Liquidity, Unemployment, and the Stock Market*

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Abstract

Interest-rate spreads and the unemployment rate vary negatively with stock prices. Liquidity plays a role in a Mortensen-Pissarides economy with a twist: households self-insure against preference shocks by accumulating equity claims. Higher stock market valuations relax liquidity constraints, creating an aggregate demand channel that strengthens firms' hiring incentives. Quantitatively, a negative shock to stocks decreases the liquidity value of equity and increases unemployment. A "perfect storm" of an increase in risk and a drop in the velocity of publicly-provided assets produces a self-fulfilling crash to an equilibrium with high unemployment and low stock prices. Reliance on privately-issued assets heightens fragility.

JEL Classification: D82; D83; E40; E50

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1 Introduction

Economic turmoil over the past decade, or more, has refocused attention on the interaction between financial and labor markets and the resulting implications for aggregate economic outcomes. Recent research, in particular, emphasizes

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the strong negative co-movement between the stock market and unemployment (see, for example, Farmer (2012) and Hall (2017)). Intuitively, the relationship can arise because high discount rates on firm profits reduce the incentive to create jobs, and also because reductions in the value of stocks might depress consumer spending. Accordingly, we incorporate both mechanisms in a simple model, which attributes observed data on stock prices, unemployment rates, and real interest rates to exogenous liquidity and productivity shocks. The framework is a standard Mortensen and Pissarides (1994) model with a single change: a limited commitment problem in the goods market. Households encounter idiosyncratic spending shocks that, because of their limited ability to commit to repaying unsecured debt, they finance by pledging the value of some assets. Consumers' liquid assets are shares in a mutual fund comprised of stocks and government bonds.

This simple twist of an otherwise standard model imparts a key role to the stock market in generating booms and busts. Higher stock market valuations relax consumers' liquidity constraints, thereby creating an aggregate demand channel that strengthens firms' hiring incentives. The creation of new firms and jobs, likewise, enhances market capitalization and feeds back into consumer demand. The key insight is that these strategic complementarities between the labor market and the stock market provide a potential explanation for the observed co-movement between stock prices and unemployment rates: a strong stock market does not just reflect but also promotes a robust labor market. Jobs create assets, and assets create jobs.

In our model, firm profits and government bonds play an insurance role analogous to capital in the Aiyagari (1994) model. In each period, markets open sequentially as in Berentsen, Menzio, and Wright (2011) and Branch, Petrosky-Nadeau, and Rocheteau (2016) with three separate stages. A frictional labor market opens first where firms with vacant positions and unemployed workers participate in a stochastic matching process. Consumption and production take place in the last two stages. In the second stage, buyers and firms trade consumption goods early in a competitive market. In the last stage, buyers and firms have a late opportunity to trade goods and assets, and wages are paid. During that second stage early-consumption market, households face idiosyncratic spending shocks and, for some purchases, a limited ability to commit to repaying debt precludes financing these purchases with unsecured credit. Instead, some buyers are liquidity constrained and finance their purchases using the value of their mutual funds as collateral. Firms, likewise, can choose to speed up the production process, subject to a convex cost, in order to meet the demand from these early-consumption households.[1] This timing structure of markets is reminiscent of Diamond and Dybvig (1983) and has been exploited extensively in the New Monetarist literature.

Our study begins by documenting the empirical relationship between outcomes in financial and labor markets. First, we estimate a version of the Mortensen-Pissarides free entry condition generalized to feature variable interest rates and (labor) hiring costs. The results indicate that a one standard deviation increase in the interest rate spread, capturing the liquidity premium, is associated with over a 3% reduction in the stock-market-capitalization to GDP ratio. Second, in a Bayesian setting, we estimate a structural vector autoregression which imposes sign/zero restrictions to identify shocks to the stock market valuation and interest rate spread. Both shocks generate negative comovement between the stock market and unemployment.

The identified impulse responses motivate the main quantitative exercise. We calibrate the model to the U.S. economy and trace out the economy's transition to a "MIT shock" to the stock market. A one-time unanticipated negative shock to stock market capitalization has a persistent effect on stock prices. Along the transition path, interest rate spreads move strongly and the unemployment rate rises sharply. In order to calibrate the model, one has to take a stand on an empirical analogue to expenditure risk. While there are various sources of unplanned consumer expenditures, we argue that unexpected health spending provides a reasonable proxy, made even more salient by the COVID-19 pandemic.

[1] The convexity of the cost function is crucial to generate the aggregate demand externality. This feature implies that firms price at marginal cost, but above average cost. Alternatively, imperfect competition can complement the aggregate demand channel in important ways. For instance, monopolistic competition generates variety effects. With more firms selling differentiated goods, consumers are better able to diversify their consumption basket, which augments the initial expenditure shock. See Bilbiie, Ghironi, and Melitz (2012) for a business cycle treatment of firm entry–which emphasizes the role of sunk entry costs–and Silva (2017) for a New Monetarist application.

A novel theoretical finding is the possibility of multiple steady states: an equilibrium with a high employment rate and real interest rate coexists with equilibria featuring low employment rates, low real interest rates, and low stock market valuations. The underlying strategic complementarity follows directly from the aggregate demand channel: high stock market capitalization reduces households' liquidity constraints, increases aggregate demand, and thereby raises prices in the early-consumption market. Firms' revenues increase and lead new firms to enter production, which further propagates the high stock market valuation. A lower need for liquidity, in turn, boosts real interest rates, which dampens firm value and entry. Conversely, the economy can be stuck at an equilibrium with low aggregate wealth, low employment, and low real interest rates where households are severely liquidity constrained.

The ability of the model to generate multiple steady-state equilibria motivates the second MIT shock quantitative exercise explored later in the paper. We consider a "perfect-storm" counterfactual where an elevated expenditure risk, perhaps because of a pandemic, leads to a strong liquidity-constrained demand simultaneously with a large decrease in the velocity of government bonds. In this perfect-storm scenario, the model predicts the existence of three steady-state equilibria, one of which is quantitatively consistent with the long-run unemployment rates in the U.S. The other two equilibria feature lower stock market values and higher unemployment. An expectations shock creates a self-fulfilling path to the equilibrium with high unemployment, low real-interest rates, and low stock prices. The results from this counterfactual highlight an important observation. The economy is most fragile – i.e. sensitive to expectations of future asset prices – at times when spending shocks are most frequent/aggregate demand is strongest, and the economy relies on privately-issued liquid assets. A policy implication reinforces the prominent role that public provision of liquidity can play in avoiding recessions and financial crises.

1.1 Related literature

The theory proposed in this paper is closely related to a class of incomplete market models where households hold assets with a precautionary savings motive to insure themselves against idiosyncratic shocks. Most closely related is Aiyagari (1994) where households self-insure by acquiring claims to physical capital. In our model, the risk arises from idiosyncratic spending shocks, rather than income shocks, and assets are claims on aggregate firm values. Unlike in Aiyagari (1994), where the price of capital is fixed, here the value of firms is endogenous and affects household liquidity. This has the effect of an additional propagation mechanism that generates a positive feedback between employment and stock market valuations.[2]

The framework in our model is inspired by monetary theory, and in particular the class of New Monetarist models that incorporate unemployment and money. The first paper to introduce stock market liquidity into a Lagos-Wright model is Geromichalos, Licari, and Suárez-Lledó (2007). The timing structure of our model comes from Berentsen, Menzio, and Wright (2011). In Berentsen, Menzio, and Wright (2011) households have access to a single liquid asset, fiat money, and trade goods with firms in a decentralized goods market characterized by search frictions. In the Appendix we explicitly show that the set of steady-states in our framework is qualitatively different from the pure currency economy. Several New Monetarist papers emphasize the dual role of assets as collateral. For instance, in Lagos (2010) consumption is financed with loans collateralized by Lucas trees (a real asset) and fluctuations in liquidity premia are shown to be important in explaining the equity premium puzzle.[3] Similarly, in Rocheteau and Wright (2013) the asset is again a Lucas tree, and with endogenous firm entry the model exhibits multiple steady-states and cycles reminiscent of recurring bubbles and crashes. Finally, Lagos and Rocheteau (2008) study the co-existence of money and capital when claims to

[2]Therefore, the aggregate demand channel generated through expenditure risk can mitigate the unemployment volatility puzzle emphasized by Shimer (2005), and can arise from shocks to either liquidity or productivity.

[3]The model under consideration here abstracts from features that may capture the equity premium. The households in our model pay a (potentially) substantial liquidity premium on stocks.

capital can collateralize consumption.

The current paper is also related to a burgeoning literature that incorporates incomplete markets into realistic business cycle models. For instance, Kaplan and Violante (2010) use a life-cycle version of a standard incomplete markets model to assess how much consumption insurance in data is derived from a precautionary motive under permanent and transitory earnings shocks. Krusell, Mukoyama, and Şahin (2010) endogenize income risk through labor market matching and assess the implications for optimal provision of unemployment insurance. Unlike most of the incomplete markets literature, for tractability we assume quasi-linear household preferences, which imply a degenerate wealth distribution.

Our model is also closely related to a literature that incorporates labor market and goods market frictions. For instance, Wasmer and Weil (2004) and Petrosky-Nadeau (2013) incorporate a frictional credit market used by investors to finance job posting costs. Bethune, Rocheteau, and Rupert (2015), like our model, incorporate a limited commitment problem in the goods market into a Mortensen-Pissarides framework. They also assume that all consumers access unsecured credit. In Branch, Petrosky-Nadeau, and Rocheteau (2016), those households who are liquidity constrained can use their home equity as an asset to serve as collateral.

There are other labor search models in which multiplicity arises due to complementarity of hiring decisions. Kaplan and Menzio (2016) assume unemployed workers spend more time searching for goods, which allows them to pay lower prices. As firms hire workers, employment rises, shopping time falls, and markups rise. They also rule out borrowing/saving. The mechanism here does not depend on cyclical variation of competition and shopping time, and focuses on the role of assets for self-insurance. Moreover, in analyzing the American Time Use Survey, Petrosky-Nadeau, Wasmer, and Zeng (2016) do not find a stable relationship between the shopping time of unemployed and employed individuals, and obtain evidence for the procyclicality of shopping time using cross-state regressions.

There are empirical papers which estimate a causal effect from the stock market capitalization to consumption and labor market outcomes. Majlesi, Di Maggio, and Kermani (2020) utilize Swedish administrative data on asset holdings to estimate the impact of consumption to stock market returns. They estimate a marginal propensity to consume out of unrealized capital gains of 23% for the bottom half of the wealth distribution and 3% for the top 30%. Importantly, for buffer-stock households–defined as those whose liquid wealth is less than 6 months of disposable income–the estimated MPC out of capital gains is nearly 40%. Chodorow-Reich, Nenov, and Simsek (2019) use IRS data to impute the county-level stock market return, and then regress employment outcomes on these returns, controlling for county and state-by-quarter fixed effects. They find that a 20% increase in stock market valuations increase aggregate hours by 0.7% and the aggregate labor bill by 1.7%. They use these estimates to discipline a two-agent New Keynesian model with geographic heterogeneity and obtain a MPC of 3.2 cents per dollar of stock wealth.

2 Motivating Evidence

This article is motivated by the co-movement of stock market capitalization, interest rate spreads, and unemployment. Farmer (2012) provides evidence, from a vector error correction model, that variations in stock prices have out-of-sample predictive power for unemployment rates. The negative relationship is apparent in Figure 1, which is based on his calculations. Since our model emphasizes the role of an endogenous real interest rate, we provide two forms of motivating evidence.

First, we present evidence from a regression of stock market capitalization on real interest rates and vacancy creation costs. The regression is motivated by a generalization of the free entry condition in Mortensen and Pissarides (1994):

$$J_t = \frac{(1 + r_t)W_t k}{q_t}$$

where J is the value of the firm, k is the vacancy posting cost, q is the vacancy rate, W is the nominal wage, and r_t is the real interest rate. This formulation proxies for variable hiring costs by assuming the vacancy posting cost depends on the wage W_t, since it is generally a labor-intensive activity, and allow for

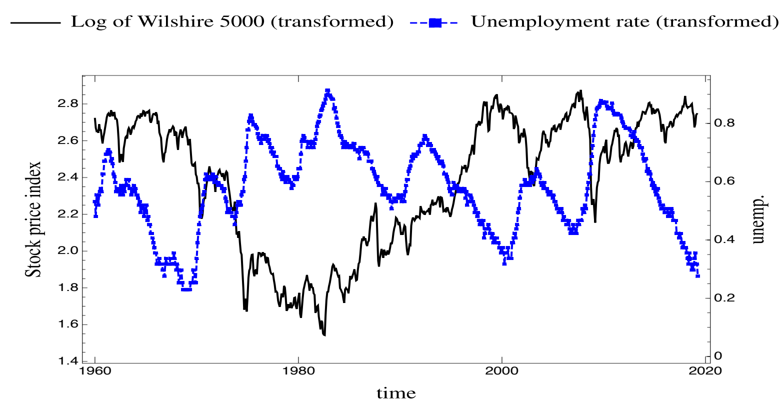


Figure 1: Stock prices (Wilshire 5000, logs) and unemployment (%), as in Farmer (2012).

interest rates to vary (which we interpret, as our model implies, as reflecting time-varying liquidity premia).[4] Dividing through by output and taking logs, the relationship can be expressed as the following regression equation:

$$\log J_{x,t} = \lambda_0 + \lambda_1 \text{spread}_t + \lambda_2 W_{x,t} + \lambda_3 \log q_t + \varepsilon_t \tag{1}$$

where we have decomposed r_t as $\rho - \text{spread}_t$, for rate of time preference ρ, which serves as the natural interest rate. A value of λ_1 statistically significant from zero is evidence that the liquidity premium is associated with firm value after controlling for hiring costs. We estimate (1) in first differences (growth rates of the firm value, labor share, and the vacancy filling rates).

To estimate (1), we construct the job finding rate from unemployment flows as in Shimer (2005). Dividing the vacancy series by the number of unemployed yields the quarterly market tightness. We use the constant returns to scale property to obtain the vacancy filling probability: $q(\theta) = f(\theta)/\theta$. This approach does not require us to impose a matching function. We calculate W_x as the aggregate nominal wage divided by nominal output. The liquidity premium is constructed following Krishnamurthy and Vissing-Jorgensen (2012) as the difference between Moody's Aaa-rated long-maturity corporate bond yields and the returns on long-term government bonds, where the latter are available until 1999. From 2000 onward, we use the yields on Treasuries with

[4] Hagedorn and Manovskii (2008), for instance, decompose the costs of vacancy creation into labor and capital costs.

20-year maturities. The data appendix describes each series.

Table 1 describes the regression results.[5] A 1 percentage point change in the interest rate spread is associated with a percentage change reduction of the stock market capitalization of -10.33%. Or, more intuitively, a one standard deviation increase in the interest rate spread ($\approx$ 30 bp) is associated with a -3.09% reduction in the stock-market capitalization to GDP ratio. The measure is statistically significant. Moreover, running the regression without the spread variable produces an adjusted R-squared of only 0.083 compared with 0.135 with the interest rate spread.

	coef	std err	t	P>|t|	0.025	0.975
Intercept	0.0098	0.005	1.929	0.055	-0.000	0.020
q	-0.1268	0.077	-1.638	0.103	-0.279	0.026
spread	-10.3305	4.822	-2.142	0.034	-19.845	-0.816
W_Y	0.1958	0.561	0.349	0.728	-0.912	1.304

No. Observations: 183 Adj. R-squared: 0.135

Table 1: Regression results.

Of course, this evidence is indirect and ignores the endogeneity of the real interest rate that motivates our model. To present evidence of the co-movement we estimate the impulse responses to an identified structural stock price shock. To motivate the identification strategy, note that the aggregate demand channel implies negative co-movement between stock market prices and the spread, whereas the interest rate channel implies positive co-movement. Both a positive shock to the stock market and the interest rate spread–a reduction in interest rates–induce more consumption and output. It is therefore natural to identify these shocks with sign restrictions. In general, sign restrictions generate set identification, in which there is a potentially large number of candidate models. Since the unemployment rate is a slow moving state variable, we also restrict the contemporaneous impact of spread and interest rate shocks on unemployment to zero. This assumption is fairly mild, but we nevertheless examine the consequences of relaxing the zero im-

[5] While endogeneity bias is a potential concern, we report only OLS estimates. The table reports robust standard errors with a small sample correction.

pact response in the appendix. Table 2 summarizes the baseline identification scheme:

	Stock mkt	Spread	Ind. pr.	Cons.	Unemployment
Stock market	+	-	+	+	0
Interest rate spread	+	+	+	+	0

Table 2: Identification assumptions. Restrictions only apply at impact.

The Appendix details the data construction and estimation strategy. Briefly, we apply the Bayesian algorithm developed by Arias, Rubio-Ramírez, and Waggoner (2018). The procedure combines the approach of imposing sign restrictions via the QR decomposition in Rubio-Ramirez, Waggoner, and Zha (2010) and uses an importance sampler to embed zero restrictions.

