Markups and Marginal Costs over the Firm Life: Implications for the Optimal Inflation Target*

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- preliminary version -

Abstract

We estimate the dynamics of relative markups, marginal costs and prices over the firm life cycle using detailed firm data from Denmark. Relative marginal costs fall strongly over the first 15 years of firm life, but relative prices fall only weakly because of a strong rise in relative markups. Relative price trends thus underestimate trends in relative productivity. This distorts recent estimates of the optimal inflation target downward by 0.2-1.2% per year. We show that relative markups increase following the introduction of new products and the discontinuation of old products, suggesting that product turnover is important driver of markup dynamics at the firm level. Only about one third of the decrease in relative marginal cost over the firm age is explained by movements in relative productivity, with the remainder being due to non-homotheticities and increasing returns in the production function.

JEL classification: E52, L11, L13

Keywords: relative markups, marginal costs, prices over the firm life, optimal inflation

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1 Introduction

A nascent literature in monetary economics estimates the optimal inflation target from relative price information (Adam et al. (2022), Adam and Weber (2023)). It exploits the fact that microeconomic fundamentals of the optimal inflation target, such as trends in relative marginal cost or relative product quality, manifest themselves in relative price trends. This allows estimating the welfare-optimal inflation target from relative price trends (Adam and Weber (2019)).

Fundamental factors such as marginal costs are not the only element influencing the dynamics of relative prices: changes in relative market power over time might also affect relative prices and thereby give rise to a wedge between the trends observed in relative prices and the trends in the underlying relative marginal costs. When these wedges are changing over time, say due to rising relative market power, then estimates of the optimal inflation rate obtained from relative price trends are biased. This is an important concern, especially since market power has been documented to have increased considerably over time in a number of advanced economies, e.g. De Loecker et al. (2020).

The present paper documents relative markup trends over the firm life cycle and quantitatively assesses the potential bias that markup trends generate in recent estimates of the optimal inflation rate. In addition, it present interesting new facts about the behavior of relative markups and relative marginal costs over the firm life cycle. To the best of our knowledge, the present paper is the first to study the dynamics of relative markup trends over the firm life cycle. It does so by leveraging a unique data set from Denmark, which contains detailed information on firm level outputs and inputs. Our data also contains information on new product introduction and the discontinuation of old products, which allows us to study the relative markup effects associated with product changes at the firm level.

Using this data, we estimate firm-level markups and how these evolve over the firm life, using the production function approach of De Loecker and Warzynski (2012). Since we observe firm level output prices, our approach can address identification issues raised in Bond et al. (2021) and De Ridder et al. (2021), which turn out to be quantitatively important for the purpose of estimating the trends in *relative* markups over the firm life cycle.

Using this approach, we can decompose relative price trends over the firm life into trends in relative marginal cost and trends in relative markups. We can thus assess to what extent firm level trends in relative prices reflect trends in underlying relative marginal costs.

We find that relative markup trends are a quantitatively important force, especially during the first 15 years of firms' life: firms' relative markup increases over this time period by 12 percentage points (p.p.), while firms relative prices fall by about 5 p.p. The drop in relative prices thus falls considerably short of 17 p.p. drop in relative marginal costs that we observe on average over the first 15 years of firms' life.

This shows that estimates of relative productivity trends that rely on relative price trends are significantly biased towards zero for firms of young age, as they underestimate the strength of relative productivity trends. This in turn causes estimates of the optimal inflation rate that rely on relative price trends to be biased downwards.

We also show that relative markups remain approximately constant for firms that are older than 15 years: relative marginal costs and relative prices thus move in lockstep for these older firms. As a result, the optimal inflation rate inferred from firm level relative price trends is underestimated – depending on the precise age distribution of firms used in the estimation – by a rate that ranges between 0.2% to 1.2%. While this is economically sizable, the fact that relative markups for older firms do not display an age trend, causes the overall biases to be nevertheless contained.

The availability of firm level price information turns out to be key for our findings. If one uses industry price deflators to compute real output at the firm level, instead of firm level price information, one erroneously finds that relative markups are flat over the *entire* firm life. This occurs despite the fact that estimates of the average/median markup over time (in the the cross section of firms) are almost unaffected when using industry deflators instead of firm price. This latter finding is in line with results reported in De Ridder et al. (2021). Yet, that fails to be true when computing relative markup trends over the firm life.

Using our estimates, we also investigate the economic forces leading to the strong decrease in relative marginal costs and the strong increase in relative markups over the first 15 years of the firm life.

In our setup, marginal cost reductions can be driven by (i) firm level trends in total factor productivity, and (ii) efficiency gains associated with size-dependent output elasticities that are due to non-homotheticities or increasing returns to scale in the production function.[1] We find that the effect of (i) accounts for about 1/3 of the decrease in relative marginal costs, while (ii) accounts for roughly 2/3 of the drop in relative marginal costs over the firm life.

[1] We use a translog specification for the production function, which the data strongly prefers against a homothetic Cobb-Douglas specification.

We then investigate the potential forces behind the increase in relative markups over the first years of firm life. To this end, we link our firm level data with information on the introduction of new products. We document that the rate of product introduction for firms with an age up to about 15 years is approximately twice as high than that for older firms. Moreover, firm markups increase significantly following the introduction of new products and are economically sizable: the relative markup rises by about 2 percentage points. Firms' relative markups also rise by a similar amount following the discontinuation of products, but the increase happens only with a delay. Taken together, these findings strongly suggest that product replacement is an important source of relative markup gains for firms.

The remainder of the paper is structured as follows. Section 2 introduces the data and section 3 presents the estimation approach. It also shows that average markups have been increasing over time and documents markup dispersion over time. Section 4 presents our main results about trends in relative prices, markups and marginal costs over the firm life, while Section 5 discusses the robustness of our main findings. In Section 6 we shed further light in the source of marginal cost and markup dynamics at the firm level, presenting amongst other things our findings on product rotation.

2 Data

To estimate markups, we combine several firm-level datasets provided by Statistics Denmark. We use price data from the Producer Price Index (PPI) survey, accounting data from the accounting statistics (FIRE) and firm demographics from the business register.

Price data We use survey data underlying the Danish Producer Price Index (PPI) to construct firm-level price deflators. The PPI microdata provides a very clear picture of the price developments of manufacturing firms. It is based on a monthly survey in which firms report prices for a persistent selection of their product portfolio. In particular, firms are asked to report prices for their most "representative" products. The microdata is available for the 2001–2016 period. On average, the data covers about 3,500 price quotes from about 500 firms. Products are classified using 8-digit Harmonized System (HS) codes. Firms also report whether goods are sold domestically or exported. In our baseline results, we pool domestic and export prices. All reported prices are transaction prices in Danish kroner and include temporary sales and discounts. One advantage is that the survey is designed to allow adjustments for quality changes and product substitutions. In the case of changes to the product, firms report both the price for the updated product as well as a hypothetical price for the same product in the previous period. Another advantage, particularly relative to unit value data, is that the dataset is strongly balanced, with very few gaps in the price series.

We use this dataset to construct firm-specific deflators that allow us to measure firm output when combined with accounting data on sales. These deflators are based on the quality-adjusted price changes over a year averaged over all products a firm reports.[2] On average, the firm-level output price deflators computed from the micro data are consistent with readily available industry-level producer price indices. Details on the construction of firm- and industry-level price indices and their correlations are provided in Appendix A.2.

Accounting data We combine firm-level price deflators with annual data on sales and inventories from the Danish accounting statistics to construct measures of inputs and output. The accounting statistics collect headline balance sheet items such as sales and assets from tax data and more detailed items such as intermediate expenses and different kinds of inventories in a large-scale survey of firms. The survey population excludes firms with less than five employees and firms in some sectors such as agriculture and finance, but includes all firms with more than 50 employees. Firms with 20 to 49 employees are included for five years every ten years, firms with 10 to 19 employees for two years every ten years, and firms with 5 to 10 employees are included every 10th year.

We calculate firms' output as annual sales plus the change in the firms' inventory of final goods, deflated with firm-specific deflators constructed from PPI prices. Firms' material input is computed as annual expenditures for intermediates minus the change in intermediate inventories deflated with sector-specific input price indices from the Danish national accounts.[3] Labor input is calculated from the firms' total wage bill deflated with an aggregate wage deflator. We prefer a measure of real labor cost rather than full-time equivalents in the production function, because it provides a quality-adjusted measure of labor input. Our results are robust to using full-time equivalents instead. We measure capital used in production as fixed assets minus real estate, plus the capitalized value of rented or leased capital, which we impute using a depreciation rate of 0.2. We deflate the value of capital with a firm-specific capital deflator that reflects the capital input mix of the firm. The construction of capital deflators is described in detail in Appendix A.2.

[2] The PPI data does not contain weights that indicate the importance of a product within a firm.

[3] We refer to Appendix A for details on all factor price deflators.

Table 1: Firm characteristics

	N	q5	q25	q50	q75	q95
Employment, average over period	764	27.18	56.00	98.21	210.82	678.59
Employment growth first to last obs.	764	-0.66	-0.35	-0.11	0.12	0.75
Age at first observation	764	2	11	21	32	59
Number of years observed	764	2	5	9	15	16
Material input share	7,103	0.26	0.41	0.51	0.61	0.78
—, - Δintermediate inventories	7,103	0.26	0.41	0.51	0.61	0.79
Labor input share	7,103	0.09	0.18	0.24	0.31	0.42

Notes: Moments of different firm characteristics at the firm and firm-year level. Factor input shares are relative to nominal sales in the given year. In the case where the material input share is corrected for the change of intermediate goods inventories, sales are equivalently corrected by the change in final goods inventories.

Firm demographics We use information on firm demographics from the Danish business register. This register contains the date of firms' registration. We calculate firm age starting with age one in the year of registration. The business register also contains a sector code for each firm, and we will define a sector as a 2-digit NACE code throughout the paper.

Final sample Our final sample contains 764 manufacturing firms that we can match between the PPI and accounting statistics. These firms operate in 16 different 2-digit NACE subsectors. We drop sectors that contain a sample of less than 20 firms. Table 1 shows that the median firm in this sample has 100 employees and is observed for 9 years out of the 2001–2016 sample period. A quarter of all firms are observed for 15 years or more, i.e., almost the entire sample period, despite sampling in both the PPI survey and the accounting data.

