H_t(j))^\nu C_t \quad (\text{B.1})$$

a consumption Euler equation

$$\frac{1}{C_t} = \beta E_t^{\mathcal{P}} \left[\frac{1}{C_{t+1}} \frac{1 + i_t}{P_{t+1}/P_t} \right], \quad (\text{B.2})$$

an optimality condition for rental units

$$\xi_t^d = R_t \frac{1}{C_t}, \quad (\text{B.3})$$

and a set of conditions determining the optimal housing demand D_t :

$$\begin{aligned} q_t^u &< \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} [q_{t+1}]^u & \text{if } D_t = D^{\max} \\ q_t^u &= \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} [q_{t+1}]^u & \text{if } D_t \in (0, D^{\max}) \\ q_t^u &> \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} [q_{t+1}]^u & \text{if } D_t = 0, \end{aligned} \quad (\text{B.4})$$

where q_t^u denotes the real price of houses in marginal utility units, defined as

$$q_t^u \equiv q_t \frac{1}{C_t}.$$

The variable q_t^u provides a measure of whether housing is currently expensive or inexpensive, in units that are particularly relevant for determining housing demand.³⁶ The price-to-rent ratio is given by

$$PR_t \equiv \frac{q_t}{R_t} = \frac{q_t^u}{\xi_t^d}.$$

With rational expectations, the upper and lower holding bounds in (B.4) never bind.³⁷ Since we are interested in how the presence of belief distortions about future housing values affect equilibrium outcomes, the bounds in equation (B.4) can potentially bind under the *subjectively* optimal plans. This explains why an internally rational household can hold subjective housing price expectations, even if she holds rational expectations about the preference shocks ξ_t^d .

Forward-iterating on equation (B.2), which holds with equality under all belief specifications, delivers a present-value formulation of the consumption Euler equation

$$\frac{1}{C_t} = \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[\frac{1}{C_T} \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1}/P_{t+k}} \right], \quad (\text{B.5})$$

³⁶In Section 3, q_t^u and q_t coincide due to risk-neutrality.

³⁷The upper bound D^{\max} has been chosen sufficiently large for this to be true. The lower bound is never reached because the housing production function satisfies Inada conditions.

which will be convenient to work with, especially under subjective belief specifications. Household choices must also satisfy the transversality constraint

$$\lim_{T \rightarrow \infty} \beta^T E_t^P \left[\frac{1}{C_T} B_T + D_T q_T^u \right] = 0. \quad (\text{B.6})$$

Optimal household behavior under potentially subjective beliefs is jointly characterized by equations (B.1) and (B.3)-(B.6).

We shall be particularly interested in the policy implications generated by subjective housing price beliefs. To ensure that an optimum exists in the presence of potentially subjective beliefs about the housing price, we require housing choices to lie in some compact choice set $D_t \in [0, D^{\max}]$, where the upper bound can be arbitrarily large. Overall, we wish to consider a minimal deviation from rational expectations, therefore keep expectations about all other variables rational to the extent possible.³⁸ Finally, to ensure that households' subjectively optimal plans satisfy the transversality condition, we assume that households hold rational housing price growth expectations in the very long run, i.e., after some arbitrarily large but finite period $\bar{T} < \infty$.³⁹ We then consider the policy problem with subjective beliefs in periods $t \ll \bar{T}$.

B.2 Goods producing firms' optimality conditions

Each differentiated good is supplied by a single monopolistically competitive producer with a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \quad (\text{B.7})$$

where A_t is an endogenously varying technology factor, and $\phi > 1$. The Dixit-Stiglitz aggregation implies that the quantity demanded of each individual good i will equal⁴⁰

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}, \quad (\text{B.8})$$

where Y_t is the total demand for the composite good, $p_t(i)$ is the price of the individual good, and P_t is the price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \quad (\text{B.9})$$

corresponding to the minimum cost for which a unit of the composite good can be purchased in period t . Total demand is given by

$$Y_t = C_t + k_t. \quad (\text{B.10})$$

³⁸In particular, household continue to hold rational expectations about all other prices, i.e., about $\{P_t, w_t(j), i_t\}$ and firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$. Furthermore, all actors continue to hold rational expectations about the exogenous fundamentals. Beliefs about profits and lump sum taxes, $\{\Sigma_t/P_t, T_t/P_t\}$ continue to be determined by equations (B.20) and (B.24), evaluated with rational output expectations and the state-contingent optimal choices for $\{H_t, k_t, B_t\}$. Rental price expectations, however, cannot be kept rational: they need to satisfy equation (B.3), which shows that they are influenced by the subjectively optimal consumption plans implied by equation (B.37).

³⁹Appendix B.6 shows that this is sufficient to ensure that subjectively optimal plans satisfy the transversality constraint (B.6).

⁴⁰In addition to assuming that household utility depends only on the quantity obtained of C_t , we assume that the government also cares only about the quantity obtained of the composite good, and that it seeks to obtain this good through a minimum-cost combination of purchases of individual goods.

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983). Producers use the representative households' subjectively optimal consumption plans to discount profits and are assumed to know the product demand function (B.8). They need to formulate beliefs about the future price levels P_T , industry-specific wages $w_T(j)$, aggregate demand Y_T , and productivity A_T .

Let $0 \leq \alpha < 1$ be the fraction of prices that remain unchanged in any period. A supplier i in industry j that changes its price in period t chooses its new price $p_t(i)$ to maximize

$$E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), P_T, w_T(j), Y_T, A_T), \quad (\text{B.11})$$

where E_t^P denotes the expectations of price setters conditional on time t information, which are identical to the expectations held by consumers. Firms discount nominal income in period T using households' subjective stochastic discount factor $Q_{t,T}$, which is given by

$$Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_C(C_T, \xi_T)}{\tilde{u}_C(C_t, \xi_t)} \frac{P_t}{P_T}.$$

The term α^{T-t} in equation (B.11) captures the probability that a price chosen in period t will not have been revised by period T , and the function $\Pi(\cdot)$ indicates the nominal profits of the firm in period T with the price set at time t , as discussed next.

Profits are equal to after-tax sales revenues net of the wage bill. Sales revenues are determined by the demand function (B.8), so that (nominal) after-tax revenue in period T with the price set at time t equals

$$(1 - \tau) p_t(i) Y_T \left(\frac{p_t(i)}{P_T} \right)^{-\eta}.$$

Here τ is a proportional and time-constant tax on sales revenues. τ is set such that the steady state is efficient with firms setting prices equal to marginal costs.