Figure 2 plots the main piece of evidence linking stock prices, liquidity premia, and unemployment rates. The figure plots point-wise median and 68 percent equal-tailed probability bands for the impulse responses of the interest rate spread, stock prices, unemployment, consumption, industrial production, and vacancies to a positive unit standard-deviation shock to stock prices. The median stock price jumps to 2%, and the median interest rate spread falls by 6 basis points. The interest rate spread is relatively persistent in responding to the stock market rise. Industrial production and consumption both rise and exhibit high persistence, and the former has a significant hump shape. Importantly, the probability bands for unemployment do not contain zero until almost 40 months, or about 3 years.

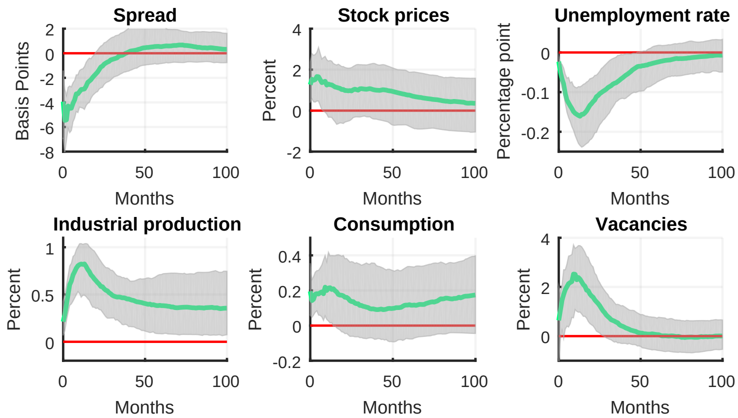


Figure 2: Impulse response to a positive stock price shock.

3 Environment

The set of agents consists of a unit measure of households, composed of one buyer and one worker. Time is discrete and is indexed by $t \in \mathbb{N}$. Each period of time is divided into three stages. The first stage stage is a frictional labor market where unemployed workers and vacant firms participate in a stochastic matching process. Consumption and production take place in the last two stages. In the second stage, buyers and firms trade consumption goods early in a Walrasian market. In the last stage, buyers and firms have a late opportunity to trade goods and assets and wages are paid. We take the late-consumption good traded in the last stage as the numéraire.

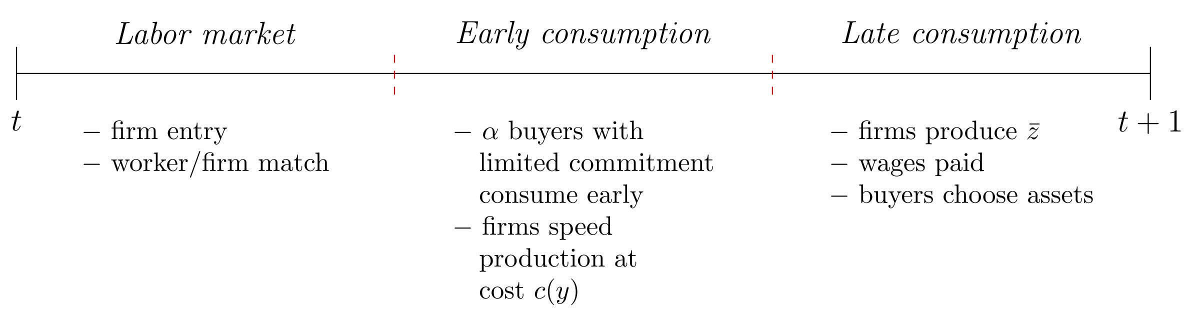


Figure 3: Timing.

The utility of a household is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\varepsilon\upsilon(y_t)+x_t^b+x_t^w\right],$$

where $\beta=(1+\rho)^{-1}\in(0,1)$ is a discount factor, $y_t\in\mathbb{R}_+$ is the buyer's early consumption, $x_t^b\in\mathbb{R}$ is the buyer's late consumption, and $x_t^w\geq 0$ is the worker's late consumption. If $x^b<0$, then the buyer is self-employed and produces the numéraire good. Because of the linear preferences in terms of the numéraire good, we can either treat the buyer and the worker as distinct agents, or as a joint entity with a consolidated budget constraint and impose conditions on primitives so that $x^b\geq 0$ holds.[6]

The utility function for early consumption, $\varepsilon\upsilon(y_t)$, is twice continuously differentiable, strictly increasing, and concave, with $\upsilon(0)=0$, $\upsilon'(0)=\infty$, and $\upsilon'(\infty)=0$. The multiplicative term, ε, is an idiosyncratic preference shock that is equal to $\varepsilon=1$ with probability α and $\varepsilon=0$ otherwise. These preference shocks correspond to liquidity shocks in the banking literature (e.g., Diamond and Dybvig (1983)) according to which some buyers have the desire for early consumption.

Each firm is a technology to produce $\bar{z}$ units of numeraire with one unit of indivisible labor (one worker) as the only input. Production takes time so that $\bar{z}$ is available in the last stage. The firm can speed up the production process and serve y units of goods to early consumers at cost $c(y)$ in terms of numeraire, where $c'>0$ and $c''\leq 0$. Unless stated otherwise, we assume $c(0)=0$, and $c'(0)=0$. There is an upper bound $\overline{y}$ such that $\overline{z}=c(\overline{y})$. One can impose conditions on fundamentals that ensures $y\in(0,\overline{y})$, so that the constraint can be ignored. The output in the last stage is $\bar{z}-c(y)$. With probability λ, the buyer can access intra-period credit. In that case, repayment can be fully enforced. With probability $1-\lambda$, a firm cannot monitor the buyer.

[6]The quasi-linearity in preferences keeps the model tractable and, in particular, implies that individual histories in the labor and goods markets are independent of asset holdings made in the third-stage; that is, the equilibrium wealth distribution is degenerate. More general preferences lead to self-insurance against employment and expenditure shocks (see Bethune and Rocheteau (2019)). Here households do have a precautionary demand for assets due to the spending shocks ϵ_t.

In order to hire a worker at time t, a firm must advertise a vacant position, which costs $k > 0$ units of the numéraire good at $t-1$. The measure of matches between vacant jobs and unemployed households in period t is given by $m(s_t, o_t)$, where s_t is the measure of job seekers and o_t is the measure of vacant firms (openings). The matching function, m, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m(0, o_t) = m(s_t, 0) = 0$ and $m(s_t, o_t) \leq \min(s_t, o_t)$. The exit probability out of unemployment for a worker is $e_t = m(s_t, o_t)/s_t = m(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. The vacancy filling probability for a firm is $q_t = m(s_t, o_t)/o_t = m\left(1/\theta_t, 1\right)$.

Employment (measured after the matching phase at the beginning of the second stage) is denoted n_t and the economy-wide unemployment rate (measured after the matching phase) is u_t. Therefore, $u_t + n_t = 1$. An existing match is destroyed at the beginning of a period with probability δ. A worker who loses her job in period t becomes a job seeker in period $t+1$. So, workers who lose their jobs must go through at least one period of unemployment, i.e. $s_{t+1} = u_t$. An employed worker in period t receives a wage in terms of the numéraire good, $w_{1,t}$, paid in the last stage. An unemployed workers enjoys w_0, which represents unemployment benefits and the value of leisure.

There is a fixed supply of one-period real government bonds A^g. Each bond issued in the third stage is a claim to one unit of the numeraire in the following period. In the second stage buyers are anonymous and cannot commit to repay their debt. There are perfectly competitive mutual funds which buy stocks and bonds and issue risk-free shares. We let r_t denote the rate of return of such claims from the last stage of $t-1$ to the last stage of t. These claims are perfectly diversified and hence free of idiosyncratic risk. Moreover, they can be authenticated and transferred at no cost. Household wealth, a_{t+1}, thus comprises shares in mutual funds that acquire existing firms or invest in new firms by creating vacant positions. To formalize that there is limited access to unsecured intra-period credit, we assume that with a probability of λ the buyer has access to a technology to enforce debt payments.[7]

[7] We focus on stock mutual funds, government bonds, and debt obligations as the assets for the following reasons. Stocks are a primitive given the fundamental role of firms in labor search models, and government bonds provide a policy instrument. Finally, probabilistic

In summary, the novelty relative to a standard Mortensen-Pissarides model is the second stage where households receive opportunities to consume early and are subject to limited commitment. As in Aiyagari (1994), households can self-insure against idiosyncratic risk by accumulating capital, here in the form of stock ownership. Relative to an Aiyagari model, the environment features both employment and expenditure risk, but it is the latter that matters for the determination of the real interest rate.

4 Equilibrium

In the following, we characterize an equilibrium by moving backward from agents' choice of asset holdings in the last stage, to the determination of prices and quantities for early consumption/production, and finally the entry of firms and the determination of wages in the labor market.

4.1 Goods and asset markets

As previously indicated, the lifetime utility of a household is the sum of the lifetime utility of the buyer and the lifetime utility of the worker. Therefore, in the following, we treat separately the two agents composing the households. Let $W_t(\omega_t)$ denote the lifetime expected discounted utility of a buyer at the beginning of the last stage with ω_t units of wealth in terms of the numeraire. Wealth ω_t is composed of shares of mutual funds net of debt obligations and tax liabilities, and assets a_{t+1} taken into the third subperiod consist solely of mutual funds since we only consider intra-period debt. Similarly, let $V_t(a_t)$ be the buyer's value function at the beginning of the second stage, before preference shocks for early consumption are realized.

access to credit by consumers enables us to characterize the space between no-and-full commitment. Though the economy is cashless, Hu and Rocheteau (2013) shows that fiat money is not essential in environments with Lucas trees. The appendix sketches out an extension to include fiat money.

The buyer's problem can be written recursively as

$$W_t(\omega_t) = \max_{x_t, a_{t+1}} \{x_t + \beta V_{t+1}(a_{t+1})\} \quad \text{s.t.} \tag{2}$$

$$a_{t+1} = (1 + r_{t+1})(\omega_t - x_t) \geq 0. \tag{3}$$

From (2), the buyer chooses its consumption, x_t, and asset holdings, a_{t+1}, in order to maximize its lifetime utility subject to a budget constraint. The budget constraint says that next-period wealth is equal to the current wealth net of consumption capitalized at the gross interest rate, $1 + r_{t+1}$. Or, combining (2) and (3) leads to

$$W_t(\omega_t) = \omega_t + \max_{a_{t+1} \geq 0} \left\{ -\frac{a_{t+1}}{1 + r_{t+1}} + \beta V_{t+1}(a_{t+1}) \right\} \tag{4}$$

From (4), W_t is linear in wealth and a_{t+1} is independent of ω_t. The Euler equation for the buyer's problem is:

$$1 = (1 + r_t)\beta V_t'(a_t).$$

The disutility cost of accumulating one unit of wealth in the last stage is equal to one. This investment yields $1 + r$ and is valued according to the buyer's discounted marginal utility of wealth in the early-consumption stage, $\beta V_t'(a_t)$.

We now turn to the goods market for early consumers. The expected discounted utility of a buyer at the start of the early-consumption stage holding assets a_t is

$$\begin{aligned} V_t(a_t) = \alpha \Big[&(1 - \lambda) \max_{p_t y_t \leq a_t} \{v(y_t) + W(a_t - p_t y_t - \tau_t)\} \\ &+ \lambda \max \{v(y_t) + W(a_t - p_t y_t - \tau_t)\} \Big] + (1 - \alpha) W_t(a_t - \tau_t) \end{aligned}$$

With probability α the buyer wants to consume early. In that case, the buyer can finance expenditures using intra-period credit with probability λ, or with assets when payment cannot be enforced. The constraint, $p_t y_t \leq a_t$, captures the inability of buyers to delay settlement. As a result, spending cannot exceed wealth. With probability, $1 - \alpha$, the buyer does not want to consume early. In general, the buyer enters the late-consumption stage with $a_t - p_t y_t - \tau_t$ units of wealth, where τ_t are lump-sum taxes. Using the linearity of W_t,

$$V_t(a_t) = \alpha[(1-\lambda)\max_{p_t y_t \leq a_t}\{v(y_t) - p_t y_t\} + \lambda \max_{y_t \geq 0}\{v(y_t) - p_t y_t\}] + a_t - \tau_t + W_t(0). \tag{5}$$

Denote the optimal early consumption under perfect credit y_t^* and without credit $\hat{y}_t$. These quantities satisfy $y_t^* = v'^{-1}(p_t)$ and $\hat{y}_t = \min\{y_t^*, a_t/p_t\}$. If the payment constraint does not bind, then the buyer equalizes marginal utility to price. Otherwise, early consumption equals the buyer's wealth.

The expected revenue of a firm in terms of the numeraire in period t is:

$$z_t = \bar{z} + \max_{y \in [0, \overline{y}]}\{p_t y - c(y)\}$$

Relative to the standard MP model, the novelty is the second term that represents the firm's profits from selling early. Assuming an interior solution, the optimal supply of goods in the early market is

$$y_t^s = c'^{-1}(p_t) \tag{6}$$

The price of early consumption is equal to the firm's marginal cost from producing early. Market clearing in the early-consumption stage requires

$$n_t y_t^s = \alpha[\lambda y_t^* + (1-\lambda)\hat{y}_t] \tag{7}$$

There is a measure n_t of active firms in the early market, each of which produces y_t^s. Household consumption is the sum of purchases by individuals with and without access to credit.

Finally, the buyer's choice of assets is obtained by substituting (5) into (4) and taking the first-order condition:

$$\frac{\rho - r_t}{1 + r_t} = \alpha(1-\lambda)\left[\frac{v'(\hat{y}_t)}{c'(y_t^s)} - 1\right] \tag{8}$$

The left side of (8) represents the cost of holding the asset, which approximately equals the difference between the rate of time preference and the real interest rate. The right side represents the expected marginal benefit from holding liquid wealth. The expected marginal benefit is the percentage increase of marginal utility with respect to marginal cost multiplied by the probability of having a liquidity shock and not being able to access credit. If buyers are not constrained by their asset holdings in the early-consumption stage, then $r_t = \rho$. Otherwise, $r_t < \rho$.

4.2 Labor market

We now turn to the second agent in a household. The lifetime expected utility of an employed worker, measured in either the second or third stage, is

$$U_{1,t} = w_{1,t} + (1-\delta)\beta U_{1,t+1} + \delta\beta U_{0,t+1}$$

The employed worker receives a wage, $w_{1,t}$, and keeps her job in the following period with probability $1-\delta$. Similarly, the Bellman equation of an unemployed worker:

$$U_{0,t} = w_{0,t} + (1-e_t)\beta U_{0,t+1} + e_t\beta U_{1,t+1}$$

The unemployed worker enjoys $w_{0,t}$ and finds a job in the following period with probability e_t. Therefore, the utility of a household in the third stage composed of a buyer with a units of wealth and a worker with employment state e is $W(a) + Ue$.

Arbitrage between acquiring existing firms or creating new ones equates the rate of return on a mutual fund, $1 + r_{t+1}$, to that of opening a vacancy, $q_{t+1}J_{t+1}/k$, so that

$$(1 + r_{t+1})k = q_{t+1}J_{t+1}$$

The rate of return from investing in a new firm in the last stage of t is the expected value of the firm in $t+1$, $q_{t+1}J_{t+1}$, net of the initial investment, k, expressed as a function of this initial investment. The value of a firm solves

$$J_t = z_t - w_1 + (1-\delta)\frac{J_{t+1}}{1 + r_{t+1}} \tag{9}$$

The value of a firm equals expected revenue net of the wage plus the expected discounted profits of the job multiplied by the survival probability $1-\delta$. Market tightness is determined by the arbitrage condition, $(1+r_{t+1})k = q_{t+1}J_{t+1}$, where J_t is given by (9):

$$\frac{(1+r_t)k}{q_t} = z_t - w_1 + (1-\delta)\frac{k}{q_{t+1}}.$$

Throughout, we take w_1 as exogenously given.[8]

In the second stage, firms evolve according to

$$n_{t+1} = (1-\delta)n_t + m(1,\theta_{t+1})(1-n_t).$$

Among the n_t existing firms in period t, a fraction $1-\delta$ survive. The measure of new firms equals the measure of job seekers in $t+1$, u_t, multiplied by the job finding probability $e_{t+1} = m(1,\theta_{t+1})$. The value of buyers' assets in the second stage is the market capitalization of firms plus the total value of government bonds.

$$a_t = n_t J_t + A_t^g = \frac{n_t(1+r_t)k}{q_t} + A_t^g. \tag{10}$$

By market clearing, the total value of the stock market and government bonds equals the value of assets held by buyers buyers when entering the early-consumption stage, a_t. Equation (10) implies a positive relationship between stock market capitalization, employment, and interest rates. Combining (6) (7), and (10) allows us to express the price as a function of assets and employment:

$$c'^{-1}(p_t) = \frac{\alpha}{n_t}\left[\lambda v'^{-1}(p_t) + (1-\lambda)\min\left\{v'^{-1}(p_t), \frac{n_t J_t + A_t^g}{p_t}\right\}\right].$$

We are now ready to define an equilibrium as a bounded sequence, $\{J_t, \theta_t, n_t, p_t, r_t\}_{t=0}^{+\infty}$,

[8]We abstract from bargaining and wage-determination considerations in the labor market. The Appendix presents an extension in which w_1 is determined via Nash bargaining.

that solves:

$$J_t = \frac{(1+r_t)k}{q(\theta_t)} = \bar{z} + \max_y \{p_t y - c(y)\} - w_1 + (1-\delta)\frac{J_{t+1}}{1+r_{t+1}} \tag{11}$$

$$c'^{-1}(p_t) = \frac{\alpha}{n_t}\left[\lambda v'^{-1}(p_t) + (1-\lambda)\min\left\{v'^{-1}(p_t), \frac{n_t J_t + A_t^g}{p_t}\right\}\right] \tag{12}$$

$$\frac{\rho - r_t}{1+r_t} = \alpha(1-\lambda)\left[\frac{v'\left(\frac{n_t J_t + A_t^g}{p_t}\right)}{p_t} - 1\right]^+ \tag{13}$$

$$n_{t+1} = (1-\delta)n_t + m(1,\theta_{t+1})(1-n_t), \tag{14}$$

for some given n_0. Equation (11) determines the value of a firm and market tightness taking the real interest rate and the early-consumption price as given. Equation (12) determines the early-consumption price by market clearing while (13) determines the real interest rate from the buyer's demand for liquid wealth. Equation (14) is the law of motion of employment.