The firms in the sample are highly heterogeneous in terms of age. We observe 50 firms at a very young age in the first and/or second year after entry, and 174 firms in their first 10 years of operation. On the other hand, we observe several firms that are well above 50 years old. The median age at first observation in the sample is 21 years. A notable feature of the firm population in our data is that it gets older over time. Over the 15 years covered by our data, the average firm age increases by almost 9 years. This feature is not unique to our data sample. In Figure A.1 in the appendix we benchmark our sample against all manufacturing firms that eventually reach 50 full-time equivalents at least once during the sample period. In that sample, firms are on average somewhat younger, but age from an average of 22 years to 32 years over the sample period as well.

3 Estimating firms' markups and marginal costs

We estimate firm markups following the production function approach of De Loecker and Warzynski (2012). Importantly, we estimate production functions using proper quantities. Bond et al. (2021) and De Ridder et al. (2021) discuss the limitations of markup estimation when production functions are estimated using sales. Our estimates are not subject to these limitations, and we show below that using quantities in the production function estimation is key to recover the correct age patterns in markups. Moreover, the combination of price data with our markup estimates allows us to recover marginal cost, and to discuss joint dynamics in markups, prices and marginal cost as firms age.

3.1 Estimation approach

The production function approach does not require assumptions on the structure of output markets or demand, but instead relies on cost minimization and the assumption of competitive input markets to identify markups. Following standard practice, we assume that ex-post observed output Y_{it} of firm i at time t is the combination of planned output Y^*_{it} and a disturbance ϵ_{it} that is realized after production decisions are made:[4]

$$Y_{it} = Y^*_{it}(K_{it}, L_{it}, M_{it}, \Omega_{it}) \exp(\epsilon_{it}) \tag{1}$$

Planned output Y^*_{it} is unobserved by the researcher and is a function of capital K_{it}, labor L_{it}, intermediate material inputs M_{it} and total factor productivity Ω_{it}. As is standard, we treat materials input M_{it} as a flexible input that can contemporaneously adjust to the current level of productivity Ω_{it}.[5] For a given output level Y^*_{it}, the firm chooses in period t material input (and perhaps other inputs) to produce at minimal cost, taking factor prices as given. The first-order condition from this minimization problem together with the definition of the markup as the ratio of the output price over marginal cost ($\mu_{it} \equiv \frac{P_{it}}{\lambda_{it}}$) yields the familiar

[4] The disturbance may simply capture measurement error in output.

[5] Other inputs may also be flexible in this sense, e.g., labor input, but this is not required for the approach to work. Also, for materials markets the assumption of competitive input markets appears most plausible.

ratio estimator for the markup[6]

$$\mu_{it} = \frac{\theta_{it}^M}{\alpha_{it}^M} \exp(-\epsilon_{it}), \tag{2}$$

where $\theta_{it}^M \equiv \frac{\partial Y_{it}^*}{\partial M_{it}} \frac{M_{it}}{Y_{it}^*}$ denotes the output elasticity with respect to the material input, $\alpha_{it}^M \equiv \frac{P_{it} Y_{it}}{P_t^M M_{it}}$ the share of expenditures for materials in total revenue, P_{it} the firm's output price, and P_t^M the input price for materials.[7]

Given an estimate for the output elasticity $\hat{\theta}_{it}^M$ and the surprise component $\hat{\epsilon}_{it}$, the estimate for the markup $\hat{\mu}_{it}$ and the marginal cost $\hat{\lambda}_{it}$ are given by

$$\hat{\mu}_{it} = \frac{\hat{\theta}_{it}^M}{\alpha_{it}^M \exp(\hat{\epsilon}_{it})}, \qquad \hat{\lambda}_{it} = \frac{1}{\hat{\mu}_{it}} P_{it}, \tag{3}$$

where α_{it}^M and P_{it} can be directly measured in the data. The second equation in (3) shows how we obtain a measure of marginal costs, given the firm level markup estimate and the observed output price. We describe below how we obtain the estimates $\hat{\theta}_{it}^M$ and $\hat{\epsilon}_{it}$.

Production function estimation We estimate the output elasticity of material inputs using a flexible translog production function (in logarithms):

$$\begin{aligned} y_{it} = &\beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \beta_{kk} k_{it}^2 + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 \\ &+ \beta_{kl} k_{it} l_{it} + \beta_{km} k_{it} m_{it} + \beta_{lm} l_{it} m_{it} + \beta_{klm} k_{it} l_{it} m_{it} + \omega_{it} + \epsilon_{it}. \end{aligned} \tag{4}$$

All variables in the estimation are real quantities. Output y_{it} is based on revenues deflated with firm-level output price deflators as described in Section 2. Deflating nominal output with firm-level prices rather than with an industry price index turns out to be crucial for uncovering the dynamics of markups over the lifetime of firms. The variables on the r.h.s. of equation (4) are deflated as described in Appendix A. We estimate separate production functions for 16 different 2-digit NACE industries. To keep the notation to a minimum, we suppress industry-level subscripts.

Given estimates of the production function coefficients, the output elasticity of material

[6] Notice that λ_{it} is also the Lagrange multiplier of the cost minimization problem.

[7] Because the researcher does not observe the expenditure share in terms of revenue anticipated by the firm at the point of the decision but only realized revenues in hindsight, the revenue share has to be corrected for $\exp(\epsilon_{it})$.

input is:

$$\hat{\theta}_{it}^{M} = \hat{\beta}_m + \hat{\beta}_{km} k_{it} + \hat{\beta}_{lm} l_{it} + 2\hat{\beta}_{mm} m_{it} + \hat{\beta}_{klm} k_{it} l_{it}. \tag{5}$$

We estimate the production function in (4) in two steps following the literature (see e.g. Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg et al. (2015)). The first step removes ϵ_{it} from observed output.[8] The second step addresses the issue of endogeneity. This is necessary because realizations of total factor productivity ω_{it} affect both the firm's choice of material input m_{it} as well as the resulting output y_{it}. Following standard practice, we address this issue using a GMM estimator (Blundell and Bond, 2000, De Loecker, 2011).

Purging ϵ_{it} in the first step: The first step uses of the fact that the firm knows ω_{it} but not ϵ_{it} when it makes production decisions. If the optimal choice of m_{it} is a monotonic and invertible function in ω_{it} and (potentially other information stacked in the vector Ξ_{it}, one can replace $\omega_{it} = m^{-1}(m_{it}, \Xi_{it})$ in Equation (4) and thereby consistently estimate ϵ_{it}. Importantly, ϵ_{it} is not part of Ξ_{it}. The variables that are included among the first-step regressors Ξ_{it} are, as in De Loecker (2011), third-order polynomials of all production function inputs and their interactions as well as time fixed effects to absorb common variation across periods, e.g. factor input trends.

The first-order condition of material inputs in the cost minimization problem contains marginal cost and hence this or prices and the markup itself should be part of Ξ_{it} if they are heterogeneous across firms, as in the case of imperfect competition (Doraszelski and Jaumandreu, 2021, De Ridder et al., 2021). To control for the markup, we include two dimensions along which we conjecture that both markups and prices are heterogeneous. First, we include a firm fixed effect to absorb variety-specific demand that is not accounted for in the data. Second, we include a polynomial in age, because the hypothesis of the paper is that the firm life cycle is an important determinant of the chosen markup.[9] The second step uses the fitted values from the first step as the dependent variable, i.e. estimates the elasticity of y_{it}^{*} with respect to m_{it}.

[8] This is necessary because without additional restrictions, productivity ω_{it} and the shock ϵ_{it} cannot be separately identified.

[9] De Ridder et al. (2021) include prices, or in their case unit values, and market shares instead in the first step instead. They find estimated log markups with their extended controls in the first step have a Pearson correlation of 0.62 with markups estimated without these additional controls for the markup. In our case, it is 0.65. Notice that due the nonparametric nature of the first step estimation, we are not required to know the parameters governing the structural relationship between our additional control and the markup.

Addressing endogeneity of ω_{it} and m_{it} in the second step: We follow the standard approach in the literature by instrumenting m_{it} with with lagged variable input $m_{i,t-1}$. The reason is that values of $m_{i,t-1}$, while predictive of m_{it}, are orthogonal to ω_{it}. To ensure that $\mathbb{E}[m_{i,t-1}, m_{it}] \neq 0$, one can assume that ω_{it} is persistent, i.e. $\omega_{it} = \rho\omega_{i,t-1} + \xi_{it}$.[10]

De Ridder et al. (2021) show that moment condition $\mathbb{E}(\hat{\xi}_{it}(\beta)m_{i,t-1}) = 0$ only correctly identifies β if y^*_{it} measures real output. In many empirical settings, however, it is difficult to estimate physical output elasticities because only observes nominal revenues are observed. For example, De Loecker (2011) deflate nominal revenues with industry-level deflators that are common across firms. Because of the inclusion of time fixed effects in the first step, this essentially estimates the revenue elasticity. In that case, the estimated production function coefficients (and by extension $\hat{\theta}^M_{it}$ and $\hat{\mu}_{it}$ will contain information on the true markup, but be biased by the covariance between the lagged input variable with output prices. The bias might be both positive or negative and wash out in the aggregate.

Our baseline specification uses firm-level deflators from PPI prices, denoted P_{it}, to deflate firm-level sales. These output prices are observed directly, and thus more precisely measured than unit values used in other datasets. We contrast our results to a version where we estimate the output elasticity using industry-level deflators P_{kt}, i.e., effectively estimate revenue elasticities. In line with De Ridder et al. (2021), we find differences in the level of estimated output and revenue elasticities, but similar aggregate time trends. With respect to our main research question, which is to study markups over the firm's life cycle, we find that estimating markups using the real output elasticity is highly important.

3.2 Results of markup estimation

It is useful to discuss some properties of our production function estimation and benchmark the resulting markup estimates to the existing literature. Table B.1 presents the average output elasticities implied by the Translog production function coefficients by sector. The values generally line up with what has been reported in previous literature. The average output elasticity of material inputs is estimated to be 0.58 in the full sample, compared to an average value of 0.64 for French manufacturing firms in De Ridder et al. (2021). The

[10] This results in the moment conditions $\mathbb{E}(\hat{\xi}_{it}(\beta)X) = 0$, where X is $(k_{it}, l_{it}, m_{i,t-1}, k^2_{it}, l^2_{it}, m^2_{i,t-1}, k_{it}l_{it}, k_{it}m_{i,t-1}, l_{it}m_{i,t-1}, k_{it}l_{it}m_{i,t-1})$. Starting with a set of values for the parameters β and ρ, we compute the implied $\hat{\omega}_{it}$, $\hat{\omega}_{i,t-1}$ and $\hat{\xi}_{it}$ and iterate until said moment conditions hold. The output elasticities implied by the optimized $\hat{\beta}$'s enters in the numerator of the ratio estimator of the markup.

average output elasticities of labor and capital inputs amount to 0.37 and 0.03 respectively.

