Disproof of John Stewart Bell

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Abstract

The proof of John Stewart Bell [1] [2] is based on wrong premises. A deterministic model for a hidden mechanism with an parameter λ is therefor possible. This model is possible by an integration of the experimental context into the wave function

1 The basic experiment

The following considerations are based on an experiment with a pair of entangled photons, each of them meets a polarisation filter. The crucial phenomenon consists of the fact, that, if the polarisation filters are adjusted identically, the photons react always identically: Either both will be absorbed or both will be transmitted. For the angular difference Φ between the polarisation filter adjustments the probability, that both photons give identical measurement results, is the square of the cosine of that angular difference (law of Malus)[3]. The set of the possible measurement results is a binary one, it can be referred to by expressions like $\{0,\ 1\}$ or $\{+1,\ -1\}$. For later considerations of serie experiments and related expected values I use the set $\{+1,\ -1\}$, for the treatment of the proof variant according to Wigner-Bell I choose the set $\{0,\ 1\}$.

I use the following assignments for measurement results and eigenstates

- $0 \leftarrow |0\rangle \rightarrow 1$
- $1 \leftarrow |1\rangle \rightarrow -1$

May α denote the measurement result of the left photon and β the one of the right photon. The upper proposition for the conditional probability P of identical measurement results given the angular difference Φ one would write like this:

$$P\left(\alpha = \beta | \Phi\right) = \cos^2 \Phi \tag{1}$$

1.1 The quantum mechanical doctrine

In the literature that I know there is a strict separation between the quantum state $|\Psi\rangle$ of the pair of photons on the one hand and the experimental context, consisting of nothing more than the angular difference of the polarisation filters or the polarizers, on the other hand. (polarisation filters prepair the transmitting photons as polarised with angle Φ – for that using the term „polarizer“ does make sense). One proposes on the one hand, the quantum state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \tag{2}$$

describes entirely the pair of the entangled photons and on the other hand the conditional probability of identical measurement results - given the angular difference Φ – is given by 1. Thereby it is noticed, that for a Φ with $0 < \Phi < \frac{\pi}{2}$ the wave function $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ does not reproduce the experimentally measured statistics. This leads to a contradiction with the Born interpretation of the wave function [4].

1.2 A first solution

Provisionally I ignore possibly existing individual polarisation of each of the entangled photons. I just not refer to it in the following. Only one sentence about it: If there would be an individual polarisation, then it would be identical for the both of the photons and it would be evenly distributed.[1]

From the strict separation of the quantum state and the experimental context follows not only the above showed violation of the Born interpretation of the wave function (i.e. the mathematical expression of the quantum state), but furthermore this separation enables the wrong argumentation of John Stewart Bell. How this works in detail I will explain soon. For now I start with getting rid of the separation between the quantum state and the experimental context (i.e. Φ). For equation 1 the eigenstates $\{|00\rangle\,,\ |11\rangle\}$ the weight $\cos^2\Phi$ is alotted to and so corresponding the eigenstates $\{|01\rangle\,,\ |10\rangle\}$ the weight $1 - \cos^2\Phi = \sin^2\Phi$ is alotted to. The easiest way is to distribute the weights among the eigenvalues in equal parts:

$$|\Psi_\Phi\rangle = \frac{\cos\Phi}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) + \frac{\sin\Phi}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) \tag{3}$$

[1] I believe, that there is a solution with respect to individual polarisation ϕ, but for now I dont talk about that

This simple wave function reproduces the statistics of serial measurements with pairs of entangled photons. It combines the quantum state of the photons with the experimental context. The relation between the correlation of the measurement results and the angular difference Φ is described correctly.

2 The proof variant according to Wigner-Bell - the 0°-30°-60°-Game

At next let us consider a single photon and a polarizer, for which there are three possible adjustments as it is described in [5]. Further I assume, that, as soon the photon is on its way to the polarizer, the measurement results for all possible adjustments of the polarizer are determined by a law. At three adjustments we get eight possible strategies, out of which our single photon can choose one:

0° (α)	30° (β)	60° (γ)	weight
0	0	0	f_1
0	0	1	f_2
0	1	0	f_3
0	1	1	f_4
1	0	0	f_5
1	0	1	f_6
1	1	0	f_7
1	1	1	f_8

