Unemployment and Labor Productivity Comovement: the Role of Firm Exit

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Introduction

Labor productivity and unemployment

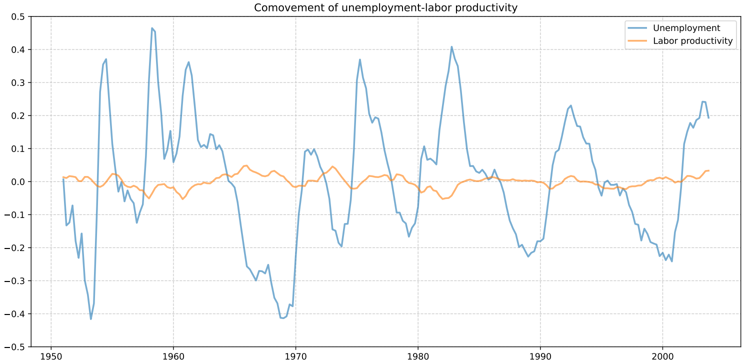

Figure: Unemployment and labor productivity. Each series series ranges from 1951M1-2003M12 and is aggregated to quarterly, logged, and HP-filtered with smoothing parameter $\lambda = 10^5$.

- Mild correlation between unemployment and labor productivity ≈ -0.4
- Under smoothing parameter $\lambda = 1600$, correlation is -0.21; under Hamilton filter, it is -0.26

Puzzle

- Labor productivity is closely tied to incentive for job creation in canonical Diamond-Mortensen-Pissarides (DMP) model
- Reasonable to expect search-theoretic labor market models to fit comovement of these series
- Yet DMP model implies a nearly perfect correlation between the two series

Contribution

- Develop model with endogenous mechanism that breaks the near-perfect correlation
- Sunk entry costs cause vacancies to be a positively valued, predetermined variable
- Under low destruction rate, then most current vacancies were created in the past

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- Under low destruction rate, then most current vacancies were created in the past
  $\Rightarrow$ depend relatively more on past productivity than current productivity

Key finding

Provided destruction shock is calibrated to match either

1. micro-level evidence on product destruction/firm exits
2. values used in growth literature

then model can mostly reproduce the mild correlation between productivity and unemployment while still maintaining the high cross-correlation between labor market variables

Model

Entry

- Fixed measure $F > 0$ of firms that can create vacancies
- Each period firms can access business opportunity at cost x (R&D, bringing product to production phase)
- $x \sim H$ (cdf)
- Let Q_t denote value of posting vacancy at time t
- Firms undertake business opportunity if and only if $x \leq Q_t$
- New vacancy creation

$$e_t = FH(Q_t)$$

Matching

- $M(u, v)$ matches given u unemployed, v vacancies
- $M(\cdot)$ is CRS, increasing and concave in each argument
- Tightness $\theta_t = v_t / u_t$ determines matching rates
  - Job finding rate $f(\theta_t) \equiv M(u_t, v_t)/u_t = M(1, \theta_t)$
  - Vacancy filling rate $q(\theta_t) \equiv M(u_t, v_t)/v_t = M(\theta_t^{-1}, 1)$
- Separation rate s: worker exits but product line continues
- destruction rate δ: product line and match are destroyed