The labor demand of firm i at a given industry-specific wage $w_T(j)$ can be written as

$$\left(\frac{Y_T}{A_T} \right)^{\phi} p_t(i)^{-\eta\phi} P_T^{\eta\phi}, \quad (\text{B.12})$$

which follows from (B.7) and (B.8). Using this, the nominal wage bill is given by

$$P_T w_T(j) \left(\frac{Y_T}{A_T} \right)^{\phi} p_t(i)^{-\eta\phi} P_T^{\eta\phi}. \quad (\text{B.13})$$

Subtracting the nominal wage bill at time T from the above expression for nominal after-tax revenue at time T , we obtain the function

$$\Pi(p_t(i), P_T, w_T(j), Y_T, A_T) = (1 - \tau) p_t(i) Y_T \left(\frac{p_t(i)}{P_T} \right)^{-\eta} - P_T w_T(j) \left(\frac{Y_T}{A_T} \right)^{\phi} p_t(i)^{-\eta\phi} P_T^{\eta\phi} \quad (\text{B.14})$$

used in (B.11).

Each of the suppliers that revise their prices in period t chooses the same new price p_t^* , that maximizes (B.11). The first-order condition with respect to $p_t(i)$ is given by⁴¹

$$E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1(p_t(i), P_T, w_T(j), Y_T, A_T) = 0.$$

The equilibrium choice p_t^* , which is the same for each firm i in industry j , is the solution to this equation. Letting p_t^j denote the price charged by firms in industry j at time t , we have $p_t^j = p_t^*$ in periods in which industry j resets its prices and $p_t^j = p_{t-1}^j$ otherwise.

Under the assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} (1-\tau) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}}. \quad (\text{B.15})$$

The price index evolves according to a law of motion

$$P_t = [(1-\alpha) p_t^{*1-\eta} + \alpha P_{t-1}^{1-\eta}]^{\frac{1}{1-\eta}}, \quad (\text{B.16})$$

as a consequence of (B.9). Inflation is characterized by

$$\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1} = \frac{1 - (1-\alpha) \left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha}. \quad (\text{B.17})$$

The welfare loss from price adjustment frictions can be captured by price dispersion, which is defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t^j}{P_t}\right)^{-\eta(1+\omega)} dj \geq 1, \quad (\text{B.18})$$

where

$$\omega \equiv \phi(1+\nu) - 1 > 0$$

is the elasticity of real marginal cost in an industry with respect to industry output.

Using equation (B.16) together with the fact that the relative prices of the industries that do not change their prices in period t remain the same, one can derive a law of motion for the price dispersion term Δ_t of the form

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}), \quad (\text{B.19})$$

with

$$h(\Delta_t, P_t/P_{t-1}) \equiv \alpha \Delta_t \left(\frac{P_t}{P_{t-1}}\right)^{\eta(1+\omega)} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{P_t}{P_{t-1}}\right)^{\eta-1}}{1-\alpha}\right)^{\frac{\eta(1+\omega)}{\eta-1}}.$$

As is commonly done, we assume that the initial degree of price dispersion is small ($\Delta_{-1} \sim O(2)$).

⁴¹Note that supplier i 's profits in (B.11) are a concave function of the quantity sold $y_t(i)$, since revenues are proportional to $y_t(i)^{\frac{\eta-1}{\eta}}$ and hence concave in $y_t(i)$, while costs are convex in $y_t(i)$. Moreover, since $y_t(i)$ is proportional to $p_t(i)^{-\eta}$, the profit function is also concave in $p_t(i)^{-\eta}$. The first-order condition for the optimal choice of the price $p_t(i)$ is the same as the one with respect to $p_t(i)^{-\eta}$; hence the first-order condition with respect to $p_t(i)$ is both necessary and sufficient for an optimum.

Equations (B.15), (B.17), and (B.19) jointly define a short-run aggregate supply relation between inflation, output, and housing prices (via the aggregate demand equation (B.10) and (B.23)), given the current disturbances ξ_t , and expectations regarding future wages, prices, output, consumption and disturbances. Equation (B.19) describes the evolution of the costs of price dispersion over time.

For future reference, we remark that all firms together make total profits equal to

$$\frac{\Sigma_t}{P_t} = (1 - \tau)Y_t - w_t H_t, \quad (\text{B.20})$$

where $w_t H_t = \int_0^1 w_t(j) h_t(j) dj$.

B.3 Housing construction firms' optimality conditions

The representative housing construction firm chooses investment in new houses, k_t , and takes housing prices q_t as given, to maximize profits

$$\Sigma_t^d = P_t \left(q_t \tilde{d}(k_t; A_t^d) - k_t \right), \quad (\text{B.21})$$

given the production function

$$\tilde{d}_t = \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}. \quad (\text{B.22})$$

This gives rise to the optimality condition characterizing optimal investment in new houses:

$$k_t = (A_t^d q_t)^{\frac{1}{1-\tilde{\alpha}}} = (A_t^d q_t^u C_t)^{\frac{1}{1-\tilde{\alpha}}} \quad (\text{B.23})$$

B.4 Government budget constraint and market clearing

The government imposes a sales tax τ , issues nominal bonds $\tilde{B}_t \equiv P_t B_t$, and pays lump-sum transfers T_t to households. The government budget constraint is given by

$$B_t = B_{t-1} \frac{1 + i_{t-1}}{P_t/P_{t-1}} + \frac{T_t}{P_t} - \tau Y_t.$$

For simplicity, we assume that lump sum transfers (taxes if negative) are set such that they keep real government debt constant at some initial level B_{-1} . This implies that government transfers are given by

$$\frac{T_t}{P_t} = \tau Y_t + B_{t-1} \left(1 - \frac{1 + i_{t-1}}{P_t/P_{t-1}} \right). \quad (\text{B.24})$$

Using the total demand equation (B.10) and the expression for optimal housing investment (B.23), one can express the market clearing condition for the consumption/investment good as

$$Y_t = C_t + \Omega_t C_t^{\frac{1}{1-\tilde{\alpha}}}, \quad (\text{B.25})$$

where

$$\Omega_t \equiv (A_t^d q_t^u)^{\frac{1}{1-\tilde{\alpha}}} > 0 \quad (\text{B.26})$$

is a term that depends on exogenous shocks and belief distortions in the housing market only, see equation (B.4). The previous two equations implicitly define a function

$$C_t = C(Y_t, q_t^u, \xi_t), \quad (\text{B.27})$$

which delivers the market clearing consumption level, for a given output level Y_t , given housing prices q_t^u and given exogenous disturbances ξ_t .

