UC San Diego
Energy Ambitions Collection

Title
Energy and Human Ambitions on a Finite Planet

Permalink
https://escholarship.org/uc/item/9js5291m

ISBN
978-0-578-86717-5

Author
Murphy, Thomas W, Jr

Publication Date
2021-03-11

DOI
10.21221/S2978-0-578-86717-5

Supplemental Material
https://escholarship.org/uc/item/9js5291m#supplemental

Data Availability
The data associated with this publication are in the supplemental files.

Copyright Information

Energy and Human Ambitions

on a Finite Planet

Assessing and Adapting to Planetary Limits

Tom Murphy

Energy and Human Ambitions on a Finite Planet

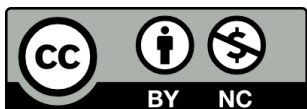## Assessing and Adapting to Planetary Limits

Thomas W. Murphy, Jr.

University of California, San Diego

This book is dedicated to Earth,
whose value is beyond measure.
May we learn to live within its bounds,
to the enduring benefit of all life.

Preface: Before Taking the Plunge

This book somewhat mirrors a personal journey that transformed my life, altered the way I look at the great human endeavor, and redefined my relationship to this planet. The transition that took a couple of decades for me is unlikely to be replicated for the reader in the short span of time it takes to absorb the content of this book. Nonetheless, the framework can be laid down so that readers might begin their own journeys and perhaps arrive at some profound realizations. This preface explains the approach and some overarching principles of the text.

We live in a physical world governed by physical law. Unlike the case for civil or criminal law, we are not even afforded the opportunity to break the laws of physics, except in fiction or entertainment. We do not need to create a physics police force or build physics jails or plead cases in front of some physics court. Nature provides perfect, automatic enforcement for free.

The domains of energy, the environment, economics, etc. are no exceptions, and can be put on a physical footing. It is worth exploring the emergent framework: reflecting on scale, efficiency, and thermodynamic limits of the human enterprise. By understanding the boundaries, we can begin to think about viable long-term plans in a way that too few are doing today. Thus far, heeding physical boundaries has not been necessary for the most part, as the scale of human endeavors has only recently become significant in a planetary context. We are now entering into a new reality: one in which our ambitions are on a collision course with natural limits on a finite planet. It is a slow-motion trajectory that has been apparent to some for an embarrassingly long time [1], but not yet acute enough to have grabbed the lasting attention of the majority.

The delirious ascent in energy and resource use witnessed over the past few centuries has been accomplished via the rapid, accelerating expenditure of a one-time inheritance of natural resources—a brief and singularly remarkable era in the long saga of human history. It has produced a dangerously distorted impression of what "normal" looks like on this planet. The fireworks show on display today is spectacular, fun, and inspirational, but also exceptionally unusual. Just as a meteorologist somehow born and trained within a 15-minute fireworks display likely cannot make useful predictions about weather and sky conditions over the next week, we are ill-equipped to intuitively understand what comes after the present phase. Luckily, science offers tools by which to transcend our narrow, warped perspectives, and can assist in discerning likely from wishful visions. The aim of this textbook is to set quantitative bounds on the present era as a way to better prepare for the possibility of a much different future. Our eventual success depends on serious attention to planetary limits.

This book is written to support a general education college course on energy and the environment. It was formulated as a physics course, but is written in the hope that it may also be accessible beyond this narrow setting. Physics is built on a mathematical foundation, and the domain of energy demands quantitative assessment. As a consequence, the book does not shy away from numbers. The math that is covered is presented in a way that aims to integrate intuition and the formality of equations. While math and quantitative elements are present throughout the book, Chapters 1, 3, and 6 are perhaps the most math-intense, featuring exponential functions, logarithms, and the lightest exposure to differential equations. But students need not master math beyond simple arithmetic operations, being able to rearrange equations, compute logarithms and exponentials, and raise a number to a power. Appendix A may serve as a useful math refresher.

An attempt is made to prevent students from equation-hunting, promoting instead development of a core understanding and intuition. This can require an adjustment on the part of students, who often treat equations as algorithmic tools to file away for use later when solving problems rather than as the embodiment of concepts to be internalized. Students often want a clear recipe so that when presented with a problem for homework, they can mimic a parallel example clearly laid out in the book. Doing so may be convenient and time-efficient, but short-circuits actual learning—bypassing the neural development that would accompany *mastering* the mental processes that are involved in solving a problem. Only the student can form these neural connections, and only through some struggle and effort. In this sense, learning is like climbing a hill: the only way to get to the top is by investing the effort to gain elevation—no shortcuts can bypass the inevitable climb.

Problems in this book are formulated to emphasize understanding the underlying concepts, rather than execution of a mathematical recipe. When students say they have math difficulties, it is usually not a problem carrying out the operations ($+$, $-$, $\times$, $\div$), but in *formulating* an approach. Therefore, the main difficulty is a conceptual one, but blamed on math because casting a problem in a mathematical framework forces a mastery of the conceptual underpinning: nowhere to hide. Given two numbers, should one divide or multiply them to get the answer sought? Resolving such questions requires a deeper understanding of the meaning *behind* the numbers in the problem (and associated units, often). By focusing on what the numbers represent and how they relate to each other, problems aim to build a more meaningful and permanent understanding of the content.

In soliciting feedback from students about problems, comments frequently pointed out that "Problem X used exactly the same math approach as Problem Y, so was redundant." This exposes a glaring difference in how students and instructors might view a problem. To the student providing such feedback, the problem seems to merely mirror an algorithm, devoid of contextual meaning. To the instructor, it is a window into a richer world: insight and personal ownership of the material is at stake. Problems are an opportunity to *learn*, as students are perhaps most actively engaged, mentally, when attempting to solve them. Instructors are trying to recreate their own learning experiences for students, through the imperfect mechanism of assigned work.