In serial experiments the weights f_i correspond to the relative frequencies, and it applies $0 \leq f_i \leq 1,\ \sum_{i=1}^{8} f_i = 1$

Presuming further, that entangled photons choose always identical strategies and denoting with **a** the adjustment at the left polarizer and with **b** the one at the right one, so let us look at the following probabilities:

$$\begin{aligned} P\left(\alpha=1,\ \beta=0|a=0°,\ b=30°\right) &= P\left(\alpha=1,\ \beta=0,\ \gamma=0|a=0°,\ b=30°\right) \\ &+ P\left(\alpha=1,\ \beta=0,\ \gamma=1|a=0°,\ b=30°\right) = f_5+f_6 \end{aligned}$$

$$\begin{aligned} P\left(\beta=1,\ \gamma=0|a=30°,\ b=60°\right) &= P\left(\alpha=0,\ \beta=1,\ \gamma=0|a=30°,\ b=60°\right) \\ &+ P\left(\alpha=1,\ \beta=1,\ \gamma=0|a=30°,\ b=60°\right) = f_3+f_7 \end{aligned}$$

$$\begin{aligned} P\left(\alpha=1,\ \gamma=0|a=0°,\ b=60°\right) &= P\left(\alpha=1,\ \beta=0,\ \gamma=0|a=0°,\ b=60°\right) \\ &+ P\left(\alpha=1,\ \beta=1,\ \gamma=0|a=0°,\ b=60°\right) = f_5+f_7 \end{aligned}$$

$\alpha,\ \beta$ and γ are denoting here the measurement results at 0°, 30° and 60°.

From these three equations it is obvious, that the following inequality must apply

$$\begin{aligned} &P\left(\alpha=1,\ \beta=0|a=0^\circ,\ b=30^\circ\right)+P\left(\beta=1,\ \gamma=0|a=30^\circ,\ b=60^\circ\right)\\ &\quad=f_5+f_6+f_3+f_7\geq f_5+f_7=P\left(\alpha=1,\ \gamma=0|a=0^\circ,\ b=60^\circ\right) \end{aligned} \tag{4}$$

The measurement results for both channels

- $(\alpha=1,\ \beta=0)$ in the 0°-30°-experiment
- $(\beta=1,\ \gamma=0)$ in the 30°-60°-experiment
- $(\alpha=1,\ \gamma=0)$ in the 0°-60°-experiment

correspond to the eigenstate $|10\rangle$, representing the measurement result, that the left photon will be absorbed, the right one will be transmitted. We know, that this eigenstate for an angular difference Φ the half of the weight is alotted to, which is alotted to the eigenstates $\{|01\rangle,\ |10\rangle\}$ and that is $\frac{\sin^2\Phi}{2}$. So we get the Wigner-Bell-inequality [7]

$$\begin{aligned} \frac{\sin^2 30^\circ}{2}+\frac{\sin^2 30^\circ}{2}&\geq\frac{\sin^2 60^\circ}{2}\\ \frac{1}{8}+\frac{1}{8}&\geq\frac{3}{8} \end{aligned} \tag{5}$$

which can not at all hold true. From that one deduces the falsehood of one of the premises. Especially the one, that there are predetermined photons strategies $(\alpha,\ \beta,\ \gamma)$ ect.

3 Introduction of the χ_Φ-Model

Lets consider the map

$$\begin{aligned} \chi_\Phi:\ \left[0,\ \frac{\pi}{2}\right]\times[0,\ 1]&\rightarrow\{-1,\ 1\}\\ (\theta,\ \lambda)&\mapsto\chi_\Phi\left(\theta,\ \lambda\right) \end{aligned}$$

with $\theta=0$ and $\theta=\Phi$. The case $\theta=0$ represents the left photon channel, the case $\theta=\Phi$ represents the right one respectively. Further for a more general wave function in the product space of both channels given by the wave function

$$\left|\Psi_\Phi\right\rangle = a_\Phi \left|00\right\rangle + b_\Phi \left|01\right\rangle + c_\Phi \left|10\right\rangle + d_\Phi \left|11\right\rangle \tag{6}$$

the corresponding functions for the channels are given by

$$\chi_\Phi\left(0,\ \lambda\right) = \begin{cases} 1,\ 0 \le \lambda \le a_\Phi^2 + b_\Phi^2 \\ -1,\ a_\Phi^2 + b_\Phi^2 \le \lambda \le 1 \\ 0 \text{ else} \end{cases}$$

for the left channel and respectively

$$\chi_\Phi\left(\Phi,\ \lambda\right) = \begin{cases} 1,\ b_\Phi^2 \le \lambda \le a_\Phi^2 + b_\Phi^2 + c_\Phi^2 \\ -1,\ \lambda \in \left[0,\ b_\Phi^2\right] \cup \left[a_\Phi^2 + b_\Phi^2 + c_\Phi^2,\ 1\right] \\ 0 \text{ else} \end{cases}$$

for the right channel.