The market clearing condition for housing is

$$D_t - (1 - \delta)D_{t-1} = \tilde{d}(k_t; A_t^d), \quad (\text{B.28})$$

and rental market clearing requires

$$D_t^R = 0. \quad (\text{B.29})$$

Labor market clearing requires that the supply of labor of type j in (B.1) is equal to labor demand of industry j , as all firms in the industry charge the same price. This delivers

$$w_t(j) = \lambda \left(\frac{Y_t}{A_t} \right)^{\nu\phi} C_t \left(\frac{p_t^j}{P_t} \right)^{-\nu\eta\phi}. \quad (\text{B.30})$$

B.5 Equilibrium

We now define the *Internally Rational Expectations Equilibrium (IREE)*, which is a generalization of the notion of a Rational Expectations Equilibrium (REE) to settings with subjective private sector beliefs:

Definition 1 *An Internally Rational Expectations Equilibrium (IREE) is a bounded stochastic process for $\{Y_t, C_t, k_t, D_t, \{w_t(j)\}, p_t^*, P_t, \Delta_t, q_t^u, i_t\}_{t=0}^\infty$ satisfying the law of motion of the aggregate price index, the firms' optimal price setting behavior, the law of motion for the evolution of price distortions, the household optimality conditions (B.4), (B.37), the housing sector's optimal investment (B.23), and the market clearing conditions (B.25), (B.28) and (B.30) for all j .*

The equilibrium features ten variables (counting the continuum of wages as a single variable) that must satisfy nine conditions, leaving one degree of freedom to be determined by monetary policy.⁴² In the special case with rational beliefs ($E_t^P[\cdot] = E_t[\cdot]$), the IREE is a Rational Expectations Equilibrium (REE).

Given the equilibrium outcome, the remaining model variables can be determined as follows. Equilibrium profits are given by equations (B.20) and (B.21), and equilibrium taxes by equation (B.24). Equilibrium labor supply $H_t(j)$ follows from equation (B.1) for each labor type j . Equilibrium bond holdings satisfy $B_t = B_{-1}$ and equilibrium inflation is $\Pi_t \equiv P_t/P_{t-1}$. Equilibrium rental units are given by equation (B.29) and equilibrium rental prices by equation (B.3).

⁴²The transversality condition (B.6) must also be satisfied in equilibrium, but is not imposed as an equilibrium condition, as it will hold for all belief specifications considered.

B.6 Assumptions about long-run beliefs and validity of the transversality constraint

To ensure that the subjectively optimal consumption plans satisfy the transversality condition (B.6), we impose that households perceive housing price growth to follow the law of motion

$$\frac{q_t^u}{q_{t-1}^u} = b_t + \varepsilon_t \quad (\text{B.31})$$

for an arbitrarily long but finite amount of time $t < \bar{T} < \infty$ and that households hold rational expectations in the long-run, i.e. for all periods $t \geq \bar{T}$. Agents thus perceive

$$q_t^u = q_t^{u*} \quad \text{for all } t \geq \bar{T}, \mathcal{P} \text{ almost surely,}$$

where $q_t^{u*} = \bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$ is the rational expectations housing price.

To show that under the considered subjective belief specifications, the optimal plans satisfy the transversality constraint, we observe that since $D_t \in [0, D^{\max}]$ and $E_t^{\mathcal{P}} q_T^u = E_t \bar{\xi}_T^d$ for $T \geq T'$, we have $\lim_{T \rightarrow \infty} \beta^T E_t^{\mathcal{P}} (D_T q_T^u) = 0$. We thus only need to show that $\lim_{T \rightarrow \infty} \beta^T E_t^{\mathcal{P}} \frac{1}{C_T} B_T = 0$. Combining the budget constraint (16) with (B.20) and (B.24) we obtain

$$C_t + B_t + \left(D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t; \xi_t) \right) q_t^u C_t + k_t = Y_t + B_{t-1}.$$

For $t \geq T'$ the subjectively optimal plans satisfy market clearing in the housing market, i.e.,

$$D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t; \xi_t) = 0$$

so that the budget constraint implies

$$C_t + B_t + k_t = Y_t + B_{t-1}. \quad (\text{B.32})$$

Furthermore, for $t \geq T'$ subjectively optimal plans also satisfy market clearing for consumption goods, i.e.,

$$C_t + k_t = Y_t.$$

It thus follows that the subjectively optimal debt level B_t in the budget constraint (B.32) is constant under the subjectively optimal plan, after period $t \geq T'$. Furthermore, the expectation about Y_t in the budget constraint (B.32) is rational under the assumed lump sum transfer expectations, so that the household's subjective consumption expectations are the same as in a rational expectations equilibrium. (The subjectively optimal investment decisions k_t are driven by rational housing price expectations.) Since the limit expectations $1/C_T$ are bounded in the rational expectations equilibrium, it follows that $\lim_{T \rightarrow \infty} \beta^T E_t^{\mathcal{P}} \frac{1}{C_T} B_T = 0$.

B.7 Response of housing investment to monetary policy

To prove equation (33), start by recalling the definition of $q_t^u = q_t \tilde{u}_c(C_t; \xi_t)$ which implies

$$\log q_t^u = \log q_t + \log \tilde{u}_c(C_t; \xi_t),$$

where $\tilde{u}_c(C_t; \xi_t)$ denotes the marginal utility of consumption. We show here the case of a general CRRA utility function with relative risk aversion $\frac{1}{\sigma}$. With log utility, we have $\frac{1}{\sigma} = 1$. Under the considered belief setup in which agents learn about risk-adjusted housing price growth, the

dynamics of risk-adjusted housing price growth and beliefs are independent of monetary policy. The response of $\log q_t^u$ to a unexpected change in the path of nominal rates \mathbf{i} is thus $\frac{d \log q_t^u}{d \mathbf{i}} = 0$, so that

$$\begin{aligned}
\frac{d \log q_t}{d \mathbf{i}} &= - \frac{d \log \tilde{u}_c(C_t; \xi_t)}{d \mathbf{i}} \\
&= - \frac{d \log \tilde{u}_c(C_t; \xi_t)}{d \log C_t} \frac{d \log C_t}{d \mathbf{i}} \\
&= - \frac{\tilde{u}_{cc}(C_t; \xi_t) C_t}{\tilde{u}_c(C_t; \xi_t)} \frac{d \log C_t}{d \mathbf{i}} \\
&= \frac{1}{\bar{\sigma}} \frac{d \log C_t}{d \mathbf{i}}.
\end{aligned} \tag{B.33}$$

The optimal housing supply equation (B.23) can be written as

$$\log k_t = \frac{1}{1 - \tilde{\alpha}} (\log A_t^d + \log q_t).$$

Taking derivatives with respect to \mathbf{i} in the previous equation and using (B.33) delivers (33).