A similar revelation stems from comments that express the sentiment: "this problem has unnecessary information that is not required to solve the problem." Is the point to churn out a number, or to embed the result into a deeper context (i.e., *learn*)? It's a matter of **context** over **algorithm**. Context is where the *real* learning happens. It's where deep and lasting connections are made to the real world. The point is not to exercise a student's ability to perform mathematical operations, but to absorb a greater insight into the issue through its quantitative analysis. Math is like the airplane that delivers a skydiver to the jump. The jump/dive is the whole point, but the airplane is a necessary conveyance. When it comes time to jump, clinging to the familiar safety of the plane won't accomplish the goal. A student who bypasses the context for just the math operation has not embraced the intended experience and attendant mental growth.

The book's format sometimes weaves math and numbers into the text, which is unfamiliar to some students who are accustomed to clear delineation between math and text. Students are advised to approach sections containing mathematical developments by treating equations as statements of truth (within the appropriate context and assumptions) that help define and complete logical arguments. Or, think of equations as short-hand sentences that *encapsulate a concept*. Experts work to understand the concepts, by *reading* rather than memorizing equations. What is the equation trying to say? What truth does it impart? What relationships does it elucidate? Equations in the text are surrounded by sentences to help bring the equations to life as guides to intuition. Students who just want a step-by-step recipe to utilizing equations in an algorithmic autopilot mode are missing an opportunity

to internalize ("own") the complete argument and concept. Once the **concept** is mastered, the equation is a natural consequence, and can be generated at need *from the concept* when solving a problem.

Most textbooks on energy and the environment for a general education audience stick to dry analyses of energy resources, their implementations, and the advantages and disadvantages of each. This textbook also does so, but is less reserved about providing contextual interpretations, like saying that resources such as waves, geothermal, tidal, or ocean currents are probably not worth serious attention, due to their small scale. In this sense, the book bears some resemblance to David MacKay's fabulous and inspirational *Sustainable Energy: Without the Hot Air* [2]. In fact, the decision to make this text fully available for free in electronic form as a PDF (available at `https://escholarship.org/uc/energy_ambitions`) was completely inspired by MacKay's first doing the same. The topic is too important to allow financial interests of a publisher to limit access. The price of the print version of this book—available at `https://www.lulu.com/`—is intended to cover production costs only. If using as a textbook for a course, consider its adaptation at `https://www.kudu.com/physics_of_energy`, allowing customization and unlimited editing of the course copy.

This text also differs from others in that it attempts to frame the energy story in a broader context of other limitations facing humanity in the form of growth (physical, economic, population) and also limitations of people themselves (psychology, political barriers). In the end, students are given quantitative guidance for adaptation and encouraged to find any number of ways to reduce consumption of resources as an effective hedge against uncertainty this century. Such advice is "bad for business" and may be seen as risky in a textbook subject to financial interests and coached by market analysis—which might explain why many textbooks come off as anodyne.

The tenor of this textbook might be characterized as being pessimistic, intoning that the coming century will present many difficulties that may not be dispatched by tidy "solutions," but instead borne with resigned adaptation. We entered this century graced by a few-hundred year run of mounting prosperity—and resulting sugar high—unlike anything previously experienced in human history, but may not exit this century in such a state of privilege. This sort of message may be off-putting to some (see also the Epilogue). But the stakes are important enough that it may be worth challenging assumptions and making students uncomfortable in a way that other texts might purposefully avoid. By the time students reach the end of Chapter 8, they are perhaps a little alarmed, and desperate to know "what's the answer?" Even though the book does not completely satisfy on that front—because it *can't*, in good faith—this is arguably exactly where an instructor would like students to be: attentive and eager. Having them carry the tension into the world is one way to help humanity take its challenges seriously and work to find a better way. Soothing their discomfort so they can emerge thinking it's all in hand is perhaps at best a wasted opportunity to create a better possible future for humanity, and at worst only contributes to humanity's fall by failing to light a fire equal to the challenge.

Tom Murphy
December, 2020
San Diego, CA

How to Use This Book

This version of the book—available for free in digital form at `https://escholarship.org/uc/energy_ambitions`—is prepared for electronic viewing: not differentiating between left and right pages, and thus not ideal for printing. A two-sided version better-suited for printing is also available at the aforementioned site. Most graphics in this version are vector-based and can be safely magnified to alarming proportions.

This book makes extensive use of margin space for notes,[1] citations, figures, tables, and captions. Hopefully, the utilization of margins leads to a smooth reading experience, preventing parenthetical, contextual information from interrupting the flow of primary text.

> 1: Numbered notes point to a specific location in the text.
>
> Non-numbered notes are general asides relating to the paragraph.

References to figures and tables, glossary items, citations, chapters and sections, definitions, boxes, equations, etc. are hyperlinked within the electronic PDF document[2] allowing easy navigation, appearing as blue text. To help navigate in print versions, references to material outside the chapter also include page numbers. Note that the page numbering in this electronic version differs from that in the two-sided print versions.[3]

> 2: A link to the electronic document is in the first paragraph, above.
>
> 3: ...due to the two-side rule that new parts or chapters start on right-side pages, often leaving empty (even-numbered) pages

The electronic version of this book is *far* easier to use once figuring out how to navigate "back" within the PDF viewer,[4] so that a link—or even several in a row—may be clicked/tapped, followed by a painless return to the starting point that may be hundreds of pages away. It is therefore **strongly recommended**[5] that you figure out how to navigate using your viewing platform. For instance, viewing the document in Adobe Acrobat on a Mac, the back and forward functions are accomplished with cursor/arrow keys as ⌘-[←] and ⌘-[→] In Mac Preview, ⌘-[and ⌘-] go back and forward. Some mouses—especially those for gaming—have additional buttons that map to forward/backward navigation.