5 Deconstructing the model

For this section, set $\lambda = 0$, so that we isolate the role of liquid mutual funds and bonds. To better understand the components of the model, we deconstruct it by starting with the textbook Mortensen-Pissarides model and adding one new ingredient at a time. For sake of illustration, we use a continuous-time version of the model that allows us to represent dynamics graphically through phase diagrams.[9]

5.1 A Mortensen-Pissarides economy

The Mortensen-Pissarides economy with a single good and frictionless goods market can be obtained by shutting down the idiosyncratic preference shocks, $\alpha = 0$, so that there is no early consumption. In this case, $z_t = \bar{z}$ and $r_t = \rho$ since stocks and bonds provide no liquidity/insurance role. Hence, a change

[9] Here we employ the methodology in Choi and Rocheteau (2019). The Appendix presents results on local uniqueness of rational expectations equilibria in the discrete-time version of the model.

in A^g has no effect on interest rates or output. An equilibrium can be reduced to a pair, (J_t, n_t), which solves

$$(\rho + \delta)J = \bar{z} - w_1 + \dot{J}$$
$$\dot{n} = m\left[1, \theta(J)\right](1 - n) - \delta n,$$

where $\theta(J)$ is the solution to $J = k/q(\theta)$. It is easy to check that there is a unique steady state and, for any initial condition n_0, a unique equilibrium corresponding to the saddle path leads to the steady state. Along this equilibrium, J is constant and equal to the discounted sum of the profits, $(\bar{z} - w_1)/(\rho + \delta)$, where the effective discount rate is the sum of the rate of time preference and the depreciation rate. Similarly, market tightness is constant. Graphically, in the left panel of Figure 4, the J-isocline is horizontal. The n-isocline is upward-sloping since a higher market value of firms induces a higher market tightness, and higher employment at the steady state.

5.2 Mortensen-Pissarides with early consumption and perfect credit

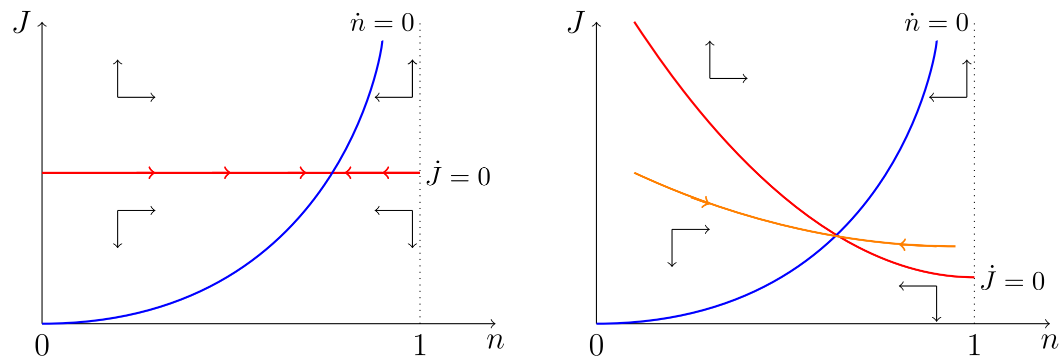
We reintroduce preference shocks for early consumption by setting $\alpha > 0$ but keep the goods markets frictionless by assuming that buyers have access to perfect credit in the early-consumption stage, i.e., $\lambda = 1$. In that case an equilibrium is a list, $\{J_t, p_t, y_t^s, n_t\}$, that solves

$$(\rho + \delta)\, J = \bar{z} + \max_y \{py - c(y)\} - w_1 + \dot{J} \tag{15}$$

$$\upsilon'\left(\frac{ny^s}{\alpha}\right) = p = c'\left(y^s\right) \tag{16}$$

$$\dot{n} = m\left[1, \theta(J)\right](1 - n) - \delta n \tag{17}$$

From (16), assuming $c'' > 0$, each firm's early-supply of goods decreases with n. As a result, the price of early consumption is a decreasing function of n. It implies that the firm's total revenue on the right side of (15) is $z = z(n)$ with $z' < 0$. As there are no liquidity constraints, a change in government bonds A^g has no effect on equilibrium.



(a) MP model.

(b) 2-good MP model with convex cost.

Figure 4: Phase diagrams: MP models.

The dynamic system, (15)-(17), can be reduced to two ODEs and two unknowns, J and n. In the right panel of Figure 4, the J-isocline is decreasing in n, since higher n means lower early-consumption prices and lower profits. As before, there is a unique steady state and a unique equilibrium starting from any initial condition n_0. Along the saddle path trajectory, the value of firms is negatively correlated with n. A positive productivity shock that raises $\bar{z}$ shifts the J-isocline upward. So the value of firms and market tightness overshoot their steady-state values. As employment increases, p_t decreases which brings J_t and θ_t back to their steady states.

5.3 Mortensen-Pissarides with limited commitment

Households accumulate wealth to self-insure against the idiosyncratic risk of early consumption. To mimic the one-good economy of the Aiyagari model, we impose a linear cost function, $c(y) = y$, so that $p = 1$ and firms are indifferent between producing early or late. As a result, the marginal product of capital, as captured by $\bar{z} - w_1$, does not depend on market capitalization. An equilibrium can now be reduced to a triple, (J, r, n),

$$\begin{aligned}(r+\delta)\,J &= \bar{z}-w_1+\dot{J}\\ \rho - r &= \alpha\left[v'\left(nJ+A^g\right)-1\right]^+ \\ \dot{n} &= m\left[1,\theta(J)\right](1-n)-\delta n.\end{aligned} \tag{18}$$

The novelty is Equation (18) that endogenizes the real interest rate. From (18) one can express r as an increasing function of $nJ+A^g$ and reduce the system to two ODEs and two unknows, (J,n). The J-isocline, such that $\dot{J}=0$, is given by $[r(nJ+A^g)+\delta]J=\bar{z}-w_1$. There is a negative relationship between J and n. Intuitively, as the measure of firms increases, market capitalization increases for given J. As households have more liquidity to finance demand shocks, r rises, which reduces the value of each firm. Thus, an increase in A^g lowers the J-isocline: raising real interest rates r, reducing firm value J, and hence depressing employment n via a reduced incentive to hire. Let $\bar{M}$ denote the market capitalization above which $r=\rho$. For all $nJ>\bar{M}$, J is constant and equal to $(\bar{z}-w_1)/(\rho+\delta)$. Let $\underline{M}$ denote the market capitalization such that $r=-\delta$, i.e., $v'\left(\underline{M}+A^g\right)=1+(\rho+\delta)/\alpha$. As nJ approaches $\underline{M}$, J tends to $+\infty$ and n tends to 0. The n-isocline gives a positive relationship between n and J. So there is a unique steady state. Moreover, for given n_0 the equilibrium is unique. Along this equilibrium J decreases over time and r increases if n_0 is less than the steady state.

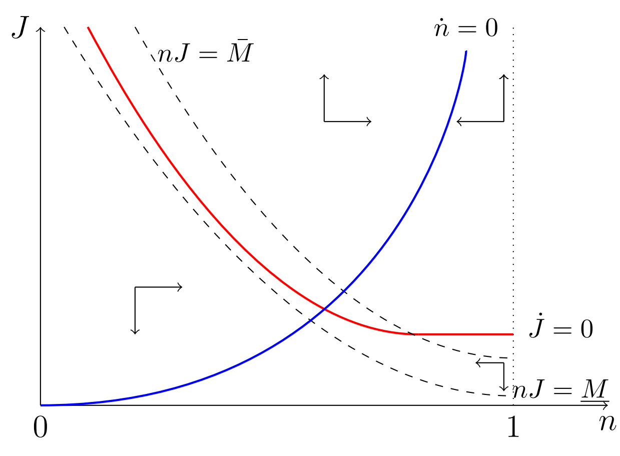


Figure 5: Phase diagram: Bewley-Aiyagari where $c(y)=y$.

A positive productity shock moves the J-isocline upward. If the initial steady state is such that households are liquidity coinstrained, then J overshoots its steady-state value. As n increases, market capitalization rises as well and the real interest rate decreases, which brings the value of firms back to their steady state.

5.4 The general case

We now combine all the ingredients: (i) households are subject to idiosyncratic preference shocks for early consumption, $\alpha > 0$; (ii) they can access credit with probability λ; (iii) and the cost of early production, $c(y)$, is strictly convex. The early-consumption price, $p = c'(y^s)$, now depends on households' liquid wealth, thereby providing another channel through which liquid wealth affects firms' revenue.[10]

An equilibrium is now a list, (J, r, p, n), that solves

$$\begin{aligned}(r+\delta)\,J &= \bar{z} + \max_y \{py - c(y)\} - w_1 + \dot{J} \\ c'^{-1}(p) &= \frac{\alpha}{n_t}\left[\lambda v'^{-1}(p) + (1-\lambda)\min\left\{v'^{-1}(p), \frac{nJ + A^g}{p}\right\}\right] \\ \rho - r &= \alpha(1-\lambda)\left[\frac{v'\left(\frac{nJ+A^g}{p}\right)}{p} - 1\right]^+ \\ \dot{n} &= m\left[1, \theta(J)\right](1-n) - \delta n.\end{aligned}$$

We can reduce these equations to a pair of ordinary differential equations by defining a sequence of functions. Households lacking credit are unconstrained if and only if $py^* \leq nJ + A^g$. Let $y^*(n)$ be the solution to $v'(y^*) = c'(\alpha y^*/n)$ and $\hat{y}(n, J, A^g)$ solve

$$\hat{y} = \frac{\alpha}{n}\left[\lambda v'^{-1}(c'(\hat{y})) + (1-\lambda)\frac{nJ + A^g}{c'(\hat{y})}\right]$$

It is easy to check that y^* is a decreasing function of n with $\lim_{n\to 0} y^* = +\infty$ and $\lim_{n\to+\infty} y^* = 0$ and $\hat{y}$ is an increasing function of J and A^g and a decreas-

[10] One further comparison, to a pure currency economy, is presented in the Appendix.

ing function of n. Thus, the buyer's liquidity constraint is more likely to bind if n is low and J is large. Let $y^s(n, J, A^g) = \min\{\hat{y}(n, J, A^g), (\alpha/n)y^*(n)\}$. We define the price, in turn, as $p(n, J, A^g) = c'[y^s(n, J, A^g)]$. The price is weakly decreasing in n and weakly increasing in J and A^g (an aggregate demand effect). The total revenue of a firm is

$$z(n, J, A^g) = \bar{z} + p(n, J, A^g)y^s(n, J, A^g) - c[y^s(n, J, A^g)].$$

Revenue is weakly decreasing in n and weakly increasing in J and A^g. The real interest can also be expressed as a function of n and J as follows:

$$r(n, J, A^g) = \rho - \alpha(1-\lambda)\left[\frac{v'\frac{nJ+A^g}{p(n,J,A^g)}}{p(n, J, A^g)} - 1\right]^+,$$

where y^b is an increasing function of $n, J.$ and A^g. So, r is a weakly increasing function of $n, J,$ and A^g. Using the functions $z(n, J, A^g)$ and $r(n, J, A^g)$ we reduce the dynamic system to two autonomous, nonlinear ODEs:

$$\dot{J} = \left[r(\overset{+}{n}, \overset{+}{J}, \overset{+}{A^g}) + \delta\right] J + w_1 - z(\overset{-}{n}, \overset{+}{J}, \overset{+}{A^g}) \equiv f(J, n)$$
$$\dot{n} = m\left[1, \theta(\overset{+}{J})\right](1-n) - \delta n \equiv g(J, n).$$

The right side of the J-ODE is monotone increasing in n but can be non-monotone in J. As a result, the J-isocline can also be non-monotone, with important consequences for the multiplicity of steady states and dynamics. An increase in government bonds A^g raises both interest rates and revenue, thus having an ambiguous effect on the J-nullcline.

In Figure 6 we provide a numerical example for the following parameter values: $m(s, o) = s^{\xi}o^{1-\xi}$ with $(1-\xi)/\xi = 0.2$, $c(y) = y^{1.9}/1.9$, $v(y) = y^{0.5}/0.5$, $\bar{z}-w_1 = -0.5$, $\rho = 0.1$, $\alpha = \delta = 1$, and $A_g = 0$. Note that for this parametrization $\bar{z}-w_1 < 0$, i.e., if the early-consumption opportunities are shut down, then firms make negative profits. This numerical example exhibits multiple active steady states. There is an equilibrium with high employment, high value for firms, and high interest rates and a different equilibrium with low employment rate, low valuation of firms, and low interest rate. The logic for this multiplic-

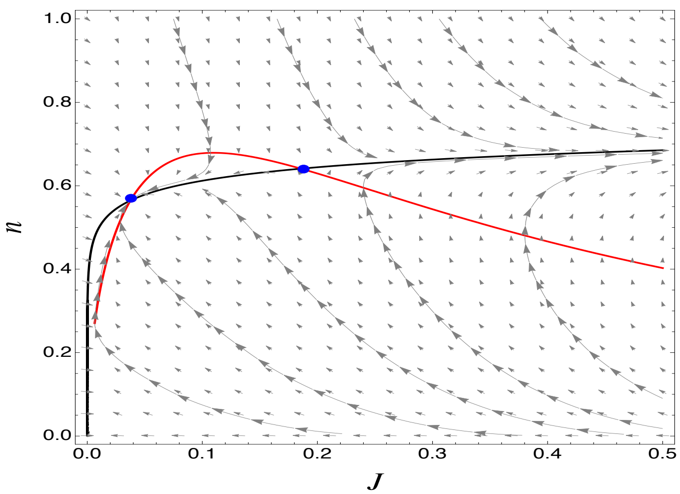


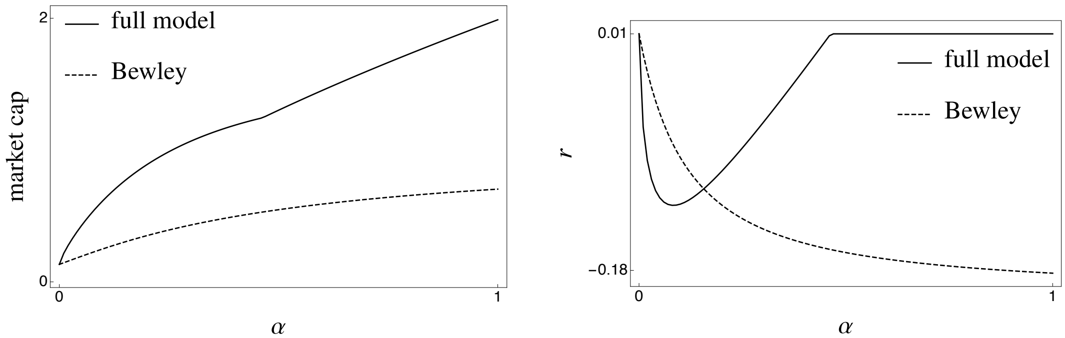
Figure 6: Multiple equilibria: full model.

ity is as follows. At the high equilibrium market capitalization is high, which relaxes the liquidity constraint faced by households in the early-consumption stage. As a result, the aggregate demand for early consumption is high, which pushes p up, raises firms' revenue, and generates entry. The real interest is high because wealth is abundant, which reduces the liquidity/insurance premium of stocks. The high p and high r have opposite effects on J, but in our example the former dominates.

At the opposite, the economy can be stuck in an equilibrium with low aggregate wealth, low employment, and low real interest rates. In this equilibrium, households are severely liquidity constrained, which reduces aggregate early-consumption and depresses the price p. It also reduces the real interest rate. Firms' profits are lower due to the lower p, which reduces entry and employment. We think of this type of equilibrium as capturing the notion of secular stagnation.