Combining firms' estimated output elasticities with material input shares, we arrive at the markup estimates. The average gross markup in our sample is 1.14. This is significantly lower than values reported for French manufacturing firms in De Ridder et al. (2021), which range between 1.3 and 1.39.

Panel (a) in Figure 1 depicts the average markup over time. We report the median, mean and employment-weighted mean. All average measures increase considerably over time: the mean markup increases by more than 10 percentage points (p.p.), the employment-weighted mean by more then 15 p.p., and the median by 9 p.p. over the sample period.[11] This increase is consistent with trends documented for other economies, often using revenue-based estimates of output elasticities (see e.g. De Loecker et al. (2020)).

Figure 1: Time trend of estimated markups

(a) Firm price deflator

(b) Industry price deflator

Notes: Average of markups estimated using the production function approach discussed in Section 3. Unbalanced sample of a total of 764 firms.

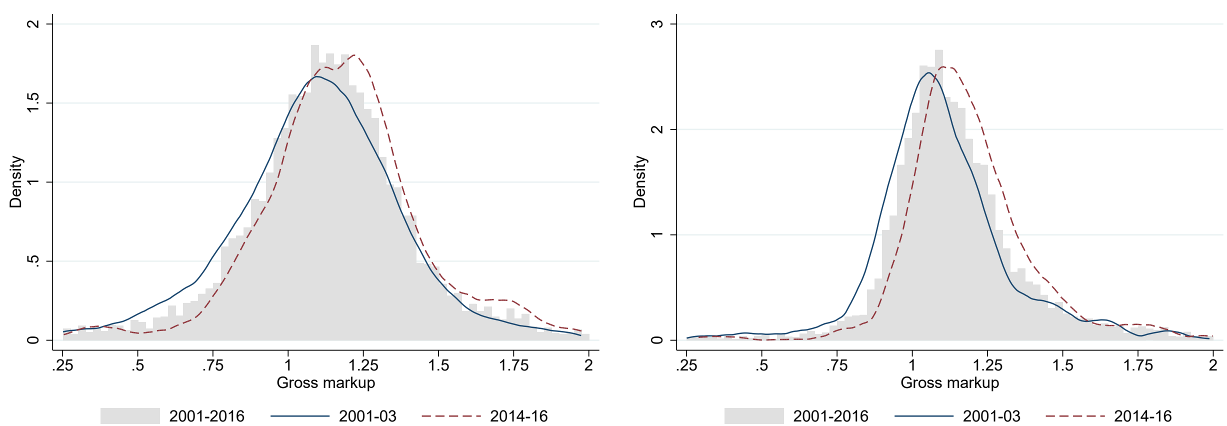
As a benchmark, panel (b) in Figure 1 illustrates trends in the same averages when we estimate markups without firm-specific price information and deflate firms' output with an industry price index instead. The resulting markup estimates are slightly higher but exhibit similar time trends to those in panel (a). This shows that firm-level price information does

[11] Markups appear to increase slightly fast after 2008 when credit conditions have become tighter for many firms. Gilchrist et al. (2017) and Renkin and Züllig (forthcoming) show that firms that lose access to external liquidity increase prices beyond credit cost, i.e., increase the markup to generate liquidity internally.

not appear crucial if one is interested in measuring trends in the average markup over time. However, the availability of firm level price information will be important for estimating the life-cycle patterns of firm's relative markups, as we show in the next section.

Figure 2(a) depicts the cross-sectional distribution of our benchmark markup estimates. We report the full distribution of markups over the entire sample (grey shaded bars) as well as kernel estimates of the markup distribution for the first three years of the sample 2001–03 (blue solid line) and for the last three sample years 2014–16 (red dashed line). The figure illustrates how the entire markup distribution has shifted toward higher markups over the sample period.[12] The same is true if markups are estimated using industry price deflator (panel (b)). Moreover, panel (b) illustrates that our baseline markup estimates are substantially more dispersed than estimates based on industry deflators that ignore price dispersion between firms.

Figure 2: Distribution of estimated markups



(a) Firm price deflator

(b) Industry price deflator

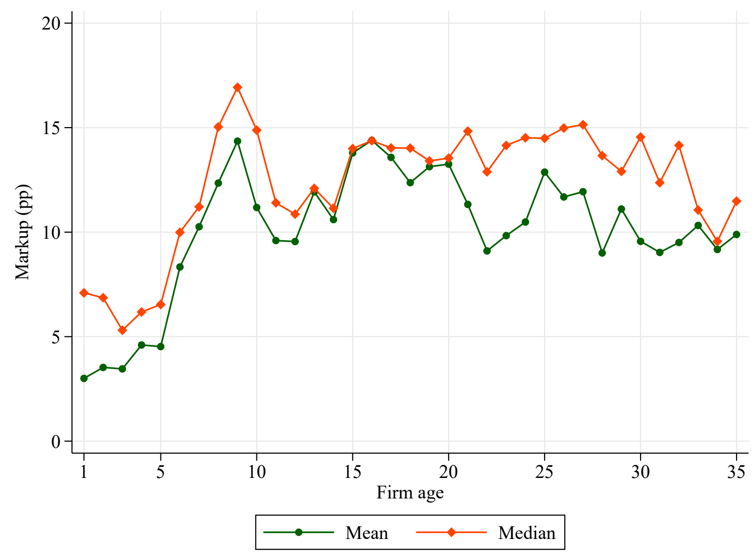
Notes: Densities of markups estimated using the production function approach discussed in Section 3. Unbalanced sample of a total of 764 firms.

[12]See Table B.2 in the Appendix for a formal test that the markup distribution has indeed shifted to the right.

4 Markups over the firm life

This section presents our main findings on age patterns in markups and subsequently prices and marginal cost. Figure 3 illustrates the average markup of firms of different ages pooled over the full sample. It is suggestive for how an age patterns in markups might look like: on average, younger firms charge lower markups. However, this age pattern might reflect cohort or time effects, and we would like to separate true age patterns from such alternative explanations.

Figure 3: Markups by firm age



4.1 Identification and estimation

Identification To identify age, time and cohort effects separately, we adopt a standard additively separable decomposition:

$$\mu_{c,a,t} = \beta_0 + \alpha_a + \tau_t + \chi_c + \varepsilon_{c,a,t} \quad . \tag{6}$$

This decomposition allows for fully non-linear age, cohort and time effects α_a, τ_t and χ_c. However, since for any firm-year observation, a firm's age is equal to the current year minus the firm's birth cohort, the *linear* components of these age, time and cohort effects cannot be separately identified without additional assumptions or restrictions. In other words, any linear age trend $\alpha \cdot a$ can be equally explained by a time trend $\alpha \cdot t$ that is offset for new firms by a negative cohort trend $-\alpha \cdot c$. It is worth noting that this is generally not the case for the *non-linear* components of age, time and cohort effects. As explained in McKenzie (2006), even though the linear components are fundamentally unidentified, non-linear age, time and cohort dynamics can be identified without restrictions.

This classic identification problem is discussed in depth in Fosse and Winship (2019) or Deaton (1997). To identify full age, time and cohort profiles, it is necessary to place a restriction on one of the linear components. Several such restrictions are common in the literature. We employ flavors of two different ones to make sure that our results are not driven by this choice. Our baseline *assumption* is that cohort effects are orthogonal to a linear cohort trend. That means markups might systematically vary between cohorts, for example because firms that enter during a recession are different to those that enter during a boom, but there is no linear trend in these cohort effects between the oldest cohorts and cohorts of younger firms. Under this assumption, we can identify unrestricted age and time effects, as well as the orthogonal cohort effects. It is worth nothing that our orthogonality assumption should hold over cohorts spanning more than 100 years (the oldest firm in our sample was founded in 1890), and that it does not rule out "local trends", e.g. more recent cohorts of firms having systematically higher markups.

For our our baseline estimation procedure described below, we will *assume* the orthogonality of cohort effects. The advantage of this approach is simplicity and flexibility in estimation— it allows us to mostly abstract from the identification issue described above in practice. Alternatively, we can impose the orthogonality assumption on our estimates, and *restrict* cohort effects to be orthogonal to a linear cohort trend. We implement this approach in robustness checks, and orthogonalize cohort effects following the algorithm suggested in Deaton and Paxson (1994).[13] It is important to note that none of our main results rest on this assumption or restriction. We contrast the results obtained from our baseline identification approach with results obtained using a completely different strategy employed in Card et al. (2013, 2016, 2018b) in the context of estimating life-cycle patterns in labor earnings. This approach restricts the age profile of markups to be flat at a certain age, and yields results that are very close to our baseline.

[13] See Deaton (2019). for a detailed description, including codes. We adapt the approach described there to allow for unbalanced panels and unequal cohort sizes.

Estimation based on relative markups Going from the general additive decompostision above to our baseline specification, we replace the age dummies α_a with a 4th-degree polynomial of age and cohort effects with firm fixed effects that fully nest cohorts. Additionally, we allow time effects to vary by 2-digit NACE industry $s(i)$:

$$\mu_{i,a,t} = \beta_0 + f^{\mu}(a_{i,t}) + \tau_{s(i),t} + \gamma_i + \varepsilon_{i,a,t} \quad . \tag{7}$$

We then subtract year-industry means from each observation and estimate our decomposition on relative markups:

$$\mu_{i,a,t} - \overline{\mu}_{s(i),t} = f^{\mu}(a_{i,t}) + \gamma_i + \varepsilon_{i,a,t} \quad . \tag{8}$$

Under our baseline identifying restriction that cohort fixed-effects are orthogonal to a linear cohort trend, the age polynomial then only identifies age effects, while non-linear cohort effects are subsumed in the firm fixed effect. In robustness checks, we estimate the conventional implementation of Deaton (2019) instead, where we impose orthogonal cohort effects directly. We compute standard errors that are clustered at the firm level to allow for persistent firm-level shocks to markups.