> 4: ...much like the "back" button in a web browser
>
> 5: ⚠ Learn how to navigate "back" in your PDF viewer to make hyperlinks within the document more attractive.

Call-out boxes indicate places where a student might enhance their understanding by engaging in personal exploration. An information symbol ((i)) in the margin is occasionally used—mostly in Problems sections—to denote supplemental content that build useful contextual links to the real world but may otherwise obscure what the problem seeks. A caution symbol (⚠) appears when an argument is being put forth that is likely faulty or nuanced. Students should be careful to avoid literal acceptance of these points and work to understand the subtleties of the argument.

> invitation to explore
>
> (i) contextual information
>
> ⚠ Beware of dog.

This electronic version contains a Changes and Corrections section that lists any modifications since the original release—the relevant dates being provided on page ii. Within the text, a red square ■ marks the location of a change, the square being hyperlinked to the corresponding entry describing the change, which itself has a hyperlinked page reference

> Try it! Click on the red square!

facilitating a quick return to the text. Corrections and other feedback can be left at https://tmurphy.physics.ucsd.edu/energy-text/.

A number of the examples (set apart in yellow boxes) are not posed in the form of a traditional question, but rather appear as a statement that captures a quantitative instance of the concept at hand. From this, students can construct multiple different questions that omit one piece of information and then solve for it using the remaining numbers. In this way, the example becomes a versatile guide to understanding. Note also the convention that ≈ is used to indicate "quantitatively very close to," while ~ represents a less precise numerical value that might be read "in the neighborhood of," or "roughly."

After the primary presentation of material, an Epilogue attempts to put the tone of the textbook in context as it relates to the challenges ahead.

Appendices include supplemental information on math, chemistry, worthy tangents, and a partial answer key for problems. The answer key is constructed to discourage shortcuts. Rather than give direct numerical answers, for instance, it often provides a range within which the answer is expected to lie. In this sense, it acts more like an intuition transplant, guiding more likely correct analysis in a manner closer to the way an expert operates.[6] This approach allows students to at least catch any glaring errors in approach, like dividing rather than multiplying, for instance.

6: Experts often have a pretty good sense of the answer even before a detailed analysis, which serves as a way to identify possible mistakes and force a closer look.

The Bibliography at the end applies to the whole book. Clicking on a green reference number navigates to the reference entry. Each reference indicates the (hyperlinked) page or pages on which the reference is cited. Many of the entries contain hyperlinks to online resources. The ability to navigate "back" from these explorations is very useful.

References often provide convenient links to data collections in the form of Wikipedia pages. Primary references are easy enough to track from there.

A Notation section after the bibliography provides an overview of physical constants and symbols used in the text, unit pre-factors, and a table of Greek letters.

A Glossary provides a collection of important terms encountered in the text. Hyperlinked words in the text (appearing blue) lead to the glossary, whose entries are also often internally linked to other glossary items. Additionally, page numbers where entries appear in the text are linked to easily navigate to them. The "back" navigation feature is extremely helpful in this context for returning from a rabbit hole of exploration.

Finally, a full alphabetical Index appears at the end to facilitate finding information in non-electronic versions of this text.

The graphic below reflects all chapters by page count,[7] suggesting a few different ways to approach this book. Just reading the "Upshot" for each chapter, and possibly the first few introductory paragraphs for each will convey a decent overview of the book's contents. The core message can also be picked out based on the guide below. The book has a definite quantitative slant, so it will not be possible to avoid math-laden

7: . . . and in some cases singles out individual sections by size and position within the chapter

	1. Exponential	2. Economic	3. Population	4. Space	5. Units	6. Thermal	7. Energy Use	8. Fossil Fuels	9. Climate	10. Renewables	11. Hydro	12. Wind	13. Solar	14. Biomass	15. Nuclear	16. Misc.	17. Matrix	18. Humanity	19. Plan?	20. Strategies	Epilogue	A. Math	B. Chemistry	D. Tangents
Lightning Skim																								
Core Message																								
Less Mathy																								
Quarter Class																								
Semester Class																								
Non-Physics Class																								

sections completely, but the guide can help find the less numerically intense sections. Finally, different class formats might take various routes through the material, as suggested above.

Acknowledgments

Contents

Part I

SETTING THE STAGE: GROWTH AND LIMITATIONS

We arrived late.
All the good stuff was gone.
Oh, what we would have paid.
But no amount of money could bring it back.

1 Exponential Growth

Humans have amazing strengths, but also significant weaknesses. Chief among them, perhaps, is our collective difficulty in grasping the mathematical consequences of exponential growth.[1] This is an ironic state, given that our economic and political goals are often geared explicitly to support continued growth. The degree to which an expectation and desire for continued growth is woven into our society makes it important to examine the phenomenon carefully, so that we might avoid building upon a shaky foundation. In this chapter, we explore the general nature of exponential growth, in order to understand the *impossibility* of its long-term continuance by way of exposing various absurd consequences that uninterrupted growth prescribes. The upshot[2] is that our societal framework eventually must face a mandatory departure from the current model—a piece of knowledge we should all lodge into the backs of our minds. Subsequent chapters will address applications to economic and population growth—including more realistic logistic growth curves, then pivot toward nailing down limits imposed by our finite planet.

1: . . . a nod to Al Bartlett, who worked to raise awareness about exponential growth

2: The word "upshot" means final result or bottom-line. Each chapter has an Upshot at the end.