Timing

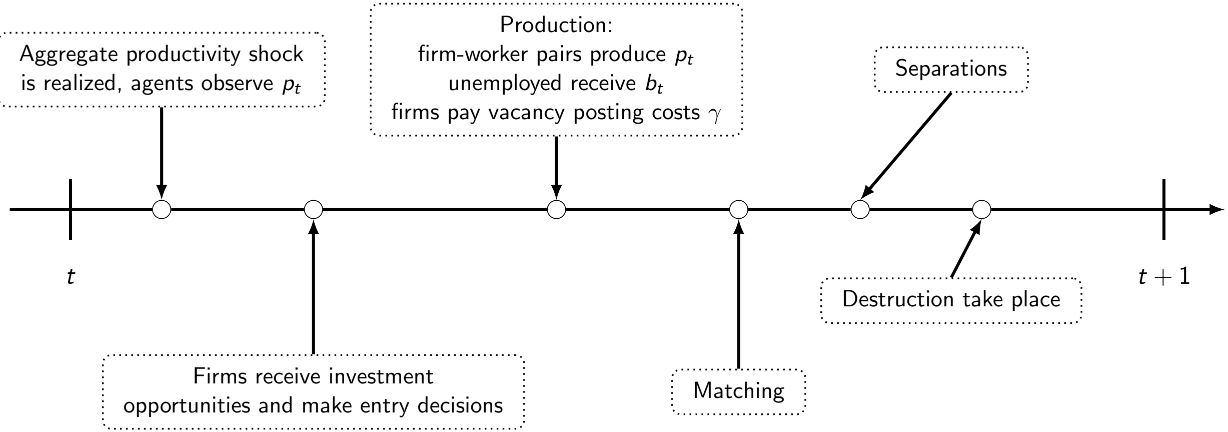

Figure: Labor Market Timing

Value functions

- Vacancy Q_t and filled job J_t

$$Q_t = -\gamma + \beta(1-\delta)\mathbb{E}_t[q(\theta_t)J_{t+1} + (1-q(\theta_t))Q_{t+1}]$$
$$J_t = p_t - w_t + \beta(1-\delta)[(1-s)\mathbb{E}_t J_{t+1} + s\mathbb{E}_t Q_{t+1}]$$

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- Unemployed worker U_t and employed worker W_t

$$U_t = b + \beta[(1-\delta)f(\theta_t)\mathbb{E}_t W_{t+1} + [1-(1-\delta)f(\theta_t)]\mathbb{E}_t U_{t+1}]$$
$$W_t = w_t + \beta[(1-\tau)\mathbb{E}_t W_{t+1} + \tau\mathbb{E}_t U_{t+1}]$$

where $\tau = 1-(1-\delta)(1-s) \approx \delta + s$ is the aggregate separation rate

Law of motion for vacancies and entry

- Vacancies

$$v_t = \overbrace{(1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})]}^{\text{Predetermined}} + e_t$$

Law of motion for vacancies and entry

- Vacancies

$$v_t = \overbrace{(1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})]}^{\text{Predetermined}} + e_t$$

- Vacancies are the sum of three flows
  1. $(1-\delta)(1-q(\theta_{t-1}))v_{t-1}$: unmatched vacancies surviving destruction shock
  2. $(1-\delta)s(1-u_{t-1})$: filled jobs that experienced a separation shock but survived destruction
  3. e_t: new entrants

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- Set $H(Q) = Q^{\xi}$, similar to Beaudry, Green, and Sand (2018) and Potter (2022)
- New entrants determined by free entry

$$e_t = FH(Q_t) \Rightarrow$$
$$Q_t = (e_t/F)^{1/\xi}$$

Law of motion for unemployment and productivity shock

- Unemployment

$$u_t = [1 - (1 - \delta) f(\theta_{t-1})] u_{t-1} + \tau (1 - u_{t-1})$$

- Technology

$$\log p_t = \rho \log(p_{t-1}) + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma)$$

Job creation condition

$$\overbrace{\frac{\gamma + K_t}{q(\theta_t)}}^{\text{New congestion effect}} = \beta(1-\delta)\mathbb{E}_t\left[p_{t+1} - w_{t+1} - K_{t+1} + (1-s)\frac{\gamma + K_{t+1}}{q(\theta_{t+1})}\right]$$

where

$$K_t \equiv \mathbb{E}_t[e_t^{1/\xi} - \beta(1-\delta)e_{t+1}^{1/\xi}]/F^{1/\xi}$$

is the expected flow entry cost

$\Rightarrow$ difference between entry cost firms face today and discounted expected entry cost tomorrow

- Now incorporates congestion effect: incentive to delay entry if many others enter
- Smoothing mechanism helps yield a hump-shaped response of vacancies as by Fujita and Ramey (2007)

Alternative parameterization of product development cost

Wage setting

- Nash bargaining yields standard surplus sharing under linearity

$$\alpha(J_t - Q_t) = (1-\alpha)(W_t - U_t)$$

where α is the worker's bargaining power

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$$w_t = \alpha[p_t - K_t + \frac{\theta_t}{1-\delta}(\gamma + K_t)] + (1-\alpha)b$$

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Two effects of flow entry cost K_t:

1. $(-)$ lower surplus: $p_t - K_t$
2. $(+)$ Higher value of current vacancy

Equilibrium

An equilibrium is an infinite, bounded sequence of productivity, wages, entrants, vacancies, and unemployment $\{p_t, w_t, e_t, v_t, u_t\}_{t=0}^{\infty}$ consistent with

- Job creation curve: e_t
- Wage setting rule: w_t
- Unemployment law of motion: u_t
- Vacancy law of motion: v_t
- AR(1) process for productivity: p_t

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Q_t follows from free entry and $\theta_t = v_t / u_t$