Given our markup estimates and firm-level price deflators, we can back out marginal cost as $mc_{i,a,t} = p_{i,a,t} - \mu_{i,a,t}$. Analogous to equation (8), we can estimate age profiles in marginal cost and prices:

$$p_{i,a,t} - \overline{p}_{s(i),t} = \gamma^{p}_{iz} + f^{p}(a_{i,t}), \tag{9}$$

$$mc_{i,a,t} - \overline{m}c_{s(i),t} = \gamma^{mc}_{iz} + f^{mc}(a_{it}). \tag{10}$$

We do not impose any cross-equation restrictions on the parameters of markup, marginal cost and price decompositions, e.g. we do not restrict the three sets of age polynomials and fixed effects to add up exactly.

4.2 Main Result

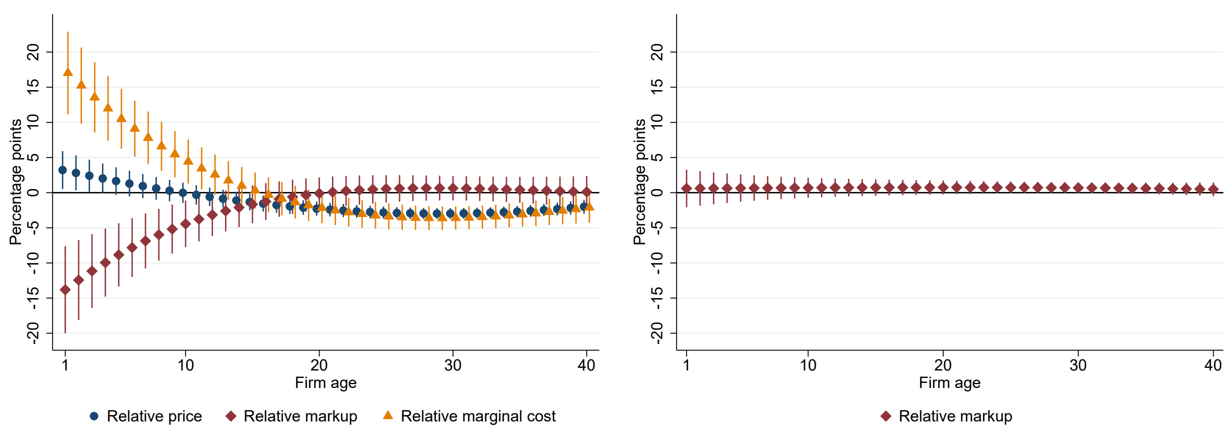
Panel (a) of Figure 4 reports the estimated age profiles of relative markups, prices and marginal costs. Estimates are based on a fourth-order polynomial in age.[14] Firms enter with markups that are about 13pp lower than the average markup. Over the first 15 years of a

[14]Using a second-order polynomial leads to very similar outcomes. The same is true if one uses age dummies for narrowly defined age groups, as we show in the appendix. The curves are normalized such that at age zero, the value are equal to the (cross-sectional) average of the firm fixed effects.

firm's life, markups increase until they converge to the average markup.

The rise in markups is not accompanied by an increase in prices over the first 15 years of a firm's life. Prices start out slightly above the (industry) average and decrease by about 5 percent over the course of firms' first 20 years. Instead, the increase in markups is driven by incomplete pass-through of a strong age profile in marginal costs. Firms start out with marginal costs that are about 18% above average. Costs fall over the first 20 years of firm's life, until they are slightly below industry average. For firms older than age 20, markups, prices and marginal costs do not exhibit a significant an age profile. As a result, the slight relative price trends reflect relative marginal cost trends for older firms. This result is consistent with complementarities in price-setting that result in a flexible markup and modest price differences between entrants and older incumbent firms despite a large difference in marginal cost.

Figure 4: Age trends in relative prices, markups and marginal costs



(a) Firm price deflator

(b) Industry price deflator

Notes: Estimates of (??)-(10), where left-hand side variables are defined relative to the industry-time average. $f^{(\cdot)}(\text{age}_{izt})$ is a fourth-order polynomial in firm age. Markups are estimated as described in Section 3, the construction of firm-level price deflators is described in Section 2 and marginal costs are defined as the log deviation between the two.

We show in Section 5 that these results are robust to (i) using age dummies instead of using age polynomials to estimate the age profile, and (ii) to using only within firm changes in relative markups to estimate the age profile.[15] While Figure 4 reports the average pattern or

[15]This differs strongly from the case of absolute markups shown in Figure 3.

relative markups, prices and marginal costs across industries, we show in Appendix C that the same patterns are present in the vast majority of the underlying industries.

While our findings turn out to be robust across alternative specifications, it is key to use firm level prices to estimate firm markups instead of using the industry price deflator. Panel (b) in Figure 4 reports the estimated age profile of relative markups using markups that are estimated with the industry price deflator, as for example in De Loecker and Warzynski (2012). These estimates completely miss the age profile of relative markups over the first 15 years of the firm life, despite the fact that they accurately capture the evolution of the cross-sectional average over time, see Figure 1.

4.3 Implications for estimates of the optimal inflation rate

We now investigate how severely trends in relative prices mis-measure trends in underlying relative marginal costs. This is of interest because existing studies that estimate the optimal inflation target (Adam et al. (2022), Adam and Weber (2023)) use estimate of relative price trends to proxy unobserved trends in relative marginal costs.

Since this literature estimates linear trends in relative prices, which effectively average firm trends over the firm lifetime, we similarly consider linear age trend regressions of the form

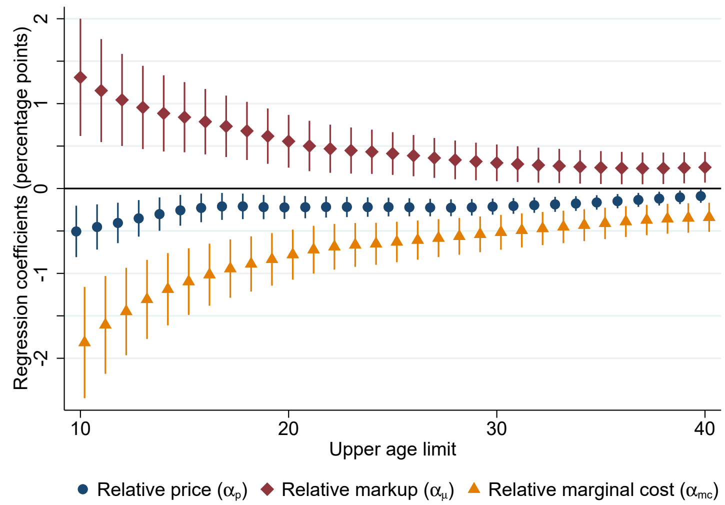
$$\begin{aligned} \ln \frac{\mu_{izt}}{\mu_{zt}} &= \alpha_{iz}^{\mu} + \alpha_{\mu} \cdot \text{age}_{izt} \\ \ln \frac{P_{izt}}{P_{zt}} &= \alpha_{iz}^{p} + \alpha_{p} \cdot \text{age}_{izt} \\ \ln \frac{mc_{izt}}{mc_{zt}} &= \alpha_{iz}^{mc} + \alpha_{mc} \cdot \text{age}_{it}. \end{aligned} \tag{11}$$

Since the true underlying age trends are nonlinear, the estimates of α_{μ}, α_p and α_{mc} will depend on the firm age distribution.

Figure 5 reports the estimates of $(\alpha_{\mu}, \alpha_p, \alpha_{mc})$ as function of the maximum firm age included in the regression. It shows that relative price trends have the same sign as the trend in relative marginal cost, but severely underestimated the strength of the relative marginal cost trend.

Depending on the upper age limit for firms used in the estimation, the trend in relative marginal costs between 0.2%-1.2% stronger than the relative price trend. This implies that the optimal inflation target is underestimated by this amount.

Figure 5: Linear age trends for different firm age limits



Notes: Estimates of linear age trends α_μ, α_p and α_{mc} in (11). Because age is highly skewed in the data, we winsorize firm age at different cutoff dates. Given the nonlinear path of relative outcomes shown in Figure 4, the lower that cutoff date, the steeper are the result linear estimates.

5 Robustness checks

Our key findings are that firms' markups start below the industry average and increase over the first 15 years of firms' lives. This is driven by marginal costs that start out high and decrease over the same time period. This decrease in markups is passed through incompletely to prices, resulting in the observed markup age pattern. In this section, we show that these results are robust to numerous alternative sample restrictions and choices in the production function and decomposition steps.

Sample Due to the sampling decisions in the PPI and accounting statistics, our sample might not be representative for the population of Danish manufacturing. In particular, our sample might contain more fast-growing young firms that cross sampling thresholds quicker than others. To alleviate the concern that this drives our results, we re-estimate the Equations (**??**)-(10) with sampling weights that adjust the weight of age-size cells to the correct weight in the population of Danish manufacturing firms. As shown in Figure C.1, panels (a) and (b) in the appendix, this sampling correction reduces the magnitude of age trends in markups and marginal cost by roughly one third, but does not alter our main conclusions. We also make sure that our results are not specific to particular large sectors by excluding the two largest sectors—food production and machinery production—from our sample. As shown in C.1 panel (c), excluding these two sectors increases the magnitudes of the age patterns in markups and marginal costs slightly.

Finally, in panel (d), we address outliers in our sample of estimated markups. We winsorize relative markups and marginal cost at the 5th and 95th percentiles. This results in more precisely estimated coefficients and reduces the magnitudes of the age patterns slightly. According to the winsorized estimation, the markup increases by approximately 10pp and relative marginal costs decrease by 15pp over the first 20 years of the firm life.

Estimation of age trends In our baseline specification, we estimate age patterns using a 4th-order polynomial in age. In Figure C.2 panel (a), we replace this polynomial with age dummies to allow a fully unrestricted age pattern in markups. This flexibility comes at the cost of lower precision. However, the pattern estimated from the dummy specification remains very similar to the polynomial specification. Markups start our below average and increase by about 12pp over the first 15 years of firms' life. Marginal cost start above average and decline by about 15pp. As in the polynomial specification, prices exhibit a small decline over the first 15–20 years of life.

Identification of age trends In our baseline estimation, we assume firm (or cohort) fixed effects are orthogonal to a linear cohort trend to identify unrestricted age and time effects. This is a critical assumption and we make sure that our results are robust to variations. First, instead of assuming that fixed effects are orthogonal to a linear trend, we can impose this in a restriction in our estimation. We impose this assumption over the full span of observed cohorts—the oldest firm in the data was started in 1890—and "local" trends in cohort effects, such as an increase of markups for firms started after 1990 are still a possibility. We follow the approach of Deaton (2019) to implement this restriction. The results are shown in Figure C.2 panel (b). As in our baseline, we find an increase in markups over the first 15 years of a firms life, a decrease in marginal cost and a smaller decrease in prices.