1.1 Bacteria in a Jar

One hallmark of exponential growth is that the time it takes to double in size, or the doubling time, is constant. An important and convenient concept we will repeatedly use in this chapter is the rule of 70:

> **Definition 1.1.1** *Rule of 70: The doubling time associated with a percentage growth rate is just 70 divided by the percentage rate. A 1% growth rate doubles in 70 years, while a 2% rate doubles in 35 years, and a 10% rate doubles in 7 years. It also works for other timescales: if pandemic cases are increasing at a rate of 3.5% per day, the doubling time is 20 days.*

Note that *any* growth, however slow, can be characterized by a doubling time, even if the process does not involve discrete steps of doubling.

NGC 253 photo credit: Dylan O'Donnell.

We will see how the rule of 70 arises mathematically later in this chapter. But first, it is more important to understand the *consequences*. To make the math simple, let's say that a town's size doubles every 10 years (which by the rule of 70 corresponds to a 7% growth rate, incidentally). Starting in the year 1900 at 100 residents, we expect town population to be 200 in 1910, 400 in 1920, 800 in 1930, eventually climbing to over 100,000 by the year 2000 (see Table 1.1). Unabated 7% growth would result in the town reaching the current world population just 260 years after the experiment began.

But let's explore an example that often reveals our faulty intuition around exponential growth. Here, we imagine a jar rich in resources, seeded with just the right number of bacteria so that if each bacterium splits every 10 minutes, the jar will become full of bacteria in exactly 24 hours. The experiment starts right at midnight. The question is: at what time will the jar be half full?

Think about this on your own for a minute. Normal intuition might suggest a half-full jar at noon—halfway along the experiment. But what happens if we work backwards? The jar is full at midnight, and doubles every ten minutes. So what time is it half full?

The answer is one doubling-time before midnight, or 11:50 PM. Figure 1.1 illustrates the story. At 11 PM, the jar is at one-64th capacity, or 1.7% full. So, for the first 23 of 24 hours, the jar looks basically empty. All the action happens at the end, in dramatic fashion.

Table 1.1: Example 7% growth progression.

Year	Population
1900	100
1910	200
1920	400
1930	800
1940	1,600
⋮	⋮
2000	102,400

10 minutes is perhaps a little fast for biology, but we're looking for easy understanding and picking convenient numbers. In practice, 20–30 minutes may be more realistic. We will also ignore deaths for this "toy" example, although the net effect only changes the rate and not the overall behavior.

Figure 1.1: The last 90 minutes in the sequence of bacteria (green) growing in a jar, doubling every 10 minutes. For the first 22.5 hours, hardly anything would be visible. Note that the upward rise of green "bars" makes an exponential curve.

Now let's imagine another illustrative scenario in connection with our jar of bacteria. The time is 11:30 PM: one-half hour before the end. The jar is one-eighth full. A thoughtful member of the culture projects the future and decides that more uninhabited resource-laden jars must be discovered in short order if the culture is to continue its trajectory. Imagine for a second the disbelief expressed by probably the vast majority of other inhabitants: the jar is far from full, and has served for 141 generations—a seeming eternity. Nonetheless, this explorer returns reporting three other equal-sized food-filled jars within easy reach. A hero's welcome! How much longer will the culture be able to continue growing? What's your answer?

The population doubles every ten minutes. If the original jar is filled at 12:00, the population doubles to fill the second jar by 12:10. Another doubling fills all four by 12:20. The celebration is short-lived.

Now we draw the inevitable parallels. A planet that has served us for countless generations, and has seemed effectively infinite—imponderably large—makes it difficult for us to conceive of hitting limits. Are we half-full now? One-fourth? One-eighth? All three options are scary, to different degrees. At a 2% rate of growth (in resource use), the doubling time is 35 years, and we only have about a century, even if at 1/8 full right now.[3]

3: If we're at 1/8 right now and double every 35 years, we will be at 1/4 in 35 years, 1/2 in 70 years, and full in 105 years.

In relation to the bacteria parable, we've already done a fair bit of exploring. We have no more jars. One planet rhymes with jars, but it is hostile to human life, has no food, and is not within easy reach. We have no meaningful outlet.[4] And even if we ignore the practical hardships, how much time would a second planet buy us anyway for uninterrupted growth? Another 35 years?

4: Chapter 4 addresses space realities.

1.1.1 Exponential Math

Box 1.1: Advice on Reading Math

This section is among the most mathematically sophisticated in the book. Don't let it intimidate you: just calmly take it in. Realize that exponential growth obeys an unchanging set of rules, and can be covered in just a few pages. Your brain can absorb it all if you give it a chance. Read paragraphs multiple times and find that each pass can add to your comprehension. Equations are just shorthand sentences[5] capturing the essence of the concepts being covered, so rather than reading them as algorithms to file and use later when solving problems, work to comprehend the *meaning* behind each one and its reason for being a part of the development. In this way, what follows is not a disorganized jumble, recklessly bouncing between math and words, but one continuous development of thought expressed in two languages at once. The Preface offers additional thoughts related to this theme, and Appendix A provides a math refresher.

Experts habitually read complicated passages multiple times before the material sinks in. Maybe it's this calm habit that turns them into experts!

5: Unlike words/language, the symbols chosen for equations are just labels and carry no intrinsic meaning—so electing to use x, n, t, b, M, etc. reflect arbitrary choices and can be substituted at will, if done consistently. The content is in the *structure* of the equation/sentence.