B.8 Derivation of the housing price gap

To derive the housing price gap under subjective beliefs, we start by deriving the efficient housing price. This first step closely follows Adam and Woodford (2021). Let $U(Y_t, \Delta_t, q_t^u, \xi_t)$ be the flow utility given by

$$U(Y_t, \Delta_t, q_t^u, \xi_t) = \log(C(Y_t, q_t^u, \xi_t)) - \frac{\lambda}{1 + \nu} \left(\frac{Y_t}{A_t} \right)^{1 + \omega} \Delta_t + \frac{A_t^d \bar{\xi}_t^d}{\tilde{\alpha}} \Omega(q_t^u, \xi_t)^{\tilde{\alpha}} C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1 - \tilde{\alpha}}},$$

where $\bar{\xi}_t^d$ is the efficient housing price given by

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1 - \delta)^{T-t} \beta^{T-t} \xi_T^d]. \tag{B.34}$$

To see that this is indeed the efficient housing price, note that we need that $\partial U(Y_t, \Delta_t, q_t^u, \xi_t) / \partial q^u \equiv U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t) = 0$.

This yields

$$\begin{aligned}
U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t) &= C_{q^u}(Y_t, q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{-1} \\
&\quad + A_t^d \bar{\xi}_t^d \frac{\partial \Omega(q_t^u, \xi_t)}{\partial q_t^u} \Omega(q_t^u, \xi_t)^{\tilde{\alpha}-1} C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1 - \tilde{\alpha}}} \\
&\quad + \frac{1}{1 - \tilde{\alpha}} A_t^d \bar{\xi}_t^d \Omega(q_t^u, \xi_t)^{\tilde{\alpha}} C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1 - \tilde{\alpha}}-1} C_{q^u}(Y_t, q_t^u, \xi_t) = 0,
\end{aligned}$$

where

$$\frac{\partial \Omega(q_t^u, \xi_t)}{\partial q_t^u} = \frac{1}{q_t^u} \frac{1}{1 - \tilde{\alpha}} \Omega(q_t^u, \xi_t),$$

and when defining $\chi \equiv \frac{1}{1 - \tilde{\alpha}} - 1$, we get

$$C_{q^u}(Y_t, q_t^u, \xi_t) \equiv \frac{\partial C(Y_t, q_t^u, \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u} \frac{1}{1 - \tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}}.$$

Taking everything together, we get

$$U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t) = \frac{\frac{1}{q_t^u} \frac{1}{1-\alpha} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^\chi} \left(\frac{\bar{\xi}_t^d}{q_t^u} - 1 \right).$$

In order for U_{q^u} to be zero, we need to have that

$$q_t^{u*} = \bar{\xi}_t^d.$$

Note, that under rational expectations the asset-pricing equation (B.4) always holds with equality (also in expectations). Iterating forward, we obtain $q_t^{u, RE} = q_t^{u*} = \bar{\xi}_t^d$.

We now linearize the efficient housing price q_t^{u*} . We have

$$\widehat{q}_t^{u*} = \widehat{\xi}_t^d,$$

and log-linearizing the AR(1) process for the housing preference shock (13) delivers

$$\widehat{\xi}_t^d = \rho_\xi \widehat{\xi}_{t-1}^d + \varepsilon_t^d.$$

Since the steady-state value of $\underline{\xi}^d$ is

$$\underline{\xi}^d = \frac{\xi^d}{1 - \beta(1 - \delta)},$$

the log-linearization of $\bar{\xi}_t^d$ delivers

$$\begin{aligned} \widehat{\xi}_t^d &= (1 - \beta(1 - \delta)) \left[\widehat{\xi}_t^d + \beta(1 - \delta) E_t \widehat{\xi}_{t+1}^d + \dots \right] \\ &= (1 - \beta(1 - \delta)) \left[\widehat{\xi}_t^d + \beta(1 - \delta) \rho_\xi \widehat{\xi}_t^d + \dots \right] \\ &= (1 - \beta(1 - \delta)) \sum_{T=t}^{\infty} (\beta(1 - \delta) \rho_\xi)^{T-t} \widehat{\xi}_t^d \\ &= \widehat{\xi}_t^d \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) \rho_\xi}. \end{aligned}$$

From equation (B.4), which has to hold with equality in equilibrium, and equation (9) we get

$$q_t^u = \frac{1}{1 - \beta(1 - \delta) \gamma_t^u} \xi_t^d$$

The percent deviation of housing prices from the steady state, in which $\gamma_t^u = 1$ and $\xi_t^d = \underline{\xi}^d$, is then given by

$$\begin{aligned} \widehat{q}_t^u &= \frac{\frac{1}{1 - \beta(1 - \delta) \gamma_t^u} \xi_t^d - \frac{1}{1 - \beta(1 - \delta)} \underline{\xi}^d}{\frac{1}{1 - \beta(1 - \delta)} \underline{\xi}^d} \\ &= \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) \gamma_t^u} \frac{\xi_t^d}{\underline{\xi}^d} - 1 \\ &= \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) \gamma_t^u} \left(1 + \widehat{\xi}_t^d \right) - 1 \\ &= \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) \gamma_t^u} \widehat{\xi}_t^d + \frac{\beta(1 - \delta)(\gamma_t^u - 1)}{1 - \beta(1 - \delta) \gamma_t^u} \end{aligned} \tag{B.35}$$

Note, that by adding and subtracting the efficient housing price, we can decompose the housing price under subjective beliefs into the efficient part and terms that are driven by beliefs:

$$\widehat{q}_t^u = \widehat{q}_t^{u*} + \frac{\beta(1-\delta)(\gamma_t^u - 1)}{1 - \beta(1-\delta)\gamma_t^u} + \frac{(1-\beta(1-\delta))(\beta(1-\delta)(\gamma_t^u - \rho_\xi))}{(1-\beta(1-\delta)\gamma_t^u)(1-\beta(1-\delta)\rho_\xi)} \widehat{\xi}_t^d. \quad (\text{B.36})$$

Rearranging to obtain the wedge between actual and efficient housing price yields the housing price gap. Equation (27) in the main text follows from a log-linearization of the optimal housing investment equation (B.23), which can be written once for actual housing prices and once for efficient housing prices. Taking the difference between the two delivers equation (27).

