A Parametric Model of an Air Sanitizer

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Abstract

This work estimates performance of a system for UV inactivation of SARS-CoV-2 (Covid19) in aerosols. UV inactivation is a maintenance free, power efficient, lightweight, quiet and affordable solution when compared to HEPA filter based aerosol removal. For a theoretical sanitizer we expect single-pass survival rate of 16% of Covid19 with an air volume flow of $50m^3/h$, two systems in series inactivate 97%. A single sanitizer is sufficient to reduce active Covid19 RNA by 57% in aerosols in a room of $30m^2 \times 3m$, which is equivalent to fully opening a window of $0.8m^2$ for $5min - 6min$ every hour. The system is open to optimizations.

1 Covid19 Inactivation

Lifetime of virus RNA under radiation is typically expressed as Z-Value, D_{90} or D_{10} dose. These describe active RNA decay constant, dose required for 90% inactivation or 10% survival rate respectively. With N_D denoting active RNA count after having applied a dose D to an initial RNA count of N_0 they are defined as

$$Z = -\ln(\frac{N_D}{N_0})/D$$

and

$$D_{90} = D_{10} = -D/\ln_{10}(\frac{N_D}{N_0}) = \frac{\ln(10)}{Z}$$

in absence of other relevant inactivation mechanisms.

For UV-C light between $250nm$ and $290nm$ literature suggest $D_{90} \approx 21J/m^2$ in liquids, however, values between $12.3J/m^2$ and $41.7J/m^2$ were reported[1, 2, 3]. D_{90} values for viruses in aerosols have consistently been found to be less by factors between 16 and 281[4, 5, 6]. Consequently we expect susceptibility of Covid19 in aerosols to UV radiation of $D_{90,\text{Covid19}}^{\text{aerosol, extrap}} < 41.7J/m^2/16 \approx 2.6J/m^2$. Literatur indicates, however, that Covid19 could be similarly susceptible to UV radiation as SARS-CoV-1[1, 7], thus we here calculate conservatively with Walker and Ko findings on SARS-CoV-1[5]:

$$D_{90,\text{Covid19}}^{\text{aerosol, experimental}} \approx 6.6\frac{J}{m^2}. \quad (1)$$

2 Effective Air Exchange Rate

We here model the number n of active RNA and its time derivative $\dot{n}$ in a volume V over a time-span t to determine concentration $c(t) = n(t)/V$. We assume three sets of sources and sinks for active RNA in aerosol:

- $\dot{n}_{\mathrm{const}}$: sources and sinks with constant RNA absorption/emission rate, for example an infected person.
- $\dot{n}_{\alpha}(t) = \alpha n$: sources and sinks which scale with number of active RNA, for example sedimentation or limited lifetime of RNA.
- $\dot{n}_{c}(t) = \beta \dot{V} c$: sources and sinks with concentration effect, for example fresh air ventilation, filters or air sanitizers, where $\dot{V}$ denotes volume flow per time unit.

Mathematically:

$$
\begin{aligned}
\dot{n}_{\mathrm{const}} &= \dot{n}_{\mathrm{Infected\ Person}} + \dot{n}_{\mathrm{Environment,\ in}} + \ldots \\
\dot{n}_{c}(t) &= \frac{n(t)}{V} \left(-\dot{V}_{\mathrm{Sanitizer,\ in}} + \beta_S \dot{V}_{\mathrm{Sanitizer,\ out}} - \dot{V}_{\mathrm{Windows,\ out}} + \ldots \right) \\
\dot{n}_{\alpha}(t) &= n(t) \left(-\alpha_{\mathrm{Sedimentation}} - \alpha_{\mathrm{natural\ inactivation}} + \ldots \right)
\end{aligned}
$$

with β_S denoting survival rate of RNA in a sanitizer. We now define decay rate r by

$$r = -\left(\sum \beta_i \dot{V}_i / V + \sum \alpha_i\right)$$

and solve the differential equation $\dot{n}(t) = \dot{n}_{\mathrm{const}} - r n(t)$ to