The essential feature of exponential growth is that the scale goes as the *power* of some base (just some number) raised to the time interval. In the doubling sequence, we start at $1\times$ the original scale, then go to $2\times$, then $4\times$, then $8\times$, etc. At each time interval, we multiply by 2 (the base). After 5 such intervals, for instance, we have $2 \times 2 \times 2 \times 2 \times 2$, or $2^5 = 32$. More generally, after n doubling times, we have increased by a factor of 2^n, where 2 is the base, and n is the number of doubling times. We might formalize this as

$$M = 2^n = 2^{t/t_2}, \tag{1.1}$$

where M represents the multiplicative scale, t is the elapsed time, and t_2 is the symbol we choose to represent the doubling time—so that $n = t/t_2$ is just "counting" the number of doubling times.

For instance, doubling has $M = 2$, tripling has $M = 3$, and increasing by 29% would mean $M = 1.29$.

Box 1.2: Interest Example

The same process happens in a bank account accumulating interest. Let's consider that you deposit $100 into a bank account bearing 2% annual interest. At the end of one year, you'll have $102, which is 1.02 times the original amount. For the next year, it's 1.02 times $102, or $104.04, which is the original $100 times 1.02×1.02. Then in three years it will be $106.18, or $100 times 1.02^3. Having sussed out the pattern, after 35 years it would be $100 times 1.02^{35}, which happens to come to $199.99. Notice that doubling in 35 years at 2% exactly obeys the rule of 70. Table 1.2 summarizes this example.

Table 1.2: Interest example (2% rate).

year	b^n	dollars
0	1.00	$100.00
1	1.02	$102.00
2	1.0404	$104.04
3	1.0612	$106.12
⋮	⋮	⋮
10	1.2190	$121.90
⋮	⋮	⋮
35	1.9999	$199.99

The pattern—whether doubling, or applying interest as in Box 1.2—is that we multiply a chain of the same number, the base, over and over. This is the same as raising the base to some power—the power equaling how many times the base appears in the chain to get our overall factor. Therefore, if we designate the base as b and the number of times it appears as n, we have

$$M = b^n. \tag{1.2}$$

Now we're going to play a math trick that will help us compute various useful attributes of growth. The exponential and natural logarithm are inverse functions, each undoing the other. So $\ln(e^x) = x$ and $e^{\ln x} = x$. We can use this trick to express the number 2 as $e^{\ln 2}$, or any base number $b = e^{\ln b}$. For the special case of $b = 2$ (doubling), we then have:

By "trick," we do not mean to imply anything devious or untoward: just a cute manipulation that can bring additional insight or make something easier.

$$M = 2^{t/t_2} = \left(e^{\ln 2}\right)^{t/t_2} = e^{t \ln 2/t_2}, \tag{1.3}$$

Try it on a calculator for several examples of b that you concoct (make it real for yourself!).

where we started with Eq. 1.1, re-expressed the number 2, and then applied the rule that raising a power to another power is the same as multiplying the powers to form a single one.[6] By employing such tricks, we could cast any base to a power, like b^x as some exponential function $e^{x \ln b}$, and thus can transform any "power" relationship into an exponential using base $e \approx 2.7183$. Casting Eq. 1.2 in this form:

6: As an example, think of $\left(5^3\right)^4$ as $(5 \times 5 \times 5)^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5)$, which is just 12 fives multiplied, or 5^{12}. So we effectively just multiplied the two exponents—3 and 4—to get the 12. It always works. Often, one need not memorize math rules: quick experimentation reveals how and why it works.

$$M = b^n = e^{n \ln b}. \tag{1.4}$$

If we want to go backwards, and compute the time to reach a certain M factor, we can take the natural logarithm of both sides to learn that

$$\ln M = n \ln b, \tag{1.5}$$

so that the number of applications of base, b, needed to achieve multiplicative factor M is found by solving the equation above for n, in which

case we get: $n = \ln M/\ln b$.

Example 1.1.1 The time it would take to increase by a factor of 1,000 ($M = 1000$) at a rate of 1.07 (annual growth rate of 7%; $b = 1.07$) is $n = \ln M/\ln 1.07 = 102$ years.

The rule of 70 can be recovered[7] by setting the multiplicative factor, M, to 2. Comparing to interest accumulation described by $(1 + p)^t$, where p is the annual interest (0.02 for 2%, e.g.) and t is the number of years, Eq. 1.4 can be re-expressed by substituting $b = 1+p$ and $n = t$ as the number of years, then equating the result to the doubling time representation in Eq. 1.3 to form

$$M = e^{t \ln(1+p)} = e^{t \ln 2/t_2}. \tag{1.6}$$

From this expression, we can gather that $\ln(1 + p) = \ln 2/t_2$ by equating the exponents, and then see that the doubling time, t_2, can be solved as

$$t_2 = \ln 2/\ln(1 + p). \tag{1.7}$$

For small values of p (much smaller than 1), the natural log of $1 + p$ is approximately p. In other words, when $p = 0.02$, $\ln 1.02 \approx 0.02 \approx p$. This is part of the reason why we chose e as our base, as it is mathematically "natural." Since $\ln 2 \approx 0.693 \approx .70$, the doubling time, t_2, is approximately 70 divided by the annual growth rate, p, in percent. So the reason it's a rule of 70 for doubling (and not a rule of 60 or 80) is basically because the natural log of 2 (representing doubling) is roughly 0.70.

Example 1.1.2 To tie some things together, let's look at a quantitative case that can be used to validate how various pieces relate to each other. We will describe a 5% annual growth rate.

The rule of 70 (Definition 1.1.1) indicates a 14 year doubling time, so that we could define t_2 appearing in Eqs. 1.1, 1.3, 1.6, and 1.7 to be 14 years. Calculating exactly using Eq. 1.7 yields 14.2 years.