B.9 Derivation of the Euler equation

Log-linearizing the general present-value formulation of the consumption Euler equation

$$\tilde{u}_C(C_t; \xi_t) = \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1}/P_{t+k}} \right], \quad (\text{B.37})$$

around the optimal steady state, as denoted by hatted variables, delivers

$$\tilde{u}_{CC} C \widehat{c}_t + \tilde{u}_{C\xi} \widehat{\xi}_t = E_t^{\mathcal{P}} \sum_{k=0}^{\infty} \tilde{u}_C(i_{t+k} - \pi_{t+1+k}) + \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left(\tilde{u}_{CC} C \widehat{c}_T + \tilde{u}_{C\xi} \widehat{\xi}_T \right),$$

and log-linearizing the Euler equation under rational expectations gives

$$\tilde{u}_{CC} C \widehat{c}_t^* + \tilde{u}_{C\xi} \widehat{\xi}_t = E_t \sum_{k=0}^{\infty} \tilde{u}_C \check{r}_{t+k} + \lim_{T \rightarrow \infty} E_t \left(\tilde{u}_{CC} C \widehat{c}_T^* + \tilde{u}_{C\xi} \widehat{\xi}_T \right).$$

Subtracting this equation from the one above delivers

$$\widehat{c}_t - \widehat{c}_t^* = E_t^{\mathcal{P}} \sum_{k=0}^{\infty} \frac{\tilde{u}_C}{\tilde{u}_{CC} C} (i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}) + \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} (\widehat{c}_{T+1} - \widehat{c}_{T+1}^*), \quad (\text{B.38})$$

where we used $\lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \xi_T = \lim_{T \rightarrow \infty} E_t \xi_T$ and $\lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \widehat{c}_{T+1}^* = \lim_{T \rightarrow \infty} E_t \widehat{c}_{T+1}^*$, which hold because agents have rational expectations about fundamentals and in the very long run about all variables.

In all periods in which the subjectively optimal plan is consistent with market clearing in the goods sector, the plan satisfies equation (B.27). Log-linearizing equation (B.27) delivers

$$\widehat{c}_t = C_Y \widehat{y}_t + C_q \widehat{q}_t^u + C_\xi \widehat{\xi}_t, \quad (\text{B.39})$$

where $\widehat{\xi}_t$ is a vector of exogenous disturbances (involving A_t^d). Evaluating this equation at the optimal dynamics defines optimal consumption \widehat{c}_t^* :

$$\widehat{c}_t^* \equiv C_Y \widehat{y}_t^* + C_q \widehat{q}_t^{u,*} + C_\xi \widehat{\xi}_t.$$

Subtracting the previous equation from (B.39) delivers

$$\begin{aligned} \widehat{c}_t - \widehat{c}_t^* &= C_Y (\widehat{y}_t - \widehat{y}_t^*) + C_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \\ &= C_Y y_t^{gap} + C_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \end{aligned} \quad (\text{B.40})$$

Since the current consumption market in period t clears, equation (B.40) holds in period t and can be used to substitute the consumption gap on the left-hand side of equation (B.38). Similarly, since housing price expectations are rational in the limit, the consumption market also clears in the limit under the subjectively optimal plans, i.e., equation (B.27) holds for $t \geq T'$. We can thus use equation (B.40) also to substitute the consumption gap on the r.h.s. of equation (B.38). Using the fact that housing price expectations are rational in the limit ($\lim_{T \rightarrow \infty} E_t^P(\hat{q}_t^u - \hat{q}_t^{u*}) = 0$), we obtain

$$y_t^{gap} = \lim_{T \rightarrow \infty} E_t^P y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}) \right) \underbrace{- \frac{C_q}{C_Y}}_{\equiv \zeta_q} (\hat{q}_t^u - \hat{q}_t^{u*}).$$

Since we assumed that agents' beliefs about profits and taxes are given by equations (B.20) and (B.24), respectively, evaluated using rational income expectations, the household holds rational expectations about total income. This can be seen by substituting (B.20) and (B.24) into the budget constraint (16). We thus have $\lim_{T \rightarrow \infty} E_t^P y_T^{gap} = \lim_{T \rightarrow \infty} E_t y_T^{gap}$ in the previous equation, which yields the final Euler equation in terms of output gaps.

B.10 Derivation of the natural rate of interest with subjective beliefs

Under the proposed policy that sets

$$i_t - E_t \pi_{t+1} = \check{r}_t - \zeta_q ((\hat{q}_t^u - \hat{q}_t^{u*}) - E_t (\hat{q}_{t+1}^u - \hat{q}_{t+1}^{u*})) \text{ for all } t \quad (\text{B.41})$$

as in equation (30) we have

$$\begin{aligned} y_t^{gap} &= \lim_{T \rightarrow \infty} E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}) \right) + \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \\ &= \lim_{T \rightarrow \infty} E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} (\check{r}_{t+k} + \zeta_q ((\hat{q}_{t+k}^u - \hat{q}_{t+k}^{u*}) - E_{t+k} (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*})) - \check{r}_{t+k}) \right) \\ &\quad + \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \\ &= \lim_{T \rightarrow \infty} E_t y_T^{gap} + E_t \left(\sum_{k=0}^{\infty} (-\zeta_q ((\hat{q}_{t+k}^u - \hat{q}_{t+k}^{u*}) - (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*}))) \right) + \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \\ &= \lim_{T \rightarrow \infty} E_t y_T^{gap} + E_t \left(-\zeta_q ((\hat{q}_t^u - \hat{q}_t^{u*}) - \lim_k E_t (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*})) \right) + \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \\ &= \lim_{T \rightarrow \infty} E_t y_T^{gap} + (-\zeta_q (\hat{q}_t^u - \hat{q}_t^{u*})) + \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \\ &= \lim_{T \rightarrow \infty} E_t y_T^{gap}, \end{aligned}$$

which proves that with this policy, the output gap is indeed constant.

B.11 Derivation of the linearized New Keynesian Phillips Curve

Recall from Appendix B.2, the three equilibrium equations arising from the firm optimality conditions and labor market clearing that are given by:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} (1-\tau) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \quad (\text{B.42})$$

$$w_t(j) = \lambda \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C(Y_t, q_t^u, \xi_t) \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \quad (\text{B.43})$$

$$(P_t/P_{t-1})^{\eta-1} = \frac{1 - (1-\alpha) \left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha} \quad (\text{B.44})$$

We now show how linearizing Equations (C.2)-(C.4) delivers the linearized Phillips curve. The condition for the equilibrium wage (C.3) in period T in industry j in which firms last updated their prices in period t is given by

$$w_T(j) = \tilde{w}_T(j) \left(\frac{p_t^j}{P_t}\right)^{-\eta\phi\nu} \left(\frac{P_T}{P_t}\right)^{\eta\phi\nu},$$

where

$$\tilde{w}_T(j) \equiv \lambda \left(\frac{Y_T}{A_T}\right)^{\phi\nu} C(Y_T, q_T^u, \xi_T).$$

Since firms' expectations about $w_T(j)$ and P_T are rational, their expectations about $\tilde{w}_T(j)$ are rational as well. Using the expression for $w_T(j)$, noting that $p_t(i) = p_t^j = p_t^*$, and writing out $Q_{t,T}$, it follows that