Remark: For a distribution

$$\rho\left(\lambda\right) = \begin{cases} 1,\ 0 \le \lambda \le 1 \\ 0 \text{ else} \end{cases}$$

the expected values for the measurement results at the left and at the right channel are

$$\begin{aligned} E_\Phi\left(\alpha\right) = E\left(\chi_\Phi\left(0\right)\right) &= \int_0^1 d\lambda\ \rho\left(\lambda\right)\ \chi_\Phi\left(0,\ \lambda\right) = 0 \\ E_\Phi\left(\beta\right) = E\left(\chi_\Phi\left(\Phi\right)\right) &= \int_0^1 d\lambda\ \rho\left(\lambda\right)\ \chi_\Phi\left(\Phi,\ \lambda\right) = 0 \end{aligned} \tag{7}$$

and they are both zero, according to the fact, that for each channel we get a 50%-50%-measurement series, that means 50% of the photons in each channel will be absorbed and 50% will be transmitted.[2]

Soon it will turn out, that the expected value $E\left(\alpha \cdot \beta\right)$ of the product of the measurement results – out of the set $\{-1,\ 1\}$ – is not zero, so $E\left(\alpha \cdot \beta\right) = E\left(\alpha\right) \cdot E\left(\beta\right)$ does not apply.

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[2] $\int_0^1 d\lambda\ \rho\left(\lambda\right)\ \chi_\Phi\left(0,\ \lambda\right) = a_\Phi^2 + b_\Phi^2 - c_\Phi^2 - d_\Phi^2 = \frac{cos^2\Phi}{2} + \frac{sin^2\Phi}{2} - \frac{sin^2\Phi}{2} - \frac{cos^2\Phi}{2} = 0$

Let us continue with the χ_Φ-Model: It is clear, that the general wave function 6 with coefficients $a_\Phi = d_\Phi = \frac{\cos\Phi}{\sqrt{2}},\ b_\Phi = c_\Phi = \frac{\sin\Phi}{\sqrt{2}}$ turns into the special symmetric weighted wave function 3.

Now the central proposition:

If λ spreads evenly distributed over the interval [0, 1], the pair $(\chi_\Phi(0,\ \lambda),\ \chi_\Phi(\Phi,\ \lambda))$ reproduces the experimental statistics of the measurement results $(\alpha,\ \beta)$ where the angular difference is Φ.

At next I consider the angular pairs (left channel: 0°, right channel: 30°), (left channel: 30°, right channel: 60°) and (left channel: 0°, right channel: 60°) and for each of it I look at the weight alotting to the eigenstate $|10\rangle$ corresponding to the measurement results (left channel: -1, right channel: 1):

For the pair (0°, 30°) the interval of those λ, so that $(\chi_{30°}(0,\ \lambda),\ \chi_{30°}(30°,\ \lambda)) = (-1,\ 1)$ applies, is $[a^2_{30°} + b^2_{30°},\ a^2_{30°} + b^2_{30°} + c^2_{30°}]$, this interval has the size $c^2_{30°} = \frac{sin^2 30°}{2} = \frac{1}{8}$

For the pair (30°, 60°) we get the same size, which means the same weight.

For the pair (0°, 60°) the interval of those λ, so that $(\chi_{60°}(0,\ \lambda),\ \chi_{60°}(60°,\ \lambda)) = (-1,\ 1)$ applies, is $[a^2_{60°} + b^2_{60°},\ a^2_{60°} + b^2_{60°} + c^2_{60°}]$, this interval has the size $c^2_{60°} = \frac{sin^2 60°}{2} = \frac{3}{8}$

So far there is no difference between me and the Wigner-Bell proof. To get it clearer, I give now a table for the three experiments with the photons strategies and the corresponding weights alotting to them.