To evaluate growth in 10 years, we could use Eq. 1.1 with $t = 10$ and $t_2 = 14.2$ to suggest $M = 1.63$, meaning a 63% increase in size (1.63 times as large as at the start). Or we could apply Eq. 1.2 using $b = 1.05$ and $n = 10$ to get the exact same result. Note that we have freedom to define the base as 1.05 or 2, and the corresponding number of steps (n) as 10 or $t/t_2 = 0.704$, respectively, and get the same answer. In terms of the exponential form in Eq. 1.4, either pair of b and n produces $e^{0.488}$.

If we wanted to "work backwards" and ask when the amount is 3 times the original ($M = 3$), we could use Eq. 1.5 to find that n is 22.5 steps at $b = 1.05$ (thus 22.5 years, since this base is the yearly increase). Had we used $b = 2$, we would compute $n = 1.58$, meaning that the scale would reach 3× after 1.58 doubling times, or $1.58t_2 = 22.5$ years.

The same result happens if using log instead of ln: try it!

7: What follows is a high-brow symbolic approach, but the same effective result can be achieved by setting $M = 2$ in Eq. 1.5 and solving for n.

Try it yourself to verify on a calculator, by sticking in various small amounts for p.

Don't view this as a recipe for solving problems, but as a way to romp through the section and help piece it together.

More generally, we are not confined to any particular base, b, having just seized upon two convenient and relevant possibilities. If we wanted $b = 10$, we would have $n = 0.211$, for example. In this case, the interpretation is that our ten-year point is 21.1% of the way to a factor-of-ten multiplication, so that 47.4 years at 5% growth results in a factor of 10 growth.

We can check the result using Eq. 1.6 by putting in $t = 22.5$ and $p = 0.05$ or $t_2 = 14.2$ in the latter form.

1.2 Exponential Energy Extrapolation

Having established some basic principles of exponential growth, it's time for a first look at how we can use the math to argue about limits to our expectations. We'll concentrate on energy use. The United States Energy Information Administration (EIA) provides information on energy use from 1949 to the present. An appendix (E1: [3]) presents an approximate account of energy use from 1635–1945. Figure 1.2 displays the more recent portion of this history.
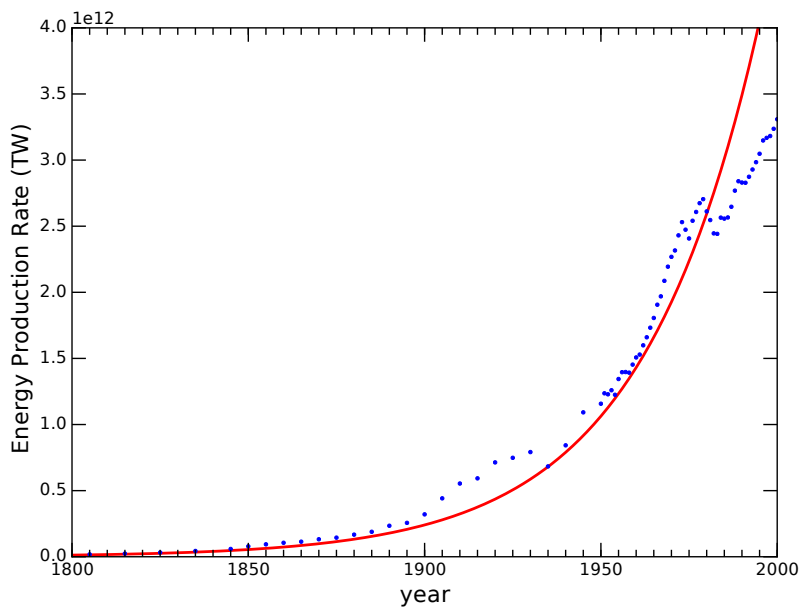
Lacking comparable data for the world, we use U.S. data simply to illustrate the more broadly applicable global growth trend. Even countries far behind are *growing* energy use—often faster than the 3% characteristic of U.S. history.

Figure 1.2: U.S. energy over 200 years, showing a dramatic rise due almost entirely to fossil fuels. The red curve is an exponential fit tuned to cover the broader period shown in Figure 1.3.

Note that the energy rate at the left edge of Figure 1.2 becomes almost invisibly small. Presenting the data on a logarithmic plot, as in Figure 1.3, we can better see the entire trajectory. On such a plot, exponentials become straight lines. The trend is remarkably consistent with an exponential (red line) for most of the history, at a rate just shy of 3% per year. Note that this total effect includes population growth, but population has not grown as fast as energy, so that per-capita energy has also risen. This makes sense: our lives today are vastly more energetically rich than lives of yesteryear, on a per-person basis.

Having established that energy growth over the past several centuries is well-described by an exponential, we can explore the implications of continuing this trend forward. Starting at a present-day global energy production rate of 18×10^{12} Watts (18 TW), we adopt a convenient growth rate of 2.3% per year for this exercise. We pick this for two reasons: 1) it is more modest than the historical trend, so will not over-exaggerate the

The astute reader might note a departure from the exponential fit in recent years. This only reinforces the primary point of this chapter that sustaining exponential growth indefinitely is absurd and will not happen. If growth is destined to stop, perhaps we are beginning to experience its limits well before the theoretical timescales developed in this chapter.