Second, we estimate our baseline decomposition using an alternative identification restriction that imposes a flat age polynomial at age 30. This is motivated by our baseline results as well as the simple average of markups by age shown in Figure 3. Similar identification assumptions have been used extensively in the context of estimating life cycles in earnings (see Card et al. (2013, 2016, 2018b)). The assumption is still compatible with flexible age profiles that can take any non-linear shape allowed our the polynomial, including monotonic increasing or decreasing ones. We illustrate the age pattern obtained this way in C.2 panel (c). As under our baseline assumption, markups start out below average and increase by roughly 16pp over the course of the first 20 years of firms' lives. Marginal cost start out above average and decrease by roughly 20pp. Prices exhibit only a slight downward trend. Importantly, the unrestricted cohort effects we estimate under this restriction are also similar to the restricted cohort effects (the average firm effects by cohort) we estimate under our baseline assumption.

Markup estimation Finally, we implement a series of robustness checks related to choices in the production function estimation. Our baseline measures of material input and physical output are adjusted for changes in intermediate and final goods inventories, to reflect production in a period, rather than sales. As shown in Figure C.3 panel (a) using unadjusted material expenditures and sales from the accounting statistics instead has little impact on our results. In our baseline estimation, we use deflated payroll expenditures as a measure of labor input. Alternatively, we can use full-time equivalent employment, which has the advantage that we directly measure it as a real variable, but the disadvantage that it is not adjusted for the quality of labor input. In Figure C.3 panel (b), we show that the magnitude of the age pattern in markups is about one third lower when we use full-time equivalent employment as a measure of labor input.

Our results are also robust to different specifications for the first stage of the markup estimation, in which we purge measurement error and shocks that occur after production decisions are made—ϵ in the notation of section 3—from output. Doraszelski and Jaumandreu (2021) show that under certain conditions, the markup itself should be part of the control variables to help identify this term. Because the markup is unobserved in the first stage, De Ridder et al. (2021) propose to include a firms' log price as well as its market share as proxies instead. As we describe in Section 3, we include a richer set of proxies that also includes a firm fixed effect as well as a fourth-order polynomial in age. However, our results are robust to following the De Ridder et al. (2021) approach instead, as we show in panel (c) of Figure C.3.

6 Sources of age patterns in markups and marginal cost

In this section we explore several possible sources for the observed age patterns in markups and marginal cost. Our results suggest that the increase in markups over the first 15 years of a firm's life is closely linked to a strong decrease in marginal cost that is not passed through to customers. Under the assumptions inherent in our production function estimation, such a decrease in marginal cost must result from increasing returns to scale—i.e. age might have an effect through firm size—or from an increase in TFP[16]. We explore both of these possibilities in Section 6.2 below. Moreover, younger firms exhibit a larger turnover in the products they produce. We explore the effect of changes in firms' product portfolio on the average price and markup that a firm charges in Section 6.1.

6.1 Product turnover

We link our firm-level data with survey data on product-level sales of Danish manufacturing firms. In this survey (which is the basis of Danish PRODCOM statistics), manufacturing firms with more than 10 employees report their quarterly sales at the level of 8-digit CN codes. We define a new product introduction whenever positive sales of a new 8-digit CN code are reported at the firm level[17] We define a product discontinuation whenever a CN code ceases to be reported at the firm level. We find that product turnover is substantially higher for younger firms—both in terms of new and discontinued products.

The left panel of Figure 6 reports the share of firms that introduced new products and the share of firms that discontinued old products by firm age. Introductions/discontinuation are measures relative to the products recorded in the previous year.[18] The share of young firms that introduce new and/or discontinues old products is substantially higher than that of old firms: for very young firms, the share of firms is almost twice as high as for old firms.

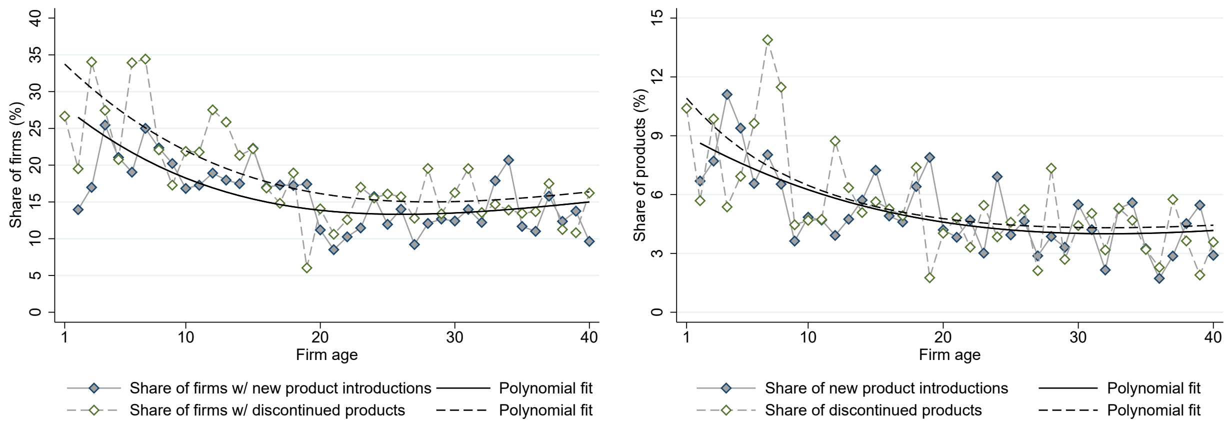
The right-hand side panel of Figure 6 depicts the average share of products (relative to the number of products present in the previous year) that gets newly introduced or discontinued,

[16]Other potential explanations, such as market power in input markets could be incorporated in the production function estimation, but we lack data on firm-specific input prices.

[17]We exclude new 8-digit CN codes that might appear due to changes in the CN classification.

[18]Since we must observe firms in the previous to detect introductions/discontinuations, the age effect is a within-firm effect that is not affected by firm entry/exit.

Figure 6: Product rotations by firm age



(a) Share of firms

(b) Share of products

Notes: Share of firms (panel (a)) or products (b) that have introduced or discontinued one or several products by firm age.

for firms of different age. We find again a downward sloping relationship, with young firms having a twice as high share of product introductions/discontinuations.

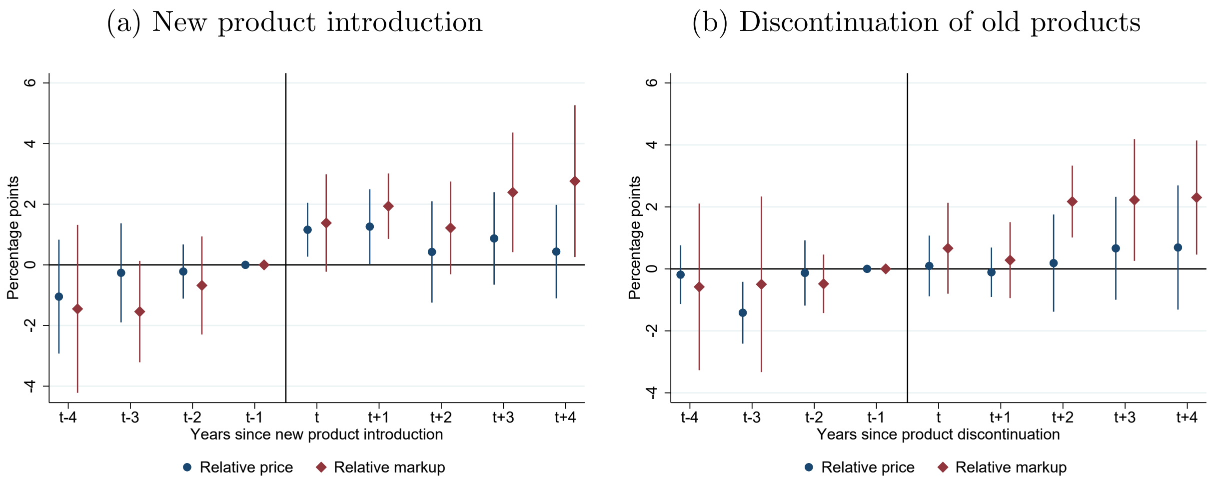
The previous findings suggest that a higher rate of product rotation could be a source of markup increases for young firms. Young firms could experiment with different products, and keep those which they are either effective at producing, or where demand conditions allow them to charge high markups. In both cases, we would expect average firm level markups to increase on average after a product has been either discontinued or introduced. To investigate this hypothesis further, we run local projections of the form

$$\ln x_{iz,t+h} = c_{iz} + \beta_h^{int} \cdot d_{izt}^{int} + \beta_h^{dis} \cdot d_{izt}^{dis} \quad , \tag{12}$$

where $x_{iz,t+h}$ denotes either the relative markup $\mu_{iz,t+h}/\mu_{z,t+h}$ or the relative price $P_{iz,t+h}/P_{zt+h}$, d_{izt}^{int} a product introduction dummy, and d_{izt}^{dis} a product discontinuation dummy. The coefficients β_h^{int} and β_h^{dis} trace out the impulse reponse following a product introduction/discontinuation.

Figure 7 depicts the relative price and markup responses. The left panel display product introductions, the right panel product discontinuations. Both events lead to a sizable increase in relative markups in the order of 2 percentage points over the medium term. The response of the relative prices is more muted and often not statistically significant. The weaker

Figure 7: Relative markups and prices following product changes



Notes: Estimates of local projections (12) with a dummy for whether or not the firm has introduced or discontinued a product, relative to the previous period. Vertical bars show 95% confidence intervals based on Driscoll-Kraay standard errors.

response of relative prices implies that relative marginal costs fall, following the introduction of new products. In fact, the marginal costs fall in a statistically significant way.

The picture looks very similar for product discontinuations, although markup increase happens there only with a considerable delay. Again, relative marginal costs, which are given by the difference in relative prices and relative markups, fall in a statistically significant way.

These results suggest that the higher rate of product introduction and discontinuation for younger firms is an important factor driving the increase in relative markups and the fall in relative marginal costs.