5.5 The aggregate demand channel: comparison of general case with Bewley-Aiyagari

In the Bewley-Aiyagari version of the model, if spending shocks occur more frequently, and consumers are liquidity constrained, then the real interest rate is lower and firm values are higher (less discounting of expected future profits).



(a) Market capitalization.

(b) Real interest rate.

Figure 7: Comparing the general case to Bewley-Aiyagari case.

In addition to this interest rate channel, the general version of the model features an aggregate demand channel whereby spending shocks impact firm revenues in early consumption markets. This section further compares the steady-state properties of these two cases.

Figure 7 plots stock market capitalization (nJ) and real interest rates as a function of the frequency of spending shocks α in both the Bewley-Aiyagari version of the model (dashed line) and the full model with the aggregate demand channel. To generate this figure, we set $\bar{z} - w_1 = 0.055, \sigma = 0.9, \gamma = \xi = k = 0.5$, and $\delta = 0.25$. The Bewley-Aiyagari version arises when $\sigma = 0$. The top panel plots the steady-state market capitalization as a function of α. In both versions of the model, greater expenditure risk increases market capitalization, through more firm-entry (n) and firm value (J). However, in the full model the market value of firms increases substantially more as $\alpha \to 1$. Notice that the full model generates a kink at the point that households are no longer liquidity constrained. For α greater than the kink point, stock market capitalization increases linearly with α.

The bottom panel of Figure 7 plots the associated real interest rate. When $\alpha = 0$, there is no expenditure risk and $r = \rho$ in both versions of the model. For small $\alpha > 0$, households are liquidity constrained and further increases in α tighten those liquidity constraints and reduce the real interest rate. This pattern continues for all values of α in the Bewley-Aiyagari version. In the full version of the model, the aggregate demand channel causes stock market capitalization to increase substantially, which loosens consumers' liquidity constraints and the real interest rate increases. Eventually, at the kink point in the top panel, consumers are no longer liquidity constrained and $r = \rho$.

The strong strategic complementarities between labor markets and goods markets renders the creation of private liquid assets elastic with respect to spending shocks. Figure 7 highlights the unique unique implications relative to a Bewley-Aiyagari model. These insights are important for interpreting the quantitative experiments that follow in the remainder of the paper.

6 Quantitative analysis

The frequency is monthly and the time range is 1948-2018. The matching function is taken from from den Haan, Ramey, and Watson (2000): $m(s,o) = so/(s^{\xi} + o^{\xi})^{1/\xi}$. The cost of early production is $c(y) = y^{1+\sigma}/(1+\sigma)$, and the utility function is $v(y) = y^{1-\gamma}/(1-\gamma)$. We choose $\sigma = 0.2$, which represents a 20% markup of price to average cost. The construction of empirical separation rates s_t and job finding rates e_t uses unemployment data as in Shimer (2005). We set δ and e according to their respective means. The parameter ξ target the mean job finding rate e. The implied rate of employment from the Beveridge curve is $n = e/(s+e)$

We assume a liquidity premium or convenience yield of liquid assets of 75 basis points, which is close to the spread estimated by Krishnamurthy and Vissing-Jorgensen (2012) for Treasury securities.[11] This spread is also similar to the baseline results of several incomplete market models, e.g., Aiyagari (1994) and Angeletos (2007). We also assume an annual risk-free interest rate of 4%, which implies a monthly target for r_t. We choose ρ to attain a risk-free rate of 4%, γ to target the interest rate spread, and A^g to match the semi-elasticity of the spread with respect to the debt-to-GDP ratio of -0.746 calculated in Krishnamurthy and Vissing-Jorgensen (2012), and k to satisfy the arbitrage condition $J = (1+r)k/q$.

The calibration of α and w_1 depends on evidence on health expenditure

[11] They consider the percentage spread between Moody's Aaa-rated long-maturity corporate bonds yields and yields on long-term maturity Treasury bonds.

shocks from the *Federal Reserve Report on the Economic Well-being of U.S. Households in 2015*. As previously mentioned, the share in the sample reporting a major health expense ranges from 22% − 30% and so we take 26% as a midrange value along with a mean expense of \$2,383. Hence, α is chosen to match health expenditures shocks of 26% annually, and w_1 is chosen to target the fraction of the wage spent on unexpected health costs. Finally, we choose λ to represent the fraction of households with revolving credit sufficient to replace income, as reported in Braxton, Herkenhoff, and Phillips (2019). The appendix provides explicit details of how the calibration procedure was implemented. Table 3 summarizes the parameters, values, and respective targets, while Figure 8 plots the steady-state relationship.

Parameter	Values	Calibration Strategy
δ	0.028	mean separation rate
γ	3.000	interest rate spread
$\overline{z}$	1.000	normalization
w_1	0.989	health expenditure shocks
w_0	0.396	replacement ratio
ρ	0.003	risk free rate
α	0.025	frequency of health shocks
σ	0.200	ratio of price to average cost
k	0.347	consistency with market tightness
ξ	1.326	consistency of tightness with job finding probability
A^g	0.225	Semi-elasticity of interest rate spread
λ	0.430	Frac. h.h.'s with revolving credit sufficient to replace income

Table 3: Parameterization

6.1 Economic Response to Stock Market Shock

We turn now to the main quantitative exercise, exploring how the model captures the dynamic response to a stock market shock as documented in Section 2. Our approach is to compute the model-implied impulse response to a MIT shock to stock market capitalization $M_t \equiv n_t J_t$. That is, beginning from steady-state, at $t = 1$ stock market capitalization M_1 is subject to a one-time shock from which we compute the perfect foresight path back to the unique

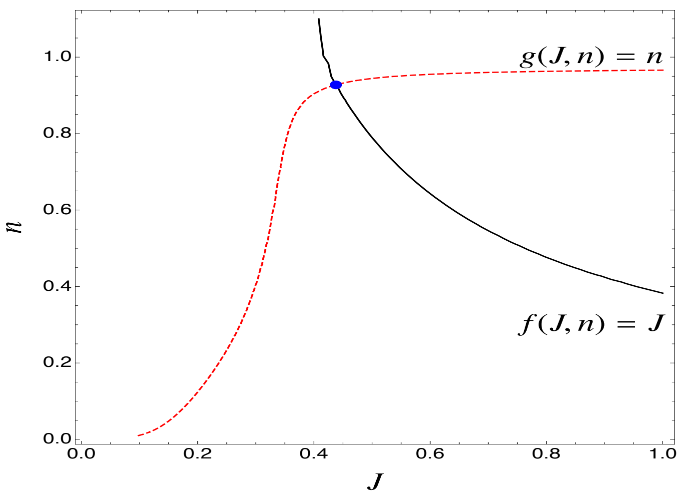


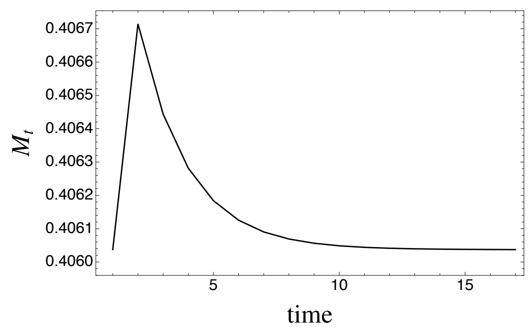
Figure 8: Steady-state at calibrated parameter values

steady-state. First, we report results for the calibrated parameter values. Second, we show sensitivity to different values for liquidity shocks (α), public liquidity (A_g), and labor-market matching efficiency (ξ).

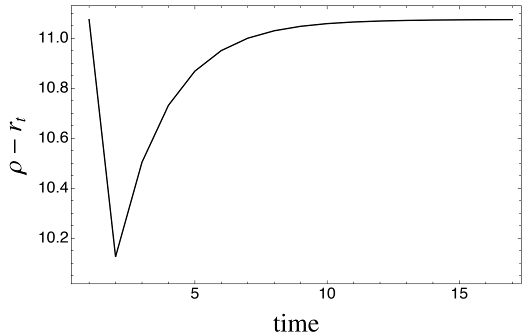
6.1.1 Benchmark

Figure 9 plots the benchmark impulse response to the stock-market MIT shock. The economy begins at $t = 0$ in the unique steady-state with parameters set to the values reported in Table 3. In line with the structural VAR evidence, we perturb stock market capitalization at $t = 1$ by 0.02/12, i.e. a 2% per annum shock. The top-left panel plots the dynamic path for stock market capitalization, the top-right panel plots the interest-rate spread $\rho - r_t$, and the bottom panel plots the unemployment rate. All variables are in levels, with the interest-rate spread expressed in basis points, and the unemployment rate in percentage points.

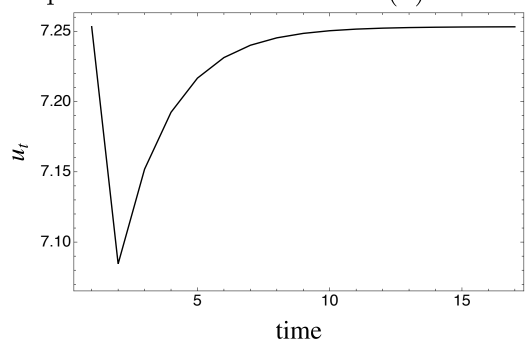
Figure 9 illustrates that the calibrated model produces the negative co-movement between stock market values and unemployment rates, and also captures the documented co-movement between interest-rate spreads and the stock market/unemployment rate. The shock to the stock market produces a persistent impact that declines monotonically back to its steady-state value. The shock to the stock market produces a contemporaneous decrease in the



(a) Stock market capitalization.



(b) Interest-rate spread.



(c) Unemployment.

Figure 9: Impulse response to a MIT shock at $t = 1$ to stock market capitalization $M_t \equiv n_t J_t$. Parameters are calibrated according to Table 3. At $t = 1$ M_1 is perturbed by a 2% per annum shock, or .02/12 at the monthly rate. The figures plot the perfect foresight path back to the steady-state.

interest-rate spread, that is, an increase in the real interest rate r_t. The unemployment rate declines as the net effect from M_t and r_t produces a higher present-value of stock market capitalization – recalling, firms discount at rate r_t – that induces greater firm entry. The role of the real-interest rate in attenuating or reinforcing the increase in M is explored in more detail below.

While Figure 9 captures the co-movements in the structural VAR displayed in Figure 2, it does not qualitatively capture the persistence. This, however, is a consequence of the MIT shock experiment in the non-stochastic model and focusing on the perfect foresight equilibrium. As an alternative to perfect foresight, we also solved for the impulse response paths where expectations are formed from an adaptive-learning rule. It is well-known that adaptive learning can introduce inertia into an economy. With this alternative theory of expectation formation, we find a hump-shaped response in both stock market capitalization and the interest-rate spread, as well as a more persistent effect from the shock. Both features that better capture the qualitative properties of Figure 2

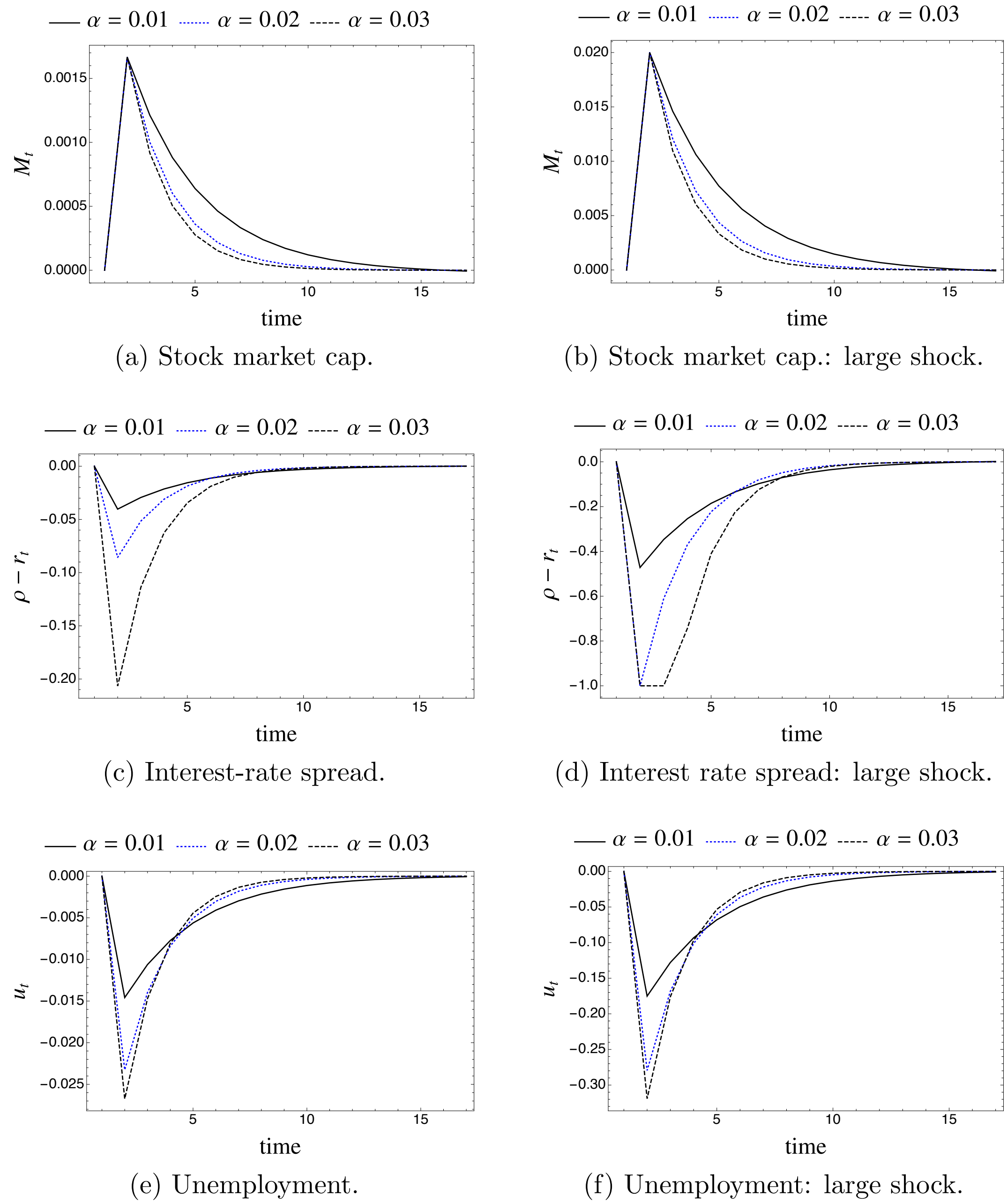
6.1.2 Sensitivity

To gain further insights into the model mechanisms behind the results in Figure 9, we present a sensitivity analysis by comparing the dynamic effects of the stock market MIT shock across economies that differ by the frequency of spending shocks (α), the extent of public liquidity (A_g), and labor-market matching efficiency (ξ). To make the comparisons comparable, the graphs below plot the impulse responses in log deviation from (unique) steady state.

Figure 10 compare the impulse responses for $\alpha = 0.01, 0.02$, and 0.03. Thus, the comparison is between the calibrated economy and one with a higher or lower expenditure risk. Figure 10 clearly demonstrates that the short-term unemployment response to the stock market innovation is increasing in the expenditure shock frequency. On impact, as before, there is a decline in the interest rate spread as the increase in stock market capitalization reduces the liquidity premium and increases the real-interest rate. The level of the real interest rate in the low α economy is higher and so the present-value effect from the stock market jump is relatively lower. Hence, the unemployment response is not as strong in the short-run. However, the stock market shock is more persistent in low α economies, but not the spread, and so the impact on the unemployment rate is also more persistent.

The right panel of plots conducts the same exercise but with a large MIT shock of a 2%, at a monthly rate, to the stock market capitalization. While the qualitative features are very similar between the small and large shocks, there is an additional non-linearity captured in panel (d.). In the large α economies, the interest rate spread effect is large and the liquidity premium vanishes in the short-run with $r_t = \rho$. In especially large α economies the interest-rate spread may be zero for an extended period of time until stock prices return sufficiently close to steady-state.

Figure 11 comparies economies that differ by the supply of government



(a) Stock market cap.

(b) Stock market cap.: large shock.

(c) Interest-rate spread.

(d) Interest rate spread: large shock.

(e) Unemployment.

(f) Unemployment: large shock.

Figure 10: Liquidity effects. The vertical axes measure log-deviations from steady-state. Frequency of expenditure shocks set to $\alpha = 0.01, 0.02, 0.03.$, remaining parameters are calibrated according to Table 3. The left columns perturb M_1 by .02/12, the large shock columns on the right perturb by 0.02. The figures plot the perfect foresight path back to the steady-state.