Watts is a unit of *power*, which is a rate of energy. Chapter 5 will cover the concept and units more thoroughly.
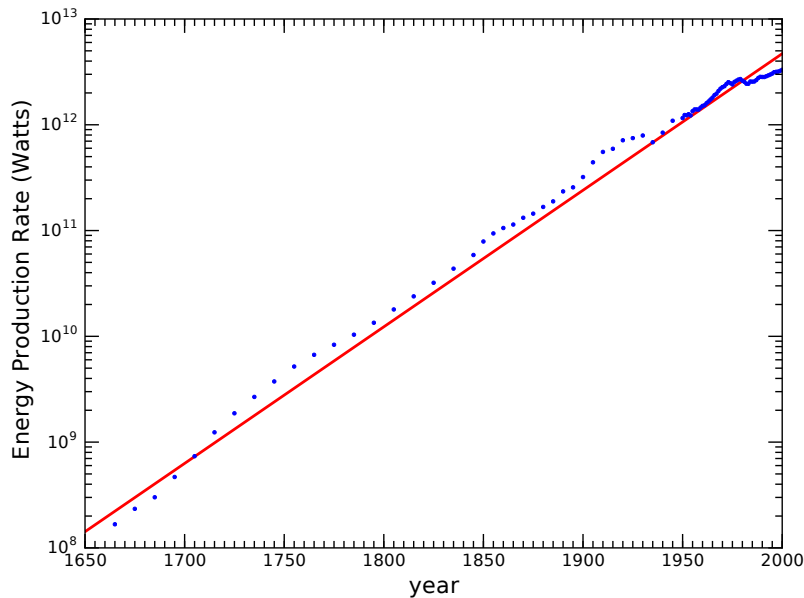
result; 2) this rate produces the mathematical convenience of a factor of 10 increase every century.[8]

What follows is a flight of fancy that quickly becomes absurd, but we will chase it to staggering levels of absurdity just because it is fun, instructive, and mind-blowing. Bear in mind that what follows should not be taken as predictions[9] of our future: rather, we can use the absurdity to predict how our future will *not* look!

The sun deposits energy at Earth's surface at a rate of about 1,000 W/m^2 (1,000 Watts per square meter; we'll reach a better understanding for these units in Chapter 5). Ignoring clouds, the projected area intercepting the sun's rays is just $A = \pi R_\oplus^2$, where $R_\oplus$ is the radius of the earth, around 6,400 km. Roughly a quarter of the earth's surface is land, and adding it all up we get about 30×10^{15} W hitting land. If we put solar panels on every square meter of land converting sunlight to electrical energy at 20% efficiency,[10] we keep 6×10^{15} W. This is a little over 300 times the current global energy usage rate of 18 TW. What an encouraging number! Lots of margin. How long before our growth would get us here? After one century, we're 10 times higher, and 100 times higher after two centuries. It would take about 2.5 centuries (250 years) to hit this limit. Then no more energy growth.

But wait, why not also float panels on all of the ocean, and also magically improve performance to 100%? Doing this, we can capture a whopping 130×10^{15} W, over 7,000 times our current rate. Now we're talking about maxing out in just under 400 years. Each factor of ten is a century, so a factor of 10,000 would be four factors of ten (10^4), taking four centuries.

So within 400 years, we would be at the point of using every scrap of

8: Fundamentally, this relates to the fact that the natural log of 10 is 2.30. The analog of Eq. 1.7 using 10 in place of 2 and $p = 0.023$ for 2.3% growth rate will produce a factor-of-ten timescale $t_{10} \approx 100$ years.

9: ⚠ Do not interpret this section as predictions of how our future *will* go.

Approximate numbers are perfectly fine for this exercise.

10: 20% is on the higher end for typical panels.

The merits of various alternative energy sources will be treated in later chapters, so do not use this chapter to form opinions on the usefulness of solar power, for instance.

⚠ In defiance of physical limits.

10,000 is not too different from 7,000, and the "rounding up" helps us conveniently make sense of the result, since a factor of 10,000 is easier to interpret as four applications of 10×, and thus 400 years.

solar energy hitting the planet at 100% efficiency. But our planet is a tiny speck in space. Why not capture *all* the light put out by the sun, in a sphere surrounding the sun (called a Dyson sphere; see Box 1.3)? Now we're talking some real power! The sun puts out 4×10^{26} W. If it were a light bulb, this would be its label (putting the 100 W standard incandescent bulb to shame). So the number is enormous. But the math is actually pretty easy to grasp.[11] Every century gets another factor of ten. To go from where we are now (18×10^{12} W) to the solar regime is about 14 orders-of-magnitude. So in 1,400 years,[12] we would be at 18×10^{26} W, which is about 4.5 times the solar output. Therefore we would use the entire sun's output in a time shorter than the 2,000-year run of our current calendar.

11: Math becomes easier if you blur your vision a bit and do not demand lots of precision. In this case, we essentially ignore everything but the exponent, recognizing that each century will increment it by 1, at our chosen 2.3% rate.

12: In this case, the "real" answer would be 1,335 years, but why fret over the details for little gain or qualitative difference in the outcome?

Box 1.3: Dyson Sphere Construction

If we took the material comprising the entire Earth (or Venus) and created a sphere around the sun at the current Earth-Sun distance, it would be a shell less than 4 mm thick! And it's not necessarily ideal material stock for building a high-tech sphere and solar panels. The earth's atmosphere distributed over this area would be 0.015 m thick. Don't hold your breath waiting for this to happen.

Bypassing boring realism, we recognize that our sun is not the only star in the Milky Way galaxy. In fact, we estimate our galaxy to contain roughly 100 billion stars! This seems infinite. A billion seconds is just over 30 years, so no one could count to 100 billion in a lifetime. But let's see: 100 billion is 10^{11}. Immediately, we see that we buy another 11 centuries at our 2.3% rate. So it takes 1,100 years to go from consuming our entire sun to all the stars in our galaxy! That's 2,500 years from now, adding the two timescales, and still a civilization-relevant time period. Leave aside the pesky fact that the scale of our galaxy is 100,000 light years, so that we can't possibly *get* to all the stars within a 2,500 year timeframe. So even as a mathematical exercise, physics places yet another limit on how long we could conceivably expect to maintain our current energy growth trajectory.