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\eta}{\eta-1} \phi C_T^{-1} \tilde{w}_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta(1+\omega)}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} C_T^{-1} (1-\tau) Y_T \left(\frac{P_T}{P_t}\right)^{\eta-1}} \right)^{\frac{1}{1+\omega\eta}}. \quad (\text{B.45})$$

Log-linearizing equation (B.45) delivers⁴³

$$\hat{p}_t^* - \hat{P}_t = \frac{1-\alpha\beta}{1+\omega\eta} \left\{ \hat{\tilde{w}}_t(j) + \phi(\hat{y}_t - \hat{A}_t) - \hat{y}_t + \alpha\beta E_t^{\mathcal{P}} \left[\frac{1+\omega\eta}{1-\alpha\beta} (\hat{p}_{t+1}^* - \hat{P}_{t+1} + \pi_{t+1}) \right] \right\}. \quad (\text{B.46})$$

As the expectation in (B.46) is only about variables about which the private agents hold rational expectations, we can replace $E_t^{\mathcal{P}}[\cdot]$ with $E_t[\cdot]$.⁴⁴ Therefore, (C.4) can be used in period t

⁴³This follows from the the fact that in steady state, we have $p^* = P$, so that

$$\frac{\eta}{\eta-1} \phi C^{-1} \tilde{w}(j) \left(\frac{Y}{A}\right)^{\phi} = C^{-1} (1-\tau) Y.$$

The steady state value of the numerator in (B.45) is thus given by $\frac{1}{1-\alpha\beta} \frac{\eta}{\eta-1} \phi C^{-1} \tilde{w}(j) \left(\frac{Y}{A}\right)^{\phi}$ and the steady state value of the denominator by $\frac{1}{1-\alpha\beta} C^{-1} (1-\tau) Y$.

⁴⁴The subjective consumption plans showing up in the stochastic discount factor drop out at this order of approximation.

and $t + 1$, which in its linearized form is given by

$$\widehat{p}_t^* - \widehat{P}_t = \frac{\alpha}{1 - \alpha} \pi_t.$$

Substituting $\widehat{w}_t(j)$ by the linearized version of the equilibrium condition (C.3) delivers the linearized New Keynesian Phillips Curve:

$$\pi_t = \kappa_y y_t^{gap} + \kappa_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) + \beta E_t \pi_{t+1}, \quad (\text{B.47})$$

where the coefficients κ_q and κ_y are given by

$$\begin{aligned} \kappa_y &= \frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + \omega\eta} (k_y - f_y) > 0 \\ \kappa_q &= -\frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + \omega\eta} f_q < 0, \end{aligned}$$

with $k_y = \partial \log k / \partial \log y$, $f_y = \partial \log f / \partial \log y$, $f_q = \partial \log f / \partial \log q^u$, such that

$$\begin{aligned} k_y - f_y &= \omega + \frac{\underline{Y}}{\underline{C} + \frac{1}{1-\bar{\alpha}} \underline{k}} = \omega + C_Y > 0 \\ f_q &= \frac{\frac{\underline{k}}{1-\bar{\alpha}}}{\underline{C} + \frac{1}{1-\bar{\alpha}} \underline{k}} = -C_q > 0, \end{aligned}$$

where $C_q \equiv \frac{q^u}{C} \frac{\partial C}{\partial q^u}$ and $C_Y \equiv \frac{Y}{C} \frac{\partial C}{\partial Y}$, and where the functions $f(Y, q^u; \xi) \equiv (1 - \tau) Y C(Y, q^u; \xi)^{-1}$ and $k(y; \xi) \equiv \frac{\eta}{\eta-1} \lambda \phi \frac{1}{A^{1+\omega}} Y^{1+\omega}$ are the same as in Adam and Woodford (2021), for the current period in which markets clear and the internally rational agents observe this.

As in the standard New Keynesian model, a linearization of the law of motion of price dispersions

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}) \quad (\text{B.48})$$

implies that the state variable Δ_t is zero to first order under the maintained assumption that initial price dispersion satisfies $\Delta_{-1} \sim O(2)$. This constraint, together with the assumption that the Lagrange multipliers are of order $O(1)$, thus drops out of the quadratic formulation of the optimal policy problem. The second-order approximation of (C.5) is, however, important to express the quadratic approximation of utility in terms of inflation.

B.12 Calibration of ζ_q

To calibrate $\zeta_q \equiv -C_q/C_Y$, the negative ratio of the consumption elasticities to housing prices and income, respectively, note that from appendix "Second-Order Conditions for Optimal Allocation" in Adam and Woodford (2021, p. 49), we have

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u} \frac{1}{1-\bar{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^\chi}$$

where $\chi \equiv \frac{1}{1-\bar{\alpha}} - 1$. In our formulation, we have defined

$$C_q \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial \ln q_t^u} = \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q_t^u} \frac{\partial q_t^u}{\partial \ln q_t^u} = C_{q^u}(Y_t, q_t^u; \xi_t) \frac{q_t^u}{C_t}$$

so that we have

$$C_q = -\frac{\frac{1}{1-\alpha}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

From [Adam and Woodford \(2021, p.50\)](#), we also have

$$C_Y(Y_t, q_t^u, \xi_t) \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial Y_t} = \frac{1}{1 + \Omega(q_t^u, \xi_t) (1 + \chi) C(Y_t, q_t^u, \xi_t)^\chi}$$

so that in our notation

$$C_Y \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial \ln Y_t} = \frac{Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t) (1 + \chi) C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

We then have

$$\zeta_q = \frac{\frac{\frac{1}{1-\alpha}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}}{\frac{Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)(1+\chi)C(Y_t, q_t^u, \xi_t)^{\chi+1}}} = \frac{1}{1-\tilde{\alpha}} \frac{\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{Y_t}.$$

In the steady state, we have $\bar{Y} = \bar{C} + \bar{\Omega}\bar{C}^{\chi+1}$, which says that output \bar{Y} is divided up into consumption \bar{C} and resources invested in the housing sector, $\bar{\Omega}\bar{C}^{1+\chi}$. We thus have that

$$\frac{\bar{\Omega}\bar{C}^{\chi+1}}{\bar{Y}} = 1 - \frac{\bar{C}}{\bar{Y}} = 1 - \frac{\bar{C}}{\bar{C} + \bar{\Omega}\bar{C}^{\chi+1}} = 1 - \frac{1}{1 + \bar{\Omega}\bar{C}^\chi}.$$