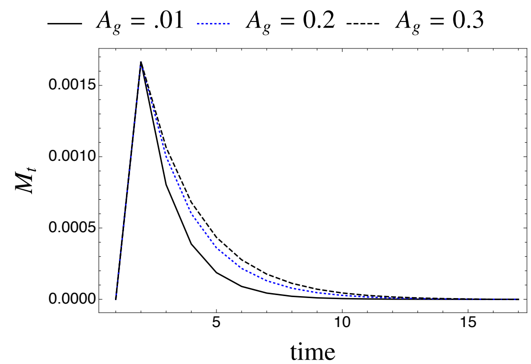
bonds A_g and the labor-market matching efficiency parameter ξ. Recall that smaller values of A_g imply a greater liquidity role to private assets and, hence, a larger liquidity premium. The right panels of Figure 11 demonstrates that this manifests in a stronger unemployment response to stock market shocks. Similarly, larger values of ξ imply a stronger matching elasticity from the stock market crash. So for large ξ economies the innovation to unemployment and the interest-rate spread is much stronger but also very close to a purely transitory shock. When $\xi = 2.3$ the economy returns to steady-state just 3 months after the shock.

6.2 Counterfactual: a perfect storm

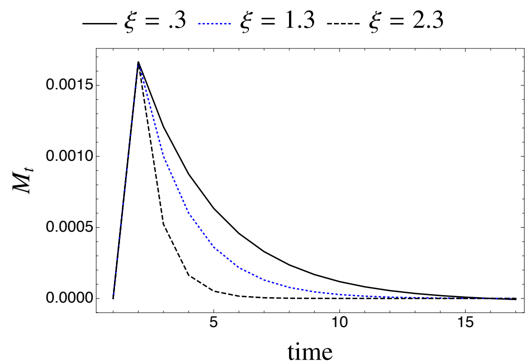
Section 5 demonstrated that when the strategic complementarities in the model are strong, then there can exist multiple steady-states including a low employment/low market capitalization equilibrium. This sections presents results from an MIT "expectations" shock in a counterfactual where aggregate demand effects are strong and there is a multiplicity of equilibria. The main result is that when liquidity effects are at their strongest the economy is most vulnerable to a self-fulfilling collapse to an equilibrium with low stock market capitalization and high unemployment.

The strategic complementarities are strongest when α is high, $\bar{z} - w_1 < 0$, and the exogenous liquidity in the form of government bond supply A^g is low. In this section, we consider a particular counterfactual where expenditure shocks are high, perhaps because of a pandemic, productivity is low, and financial frictions are high. In particular, we set $(\alpha, \bar{z}) = (0.21, -.06)$ and we decrease the value of A^g by 30%. The decrease in A^g is formally equivalent to introducing a pledgability constraint on public bonds that can serve as collateral and consumers can only pledge up to 70% of A^g. We interpret the latter as an exogenous decrease in the velocity of government bonds. This particular value is guided by data on government bond velocity. For instance, Figure 12 plots the government bond velocity in the U.S. over time and shows that post-2000 until 2006 there was a 30% decrease in the velocity.[12]

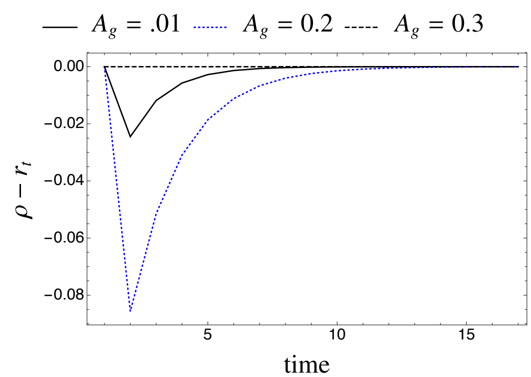
[12] We measure the government bond velocity as the ratio of nominal GDP to the nominal value of "safe" government bonds, using the methodology in Gorton, Lewellen, and Metrick (2012). A similar magnitude is seen if velocity is defined as nominal consumption to the safe bond supply.



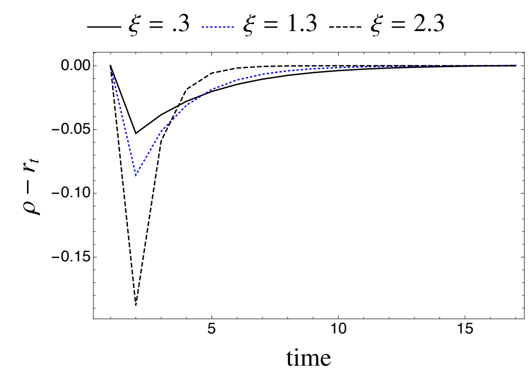
(a) Stock market cap.: A_g.



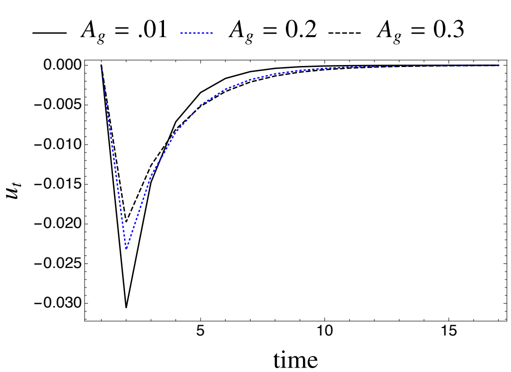
(b) Stock market cap.: ξ.



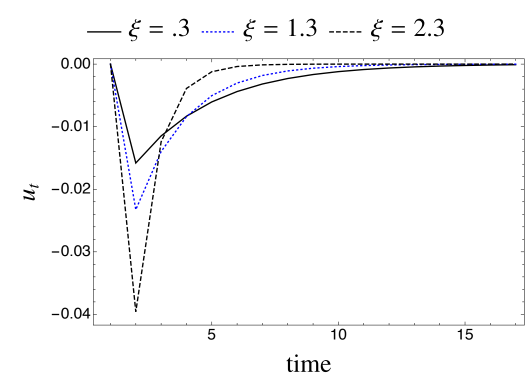
(c) Interest-rate spread: A_g.



(d) Interest rate spread: ξ.



(e) Unemployment: A_g.



(f) Unemployment: ξ.

Figure 11: Comparing IRF's across different levels of public liquidity (A_g) and labor-market matching efficiency (ξ). The vertical axes measure log-deviations from steady-state.

The "perfect storm" counter-factual assumes (i.) an unanticipated permanent shock to $(\alpha, \bar{z}, A^g)$ and (ii.) examines the impulse response to an

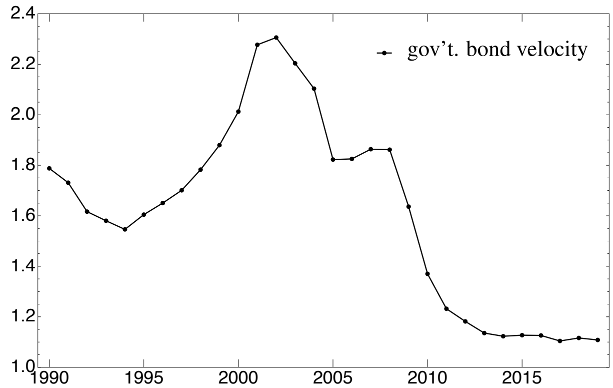


Figure 12: Government bond velocity

expectations shock.[13] In this counter-factual, there are now three steady-state equilibria, with differing levels of employment rates and market capitalization: see Figure 13. While not shown, the high and low steady-states are determinate and the middle steady-state is indeterminate.[14]

Figure 14 plots the results. In this perfect-storm counterfactual the economy converges to the middle (indeterminate) steady-state, exhibiting a lower employment rate and lower stock market capitalization. Under rational expectations, the expectations shock produces an immediate and large decrease in the stock market, slightly overshooting the intermediate steady-state. The interest-rate spread increases, more than doubling its original value, overshooting the new equilibrium which is about double the original spread. The combination of lower firm values and a higher real interest rate, leads to substantially higher unemployment rates that top out with a steady-state unemployment rate of roughly 11%. The perfect storm, with strong strategic complementarities, renders the economy susceptible to self-fulfilling crashes

[13]Although we technically decrease the lump-sum financed bonds A^g, there are alternative interpretations. We could instead imagine that various frictions may limit the number of these bonds that can be used to finance consumption purchases and our liquidity shock would arise from a further limiting of them. We find decreasing A^g, though, to be a convenient formalization of an exogenous decrease in liquidity that, in order to maintain the same level of economic activity, requires an expansion of endogenous privately-generated liquidity in the form of claims on firms.

[14]In the counterfactual experiment, we hold the steady-state employment rate in the high steady-state fixed at its calibrated value. This necessitates changing the parameter values for γ, δ, at the expense of no longer matching the moments used earlier when calibrating these parameters.

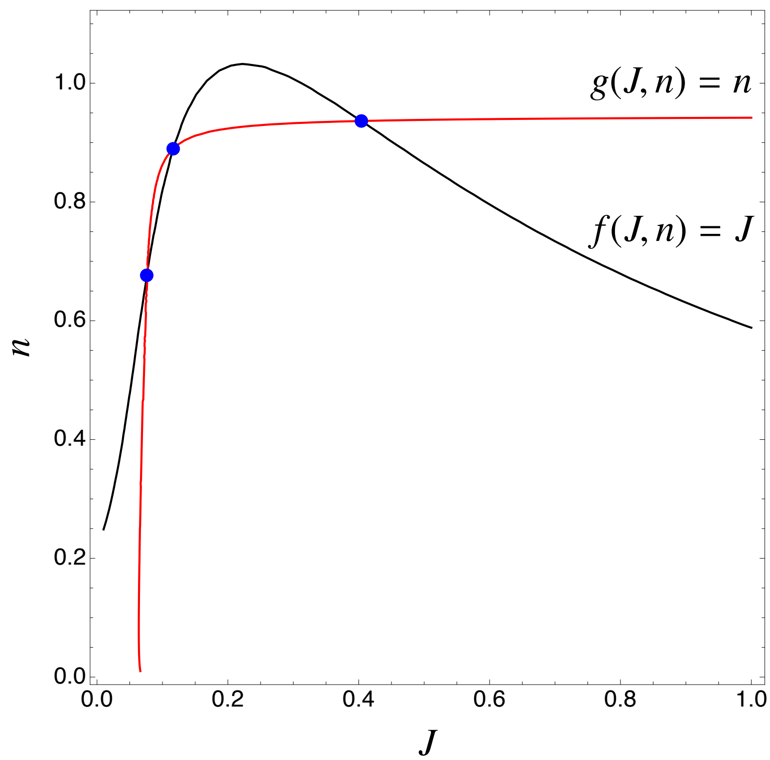
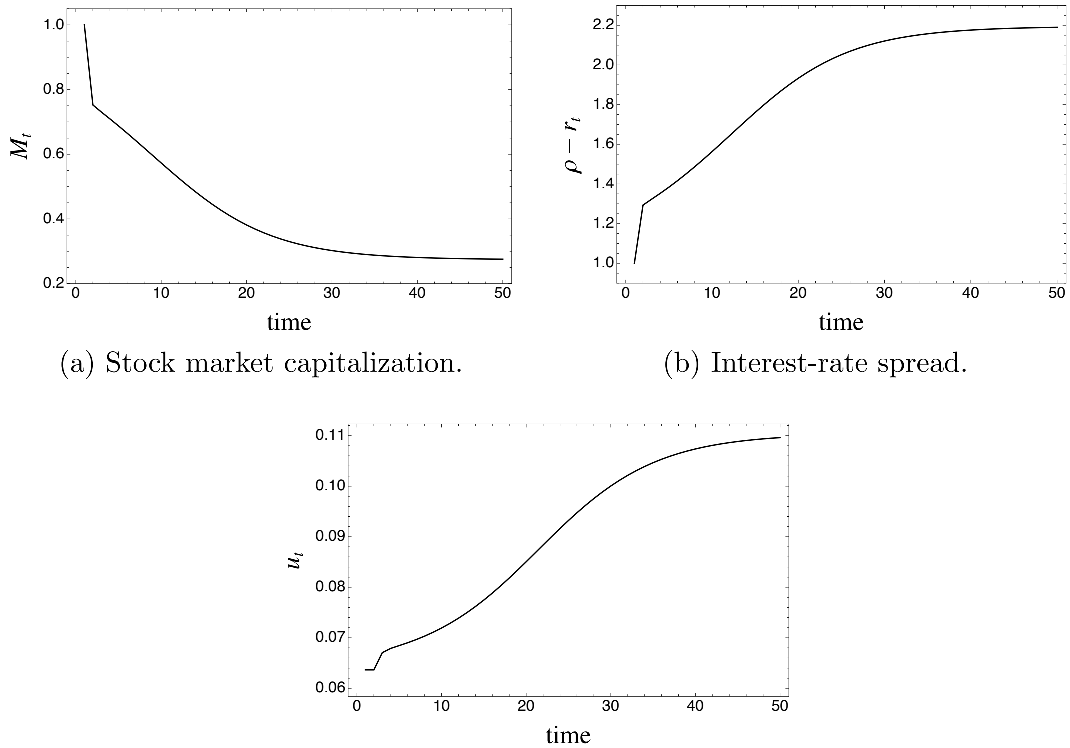


Figure 13: Multiple equilibria in the counterfactual.

precisely because of the existence of multiple steady-states.

7 Conclusion

We have studied the effects of changes in household liquidity constraints on the labor and stock markets. We generalized the Mortensen-Pissarides model along a single dimension: idiosyncratic expenditure risk introduces a limited commitment problem. For some consumption shocks, households can finance their purchases with unsecured debt but in others they must use their liquid assets, in the form of a mutual fund composed of stocks and government bonds, as collateral for intraperiod loans. This single twist of an otherwise standard model introduces strong complementarities into the economy. High stock market valuations relax household liquidity constraints and induce firms to create more jobs. The stock market consequently rises further and propagates consumption demand. A novel finding is that these complementarities



(a) Stock market capitalization.

(b) Interest-rate spread.

(c) Employment.

Figure 14: Perfect storm counterfactual: increase in demand/supply for early consumption good, a decrease in exogenous liquidity, and a pessimistic expectations shock. Stock market capitalization and interest-rate spreads are computed relative to the calibrated steady-state.

can produce multiple steady-state equilibria with high employment/high stock market capitalization co-existing with low employment/low stock market values.

We calibrated the model to the long-run properties of the U.S. economy and exploited the multiple steady-states in the model in a counterfactual exercise. Our quantitative analysis captures well the documented evidence on the co-movement between stock market capitalization, interest rates, and unemployment. We also presented results from a counterfactual experiment where there is a perfect storm of increased consumption risk and a decrease in the velocity of government bonds. This scenario coincides with an economy that has low unemployment, high stock market capitalization, and high real interest rates but that is also dependent on the private provision of liquid assets. We show in this case that multiple steady-states exist, making the economy fragile and susceptible to self-fulfilling collapses in employment and stock market values. A fragile economy collapses to a secular stagnation equilibrium with high unemployment, low stock prices, and low real interest rates.

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Appendix: For Online Publication

A Data appendix

Variable	Source
Stock market capitalization	Wilshire 5000 (in logs), FRED code WILL5000INDFC
Buffet measure of market capitalizaton	FRED code NCBEILQ027S
Nominal wage	A576RC1
Total unemployed	FRED code UNEMPLOY
Total employed	FRED code CE160V
Average hourly earnings of all employees	Fred code CES0500000003
Average weekly hours of all employees	Fred code AWHAETP
Unemployed less than 5 weeks	FRED code UNEMPLT5
Vacancies	https://www.briancjenkins.com/dmp-model
Moody's AAA	FRED code AAA
Long-term government bonds	FRED code GS20 LTGOVTBD
Treasury bonds with 20-year maturities	FRED code GS20

Table 4: Data sources used in motivating evidence and model calibration.

B Further details on identifying the stock price shock

Each series is monthly from January 1959 to October 2016. The frequency motivates the choice of industrial production in lieu of overall output. Consumption is measured in personal consumption expenditures and is normalized by the population and the consumer price index. Industrial production is also normalized by population. We use the longer horizon of stock price data available from Robert Shiller's webpage and normalize it by the nominal wage. The stock market valuation, industrial production, consumption, and vacancies enter the VAR in log levels. [15]

[15] Robert Shiller's webpage, http://www.econ.yale.edu/ shiller/data.htm contains the stock market data. In general, the spurious regression problem does not apply to a VAR with nonstationary series provided there are enough lags. Sufficient lags induce a cointegra-

We apply the algorithm developed by Arias, Rubio-Ramírez, and Waggoner (2018). The authors provide a theory in a Bayesian setting to independently draw from a family of conjugate posterior distributions with sign and zero restrictions. They leverage the fact that a SVAR can be written as a product of the reduced form and a set of orthogonal matrices. There is a conjugate uniform normal inverse Wishart density for the reduced form parameters. There is also a uniform conjugate density over the set of orthogonal matrices conditional on the reduced-form parameters. The method draws from the conjugate uniform-normal-inverse Wishart posterior over the orthogonal reduced-form parameterization and maps the draws into the structural parameterization.[16] The procedure combines the approach of imposing sign restrictions via the QR decomposition in Rubio-Ramirez, Waggoner, and Zha (2010) and uses an importance sampler to embed zero restrictions. Another another common way of imposing restrictions on signs and zeros the penalty function approach by Mountford and Uhlig (2009). However, this approach imposes additional constraints to zeros and signs and thereby distorts inference. For instance, Beaudry, Nam, and Wang (2011) provide use the penalty function approach to argue for the importance of optimism shocks in business cycles, but Arias, Rubio-Ramírez, and Waggoner (2018) show that the results largely depend on the additional constraints. Finally, we take $40,000$ draws of the orthogonal reduced form, each of which consists of the coefficient vector, the covariance-variance matrix, and the orthgonal matrix.