$$n(t) = \frac{\dot{n}_{\mathrm{const}}}{r} \left(1 - e^{-rt}\right) + N_0 e^{-rt} \tag{2}$$

with initial active RNA amount $n(0) = N_0$. $\dot{n}_{\mathrm{const}}/r$ expresses active RNA amount in equilibrium state. Often dominant factor of r is air exchange rate $r_{\mathrm{ex}} = \dot{V}_{\mathrm{out}}/V$ with $\dot{V}_{\mathrm{in}} = \dot{V}_{\mathrm{out}}$ which describes incoming volume flow of non-contaminated air as fraction of room volume in a model of perfect, instantaneous air mixture. Open windows, fresh air ventilation systems or highly effective HEPA filters may contribute to r_{ex} among others. For better comparison with these models we define an effective exchange rate or decay rate associated with the sanitizer to

$$r_S = (1 - \beta) \dot{V}_S / V \tag{3}$$

with volume flow $\dot{V}_S$ through sanitizer and sanitizer efficiency $(1 - \beta)$.

3 Exposure of Air in Sanitizer

We now want to find the relationship between exposure of air passing the sanitizer and the sanitizers design parameters such as physical dimensions, LED power or volume flow.

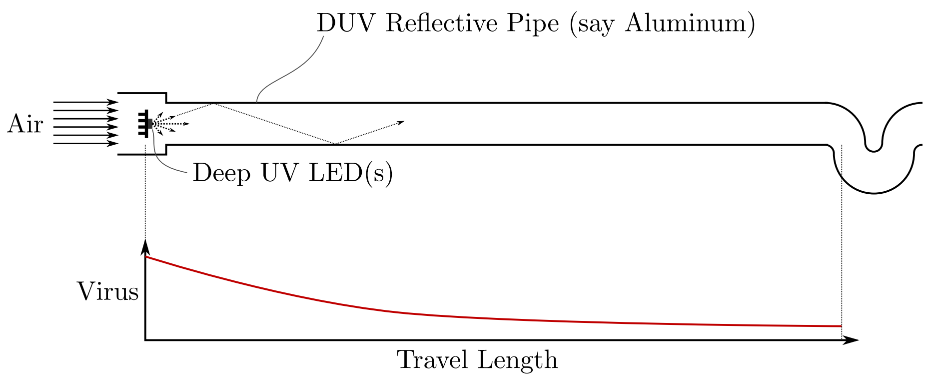

Figure 1: Working principle of air sanitizer. Contaminated air is entering from the left, passing a heat sink which carries an LED emitting deep ultraviolet light (wavelength about 275nm). It then passes a pipe which is illuminated by that LED until it reaches an UV absorbing blocking element which prevents UV light from leaving the air sanitizer. Reflection on the pipes inner surface maintains UV power (radiant flux) within the pipe.

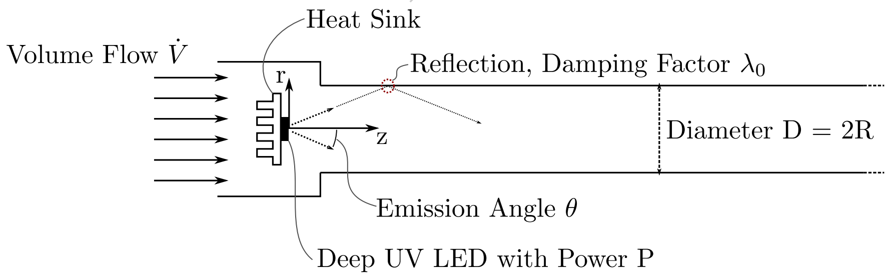

Figure 2: A volume flow $\dot{V}$ is passing a pipe with length L and diameter D, which is internally illuminated by an LED with emission power (radiant flux) P_0. The coordinate system is rooted in center of the LED on the rotation axis of the reflective pipe with radius R. A z-direction aligns with the rotation axis of the pipe, the r-direction is perpendicular to the rotation axis and normal to pipe boundary. An angle $\theta \in [0..\pi/2]$ relative to the z-axis describes direction of the emitted light of the LED which is assumed to have rotational symmetric emittance behavior. Direction of the emitted light in a plane normal to z can be described by an angle φ which goes from 0 to 2π (not shown here). Upon reflection a fraction $1 - \lambda$ of the light is being absorbed.