Remark: The fact of the entanglement is reflected by the fact, that for entangled photons the parameter λ is identical and for an angular difference $\Phi = 0$ we get for the measurement results in the left and the right channel $(\alpha,\ \beta) = (\chi_\Phi(0,\ \lambda),\ \chi_\Phi(\Phi,\ \lambda)) = (\chi_0(0,\ \lambda),\ \chi_0(0,\ \lambda))$, so for $\Phi = 0$ one gets − according to the observation − identical measurement results in the left and in the left and in the right channel.

0° (α)	30° (β)	60° (γ)	weight(0°, 30°)	weight(30°, 60°)	weight(0°, 60°)
0	0	0	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
0	0	1	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{3}{8}$
0	1	0	$\frac{1}{8}$	$\mathbf{\frac{1}{8}}$	$\frac{1}{8}$
0	1	1	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
1	0	0	$\mathbf{\frac{1}{8}}$	$\frac{3}{8}$	$\mathbf{\frac{3}{8}}$
1	0	1	$\mathbf{\frac{1}{8}}$	$\frac{1}{8}$	$\frac{1}{8}$
1	1	0	$\frac{3}{8}$	$\mathbf{\frac{1}{8}}$	$\mathbf{\frac{3}{8}}$
1	1	1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

It is obvious, that the distribution of the weights depends of the choosen angular pair. If I look now again at the inequalities 4, 5 according to this table...

$$\begin{aligned} &P\left(\alpha=1,\ \beta=0|a=0°,\ b=30°\right)+P\left(\beta=1,\ \gamma=0|a=30°,\ b=60°\right)\\ &=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}<\frac{3}{16}+\frac{3}{16}=P\left(\alpha=1,\ \gamma=0|a=0°,\ b=60°\right)\end{aligned} \tag{8}$$

we get the result: The inequalities 4, 5 do not occur. The weights of the considered sets, which are created by the experimental series, depend of the angular pairs and of the angular difference. Actually we deal with two different wave functions:

- (0°, 30°), (30°, 60°): $|\Psi_{30°}\rangle = \frac{3}{8}\left(|00\rangle + |11\rangle\right) + \frac{1}{8}\left(|01\rangle + |10\rangle\right)$
- (0°, 60°): $|\Psi_{60°}\rangle = \frac{1}{8}\left(|00\rangle + |11\rangle\right) + \frac{3}{8}\left(|01\rangle + |10\rangle\right)$

instead of the so called singulet state with no regard to the experimental context. More obvious it gets, if I enter instead of the numbers the coefficients indexed with the angular difference Φ:

0° (α)	30° (β)	60° (γ)	weight(0°, 30°)	weight(30°, 60°)	weight(0°, 60°)
0	0	0	$\frac{a_{30°}^2}{2}$	$\frac{a_{30°}^2}{2}$	$\frac{a_{60°}^2}{2}$
0	0	1	$\frac{a_{30°}^2}{2}$	$\frac{b_{30°}^2}{2}$	$\frac{b_{60°}^2}{2}$
0	1	0	$\frac{b_{30°}^2}{2}$	$\boldsymbol{\frac{c_{30°}^2}{2}}$	$\frac{a_{60°}^2}{2}$
0	1	1	$\frac{b_{30°}^2}{2}$	$\frac{d_{30°}^2}{2}$	$\frac{b_{60°}^2}{2}$
1	0	0	$\boldsymbol{\frac{c_{30°}^2}{2}}$	$\frac{a_{30°}^2}{2}$	$\boldsymbol{\frac{c_{60°}^2}{2}}$
1	0	1	$\boldsymbol{\frac{c_{30°}^2}{2}}$	$\frac{b_{30°}^2}{2}$	$\frac{d_{60°}^2}{2}$
1	1	0	$\frac{d_{30°}^2}{2}$	$\boldsymbol{\frac{c_{30°}^2}{2}}$	$\boldsymbol{\frac{c_{60°}^2}{2}}$
1	1	1	$\frac{d_{30°}^2}{2}$	$\frac{d_{30°}^2}{2}$	$\frac{d_{60°}^2}{2}$

The weights belonging to the eigenstate $|10\rangle$ are set bold.