While omitted from the main text, we also estimated the response to a spread shock. Figure 15 plots the impulse responses to an identified spread shock. The median response of the spread is 3 basis points, and it exhibits low persistence. Median stock prices rise by 1.5%, peak after 15 months, and then slowly dissipate. The probability bands of unemployment are below zero for the first three years with a median peak decline of 0.15 percentage points.

tion relationship, which generates consistent estimates of the impulse response functions, as stressed by Hamilton (1994). Many recent applications of VAR's, such as Arias, Caldara, and Rubio-Ramirez (2019), enter variables in levels.

[16]The procedure thus involves transforming densities from the orthogonal reduced-form parameterization to the structural form and relies on change-of-variable theorems. Baumeister and Hamilton (2015) directly draw on the structural parameterization but require the Metropolis Hastings algorithm to draw from the posterior density.

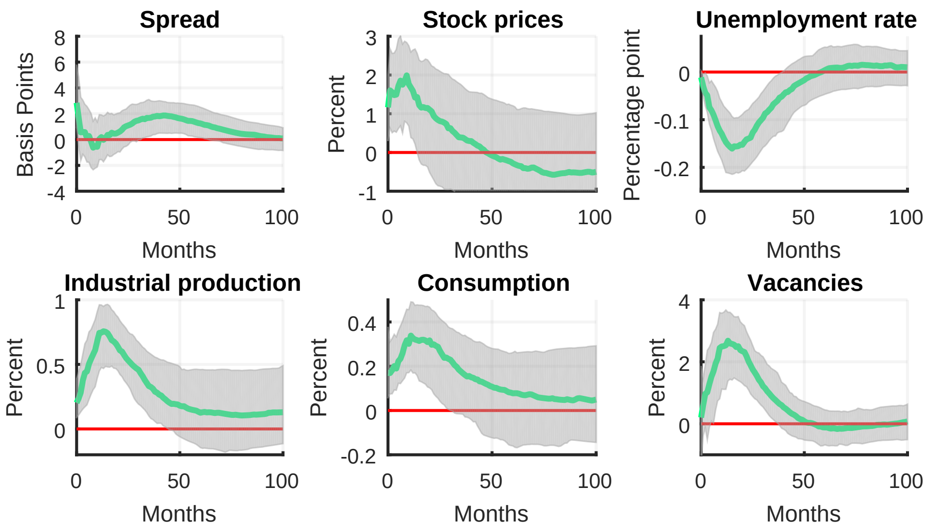

Figure 15: Impulse response to a positive unit standard-deviation shock to the interest rate spread. The identifying restrictions follow Table 2.

C Role of the zero impact response of unemployment for inference

This section relaxes the zero restriction of the response of unemployment in the first month. The sign restrictions suffice to distinguish shocks to interest rate spreads from those on firm values. Table 5 describes the relaxed identification scheme.

	Stock market	Spread	Industrial production	Consumption
Stock market	+	-	+	+
Interest rate spread	+	+	+	+

Table 5: Identification assumptions. Restrictions only apply to the impact response.

Figure 16 examines the 16th, 50th, and 84th probability bands for the impulse responses from a shock to the interest rate spread. In general, the median responses are very similar to the case with the zero restriction but the area between the probability bands is wider. In particular, the time in which the probability bands for unemployment do not contain zero shrinks to between 12 and 35 months. Nevertheless, the mass between the probability bands remains predominantly below zero. In addition, industrial production has some probability mass below zero after 30 months and, similarly, for consumption after 20 months, but the vast majority of the area remains positive.

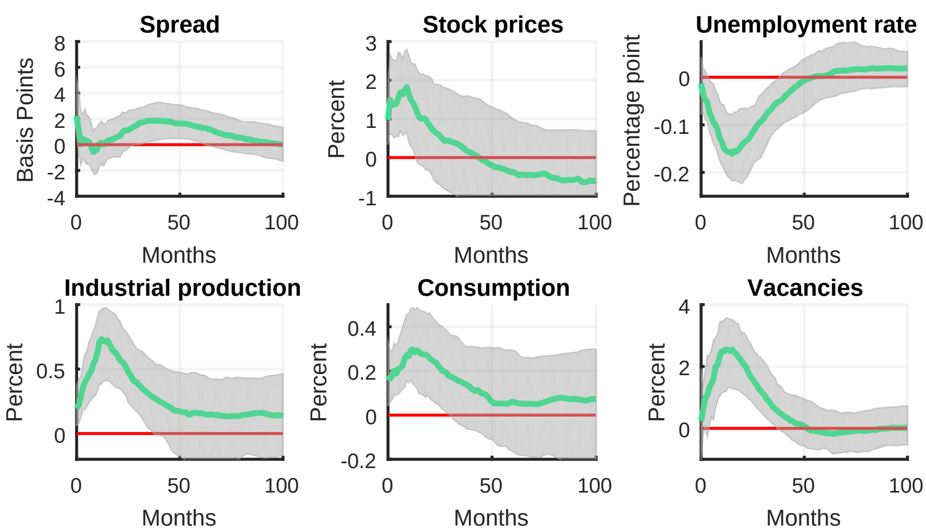

Figure 16: Impulse response of a positive unit standard-deviation shock to the interest rate spread. The identifying restrictions follow Table 5.

Figure 17 examines a shock to the stock market valuation without a zero restriction. Again, the median responses are similar, and now it takes about two quarters for the probability bands associated with the unemployment rate to not contain zero. The probability bands for industrial producton remain above zero throughout, but consumption now has significant probability mass below zero after 15 months.

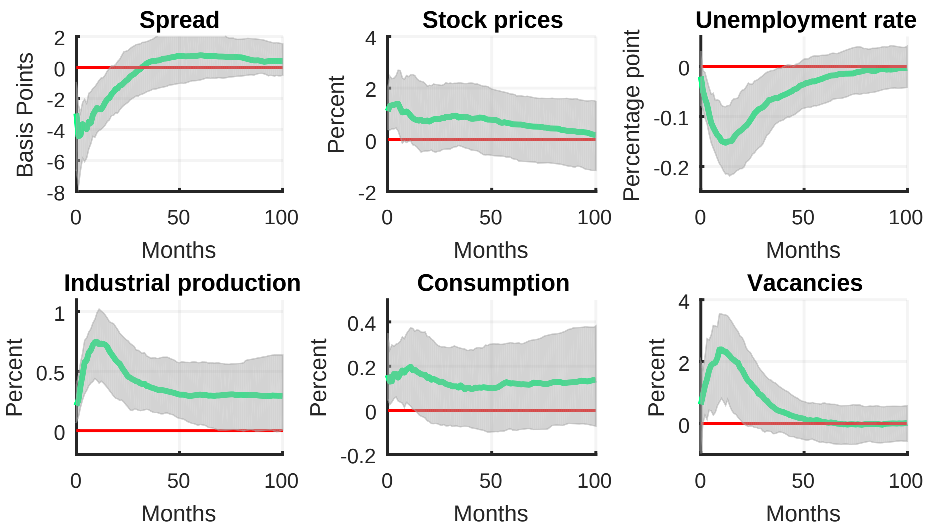

Figure 17: Impulse response to a positive unit standard-deviation shock to the stock-market valuation. The identifying restrictions follow Table 5.

Though relaxing the zero restriction on unemployment increases the area between the probability bands and makes inference less precise, there is still resonable evidence of a negative relationship between the stock market and unemployment pertaining to the aggregate demand and interest rate channels.

D Details on Equilibrium Determinacy

Section 5 presented results on the set of equilibria in a continuous time approximation of the model. While the continuous time formulation enhances tractability, in this Appendix we provide additional details on the set of rational expectations equilibria in the discrete-time version of the model.

Mortensen-Pissarides Economy

Here we shut-down the early consumption preference shocks: $\alpha = 0$. An equilibrium is a pair (J_t, n_t) that is non-explosive solution to the following pair of non-linear difference equations:

$$J_{t+1} = -\frac{1+\rho}{1-\delta}z + \frac{1+\rho}{1-\delta}J_t$$
$$n_{t+1} = (1-\delta)n_t + \left(\frac{J_{t+1}}{(1+\rho)k}\right)^{\frac{1-\xi}{\xi}}(1-n_t)$$

There is a unique steady-state $\bar{J} = \frac{1+\rho}{\delta+\rho}z$ and $\bar{n} = 1/(1+\delta\left((\delta+\rho)k/z\right)^{(1-\xi)/\xi})$. Moreover, the eigenvalues of the Jacobian matrix, evaluated at the unique steady-state, are

$$\lambda_1 = \frac{1+\rho}{1-\delta}$$
$$\lambda_2 = 1-\delta-\left(\frac{z}{(\delta+\rho)k}\right)^{(1-\xi)/\xi}$$

We can summarize the results as follows.

Proposition 1 *Let $\alpha = 0$. There exists a unique steady-state. For $z/k < (2-\delta)^{\xi/(1-\xi)}(\delta+\rho)$, there exists a unique perfect foresight equilibrium path to the steady-state for any given n_0. Else, there is a degenerate equilibrium with a unique perfect foresight path for $n_0 = \bar{n}$, and no equilibrium otherwise.*

Perfect Credit

Now we consider the case of a DMP model with two goods, and perfect credit for the early-consumption good. In this case, $r_t = \rho$, and equilibrium is a (non-explosive) solution to the difference equations

$$J_{t+1} = -\frac{1+\rho}{1-\delta}z + \frac{1+\rho}{1-\delta}J_t - \frac{\sigma(1+\rho)}{(1+\sigma)(1-\delta)}\left(\frac{n_t}{\alpha}\right)^{-\frac{\gamma(1+\sigma)}{\sigma+\gamma}}$$
$$n_{t+1} = (1-\delta)n_t + \left(\frac{J_{t+1}}{(1+\rho)k}\right)^{(1-\xi)/\xi}(1-n_t)$$

A steady-state is a pair $(\bar{J}, \bar{n})$ that solves

$$\bar{J} = \frac{1+\rho}{1-\delta} z + \frac{\sigma(1+\rho)}{(1+\sigma)(1-\delta)} \left(\frac{\alpha}{\bar{n}}\right)^{-\frac{\gamma(1+\sigma)}{\sigma+\gamma}}$$

$$\bar{J} = (1+\rho) k \left(\frac{\delta \bar{n}}{1-\bar{n}}\right)^{\xi/(1-\xi)}$$

The first equation is monotonically decreasing in n, with $\bar{J} \to \infty$ as $\bar{n} \to 0$, while the second equation monotonically increases with n and features $\bar{J} \to \infty$ as $\bar{n} \to 1$, while also $\bar{J} = 0$ when $\bar{n} = 0$. Thus, there exists a unique steady-state. Similarly computing the eigenvalues of the Jacobian, we can establish the following result.

Proposition 2 *In the DMP with 2 goods and perfect credit, there is a unique steady-state. Moreover, for k sufficiently large, there exists a unique (non-explosive) perfect foresight path, for any given n_0, that converges to the steady state. For k sufficiently small, the steady-state is a source and there exists a degenerate equilibrium with $n_0 = \bar{n}$.*

Bewley-Aiyagari Economy

Now we consider the Bewley-Aiyagari version of the economy: there's two goods, a limited commitment problem for the early-consumption good, and a linear cost function for the early-consumption good. These assumptions imply that the real interest rate is endogenous, compared to the standard DMP model or the model with perfect credit. However, $py - c(y) = 0$, which is interpreted as the marginal product of a job (asset) is independent of the *market value* of the liquid assets. With these assumptions, an equilibrium is a (non-explosive) solution to the pair of equations

$$J_t = z + (1-\delta) \frac{J_{t+1}}{1+r_{t+1}}$$

$$1 + r_t = \frac{1+\rho}{1 + \alpha \left[(n_t J_t)^{-\gamma} - 1\right]^{+}}$$

$$n_{t+1} = (1-\delta) n_t + \left[\frac{J_{t+1}\left(1 - \alpha + \alpha(n_{t+1} J_{t+1})^{-\gamma}\right)}{k(1+\rho)}\right]^{(1-\xi)/\xi} (1 - n_t)$$

The first two equations, in turn, can be re-written as

$$J_t = z + \frac{(1-\delta)(1-\alpha)}{1+\rho} J_{t+1} + \frac{(1-\delta)\alpha}{(1+\rho)} n_{t+1}^{-\gamma} J_{t+1}^{1-\gamma}$$

As before, the steady-state can be calculated as a pair $(\bar{J}, \bar{n})$ that solve the equations

$$\bar{J}\left[1 - \frac{(1-\delta)(1-\alpha)}{1+\rho} - \frac{(1-\delta)\alpha}{(1+\rho)} \bar{n}^{-\gamma} \bar{J}^{-\gamma}\right] = z$$

$$\left(\bar{J}/k\right)^{(1-\xi)/\xi} \left[\frac{1-\alpha+\alpha\left(\bar{n}\bar{J}\right)^{-\gamma}}{1+\rho}\right]^{(1-\xi)/\xi} = \frac{\delta \bar{n}}{1-\bar{n}}$$

As before, the first equation implies $\bar{J}$ is decreasing in n, with $\bar{J} \to \infty$ as $\bar{n} \to 0$. The second equation implies that $\bar{J}$ is increasing in n with $\bar{J} \to \infty$ as $\bar{n} \to 1$ and $\bar{J} = 0$ at $\bar{n} = 0$. Again, it follows that there exists a unique steady-state.

Expressions for the eigenvalues of the Jacobian are complicated and analytic results are not, in general, available. However, for the following special case we can provide a uniqueness result.

Proposition 3 *In the Bewley-Aiyagari version of the model, there exists a unique steady-state. Furthermore, for ξ sufficiently large, there exists a unique (non-explosive) perfect foresight equilibrium, for any given n_0, that converges to the steady-state.*

The general case

The equilibrium path is found as the solution to the following equations:

$$J_t = z + \frac{\sigma}{1+\sigma}\max\left\{\alpha J_t, (\alpha/n_t)^{\gamma(1+\sigma)/(\gamma+\sigma)}\right\} + \frac{(1-\delta)J_{t+1}}{1+r_{t+1}}$$

$$1 + r_t = \frac{1+\rho}{1+\alpha\left[\frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)} n_t^{\gamma} J_t^{(\sigma+\gamma)/(1+\sigma)}} - 1\right]^{+}}$$

$$n_{t+1} = (1-\delta)n_t + \left[\frac{J_{t+1}\left(1+\alpha\left[\frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)} n_t^{\gamma} J_t^{(\sigma+\gamma)/(1+\sigma)}} - 1\right]\right)}{k\,(1+\rho)}\right]^{(1-\xi)/\xi} (1-n_t)$$

Too see this, note that in a constrained equilibrium,

$$py_s - c(y_s) = \frac{\sigma}{1+\sigma} y_s^{1+\sigma}$$

Since $nJ = py_s\frac{n}{\alpha}$, $\alpha J = y_s^{1+\sigma}$, the surplus is $py_s - c(y_s) = \frac{\sigma}{1+\sigma}\alpha J$. In the interest rate equation,

$$\begin{aligned} v'(nJ/p)/p &= (nJ)^{-\gamma} p^{\gamma-1} \\ &= (nJ)^{-\gamma} y_s^{\sigma(\gamma-1)} \\ &= (nJ)^{-\gamma} (\alpha J)^{\sigma(\gamma-1)/(1+\sigma)} \\ &= \frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)} n^{\gamma} J^{(\sigma+\gamma)/(1+\sigma)}} \end{aligned}$$

Define $\bar{n} = J(\bar{J})$ as the implicit steady-state function defined from the firm's profit recursion assuming the liquidity constraint binds. It can be shown that:

1. $\bar{n} \to 0$ as $\bar{J} \to 0$;
2. $\bar{n} \to 0$ as $\bar{J} \to \infty$;
3. if $z < 0$ then $\bar{n} > 0$ for $0 < \bar{J} < \infty$.

These properties imply that the function $J(\bar{J})$ is non-monotonic. However, the second equation, as before features $\bar{n}$ increasing with $\bar{J}$. The non-monotonicity of the firm's steady-state profit equation raises the possibility of multiple steady-state equilibria. Moreover, it is apparent that the slope of the profit functions are complicated expressions which raise the possibility of bifurcations not only in the number of steady-states but also the stability of steady-states.

E Equilibrium with Nash-bargained wage

In this Appendix, we consider an extension of the analysis in Section 5.4 by allowing for the wage to be determined endogenously via Nash bargaining between firms and workers. We also briefly discuss the effects of a Nash-bargained wage on the J-isocline (the n-isocline is unaffected).