3.1 LED Emission Profile

We here assume a commercially available LED which bundles emission into a cone with opening angle $\theta_E = \frac{\pi}{4}$. For simplicity we describe the emmitance by an analytical function as follows

$$I_0(\theta, \varphi) = \begin{cases} I_p \cos(2\theta) & \text{if } \theta < \theta_E = \frac{\pi}{4} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

with θ denoting angle relative to the axis normal to LED surface (z-direction), φ denoting angle within plane of LED surface and I_p denoting peak radiant intensity (see figure 2) .

Integrating over the emitting half-cone of the LED we need to achieve total power (radiant flux) P_0 of LED, so with solid angle $d\Omega = \sin(\theta)d\theta d\varphi$ we find

$$P_0 = \int_0^{2\pi} \int_0^{\pi/4} I_p \cos(2\theta) \sin(\theta) d\theta d\varphi \tag{5}$$

and

$$I_p = \frac{3P_0}{2\pi(\sqrt{2} - 1)} \tag{6}$$

3.2 Reflections and Reflectance

In a pipe with loss-less reflectance the total power (radiant flux) passing a cross-sectional area A of the pipe would not depend on distance z of A from the LED, however, each reflection reduces power by a reflectance factor λ_0. After n reflections the residual power of a beam would calculate to λ_0^n of its initial power. Light emitted into a solid angle $d\Omega = \sin(\theta)d\theta d\varphi$ reaches the pipe boundary the first time after traveling a distance $z_1 = R/\tan(\theta)$. Subsequent reflections occur each after traveling another distance $\Delta z = 2z_1$. So we introduce a damping factor $\lambda(z)$ which calculates to

$$\lambda(z) = \lambda_0^{n(\theta,z)}, \text{ with}$$

$$n = \text{floor}(\frac{z}{D}\tan(\theta) + 0.5),$$

for diameter $D = 2R$ and function $floor()$ rounding towards zero. For simplicity we assume a continuously increasing damping, which overestimates damping to

$$\lambda(z) = \lambda_0^{\frac{z}{D}\tan(\theta)+0.5} \tag{7}$$

3.3 Flux Density

We now want to express flux density $E(z) = P(z)/A$ across the cross-sectional area $A = \pi R^2$ of the pipe in dependence of distance z from the LED. We here are interested into amplitude of the flux but not its direction. So we determine P by integrating over Intensities $I_{\text{eff}}(z, \theta)$ which depend on emittance angle θ and distance z from LED. Flux passing the cross-sectional area A at an angle $\vartheta = \pi/2 - \theta$ will travel along that area and contribute with a greater magnitude:

$$I_{\text{eff}}(z, \theta) = I_0(\theta)\frac{\lambda(z)}{\sin(\vartheta)} = I_0(\theta)\frac{\lambda(z)}{\cos(\theta)} \tag{8}$$

Combining equations 4, 7 and 8 and integrating over φ we find

$$E(z) = \frac{\tilde{P}}{A} \int_0^{pi/4} \lambda_0^{\frac{z}{D}\tan(\theta)} \cos(2\theta)\tan(\theta) d\theta, \text{ with } \tilde{P} = \frac{3P_0\sqrt{\lambda_0}}{\sqrt{2}-1} \tag{9}$$

3.4 Exposure

We finally want to determine exposure H (dose) to air passing the sanitizer with length L and volume flow $\dot{V}$, so

$$H(L, A, \dot{V}) = \int_0^T E(z(t))dt$$

for a time period T which is required to pass the sanitizers pipe. With $z(t) = vt$, $T = L/v$ and $v = \dot{V}/A$ we find

$$H(L, A, \dot{V}) = \frac{A}{\dot{V}} \int_0^L E(z)dz \tag{10}$$

Combining equations 9 and 10 and solving integral over z we find and

$$\begin{aligned} H(L, D, \dot{V}) &= \frac{\tilde{P}D}{\dot{V}\ln(\lambda_0)} \int_0^{pi/4} (\lambda_0^{\frac{L}{D}\tan(\theta)} - 1)\cos(2\theta)d\theta \\ &= \frac{\tilde{P}D}{\dot{V}\ln(\lambda_0^{-1})} \left(0.5 - \int_0^{pi/4} \lambda_0^{\frac{L}{D}\tan(\theta)} \cos(2\theta)d\theta\right) \end{aligned} \tag{11}$$