6.2 Production function origins of the trend in relative marginal cost

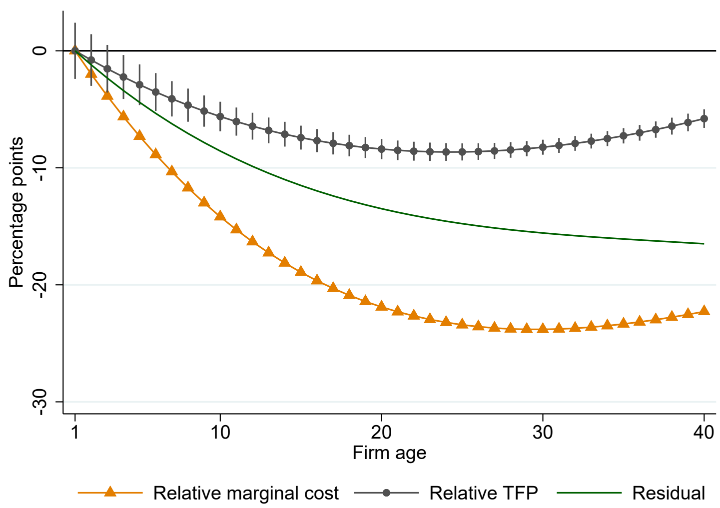
The previous section provided evidence that relative marginal cost fall around the introduction of new products and the discontinuation of old products. This section uses the production function approach to disentangle two possible sources of relative marginal cost movements for young firms: (i) movements in relative productivity and (ii) effects stemming from the non-homotheticity and increasing returns to scale (IRS) of the production function.

We can estimate TFP using our production function estimates:

$$\omega_{i,t} = y_{i,t} - \varepsilon_{i,t} - f(K, L, M) \tag{13}$$

Figure 8 reports the age profile of relative marginal cost (orange triangles), as previously shown in the left panel of figure 4.[19] The grey dotted line depicts the estimated age trend for relative firm productivity.[20] It shows that only about one third of the observed fall in relative marginal cost is explained by relative productivity trends. Two thirds of the effect arise due to non-homotheticities and increasing returns in the production function.

Figure 8: Relative TFP by firm age, contribution to marginal cost trend



Notes: Orange triangles show estimates of marginal cost, relative to the industry average, by age, estimated using a 4th-order polynomial in age (see Equation (10) and Figure 4). Grey dots show the equivalent estimates for relative TFP by firm age, including 95% confidence intervals. Estimates are inverted to denote that increasing TFP results in lower marginal costs. The residual (solid line) reflects marginal cost decreases due to the non-homotheticity of the estimated translog production functions.

[19] To ease readability of the graph in terms of the decomposition we perform, the initial value of the relative markup is now normalized to zero. The same holds true for relative productivity and the non-homotheticity term.

[20] We again use a fourth order polynominal in firm age to estimate the trend.

7 Conclusions

We estimate the evolution of relative markups, marginal costs and prices over the firm age. We find that relative markups rise strongly with firm age, while relative marginal cost fall, with both trends accelerating during periods with high product turnover. Due to the rise in relative markups, relative price trends underestimate the trends in relative marginal cost, even though both trends have the same sign.

The fact that relative markup and marginal cost changes are related to product turnover suggests that markups and marginal costs are embedded at the level of products rather than at the level of the firm. This in turn implies that estimates of the optimal inflation rate should not rely in firm level trends, as in Adam and Weber (2019), but rather on trends present at the level of individual products, as exploited in Adam et al. (2022), and Adam and Weber (2023).

References

Ackerberg, Daniel, Kevin Caves, and Garth Frazer, "Identification Properties of Recent Production Function Estimators," *Econometrica*, 2015, *86* (6), 2411–2451.

Adam, Klaus and Henning Weber, "Optimal trend inflation," *American Economic Review*, 2019, *109* (2), 702–737.

_ and _ , "Estimating the optimal inflation target from trends in relative prices," *forthcoming in: American Economic Journal: Macroeconomics*, 2023.

_ , Erwan Gautier, Sergio Santoro, and Henning Weber, "The Case for a Positive Euro Area Inflation Target: Evidence from France, Germany and Italy," *Journal of Monetary Economics (forthcoming)*, 2022.

Blundell, Richard and Stephen Bond, "GMM Estimation with Persistent Panel Data: an Application to Production Functions," *Econometric Reviews*, 2000, *19* (3), 321–340.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data," *Journal of Monetary Economics*, 2021, *121*, 1–14.

Card, David, Ana Rute Cardoso, and Patrick Kline, "Bargaining, Sorting, and the Gender Wage Gap: Quantifying the Impact of Firms on the Relative Pay of Women," *The Quarterly Journal of Economics*, May 2016, *131* (2), 633–686.

_ , _ , Joerg Heining, , and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 2018, *36* (S1), 13–70.

_ , _ , _ , and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, January 2018, *36* (S1), S13–S70.

_ , Jörg Heining, and Patrick Kline, "Workplace Heterogeneity and the Rise of West German Wage Inequality," *The Quarterly Journal of Economics*, August 2013, *128* (3), 967–1015.

De Loecker, Jan, "Product differentiation, multiproduct firms, and estimating the impact of trade liberalization on productivity," *Econometrica*, 2011, *79* (5), 1407–1451.

_ and Frederic Warzynski, "Markups and firm-level export status," *American Economic Review*, 2012, *102*, 2437–2471.

_ , Jan Eeckhout, and Gabriel Unger, "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

De Ridder, Maarten, Basile Grassi, and Giovanni Morzenti, "The Hitchhiker's Guide to Markup Estimation," *mimeo*, 2021.

Deaton, Angus, *The Analysis of Household Surveys (Reissue Edition with a New Preface): A Microeconometric Approach to Development Policy*, Washington, DC: World Bank, January 2019.

_ and Christina Paxson, "Saving, Growth, and Aging in Taiwan," in "Studies in the Economics of Aging," University of Chicago Press, January 1994, p. w4330.

Doraszelski, Ulrich and Jordi Jaumandreu, "Reexamining the De Loecker & Warzynski (2012) method for estimating markups," *CEPR Discussion Paper*, 2021, *DP16027.*

Gilchrist, Simon, Raphael Schoenle, Jae Sim, and Egon Zakrajšek, "Inflation Dynamics during the Financial Crisis," *American Economic Review*, March 2017, *107* (3), 785–823.

Levinsohn, James and Amil Petrin, "Estimating production functions using inputs to control for unobservables," *Review of Economic Dynamics*, 2003, *70* (2), 317–341.

McKenzie, David J., "Disentangling Age, Cohort and Time Effects in the Additive Model*," *Oxford Bulletin of Economics and Statistics*, 2006, *68* (4), 473–495.

Nakamura, Emi and Jón Steinsson, "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.

Olley, Steve G. and Ariel Pakes, "The dynamics of productivity in the telecommunications equipment industry," *Econometrica*, 1996, *64* (6), 1263–1297.

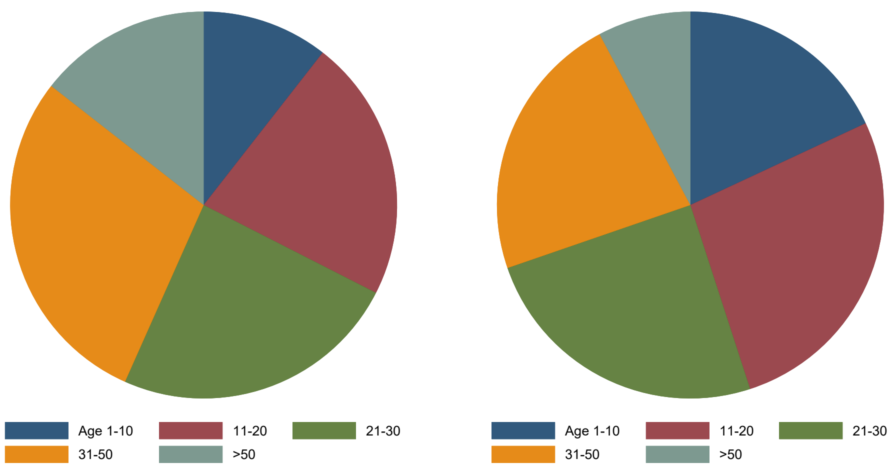
Renkin, Tobias and Gabriel Züllig, "Credit supply shocks and prices: Evidence from Danish firms," *American Economic Journal: Macroeconomics*, forthcoming.

Appendices

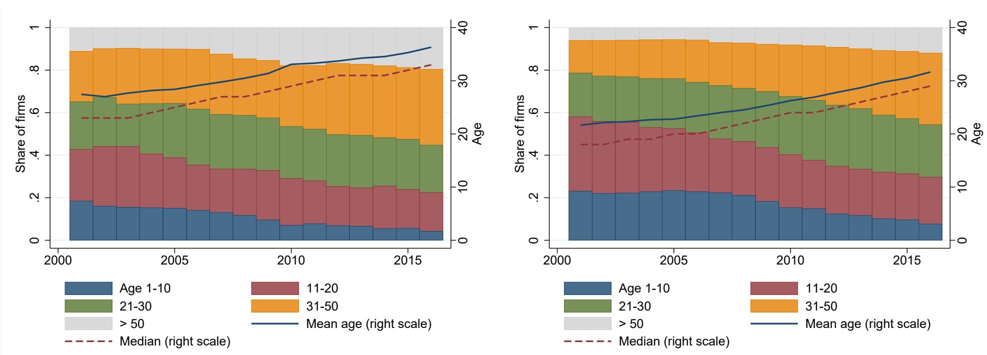
A Data

A.1 Firm demographics

Figure A.1: Firm age distribution



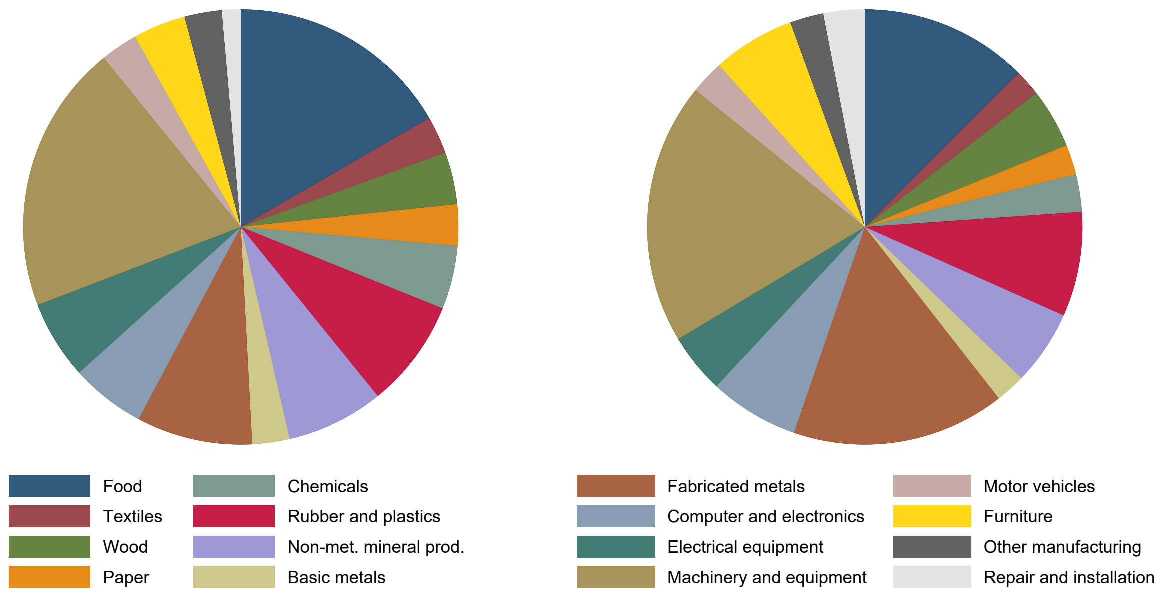
(a) in sample of markups (764 firms) (b) in benchmark sample (1,889 firms)