In general, the bargained wage solves

$$w = (1-\alpha_L)w_0 + \alpha_L(z + k\theta)$$

where α_L is the bargaining power. In the version of the model with just stocks, we can rewrite the wage equation as a function of n and J by using market clearing, the pricing relationship, and free entry:

$$w(n, J) = (1-\alpha_L)w_0 + \alpha_L\left[\overline{z} + \frac{\sigma}{1+\sigma}nJ + k\left(\frac{J}{k}\right)^{1/\xi}\right]$$

Note that the wage depends positively on n and J. From the second term in the bracket, higher nJ means greater stock market capitalization, which boosts the early-consumption price and productivity. Higher J also boosts hiring (and θ) through the free entry condition. This effect raises wages since workers can find alternative jobs more easily should wage negotiations fail. The latter effect is standard in Mortensen and Pissarides (1994) but the former effect is novel due to the aggregate demand externality.

We consider the equilibrium with no credit or bonds ($\lambda = 0, A^g = 0$) in which the wage is determined according to Nash bargaining. The set of equations governing the equilibrium path is:

$$
\begin{aligned}
(r+\delta)\,J &= \bar{z} + \max_{y}\{py - c(y)\} - w_1 + \dot{J} \\
w_1 &= (1-\alpha_L)w_0 + \alpha_L \left[\bar{z} + \max_{y}\{py - c(y)\} + k\theta\right] \\
c'^{-1}(p) &= \frac{\alpha J}{p} \\
\rho - r &= \alpha(1-\lambda)\left[\frac{v'\left(\frac{nJ}{p}\right)}{p} - 1\right]^{+} \\
\dot{n} &= m\left[1, \theta(J)\right](1-n) - \delta n
\end{aligned}
$$

We can reduce this system of equations by substituting the wage equation into the firm's Bellman:

$$(r+\delta)J = (1-\alpha_L)(\bar{z} + \max_{y}\{py - c(y)\} - w_0) + \alpha_L k\theta + \dot{J}$$

The value of the firm now depends on the weighted average of the (endogenous) match surplus and hiring costs through market tightness. The market tightness depends implicitly on firm value J through the free entry condition.

The endogenous wage affects the curvature of the J-nullcline. Suppose the levels of n and J are such that profits $z - w_1$ are the same in both cases. Then, as J rises, the wage rises, taking n as fixed. Thus, profits fall faster with the endogenous wage, which reduces job creation and hence the n associated for a given J. However, for lower J, the endogenous wage falls below the exogenous wage w_1, so the J nullcline can rise higher.

F Comparison to a pure-currency economy

In the following we show that the set of steady states arising from our model differs qualitatively from the one of a pure currency economy where fiat money is the only means of payment (Shi (1998) or Berentsen, Menzio, and Wright

(2011)).[17] Let π denote the growth of the money supply. A steady-state equilibrium of a pure currency economy is a list, (J, p, y^s, n), that solves:

$$\begin{aligned}(\rho+\delta)\,J &= \bar{z}+\max_{y}\{py-c(y)\}-w_1\\ p &= c'(y^s)\\ \rho+\pi &= \alpha\left[\frac{\upsilon'(ny^s/\alpha)}{c'(y^s)}-1\right]\\ \delta n &= m\,[1,\theta(J)]\,(1-n).\end{aligned}$$

The third equation pins down y^s as a decreasing function of n. From the second equation, p is a decreasing function of n. From the first equation, J is a decreasing function of n. From the Beveridge curve n is increasing with J. So the steady state of the pure currency economy is unique. Our economy differs from the pure currency economy in two ways. First, in our model the real interest rate is endogenous and depends on the measure of firms and their valuation. As discussed above, the logic for the determination of the real interest rate is similar to the one in the Aiyagari model. Second, the price of early consumption depends on market capitalization through a limited commitment problem. This channel links asset prices, expenditure, and employment and potentially generates multiple steady states.

G Calibration Details

The calibration targets are the replacement ratio of the unemployed of 0.4, the interest rate spread of 75 basis points, an annual frequency of liquidity shocks of 0.26, the job finding and separation rates, an annual interest rate of 4%, elasticity of marginal cost $\sigma = 0.2$, expenditure shocks relative to the wage ϵ, semi-elasticity of the interest rate spread with respect to debt-to-GDP equal to -0.746, and the mean proportion of households with at least one unsecured credit card between 2000 and 2007 from the Survey of Consumer Finances, which was 74.7%.

[17] The comparison here is not directly to Berentsen, Menzio, and Wright (2011), which includes a frictional goods market where the probability of matching is affected by matching in the labor market.

In order to determine the targets ϵ and α, we examine evidence on health expenditure shocks from the *Federal Reserve Report on the Economic Well-Being of U.S Households in 2015*. 26% of households in the sample experienced a major out-of-pocket health expense and the mean of that expense was $\$2,383$.[18] To obtain the wage figure in the data, we take hourly wages for 2015, multiply them by mean weekly hours in 2015, and multiply them by 52 to obtain the annual wage. However, as the model frequency is monthly, we multiply the percentage of the wage spent by 12.[19]

We obtain the job finding rates e and separation rates s using worker flows as in Shimer (2005). Next-period unemployment satisfy $U_{t+1} = U_t(1 - e_t) + U^s_{t+1}$, given newly unemployed U^s_{t+1}. We rearrange to isolate e_t. Given that job losers have on average half a month to find a new job before being recorded as unemployed, the newly unemployed satisfies $U^s_{t+1} = s_t(1 - U_t)(1 - 1/2)e_t$. Rearranging determines s_t.[20] We find the series' means, $\overline{e} = 41.5\%$ and $\overline{s} = 3.10\%$, and also back out the corresponding employment target: $n = \overline{e_t/(s_t + e_t)} = 92.8\%$.

We start with an initial guess of government bonds relative to GDP $x = A^g/(nz)$ as well as labor income to GDP w_1/z. Using the latter, we back out the firm value relative to GDP from the steady-state Bellman equation:

$$J/z = (1 + r)/(r + \delta)(1 - w_1/z)$$

[18]This figure excludes just one extreme outlier with an out-of-pocket expense of 1 million dollars.

[19]The FRED codes are CES0500000003 and AWHAETP for average hourly wages and and weekly earnings, respectively.

[20]We use series on aggregate unemployment (FRED code UNEMPLOY), aggregate employment (CE160V) the the aggregate number employed for less than five weeks (UEMPLT5).

and the productivity level z:

$$\begin{aligned} z &= \overline{z} + py_s - c(y_s) \\ &= \overline{z} + \frac{\sigma}{1+\sigma} py_s \\ &= \overline{z} + \frac{\sigma}{1+\sigma} \frac{\alpha p y_b}{n} \\ &= \overline{z} + \frac{\sigma}{1+\sigma} \frac{\alpha}{n} \epsilon w_1 \end{aligned}$$

Dividing through by z and rearranging yields

$$z = \overline{z} \left(\frac{1}{1 - \frac{\sigma}{1+\sigma} \frac{\epsilon \alpha w_1/z}{n}} \right)$$

Upon obtaining J/z and z, we back out J, w_1, and A^g. Note that, as $py_s = \frac{\alpha}{n} \epsilon w_1$, we can find the price as

$$p = \left(\frac{\alpha \epsilon w_1}{n} \right)^{\sigma/(1+\sigma)}$$

In a liquidity-constrained steady state equilibrium, $py_b = \lambda p y^* + (1 - \lambda)(nJ + A^g) = w\epsilon$, where ϵ is the fraction of the wage devoted to health expenditure. Dividing through by nz and applying $y^* = p^{-1/\gamma}$ yields

$$\lambda p^{(\gamma-1)/\gamma}/(nz) + (1-\lambda)[J/z + A^g/(nz)] = (w_1/z)(\epsilon/n)$$

Using this expression $\lambda p^{(\gamma-1)/\gamma}/(nz)$ and the expression for the price from above, we find

$$p = \left[(\alpha/n)(\lambda p^{(\gamma-1)/\gamma} + (1-\lambda)(nJ + A^g)) \right]^{\sigma/(\sigma+1)}$$

Given p, we solve for γ using (13):

$$\frac{\rho - r}{1+r} - \alpha(1-\lambda) \left[((nJ + A^g)/p)^{-\gamma}/p - 1 \right]$$

and recover the quantities $y^s = p^{1/\sigma}$ and $y^b = ny^s/\alpha$.

The loss is given by the market clearing differential $\mathcal{L}_1 = py^s - (\alpha/n)(\lambda p^{(\gamma-1)/\gamma}) + (1-\lambda)(nJ + A^g)$. We find the zero of $\mathcal{L}_1$ with respect to w_1/z. We obtain the vacancy posting cost k from $k = Jq/(1+r)$.

The final step is to determine the supply of government bonds A^g. The loss function $\mathcal{L}_2$ takes $x = A^g/(nz)$ as an argument and uses $\mathcal{L}_1$ to back out the remaining parameters. Given the full set of parameters, , $(\delta, \gamma, \overline{z}, w_1, w_0, \rho, \alpha, \sigma, k, \xi, A^g, \lambda)$, we compute the loss as the difference in the semi-elasticity of the spread with respect to debt-to-GDP between the model and the data:

$$\frac{\partial 100[\rho - r(A^g/(nz))]}{\partial \log(Ag/(nz))}$$

We compute the model semi-elasticity using numerical differentiation and repeated application of the chain rule. The analogue in the data -0.746, the empirical semi-elasticity computed by Krishnamurthy and Vissing-Jorgensen (2012) and reported in Table 1 of the article.[21]

We solve for x by finding the root of $\mathcal{L}_2$. Given x, we find the remaining parameters according to the steps listed above.

Additional details on the derivation of employment law of motion

We finally simplify the job finding probability $e = m(1, \theta)$. First, combining the equilibrium expression for market tightness, $J = (1+r)k/q(\theta)$, with the job finding probability under the Den Haan Ramey Watson function, $q = 1/(1+\theta^\xi)^{1/\xi}$, we obtain

$$\theta(n, J) = \left[\left(\frac{J}{(1 + r(n, J))k} \right)^\xi - 1 \right]^{1/\xi}$$

The job finding probability in terms of n, J is

$$e(n, J) = \frac{\theta(n, J)}{(1 + \theta(n, J)^\xi)^{1/\xi}}$$

[21] Their spread measure is the yield difference between Moody's Aaa-rated long-maturity corporate bonds and Treasury bonds.

which can be written explicitly as

$$e(n, J) = \frac{\left[\left(\frac{J}{(1+r(n,J))k}\right)^{\xi} - 1\right]^{1/\xi}}{\frac{J}{[1+r(n,J)]k}}$$

H A monetary economy

In the main formulation of the model, households who are liquidity constrained have access a single asset, the mutual fund composed of stocks and government bonds. We focus on stock mutual funds, government bonds, and debt obligations as the assets for the following reasons. Stocks are a primitive given fundamental role of firms, and government bonds provide a policy instrument. Finally, probabilistic access to credit by consumers enables us to characterize the space between no-and-full commitment. Though the economy remains cashless, firms' revenues and interest rates are endogenous and sensitive to the government supply of bonds.

A cashless economy is reasonable in other respects. First, Hu and Rocheteau (2013) establishes that fiat money is not essential in environments with Lucas trees. Moreover, Lagos (2010) studies a similar economy in which Lucas trees serve as the media of exchange to explain the equity premium puzzle. Second, the baseline economy endogenizes the real interest rate and firms' revenue and links them to market capitalization. The model nests Mortensen and Pissarides (1994) by shutting down the idiosyncratic preference shocks, and Bewley-Aiyagari by making firms indifferent between early and late production.

However, it is straightfoward to incorporate fiat money into the model without disrupting its fundamental insights. In this Appendix we sketch out an extension where, depending on the size of the consumption shocks, households will choose to use fiat money or liquidate their other assets.

Sketch of monetary extension

We now add fiat money and government bonds to our economy. Fiat money grows at the gross growth rate, $\mu_t = M_t/M_{t-1}$, through lump-sum transfers to buyers. There is a fixed supply, A^g, of one-period real government bonds, where each bond pays off one unit of numeraire. The preference shock in the early-consumption period is an iid draw from the cumulative distribution $F(\varepsilon)$. Whereas consumers can use fiat money for early consumption at no cost, there is a fixed cost κ of using liquid bonds and stocks. Moreover, households can liquidate all of their bond holdings but only a fraction fraction ψ of stocks on demand.

The problem of a buyer holding ω_t units of wealth is:

$$W_t(\omega_t) = \max_{x_t, \ell_{t+1}, a_{t+1}} \left\{ x_t + \beta V_{t+1}(a^s_{t+1}, a^g_{t+1}, a^m_{t+1}) \right\} \tag{19}$$

$$\text{s.t.} \quad \frac{a^s_{t+1}}{1 + r^s_{t+1}} + \frac{a^g_{t+1}}{1 + r^g_{t+1}} + (1 + \pi_{t+1})\, a^m_{t+1} = \omega_t - x_t. \tag{20}$$

The buyer's portfolio in the second stage is now composed of three types of assets: real balances, a^m, bonds, a^g, and stocks, a^s. Since they have different liquidity properties, assets offer generally different rates of return: $1/(1+\pi_{t+1})$ for fiat money, $1 + r^g_{t+1}$ for bonds, and $1 + r^s_{t+1}$ for stocks. Substituting x from (20) into (19),

$$W_t(\omega_t) = \omega_t + \max_{x_t, \ell_{t+1}, a_{t+1}} \left\{ -\frac{a^s_{t+1}}{1 + r^s_{t+1}} - \frac{a^g_{t+1}}{1 + r^g_{t+1}} - (1 + \pi_{t+1})\, a^m_{t+1} + \beta V_{t+1}(a^s_{t+1}, a^g_{t+1}, a^m_{t+1}) \right\}.$$

The buyer's value function in the AM is

$$V_t(a^s_t, a^g_t, a^m_t) = \alpha \int \max_{\chi_t \in \{0,1\}} \left\{ \varepsilon v\,(y_t) - \chi_t \kappa - p_t y_t \right\} dF(\varepsilon) + W_t(a^s_t + a^g_t + a^m_t)$$
$$\text{s.t.} \quad p_t y_t \le a^m_t + \chi_t\,(a^g_t + \psi a^s_t)$$

where $\chi_t = 1$ if the buyer chooses to liquidate stocks and bonds and $\chi_t = 0$ otherwise. The buyer wants to consume early with probability α, in which case his marginal utility of consumption is determined by a draw from $F(\varepsilon)$. The buyer's surplus is reduced by the fixed cost κ if the buyer chooses to liquidate some bonds and stocks. There is a threshold, $\tilde{\varepsilon}_t$, such that:

$$\max_{p_t y_t \le a_t^m} \{\tilde{\varepsilon}_t \upsilon(y_t) - p_t y_t\} = \max_{p_t y_t \le a_t^m + a_t^g + \psi a_t^s} \{\tilde{\varepsilon}_t \upsilon(y_t) - p_t y_t\} - \kappa.$$

For all $\varepsilon \le \tilde{\varepsilon}_t$ the buyer uses cash as means of payment whereas for all $\varepsilon > \tilde{\varepsilon}_t$ the buyer uses both cash and stocks. The first-order conditions of the buyer's portfolio problem are:

$$i_t = \alpha \int_0^{+\infty} \left\{ \frac{\varepsilon \upsilon'\left((a_t^m + \chi_t(a_t^g + \psi a_t^s))/p_t\right)}{p_t} - 1 \right\}^+ dF(\varepsilon) \quad (21)$$

$$\frac{\rho - r_t^g}{1 + r_t^g} = \alpha \int_{\tilde{\varepsilon}_t}^{+\infty} \left\{ \frac{\varepsilon \upsilon'\left[(a_t^m + a_t^g + \psi a_t^s)/p_t\right]}{p_t} - 1 \right\}^+ dF(\varepsilon) \quad (22)$$

$$\frac{\rho - r_t^s}{1 + r_{t+1}^s} = \alpha v \int_{\tilde{\varepsilon}_t}^{+\infty} \left\{ \frac{\varepsilon \upsilon'\left[(a_t^m + a_t^g + \psi a_t^s)/p_t\right]}{p_t} - 1 \right\}^+ dF(\varepsilon) \quad (23)$$

where $\{x\}^+ = \max\{x, 0\}$. The choice of real balances as given by (21) equalizes the expected marginal value of money in all trades to the cost from holding money relative to stocks. Let i^g denote the nominal interest rate on government bonds, and i^s the nominal interest rate on stocks. We have

$$\frac{\rho - r_t^g}{1 + r_t^g} = \frac{i_t - i_t^g}{1 + i_t^g} \text{ and } \frac{\rho - r_t^s}{1 + r_{t+1}^s} = \frac{i_t - i_t^s}{1 + i_t^s}.$$

So, (22) and (23) define the interest rate differential between government bonds and illiquid bonds and, stocks and illiquid bonds.

$$i_t^s = \frac{(1 - \psi) i_t}{1 + \psi i_t}$$

The clearing of the AM goods market implies

$$n p_t c'^{-1}(p_t) = \alpha \int_0^{+\infty} \min \left\{ p_t \upsilon'^{-1}\left(\frac{p_t}{\varepsilon}\right), (a_t^g + \psi a_t^s)\mathbb{I}_{\{\varepsilon \ge \tilde{\varepsilon}_t\}} + a_t^m \right\} dF(\varepsilon), \quad (24)$$

where $a_t^s = n_t J_t$ and $a_t^g = A^g$. The left hand side is the aggregate supply of assets arising from the n firms. The right hand side is aggregate demand. Buyers with a preference shock less than $\tilde{\varepsilon}_t$ spend their real balances while buyers with a preference shock larger than $\tilde{\varepsilon}_t$ spend their real balances and some of their bonds and stocks. The rest of the model is similar to that of previous sections.