The first term provides us with a conservative estimate for exposure within a pipe with infinite length

$$H_{L=\infty} > \frac{3}{2(\sqrt{2}-1)} \frac{P_0 D\sqrt{\lambda_0}}{\dot{V}\ln(\lambda_0^{-1})}$$

With $\tan(\theta) \leq \frac{\pi}{4}\theta$ within interval $\theta \in [0..\pi/4]$ we estimate the remaining integral of equation 11 to

$$\int_0^{pi/4} \lambda_0^{\frac{L}{D}\tan(\theta)} \cos(2\theta)d\theta > \int_0^{\pi/4} \lambda_0^{\frac{4L}{D\pi}\theta} \cos(2\theta)d\theta = 0.5\pi \frac{\pi\lambda_0^{L/D} + 2\frac{L}{D}\ln(\lambda_0^{-1})}{(2\frac{L}{D}\ln(\lambda_0^{-1}))^2 + \pi^2}.$$

so the overall expression for exposure in sanitizer is

$$H(L, D, \dot{V}) > 0.5\frac{\tilde{P}D}{\dot{V}\ln(\lambda_0^{-1})} \left(1 - \pi\frac{\pi\lambda_0^{L/D} + 2\frac{L}{D}\ln(\lambda_0^{-1})}{(2\frac{L}{D}\ln(\lambda_0^{-1}))^2 + \pi^2}\right) \tag{12}$$

4 Analysis

Let us consider an example configuration that can be realized with typical stock components

- LED power $P_0 = 100mW$
- reflectance of $\lambda_0 = 0.7$

- diameter $D = 20cm$
- length $L = 1m$
- volume flow $\dot{V}_S = 50m^3$

Using equation 1, 12 and 3 we find aerosols in air passing the sanitizer to be exposed to $4.7J/m^2$, we find a survival rate of Covid19 RNA to $\beta_S \approx 16\%$ and we find an effective air exchange rate $r_S \approx 0.47/h$ for room of $30m^2$ and $3m$ height.

We got used to opening windows to protect us against Covid19 indoors, so it may be helpful to compare against an open-window scenario. With windows and doors closed an air exchange rate $r_{\text{ex, 0}} \approx 0.35/h$ was reported in literature[8]. With active blow ventilation applied several times per hour an air exchange rate of $r_{\text{ex, blow}} \approx 2/h$ is expected[9]. A single open window of $0.8m^2$ may contribute $r_{\text{ex, w}} = 5m/h$ at wind velocities of $2m/s$ [8, 10].

Natural lifetime of Covid19 is about $T_0 = 1.7h$[9], which contributes another decay term $r_N = 0.59/h$. Consequently we determine reduction R of active RNA concentration in aerosols according to eq. 3 to

$$R = 1 - \frac{r_{\text{ex, 0}} + R_N}{r_s + r_{\text{ex, 0}} + R_N} \tag{13}$$

with $R \approx 57\%$ for the setting lined out above.

5 Discussion

The sanitizer promises to contribute significantly to reducing spread of Covid19 indoors. During summer opening windows became natural to reduce Covid19 infection risks. Consequently we would prefer to improve efficiency to emulate an open window with $r_{\text{ex}} \approx 5/h$.

We have not analyzed role of poor field homogeneity across the cross-sectional area of the pipe. With a centered LED we expect a strong maximum along the rotational axis of the pipe, which strongly reduces efficiency of the sanitizer. This can be mitigated, however, by placing the LED off-center, which is subject of further studies.

The results are based on careful analysis of literature on Covid19 susceptibility to UV-C and on expected air exchange rates, however, some values may require update as we learn more on Covid19 behavior.

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