(c) in sample of markups (764 firms) (d) in benchmark sample (1,889 firms)

Notes: Distribution of age brackets over the whole sample spanning 2001–16 and by year. The left-hand side panels (a) and (c) show the distribution for the 764 firms for which we are able to estimate markups, i.e. manufacturing for which we have consistent accounting and output price data (see Section 2). Panels (b) and (d) show the same information for a broader sample of firms, namely all manufacturing firms that eventually reach employment of 50 or higher at least once. This is referred to as the benchmark sample.

Figure A.2: Industry composition



(a) in sample of markups (764 firms)

(b) in benchmark sample (1,889 firms)

Notes: Distribution of 2-digit NACE industry codes in the firm sample.

A.2 Firm- and industry-level output and input deflators

A.2.1 Firm-level output deflators from PPI data

In the PPI survey, producers report each month the transaction price quote of a representative set of their products, including temporary sales.[21] Over the sample we use, the raw PPI data covers 1,138 firms and 6,908 products in total.[22]

One advantage of this data is that price of individual products is tracked for a relatively long time, i.e., we have few gaps in the individual price series. The average length of a product spell we observe is 130, i.e., longer than 10 years over the sample period.

Another advantage, for example relative to unit value data, is that in the case that any feature of the product such as size or quality is changed, firms are also asked to report

[21] When applying sales filter "B" of Nakamura and Steinsson (2008) to the raw data, 0.3% of price quotes are identified as sales. Temporary sales are not a prominent feature in the Danish PPI.

[22] This covers both the domestic and export share of the PPI survey. If a good is sold both domestically and internationally, we treat them as separate product IDs.

the hypothetical price of the exact same product in the previous month. This allows us to compute quality-adjusted firm level inflation rates.

One drawback of the data is that we do not observe quantities of each product within the firm. The mean firm reports prices for 6 products in total over the sample period. As firms are asked to report prices for a “representative” sample of products, we assign uniform weights across products within a firm when computing firm-level inflation rates.

FINISH

A.2.2 Industry-level output deflators

TO DO Gabriel

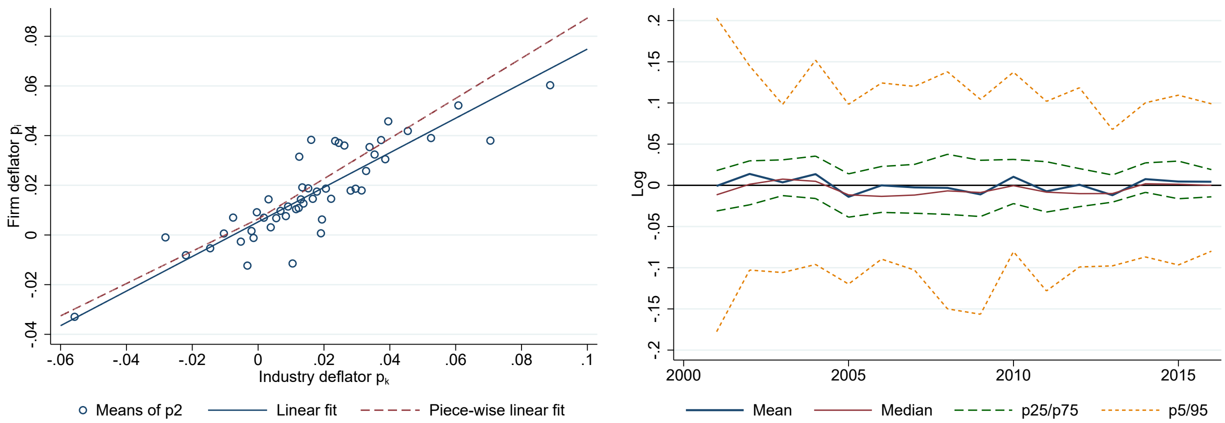
A.2.3 Verification of firm-level output deflators

As the estimation approach discussed in Section 3 hinges on a reliable measure of real output, we show that our firm-level output deflators on average accurately reflect industry price dynamics which are published by Statistics Denmark.

We first show that annual inflation rates of our own firm-level output deflators (Δp_{it}) and the sector-level equivalent which are publicly available (Δp_{kt}) have a high degree of correlation. In principle, the latter are a weighted average of the former, but our main challenge is that we do not know any weights of the micro price data used to construct the aggregate PPI and sector-level subindices. Figure A.3(a) shows a binned scatter plot and the fit of a linear regression (blue solid line). The estimated coefficient is 0.70 (with standard errors of 0.08). While not perfect, there is a clear positive relationship.

While industry-price deflators are informative, price changes in a sector a far from uniformly distributed across firms. We compute, for each firm and year, the difference between changes of firm and industry deflators. In Figure A.3(b), we show that there is no bias over time. While the mean is not zero in each year, there is no systematic bias that would lead to diverting time trends between the average firm and industry deflators. At the same time, it becomes clear that there are large deviations of firm-level price changes relative to the weighted industry average. As is shown by the dashed lines, around half the firms in most years increase or decrease their prices more than 2pp more than the average industry price. The fact that output deflators behave heterogeneously across firms highlights the importance

Figure A.3: Firm- vs. industry-level deflators



(a) Δp_{it} vs. Δp_{kt}

(b) Distribution of $\Delta p_{it} - \Delta p_{kt}$ by year

of using firm level output deflators in our production function estimation.

A.2.4 Factor input deflators

As is standard in the literature, we deflate all factor inputs before they enter the production function. In the following, we discuss the construction of the deflators for each input factor in turn.

Material

Labor Our production function features real labor cost as labor input to better reflect quality differences in labor than with simple full-time equivalents. To deflate nominal labor cost, we calculate a nominal wage index for the entire economy. To do so, we sum both the number of full-time equivalents and all wage payments in the accounting statistics for each year.

Capital We use capital deflator time series from Statistics Denmark but use firm-specific shares for the types of capital to construct firm-specific capital deflators. The time series are obtained from the fact that Statistics Denmark publishes capital stock data in real and nominal terms for different subcomponents, such as buildings, machines and immaterial assets. Our measure of capital does not include buildings. Therefore, instead of using the deflator for the aggregate capital stock, we compute the share of machinery and immaterial assets for each firm and calculate a weighted average of the deflators for the two subcomponents. The price of immaterial assets grew substantially faster and was less cyclical over the sample period. Our approach accounts for some of the heterogeneity in the kind of capital across firms, as much as the data allow.

B Descriptive statistics on estimated production functions and markups

Table B.1: Output elasticities by industry

Industry	N	$\hat{\theta}^M$	$\hat{\theta}^L$	$\hat{\theta}^K$
Food	1,191	0.72	0.26	0.06
Textiles	188	0.68	0.15	0.04
Wood	277	0.59	0.39	0.03
Paper	225	0.59	0.38	-0.11
Chemicals	323	0.55	0.27	0.16
Rubber and plastics	574	0.48	0.54	0.03
Non-met. mineral prod.	511	0.48	0.43	0.07
Basic metals	201	0.66	0.31	-0.03
Fabricated metals	606	0.43	0.35	0.04
Computer and electronics	395	0.43	0.58	-0.02
Electrical equipment	415	0.56	0.36	0.02
Machinery and equipment	1,420	0.62	0.33	0.02
Motor vehicles	210	0.63	0.39	0.03
Furniture	267	0.52	0.40	0.04
Other manufacturing	211	0.59	0.44	0.01
Repair and installation	89	0.45	0.97	0.06

Notes: Before computing means, all variables are winsorized at the 1st and 99th percentile by industry and year.

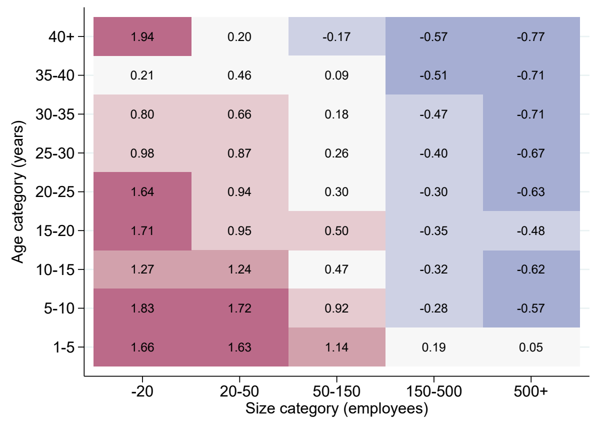
Table B.2: Tests for equality of distributions

Moment	2001-03	2014-16	Test stat.*	p-value
Markups estimated with firm price deflator				
Mean	1.091	1.201	8.682	0.0000
Standard deviation	0.304	0.338	1.240	0.0001
Skewness	-0.035	1.171		
Markups estimated with industry price deflator				
Mean	1.103	1.213	9.874	0.0000
Standard deviation	0.259	0.305	1.385	0.0000
Skewness	0.599	3.286		

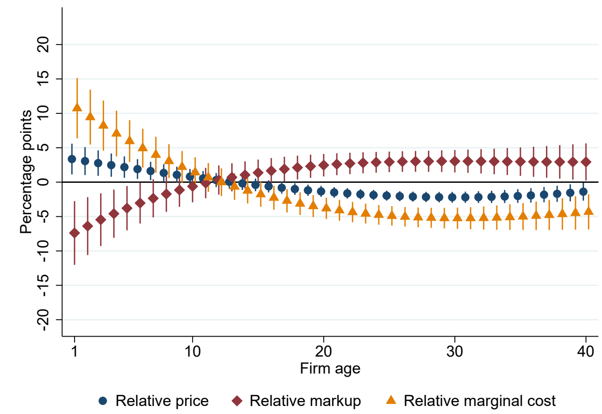
Notes: * t-test for equality of means and F-test for equality of standard deviations of two sub-samples, namely the markups in the early and later parts of the sample (2001-03 vs. 2014-16).