A third table, which deals with anonymous identifiers, provides us with the analogue of the table in section 2, but this time instead of only one column for the weights f_i it contains three columns for the weights f_i, g_i and h_i, with respect to the three angular pairs (0°, 30°), (30°, 60°) and (0°, 60°):

0° (α)	30° (β)	60° (γ)	weight(0°, 30°)	weight(30°, 60°)	weight(0°, 60°)
0	0	0	f_1	g_1	h_1
0	0	1	f_2	g_2	h_2
0	1	0	f_3	g_3	h_3
0	1	1	f_4	g_4	h_4
1	0	0	f_5	g_5	h_5
1	0	1	f_6	g_6	h_6
1	1	0	f_7	g_7	h_7
1	1	1	f_8	g_8	h_8

According to the inequality 8 we have

$$\begin{aligned} &P\left(\alpha=1,\ \beta=0|a=0°,\ b=30°\right)+P\left(\beta=1,\ \gamma=0|a=30°,\ b=60°\right) \\ &\quad = f_5+f_6+g_3+g_7 < h_5+h_7 = P\left(\alpha=1,\ \gamma=0|a=0°,\ b=60°\right) \end{aligned} \tag{9}$$

and this time the inequality is correct and there is no violation, so the contradiction of Wigner-Bell disappears.

4 The Bellian inequality by Abner Shimony

For two adjustmens $(a,\ b)$, $(a',\ b')$ Shimony [8] derives - as shown in [6] - the inequality

$$
\begin{aligned}
-2 \leq E_{b-a}(\alpha) \cdot E_{b-a}(\beta) + E_{b-a'}(\alpha) \cdot E_{b-a'}(\beta) + E_{b'-a}(\alpha) \cdot E_{b'-a}(\beta) \\
- E_{b'-a'}(\alpha) \cdot E_{b'-a'}(\beta) \leq 2
\end{aligned}
$$

For any pair $(a,\ b)$ of directions the expected values $E_{b-a}(\alpha) = E_{b-a}(\chi_{b-a}(0))$, $E_{b-a}(\beta) = E_{b-a}(\chi_{b-a}(b-a))$ both give zero as we have seen in 5, 6.

Inserting in the Shimonys inequality leads to

$$
\begin{aligned}
-2 &\leq 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 - 0 \cdot 0 \leq 2 \\
-2 &\leq 0 \leq 2
\end{aligned}
$$

But this inequality is not the wanted Bellian inequality

$$
-2 \leq E_{b-a}(\alpha \cdot \beta) + E_{b-a'}(\alpha \cdot \beta) + E_{b'-a}(\alpha \cdot \beta) - E_{b'-a'}(\alpha \cdot \beta) \leq 2 \quad (10)
$$

This wanted Bellian inequality cannot be obtained using the initial inequality by Shimony, for the basic adoption of that proof idea is the condition of the local factorizability of the expected values of the measurement results α and β:[3]

$$
\begin{aligned}
E_{\Phi}(\alpha \cdot \beta) = E(\chi_{\Phi}(0) \cdot \chi_{\Phi}(\Phi)) = \int_0^1 d\lambda\ \rho(\lambda)\ \ \chi_{\Phi}(0,\ \lambda)\ \ \chi_{\Phi}(\Phi,\ \lambda) \\
= \cos 2\Phi \neq 0 = E_{\Phi}(\alpha) \cdot E_{\Phi}(\beta)
\end{aligned}
$$

As one can see, this factorization condition is not fullfilled in case of the χ_{Φ}-Model. Thereby it follows, that on the one hand there is a possibility of a deterministic model – f.e. the χ_{Φ}-Model – which reproduces the observed statistics and which on the other hand does not fullfill the condition of factorizability of the expected values. For that condition is a necessary preadoption of Bells proof, we see, that the considered model fullfills the initial inequality and the Bell proof doesnt work for the χ_{Φ}-Model with the hidden parameter λ.