Steady state

Let $\rho = (1-\beta)/\beta$ denote discount rate

$$u = \frac{\tau}{\tau + (1-\delta) f(\theta)}$$

$$v = \frac{(1-\delta)s(1-u) + e}{1 - (1-\delta)(1-q(\theta))}$$

$$p - w - K = \frac{\gamma + K}{q(\theta)} \frac{\rho + \tau}{1-\delta}$$

$$Q = \left(\frac{e}{F}\right)^{1/\xi} = K \frac{1+\rho}{\rho + \delta}$$

Quantitative analysis

Calibration

Table: Calibration

Preferences/Technology	Parameter	Value	Calibration Strategy
Vacancy posting cost	γ	0	Coles and Moghaddasi Kelishomi (2018)
Bargaining power	α	0.6	Coles and Moghaddasi Kelishomi (2018)
Unemployment benefits	b	0.7	Coles and Moghaddasi Kelishomi (2018)
Matching function elasticity	ν	1.575	Job-finding rate
Discount factor	β	0.997	4% annual discount rate
Separation rate	s	0.0258	3.4% monthly match dissolution probability
Destruction rate	δ	0.0051	6% annual destruction rate
Population of firms	F	0.000235	Job-filling rate
Cost distribution parameter	ξ	1	Coles and Moghaddasi Kelishomi (2018)

$\Rightarrow$ Implies steady-state $u = 0.07$

Quantitative analysis

Empirical evidence on product/firm destruction rate

- Broda and Weinstein (2010): 3% product destruction rate
- Comin and Gertler (2006): 3% obsolescence using balance growth restrictions
- Estimates from Bernard, Redding, and Schott (2010): $5 - 6\%$
- Broda and Weinstein (2010) estimate firm exit to be 10% annually

$\Rightarrow$ We calibrate benchmark to be consistent with 6% annual rate and use 10% as robustness check

Impulse responses: baseline

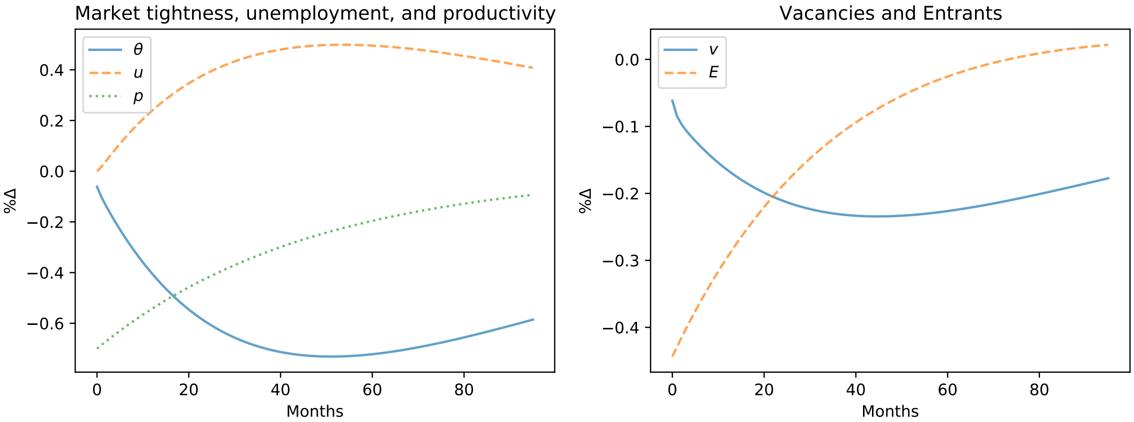

Figure: Impulse response functions in the benchmark calibration with $\delta = 0.0051$. Percentage deviations in response to a unit negative standard deviation technology shock.

 Quantitative analysis

Impulse responses: all job losses from firm destruction

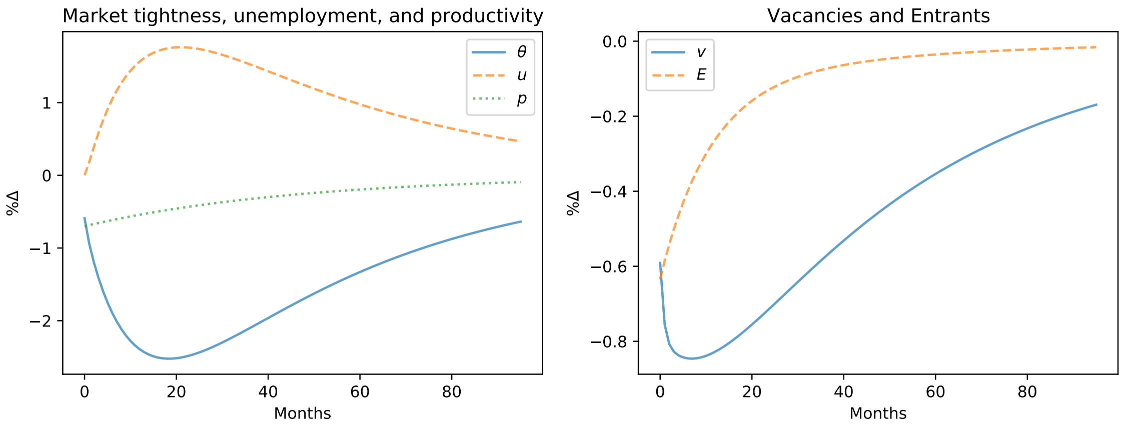

Figure: Impulse response functions in the benchmark calibration with $s = 0$ and $\delta = 0.0342$. Percentage deviations in response to a negative unit standard deviation technology shock.