Following [Adam and Woodford \(2021\)](#), we set this to the share of housing investment to total consumption, $\bar{\Omega}\bar{C}^\chi$, equal to 6.3%, so that in steady state we have

$$\zeta_q = \frac{1}{1-\tilde{\alpha}} \left(1 - \frac{1}{1.063} \right)$$

Finally, following [Adam and Woodford \(2021\)](#), we set the long-run elasticity of housing supply equal to five, which implies $\tilde{\alpha} = 0.8$, so that

$$\zeta_q = 5 \left(1 - \frac{1}{1.063} \right) \approx 0.29633.$$

From this, it follows that

$$C_Y = \frac{Y}{C + (1+\chi)\Omega C^{\chi+1}} = \frac{C+k}{C + \frac{1}{1-\tilde{\alpha}}k} = \frac{1 + \frac{k}{C}}{1 + \frac{1}{1-\tilde{\alpha}}\frac{k}{C}} = \frac{1 + 0.063}{1 + 5 \cdot 0.063} = 0.80836$$

and

$$C_q = -0.29633 \cdot 0.80836 = -0.23954.$$

Appendix C Details on optimal monetary policy

C.1 The non-linear optimal policy problem

The non-linear optimal monetary policy problem is given by

$$\max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t) \quad (\text{C.1})$$

subject to

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} (1-\tau) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \quad (\text{C.2})$$

$$w_t(j) = \lambda \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C(Y_t, q_t^u, \xi_t)^{\tilde{\sigma}-1} \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \quad (\text{C.3})$$

$$(P_t/P_{t-1})^{\eta-1} = \frac{1 - (1-\alpha) \left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha} \quad (\text{C.4})$$

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}) \quad (\text{C.5})$$

$$\tilde{u}_C(C(Y_t, q_t^u, \xi_t); \xi_t) = \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1}/P_{t+k}} \right] \quad (\text{C.6})$$

$$q_t^u = \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u, \quad (\text{C.7})$$

where the initial price level P_{-1} and initial price dispersion Δ_{-1} are given. Equation (C.3) insures that wages clear current labor markets. Similarly, by setting $C_t = C(Y_t, q_t^u, \xi_t)$ on the left-hand side of the consumption Euler equation (C.6), we impose market clearing for output goods in period t . Similarly, setting q_t^u equal to the value defined in (C.7) insures market clearing in the housing market.⁴⁵

C.2 Quadratic approximation of the policy problem

This appendix derives the quadratic loss function that the Ramsey planner minimizes (following Adam and Woodford, 2021). For this, we need to compute the second-order approximation to

$$\sum_{t=0}^{\infty} \beta^t [U(Y_t, \Delta_t, q_t^u, \xi_t) + \Gamma_t(h(\Delta_{t-1}, \Pi_t) - \Delta_t)]. \quad (\text{C.8})$$

In the optimal steady state, we have $U_Y = U_{q^u} = U_{Yq^u} = 0$, as well as $U_{\Delta} + \Gamma(\beta h_1 - 1) = 0$. Given the assumption $\Delta_{-1} \sim O(2)$, it follows $\Delta_t \sim O(2)$ for all $t \geq 0$. Additionally, we have $h_2 \equiv \frac{\partial h(\Delta, \Pi)}{\partial \Pi} = 0$ at the optimal steady state. Therefore, a second-order approximation of the contribution of the variables $(Y_t, \Delta_t, q_t^u, \Pi_t, \xi_t)$ to the utility of the household yields

$$\frac{1}{2} U_{\hat{Y}\hat{Y}} (\hat{y}_t - \hat{y}_t^*)^2 + \frac{1}{2} U_{\hat{q}^u \hat{q}^u} (\hat{q}_t^u - \hat{q}_t^{u*})^2 + \frac{1}{2} \Gamma^* h_{22} \pi_t^2 + t.i.p.,$$

where *t.i.p.* contains all terms independent of policy. Under rational expectations, we have that $(\hat{q}_t^u - \hat{q}_t^{u*}) = 0$ and is thus constant and independent of (monetary) policy. Under subjective beliefs, $(\hat{q}_t^u - \hat{q}_t^{u*})$ is purely driven by beliefs γ_t^u and housing demand shocks ξ_t^d , both independent of policy. Therefore, we include $\frac{1}{2} U_{\hat{q}^u \hat{q}^u} (\hat{q}_t^u - \hat{q}_t^{u*})^2$ in *t.i.p.*

The term $U_{\hat{Y}\hat{Y}}$ is given by $U_{\hat{Y}\hat{Y}} \equiv Y \frac{\partial}{\partial Y} (U_{\hat{Y}}) \equiv Y \frac{\partial}{\partial Y} (Y U_Y) = \underline{Y}^* U_Y + (\underline{Y}^*)^2 U_{YY}$. At the optimal steady state, we have

$$\begin{aligned} \Lambda_{\pi} &= -\frac{1}{2} \Gamma^* h_{22} > 0 \\ \Lambda_y &= -\frac{1}{2} (\underline{Y}^*)^2 U_{YY} > 0, \end{aligned}$$

⁴⁵This holds as long as D^{\max} is chosen sufficiently large, such that it never binds along the equilibrium path.

where

$$\begin{aligned}
U_{YY} &= -C(\underline{Y}, \underline{q}^u, \underline{\xi})^{-2} C_Y \frac{\underline{Y}^*}{C(\underline{Y}, \underline{q}^u, \underline{\xi})} \\
&\quad - \frac{\lambda}{1+\nu} (1+\omega) \omega \frac{1}{\underline{A}^{1+\omega}} \underline{Y}^{\omega-1} < 0 \\
h_{22} &= \frac{\alpha\eta(1+\omega)(1+\omega\eta)}{1-\alpha} > 0 \\
\underline{\Gamma}^* &= \frac{U_\Delta}{1-\alpha\beta} < 0,
\end{aligned}$$

with

$$U_\Delta = -\frac{\underline{Y}^*}{1+\omega} \frac{1}{C(\underline{Y}^*, \underline{q}^{u*}, \underline{\xi})} < 0.$$

C.3 Recursified optimal policy problem

We numerically solve the quadratically approximated optimal policy problem with forward-looking constraints. To obtain a recursive problem, we apply the approach of [Marcet and Marimon \(2019\)](#) to the following problem

$$\begin{aligned}
&\max_{\{\pi_t, y_t^{gap}, i_t \geq i\}} \min_{\{\varphi_t, \lambda_t\}} \tag{C.9} \\
&E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2 \right) \right. \\
&\quad + \varphi_t [\pi_t - \kappa_y y_t^{gap} - \kappa_q (\hat{q}_t^u - \hat{q}_t^{u*}) - \beta E_t \pi_{t+1}] \\
&\quad + \lambda_t \left[y_t^{gap} - \lim_{T \rightarrow \infty} E_t y_T^{gap} + E_t \sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+1+k} - \check{r}_{t+k}) - \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \right] \\
&\quad \left. - \varphi_{-1} \pi_0 - \lambda_{-1} (\pi_0 - y_0^{gap} + \zeta_q (\hat{q}_0^u - \hat{q}_0^{u*})) \right\},
\end{aligned}$$

where the process for $(\hat{q}_t^u - \hat{q}_t^{u*})$ can be treated as exogenous for the purpose of monetary policy and where the initial Lagrange multipliers $(\varphi_{-1}, \lambda_{-1})$ capture initial pre-commitments.