Suppose first that $c(y) = y$ and $y^s < \bar{z}$. In this case, $p_t = 1$ and firms are indifferent between selling to early buyers or late buyers. Moreover, assume $A^g = 0$ and $v = 1$. Consider equilibria in which stocks do not pay a liquidity premium: $r_t = \rho$. From (22) $y = y^*_\varepsilon$ for all $\varepsilon \geq \tilde{\varepsilon}_t$ where $\varepsilon \upsilon'(y^*_\varepsilon) = 1$. The threshold for ε above which buyers liquidate stocks solves

$$\kappa = \left[\tilde{\varepsilon}\upsilon\left(y^*_{\tilde{\varepsilon}}\right) - y^*_{\tilde{\varepsilon}}\right] - \left[\tilde{\varepsilon}\upsilon\left(a^m\right) - a^m\right].$$

The threshold $\tilde{\varepsilon}$ increases with ℓ and κ. Real balances are determined by (21),

$$i = \alpha\int_0^{\tilde{\varepsilon}(a^m)}\left[\varepsilon\upsilon'\left(a^m\right) - 1\right]dF(\varepsilon).$$

Because $r = \rho$ and $p = 1$ market capitalization, nJ, is determined independently of ℓ. The condition for this equilibrium to occur is $nJ \geq y^*_{\bar{\varepsilon}} - \ell$. The buyer's total wealth is large enough to finance the early consumption for the largest value of ε. If this condition does not hold, then r falls below ρ and stocks pay a liquidity premium.

Suppose next that buyers receive a high preference shock, ε_H, with probability α_H, a low preference shock, $\varepsilon_L < \varepsilon_H$, with probability α_L, and no preference shock with complementary probability $1 - \varepsilon_H - \varepsilon_L$. Moreover, we consider equilibria where $nJ + \ell < y^*_{\varepsilon_H}$ and we assume that κ is neither too low nor too large so that $\varepsilon_L < \tilde{\varepsilon} < \varepsilon_H$. Liquidity and interest rates are determined by:

$$i = \alpha_L\left\{\varepsilon_L\upsilon'\left(a^m\right) - 1\right\}^+ + \alpha_H\left\{\varepsilon_H\upsilon'(\ell + nJ) - 1\right\}$$
$$\frac{\rho - r}{1 + r} = \alpha_H\left\{\varepsilon_H\upsilon'(a^m + nJ) - 1\right\}.$$

If i is not too large, $a^m \geq y^*_{\varepsilon_L}$ and aggregate liquidity is determined by $i = \alpha_H\left\{\varepsilon_H\upsilon'(a^m + nJ) - 1\right\}$. In this case the nominal interest rate on stocks is zero and the real interest rate is determined by $r = -\pi/(1 + \pi)$. This equilibrium corresponds to a liquidity trap. If i is sufficiently large so that

$\ell < y^*_{\varepsilon_L}$, then the nominal interest rate on stocks is positive and is affected by monetary policy.

Finally, if $A^g > 0$ and $\psi < 1$, then there exists liquidity trap equilibria where $i^g = 0$ and $i^s_t = (1-\psi)i_t/\left(1+\psi i_t\right) > 0$. The value of money solves:

$$\frac{i - i^s}{v\left(1+i^s\right)} = i = \alpha_H \left\{\varepsilon v'\left(a^m + A^g + \psi n J\right) - 1\right\}.$$

Note, however, that such liquidity trap equilibria do not exist if the distribution is continuous because it would require $\tilde{\varepsilon} v'\left(a^m_t/p_t\right) = 1$, which is inconsistent with $\kappa > 0$.

Evaluating (24) requires us to establish whether the consumer is constrained given an ε shock and for what shock an individual chooses to liquidate bonds and stocks. The following help us characterize the demand for assets.

Lemma 4 *There is an interval* $[0, \varepsilon_m]$ *in which an individual can reach the first best using just money and an interval* $(\varepsilon_m, \varepsilon_s]$ *in which an individual can reach the first best using stocks and bonds but not money alone.*

Proof. From the Inada condition on $v(\cdot)$, as $\varepsilon \to 0$, $pv'^{-1}(p/\varepsilon) \to 0 < a^m$. For ε sufficiently large, $pv'^{-1}(p/\varepsilon) > a^m$. By continuity, there is a value ε_m such that $pv'^{-1}(p/\varepsilon_m) = a^m$, which satisfies $\varepsilon_m = p/v'(a^m/p)$. By a similar argument, there is an ε_s such that an individual can just afford the first best using stocks and bonds, given by $\varepsilon_s = p/(v'((a^m+nJ+A^g)/p))$. It immediately follows that $\varepsilon_s > \varepsilon_m$. ■

Lemma 5 $\varepsilon_m < \tilde{\varepsilon}$. *The preference shock at which an individual ceases to be able to finance the first best using cash alone is strictly below the threshold at which the individual chooses to liquidate stocks and bonds.*

Proof. Suppose instead that $\varepsilon_m \geq \tilde{\varepsilon}$. An individual in the range $[\tilde{\varepsilon}, \varepsilon_m]$ choose to liquidate stocks and bonds at cost κ but can afford the first best with cash. Since liquidating stocks and bonds does not change the consumption profile and imposes a cost, it is suboptimal. Hence, $\varepsilon_m < \tilde{\varepsilon}$. ■

Lemmas 4 and 5 imply that there are only two possibilities: $\varepsilon_m < \varepsilon_s < \tilde{\varepsilon}$, or $\varepsilon_m < \tilde{\varepsilon} < \varepsilon_s$. In the first case, an individual liquidates stocks and bonds only after being unable to finance the first best even with stocks and bonds. In the second case, the individual liquidates stocks and bonds and is able to finance the first best until the preference shock rises sufficiently. The following lemma characterizes which case holds.

Lemma 6 *Provided that*

$$\frac{p}{v'(a^m + \psi nJ + A^g)} < \frac{\psi nJ + A^g + \kappa}{v([a^m + \psi nJ + A^g]/p) - v(a^m/p)} \tag{25}$$

then $\varepsilon_m < \varepsilon_s < \tilde{\varepsilon}$. Under CRRA preferences, (25) simplifies to

$$(a^m + \psi nJ + A^g)^\gamma < \frac{(\psi nJ + A^g + \kappa)(1-\gamma)}{(a^m + \psi nJ + A^g)^{1-\gamma} - (a^m)^{1-\gamma}} \tag{26}$$

which depends only on holdings of liquid assets (independent of the price).

Proof. In order to check whether $\varepsilon_m < \varepsilon_s \leq \tilde{\varepsilon}$, note that once the consumer liquidates stocks and bonds, then he will use all his assets. Accordingly,

$$\tilde{\varepsilon} v(a^m/p) - a^m = \tilde{\varepsilon} v([a^m + \psi nJ + A^g]/p) - (a^m + \psi nJ + A^g) - \kappa \tag{27}$$

Rearranging (27) for $\tilde{\varepsilon}$ yields the right hand side of (25). The left hand side of (25) follows immediately from Lemma 4. Equation (26) results from substitution of the CRRA form. ■

Further details in model with money

Pricing relationship from market clearing

Market clearing implies that

$$npc'^{-1}(p) = \alpha \int_0^\infty \min\left\{pv'^{-1}(p/\varepsilon), \chi(a^g + \psi a^s) + a^m\right\} dF(\varepsilon)$$

If Case 1 holds, then $\varepsilon_m < \varepsilon_s < \tilde{\varepsilon}$ and

$$\begin{aligned} np^{(\sigma+1)/\sigma} &= \alpha \int_0^{\varepsilon_m} p \left(\frac{p}{\varepsilon}\right)^{-1/\gamma} dF(\varepsilon) + \alpha \int_{\varepsilon_m}^{\tilde{\varepsilon}} l dF(\varepsilon) \\ &+ \alpha \int_{\tilde{\varepsilon}}^{\infty} l + \psi n J + A^g dF(\varepsilon) \end{aligned}$$

If Case 2 holds, then $\varepsilon_m < \tilde{\varepsilon} < \varepsilon_s$ and

$$\begin{aligned} np^{(\sigma+1)/\sigma} &= \alpha \int_0^{\varepsilon_m} p^{(\gamma-1)/\gamma} \varepsilon^{1/\gamma} dF(\varepsilon) + \alpha \int_{\varepsilon_m}^{\tilde{\varepsilon}} l dF(\varepsilon) + \alpha \int_{\tilde{\varepsilon}}^{\varepsilon_s} p^{(\gamma-1)/\gamma} \varepsilon^{1/\gamma} dF(\varepsilon) \\ &+ \alpha \int_{\varepsilon_s}^{\infty} l + \psi n J + A^g dF(\varepsilon) \end{aligned}$$

Letting $\varepsilon_{max} = \max\{\varepsilon_s, \tilde{\varepsilon}\}$, we can express both cases as

$$\begin{aligned} np^{(\sigma+1)/\sigma} &= \alpha \int_0^{\varepsilon_m} p^{(\gamma-1)/\gamma} \varepsilon^{1/\gamma} dF(\varepsilon) + \alpha \int_{\tilde{\varepsilon}}^{\varepsilon_{max}} p^{(\gamma-1)/\gamma} \varepsilon^{1/\gamma} dF(\varepsilon) + \alpha \int_{\varepsilon_m}^{\tilde{\varepsilon}} l dF(\varepsilon) \\ &+ \alpha \int_{\varepsilon_{max}}^{\infty} l + \psi n J + A^g dF(\varepsilon) \end{aligned}$$

We assume that the preference shock follows a Pareto distribution, so that

$$F(\varepsilon) = 1 - \left(\frac{b}{\varepsilon}\right)^{\lambda}, \quad \varepsilon \geq b$$

We use

$$\begin{aligned} \int \varepsilon^{1/\gamma} dF(\varepsilon) &= \int \varepsilon^{1/\gamma} \frac{\lambda b^{\lambda}}{\varepsilon^{1+\lambda}} d\varepsilon \\ &= \lambda b^{\lambda} \frac{\varepsilon^{1/\gamma - \lambda}}{1/\gamma - \lambda} \end{aligned}$$

so that

$$\begin{aligned} np^{(\sigma+1)/\sigma} &= \alpha b^{\lambda} \left[\lambda p^{(\gamma-1)/\gamma} \frac{\varepsilon_m^{1/\gamma-\lambda} - b^{1/\gamma-\lambda} + \varepsilon_{max}^{1/\gamma-\lambda} - \tilde{\varepsilon}^{1/\gamma-\lambda}}{\frac{1}{\gamma} - \lambda} + l(\varepsilon_m^{-\lambda} - \tilde{\varepsilon}^{-\lambda} + \varepsilon_{max}^{-\lambda}) \right. \\ &+ \left. (\psi n J + A^g)(\varepsilon_{max}^{-\lambda}) \right] \end{aligned} \tag{28}$$

Equation (28) implicitly defines the price $p(n, J, l)$ implicitly as a function of n, J, and l.

Firm revenue

Firm revenue z satisfies

$$\begin{aligned} z &= \overline{z} + py_s - c(y_s) \\ &= \overline{z} + y_s^{1+\sigma} - \frac{1}{1+\sigma} y_s^{1+\sigma} \\ &= \overline{z} + \frac{\sigma}{1+\sigma} y_s^{1+\sigma} \\ &= \overline{z} + \frac{\sigma}{1+\sigma} p(n, J, l)^{\frac{\sigma+1}{\sigma}} \end{aligned}$$

Liquidation threshold $\tilde{\varepsilon}$

The liquidation threshold $\tilde{\varepsilon}$ satisfies

$$\max_{py \le a^m} \{\tilde{\varepsilon} v(y) - py\} = \max_{py \le a^m + a^g + \psi a^s} \{\tilde{\varepsilon} v(y) - py\} - \kappa$$

In Case 1, $\varepsilon_m < \varepsilon_s < \tilde{\varepsilon}$, then the first best is not attained even with stocks and bonds, and then $\tilde{\varepsilon}$ satisfies

$$\begin{aligned} \tilde{\varepsilon} \frac{(l/p)^{1-\gamma}}{1-\gamma} - l = \tilde{\varepsilon} \frac{[(l + \psi nJ + A^g)/p]^{1-\gamma}}{1-\gamma} - (l + \psi nJ + A^g) - \kappa \Leftrightarrow \\ \tilde{\varepsilon}[(l + \psi nJ + A^g)/p]^{1-\gamma} - \tilde{\varepsilon}(l/p)^{1-\gamma} = (\psi nJ + A^g + \kappa)(1-\gamma) \end{aligned}$$

so that

$$\tilde{\varepsilon} = \frac{(\psi nJ + A^g + \kappa)(1-\gamma) p^{1-\gamma}}{(l + \psi nJ + A^g)^{1-\gamma} - l^{1-\gamma}}$$

Otherwise in Case 2, $\varepsilon_m < \tilde{\varepsilon} < \varepsilon_s$; once the household is indifferent between liquidating stocks or not, then liquidating suffices to finance the first best.

$$\tilde{\varepsilon} \frac{(l/p)^{1-\gamma}}{1-\gamma} - l = \tilde{\varepsilon} v(y_{\tilde{\varepsilon}}{}^*) - p y_{\tilde{\varepsilon}}{}^* - \kappa$$

where $y^*_{\tilde{\varepsilon}} = (\tilde{\varepsilon}/p)^{1/\gamma}$. Substitution yields

$$\tilde{\varepsilon}\frac{(l/p)^{1-\gamma}}{1-\gamma} - l = \frac{\tilde{\varepsilon}^{1/\gamma}}{p^{(1-\gamma)/\gamma}}\frac{\gamma}{1-\gamma} - \kappa$$

Portfolio problems

Cash holdings generally satisfy

$$\begin{aligned} i &= \alpha \int_0^\infty \left\{ \varepsilon \frac{v'([l + \chi(a^s + a^g)]/p)}{p} - 1 \right\} dF(\varepsilon) \\ &= \alpha \int_0^\infty \left[\varepsilon (l + \chi(a^s + a^g))^{-\gamma} p^{\gamma - 1} - 1 \right] dF(\varepsilon) \end{aligned}$$

In Case 1, $\varepsilon_m < \varepsilon_s < \tilde{\varepsilon}$, the integral simplifies to

In Case 2 ($\varepsilon_m < \tilde{\varepsilon} < \varepsilon_s$), the integral evaluates to

$$\begin{aligned} i &= \alpha \int_{\varepsilon_m}^{\tilde{\varepsilon}} [\varepsilon (l)^{-\gamma} p^{\gamma-1} - 1] dF(\varepsilon) + \alpha \int_{\varepsilon_s}^{b} [\varepsilon (l + nJ + A^g)^{-\gamma} p^{\gamma - 1} - 1] dF(\varepsilon) \\ &= \alpha b^\lambda \left[\frac{\lambda}{1-\lambda} l^{-\gamma} p^{\gamma-1} (\tilde{\varepsilon}^{1-\lambda} - \varepsilon_m^{1-\lambda}) + \tilde{\varepsilon}^{-\lambda} - \varepsilon_m^{-\lambda} \right] \\ &+ \alpha b^\lambda \left[\frac{\lambda}{1-\lambda} (l + \psi nJ + A^g)^{-\gamma} p^{\gamma-1} (-\varepsilon_s^{1-\lambda}) - \varepsilon_s^{-\lambda} \right] \end{aligned}$$

We can encompass both cases as follows:

$$i = \alpha b^\lambda \left[\frac{\lambda}{1-\lambda} l^{-\gamma} p^{\gamma-1} (\tilde{\varepsilon}^{1-\lambda} - \varepsilon_m^{1-\lambda}) + \tilde{\varepsilon}^{-\lambda} - \tilde{\varepsilon_m}^{-\lambda} + \frac{\lambda}{1-\lambda} (l + \psi nJ + A^g)^{-\gamma} p^{\gamma-1} (-\varepsilon_{max}^{1-\lambda}) - \varepsilon_{max}^{-\lambda} \right]$$

Similarly, bond holdings satisfy

$$\frac{\rho - r^g}{1 + r^g} = \alpha b^\lambda \left[\frac{\lambda}{1-\lambda} (l + \psi nJ + A^g)^{-\gamma} p^{\gamma-1} (-\varepsilon_{max}^{1-\lambda}) - \varepsilon_{max}^{-\lambda} \right]$$

and stock holdings satisfy

$$\frac{\rho - r^s}{1 + r^s} = \psi \frac{\rho - r^g}{1 + r^g}$$

With these sets of equilibrium conditions, one can proceed as in the main text.