Contemporaneous correlations

		$Corr(X, p)$				
Var X	Data	Benchmark (6% dest.) ($\delta = 0.0051$)	10% dest. ($\delta = 0.00874$)	CM (34% dest.) ($\delta = 0.0342$)	FR/MS (24% dest.) ($\delta = 0.0222$)	SS (20% dest.) ($\delta = 0.018$)
u	-0.408	-0.329	-0.462	-0.77	-0.686	-0.648
θ	0.396	0.419	0.554	0.861	0.78	0.736
v	0.364	0.593	0.721	0.975	0.924	0.887

Table: Correlations between unemployment and productivity under different specifications of the destruction rate δ. The remaining parameters are recalibrated. Moments are based on quarterly averages of $100,000$ monthly observations. Each observable series ranges from 1951M1-2003M12 and is logged and HP-filtered with smoothing parameter $\lambda = 10^5$.

Robustness to ξ

Dynamic correlations

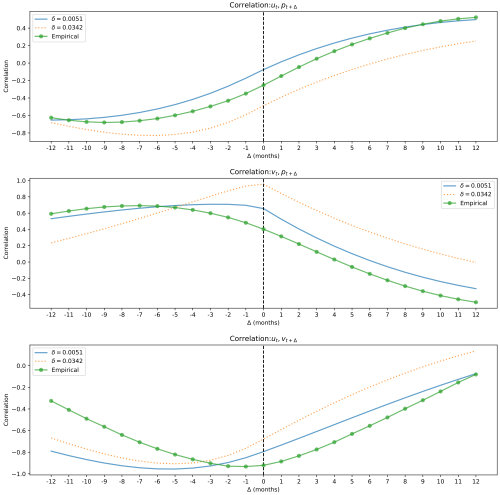

Figure: Dynamic correlations. The horizontal axis in each period depicts the time-shift Δ, measured in months, the vertical axis the correlation coefficient.

Conclusion

- DMP model produces a near perfect correlation between unemployment and productivity, whereas it is mild in the data
- Sunk entry costs and congestion in entry with a mild destruction shock can approximately fit data
- Key
  1. only vacancies from new entrants are determined by current productivity shocks
  2. congestion in entry induces firms to to smooth out entry in response to a shock

Appendix

Solution method

- Algorithm for solving the model is an Euler-equation based method described in detail by Coleman, Lyon, Maliar, and Maliar (2021)
- Unknown policy functions are approximated using complete quadratic monomials of the state variables with coefficients Θ
- One exogenous state variable p_t and two endogenous states: u_t and predetermined vacancies $v_{pret,t} = (1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})]$
- Use a quasi-random grid (Sobol) on a fixed hypercube to discretize the state space
- Approximate the flow entry cost K_t and entrants e_t
- Update Θ using ordinary least squares

Table of dynamic correlations

Table: Moments

	$Corr(v_t, p_{t-i})$				
Lagged Productivity	Benchmark ($\delta = 0.0051$)	10% Destruction Rate ($\delta = 0.00874$)	CM ($\delta = 0.0342$)	FR/MS ($\delta = 0.0222$)	SS ($\delta = 0.018$)
p_{t-1}	0.667	0.784	0.95	0.933	0.917
p_{t-2}	0.709	0.809	0.87	0.902	0.898
p_{t-3}	0.735	0.816	0.78	0.849	0.861
p_{t-4}	0.749	0.81	0.69	0.785	0.812

Back to dynamic correlations

Robustness to values of ξ

Table: Alternative values of ξ

Value of ξ	0.25	0.5	1	2	4	8
$\text{Corr}(u_t, p_t)$	-0.515	-0.411	-0.329	-0.255	-0.196	-0.151

Back to moments

Alternate parameterization

- Assume firm can develop product line at sunk entry cost ke_t^{ϕ}
- Value of a vacant firm with a product line is $Q_t = ke_t^{\phi}$
- Then flow entry cost K_t becomes

$$K_t = k\mathbb{E}_t\left(e_t^{\phi} - \beta(1-\delta)e_{t+1}^{\phi}\right)$$

- Parameter mapping

$$k = 1/F^{1/\xi}$$
$$\phi = 1/\xi$$

- Nest DMP by setting $\delta \to 0$ and $k \to 0$ (which implies $F \to \infty$)

Back to job creation condition

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