To apply the approach of [Marcet and Marimon \(2019\)](#) to the problem with forward-looking constraints (C.9), we assume that the Lagrangian defined above satisfies the usual duality properties that allow interchanging the order of maximization and minimization, which we verify ex-post using the computed value function. We set the terminal value function for $t = T'$ to its RE value function $W^{RE}(\cdot)$. For $t \leq T'$ we have a value function $W_t(\cdot)$ satisfying the following

recursion:

$$\begin{aligned}
& W_t(\varphi_{t-1}, \mu_{t-1}, \check{r}_t, \gamma_t^u, \xi_t^d, q_{t-1}^u) \\
= & \max_{(\pi_t, y_t^{gap}, i_t \geq \underline{i})} \min_{(\varphi_t, \lambda_t)} -\frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2 \right) \\
& + (\varphi_t - \varphi_{t-1}) \pi_t - \varphi_t (\kappa_y y_t^{gap} + \kappa_q (\hat{q}_t^u - \hat{q}_t^{u*})) \\
& + \lambda_t \left[y_t^{gap} - \lim_{T \rightarrow \infty} E_t y_T^{gap} + \left(i_t - E_t \sum_{k=0}^{\infty} \check{r}_{t+k} \right) - \zeta_q (\hat{q}_t^u - \hat{q}_t^{u*}) \right] + \mu_{t-1} (i_t - \pi_t) \\
& + \beta E_t \left[W_{t+1}(\varphi_t, \underbrace{\beta^{-1}(\lambda_t + \mu_{t-1})}_{=\mu_t}, \check{r}_{t+1}, \gamma_{t+1}^u, \xi_{t+1}^d, q_t^u) \right]
\end{aligned} \tag{C.10}$$

where the next period state variables (γ_{t+1}^u, q_t^u) are determined by equations (10) and (23) and $(\hat{q}_t^u - \hat{q}_t^{u*})$ is determined by equation (26). Here we assume that \check{r}_t follows a Markov process, such that the term $E_t \sum_{k=0}^{\infty} \check{r}_{t+k}$ showing up in the current-period return can be expressed as a function of the current state \check{r}_t . The future state variables $(\varphi_t, \mu_t, \gamma_{t+1}^u, q_t^u)$ are predetermined in period t . The expectation about the continuation value is thus only over the exogenous states $(\check{r}_{t+1}, \xi_{t+1}^d)$. The endogenous state variable φ_{t-1} is simply the lagged Lagrange multiplier on the New Keynesian Phillips curve. The endogenous state variable μ_{t-1} is given for all $t \geq 0$ by

$$\mu_t = \beta^{-(t+1)} (\lambda_0 + \mu_{-1}) + \beta^{-t} \lambda_1 + \dots + \beta^{-1} \lambda_t.$$

The initial values (φ_{-1}, μ_{-1}) are given at time zero and equal to zero in the case of time zero-optimal monetary policy.

For periods $t < T'$, where T' is the period from which housing price expectations are rational and the lower bound constraint ceases to bind, the value functions depend on time, thereafter they are time-invariant. Likewise, for sufficiently large T' , the value functions $W_t(\cdot)$ and $W_{t+1}(\cdot)$ will become very similar.

We can numerically solve for the value function $W_t(\cdot)$ by value function iteration, starting with $W_{T'}$ which is the value function associated with the linear-quadratic problem with RE.

C.4 Optimal targeting rule

Taking the first-order conditions of the optimal policy Lagrangean (C.10), including the constraint $i_t \geq \underline{i}$ with Lagrange multiplier ζ_t , with respect to $\{\pi_t, y_t^{gap}, i_t\}$ yields:

$$\begin{aligned}
\frac{\partial W_t}{\partial \pi_t} &= -\Lambda_\pi \pi_t + (\varphi_t - \varphi_{t-1}) - \mu_{t-1} = 0 \\
\frac{\partial W_t}{\partial y_t^{gap}} &= -\Lambda_y y_t^{gap} - \varphi_t \kappa_y + \lambda_t = 0 \\
\frac{\partial W_t}{\partial i_t} &= \zeta_t + \lambda_t + \mu_{t-1} = 0 \text{ and } \zeta_t (i_t - \underline{i}) = 0.
\end{aligned}$$

Combining these, we can derive the following targeting rule which characterizes optimal monetary policy

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) + \frac{\lambda_{t-1}}{\kappa_y} + \mu_{t-1} \left(1 + \frac{1}{\kappa_y} \right) + \frac{\zeta_t}{\kappa_y} = 0.$$

If the lower bound on the nominal interest rate does not bind in the current period, we have $\zeta_t = 0$. Furthermore, if the lower bound has not been binding up to period t , the IS equation has not posed a constraint for the monetary policymaker. Thus, $\lambda_{t-1} = \lambda_{t-k} = 0$ for all $k = 0, 1, \dots, t$. For an initial value of $\mu_{-1} = 0$, it follows that $\mu_{t-1} = 0$. The targeting rule then collapses to

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) = 0,$$

which is the same as in [Clarida, Galí, and Gertler \(1999\)](#).

The Lagrange multiplier $\zeta_t \leq 0$ captures the cost of a currently binding lower bound. If $\zeta_t < 0$, the optimal policy requires a compensation in the form of a positive output gap or inflation. The multipliers λ_{t-1} and μ_{t-1} capture promises from past commitments when the lower bound was binding.

Another way to express equation (C.11) is to write it as

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) + \frac{1}{\kappa_y} \left[\zeta_t - \frac{1 + \beta + \kappa_y}{\beta} \zeta_{t-1} + \frac{\zeta_{t-2}}{\beta} \right] = 0. \quad (\text{C.11})$$

Housing prices do not enter the optimal target criterion directly but larger fluctuations in housing prices make the lower bound bind more often and for a longer period of time. The optimal policy, thus, requires larger compensations in terms of positive output gaps and inflation. To implement this, the nominal interest rate needs to be kept longer at the lower bound.