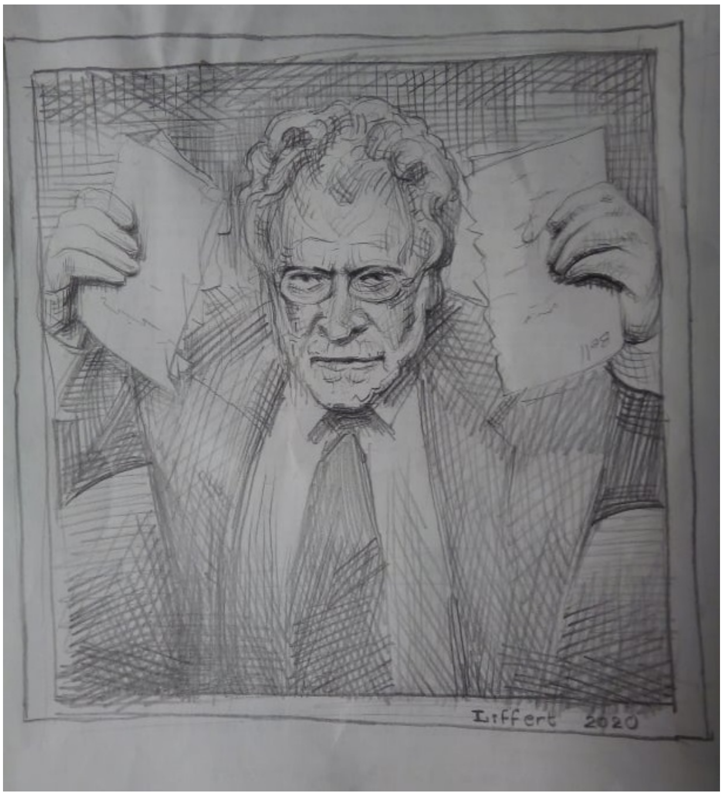
[3] $\int_0^1 d\lambda\ \rho(\lambda)\ \ \chi_{\Phi}(0,\ \lambda)\ \ \chi_{\Phi}(\Phi,\ \lambda) = a_{\Phi}^2 + d_{\Phi}^2 - b_{\Phi}^2 - c_{\Phi}^2 = \frac{cos^2\Phi}{2} + \frac{cos^2\Phi}{2} - \frac{sin^2\Phi}{2} - \frac{sin^2\Phi}{2} = \cos 2\Phi$

5 Conclusion

What is the picture of the situation now? I think, the best way, to describe it, is that:

- left channel: $\text{photon}_1 = (\lambda) \rightarrow$ left polarizer $= (\chi_\Phi (0))$
- right channel: $\text{photon}_2 = (\lambda) \rightarrow$ right polarizer $= (\chi_\Phi (\Phi))$

That pattern means, that we have the entangled photons, carrying the hidden parameter $\lambda \in [0,\ 1]$ and we have the hidden law χ_Φ at the polarizers, $\chi_\Phi (0)$ at the left polarizer and $\chi_\Phi (\Phi)$ at the right polarizer. Together with the parameter, carried by the photons, the law produces the measurement results in a deterministic way. This means especially, that I deny the conception, that the one photon transfers instantaneously its result to the other one. So the photons in my conception behave similar to two ideal identically prepaired (λ) missiles, fired on to two aims. If the aims are identically adjusted $(\Phi = 0)$, the missiles do the same. The degree of the similarity of their behaviour depends of the similarity of the adjustments at the aims. May be, Anton Zeilinger will not like it...



References

[1] Bell, John S.: *On the Einstein-Podolsky-Rosen paradox* Physics 1(3), pp 195-200, (1964)

[2] Bell, John S.: *Introduction to the hidden-variable question* In: B. d'Espagnat (Ed.). *Foundations of Quantum Mechanics: Proceedings of the International School of Physics 'Enrico Fermi', Course XLIX*, pp 171-181, (1971)

[3] https://www.quantumlab.nat.fau.de/

[4] Baumann, Kurt, Roman U. Sexl: *Die Deutungen der Quantentheorie*, (1984)

[5] Friebe C.; M. Kuhlmann; H. Lyre; P. Näger; O. Passon; M. Stöckler: *Philosophie der Quantenphysik* Einführung und Diskussion der zentralen Begriffe und Problemstellungen der Quantentheorie für Physiker und Philosophen, pp 133 - 137, (2015)

[6] ibid: p 142:

[7] Wigner, Eugene P.: *On hidden variables and quantum mechanical probabilities* American Journal of Physics **38** (1970), 10051009

[8] Shimony, Abner: *An exposition on Bell's theorem* In: A. I. Miller (Ed.). *Sixty two Years of Uncertainty: Historical, Philosophical, and Physical Inquiries Into the Foundations of Quantum Mechanics*, pp 33-43, (1990)