C Supplementary Results and Robustness Checks

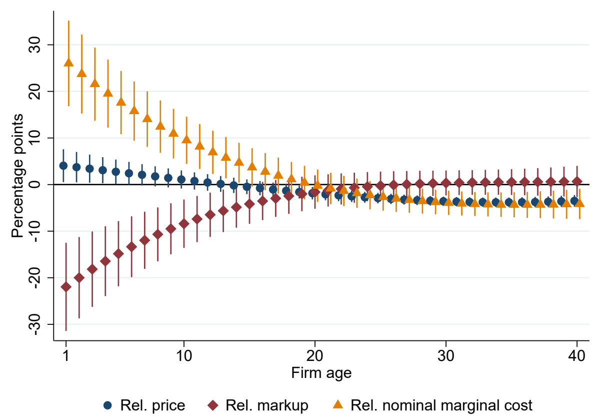
Figure C.1: Relative age trends: Robustness with respect to sample



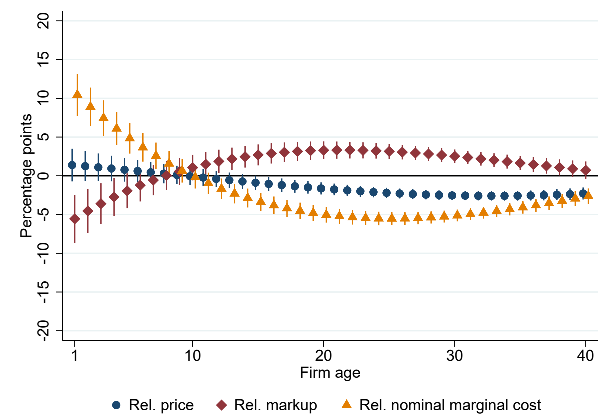
(a) Sample weights by age and size



(b) Relative trends with sample weights



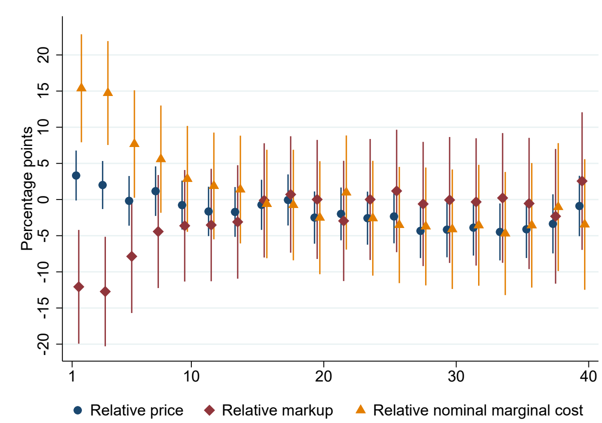
(c) Sample excl. food and machinery



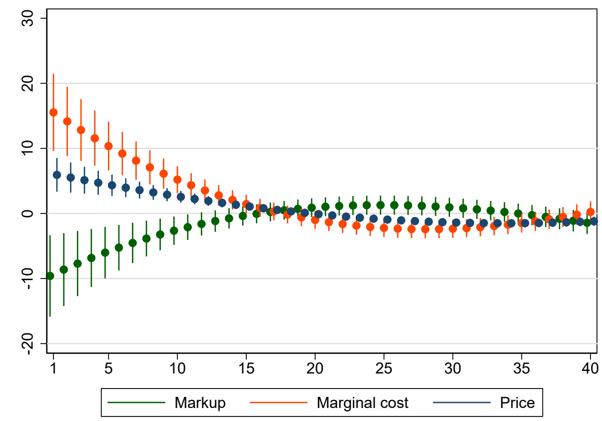
(d) Winsorized at 5th and 95th percentile

Notes: Panels (a) and (b): Our sample of firms is not representative of the manufacturing sector as a whole. In particular, our firms are larger and older then the benchmark sample, which we define as all manufacturing firms that eventually reach employment of 50 (see also Figure A.1.) We define age and size category cells in the benchmark sample and weight regressions by the fraction of firms in each cell in our sample and the benchmark sample. Panel (a) shows the log of these weights for each cell. Panel (b) shows the polynomial trend regressions in age when applying these weights to each observation. In panel (d) we winsorize markups and marginal costs for each year before computing deviations from the industry mean.

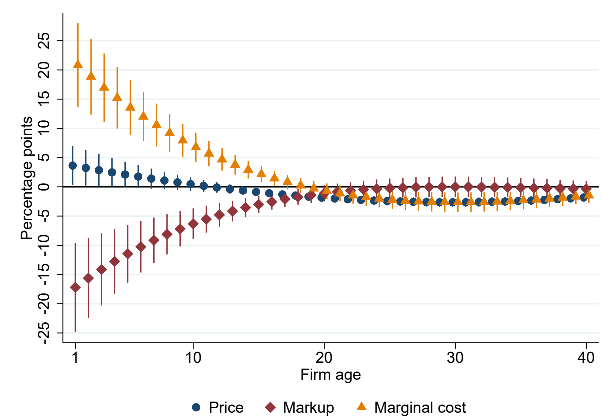
Figure C.2: Relative age trends: Robustness with respect to relative trend specification



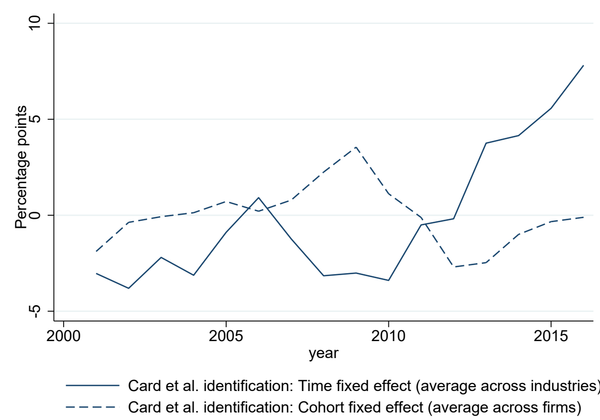
(a) Flexibility of relative age trends



(b) Restrict firm FE to be orthonal to linear cohort trend



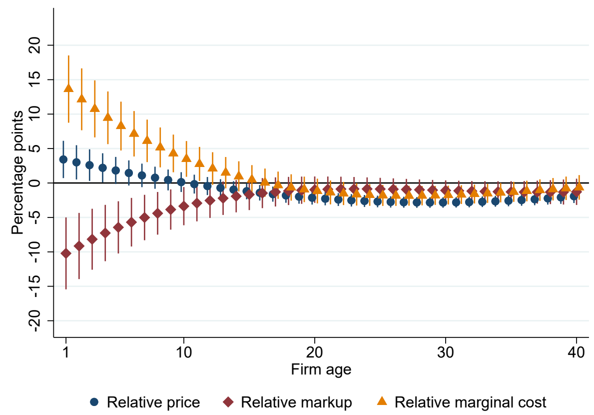
(c) Card et al. (2018a) identification assumptions



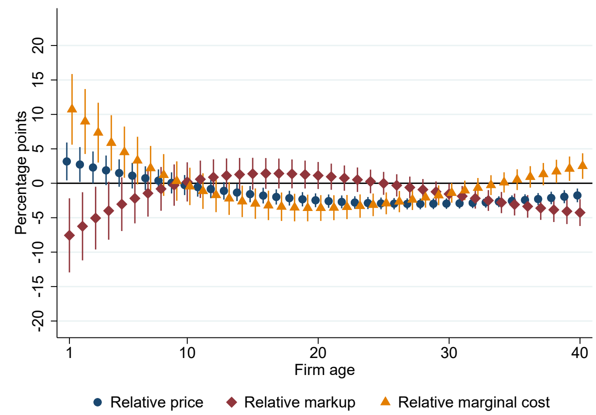
(d) Resulting markup time and cohort FE

Notes: Panel (a): Instead of including a fourth-order polynomial of age, we include a range of dummies each encompassing two years of age, i.e. the first estimate shows the mean relative markup for firms of age 1 and 2. Panel (b): In our baseline regression, we cannot identify age, time and cohort effects separately (the latter are determined by the firm fixed effects we include). To address this issue, we follow Card et al. (2018a) by assuming that the slope of the function in age is zero at one point, in our case age 30. Specifically, we estimate the following function on absolute markups: $\ln \mu_{izt} = \alpha^{\mu}_{iz} + \eta^{\mu}_{zt} + \beta_2 \text{age}^2_{izt} + \beta_3 \text{age}^3_{izt} + \beta_4 \text{age}^4_{izt}$. Because prices have no level interpretation after including cohort and time fixed effects, we normalize them to have the same level as in baseline Figure 4(a). The blue lines in panel (d) show the time fixed effect η^{μ}_{zt} (averaged across industries) and the average α^{μ}_{iz} of the firms in the sample at each point in time (cohort fixed effect).

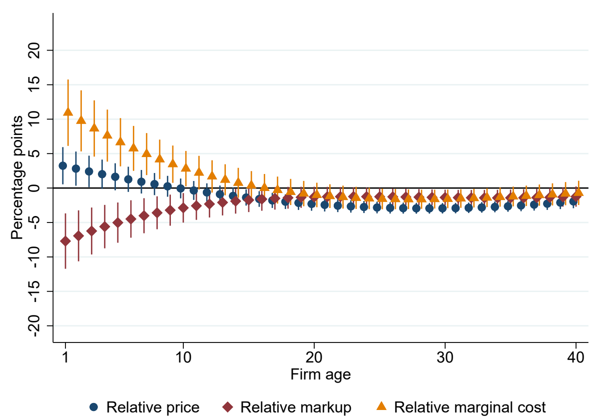
Figure C.3: Relative age trends: Robustness with respect to markup estimation



(a) No inventory correction of material inputs and sales



(b) FTE labor input



(c) De Ridder et al. (2021) first stage

Notes: Panel (a): xxxx.