The unhinged game can continue, pretending we could capture all the light put out by all the stars in all the galaxies in the visible universe. Because the visible universe contains about 100 billion galaxies, we buy another 1,100 years. We can go even further, imagining converting all matter (stars, gas, dust) into pure energy ($E = mc^2$), not limiting ourselves to only the light output from stars as we have so far. Even playing these unhinged games, we would exhaust all the matter in the visible universe within 5,000 years at a 2.3% rate. The exponential is a cruel beast. Table 1.3 summarizes the results.

The point is not to take seriously the timescales for conquering the sun or the galaxy. But the very absurdity of the exercise serves to emphasize

Table 1.3: Energy limit timescales.

Utilizing	years until
Solar, land, 20%	250
Solar, earth, 100%	390
Entire Sun	1,400
Entire Galaxy	2,500
Light in Universe	3,600
Mass in Universe	5,000

By coincidence, the visible universe has about as many galaxies as our galaxy has stars. By "visible" universe, we mean everything within 13.8 billion light years, which is as far as light has been able to travel since the Big Bang (see Sec. D.1; p. 392).

the impossibility of our continuing exponential growth in energy. All kinds of reasons will preclude continued energy growth, including the fact that human population cannot continue indefinite growth on this planet. We will address space colonization fantasies in Chapter 4.

1.3 Thermodynamic Consequences

Physics places another relevant constraint on growth rate, and that concerns waste heat. Essentially all of our energy expenditures end up as heat. Obviously many of our activities directly involve the production of heat: ovens, stoves, toasters, heaters, clothes dryers, etc. But even cooling devices are net heat generators. Anything that uses power from an electrical outlet ends up creating net heat in the environment, with very few exceptions. A car moving down the road gets you from place A to place B, but has stirred the air,[13] heated the engine and surrounding air, and deposited heat into the brake pads and rotors, tires and road. Our metabolic energy mostly goes to maintaining body temperature. But even our own physical activity tends to end up as heat in the environment. The only exceptions would be beaming energy out of the earth environment (e.g., light or radio) or putting energy into storage (eventually to be converted to heat). But such exceptions do not amount to much, quantitatively.

What happens to all of this waste heat? If it all stayed on Earth, the temperature would climb and climb. But the heat does have an escape path: infrared radiation[14] to space. The earth is in an approximate thermodynamic equilibrium: solar energy is deposited, and infrared radiation balances the input to result in steady net energy. As we will see in Chapter 5, the *rate* at which energy flows is called power, so that we can describe energy flows into and out of the earth system in terms of power. Physics has a well-defined and simple rule for how much power a body radiates, called the Stefan–Boltzmann law:[15]

$$P = A_{\text{surf}} \sigma (T_{\text{hot}}^4 - T_{\text{cold}}^4). \tag{1.8}$$

P is the power radiated, A_{surf} is the surface area, T_{hot} is the temperature of the radiating object in Kelvin[16] (very important!), T_{cold} is the temperature of the environment (also Kelvin), and σ is the Stefan–Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4.[17] Note that the law operates on the *difference* of the fourth powers of two temperatures.

> **Example 1.3.1** A table in a room in which the table and walls are all at the same temperature does not experience net radiation flow since the two temperatures to the fourth power subtract out. In this case, as much radiation leaves the table for the walls as arrives from the walls to the table. But a room-temperature object at 300 K radiates approximately 450 W per square meter to the coldness of space.

Some time, go feel the exhaust air from an air-conditioning unit, or the heat produced at the back and bottom of a refrigerator. Even though these devices perform a cooling function, they make more heat than cool.

13: Stirred-up air eventually grinds to a halt due to viscosity/friction, becoming heat.

14: . . . a form of electromagnetic radiation

15: . . . leaving out something called *emissivity*, not relevant for our purposes

16: Conversions to Kelvin from Celsius (or Fahrenheit) go like:
$T(\text{K}) = T(\text{C}) + 273.15$;
$T(\text{C}) = (T(\text{F}) - 32)/1.8$

17: It's actually an easy constant to remember: 5-6-7-8 (but must remember the minus sign on the exponent).

Because space is so cold (tens of Kelvin, effectively, unless exposed to the sun), the fourth power of such a small number pales *so much* in comparison to the fourth power of a number like 300 that we can safely ignore it for radiation to space:

$$P_{\text{space}} \approx A_{\text{surf}} \sigma T^4, \tag{1.9}$$

⚠ Temperature must be in Kelvin.

where we now just have a single temperature: that of the warm body in space.

Earth reaches an equilibrium so that power–in equals power–out.[18] If more power is dumped onto the planet, then the temperature rises until σT^4 climbs to match. The relation in Eq. 1.9 is fundamentally important to Earth's temperature balance, and applies pretty universally, as highlighted in Box 1.4.

18: Climate change is due to greenhouse gases blocking the escape of some radiation to space, presently causing a ~0.1% imbalance that Chapter 9 will address.

Box 1.4: Everything Radiates

The same relation (Eq. 1.8) governs the surface of the sun, light bulb filaments, glowing coals, and even the human body. While the human body expends metabolic energy at a similar rate to an incandescent light bulb (about 100 W), one is *much* hotter than the other because the surface areas are vastly different.

Figure 1.4: Earth—shown here in northern hemisphere summer—intercepts sunlight across the projected area of the Earth's disk (πR^2), while radiating from the entire surface area, which is four times larger ($4\pi R^2$).

To evaluate the expected temperature of the earth, we know that the sun delivers 1,360 W/m² to the *top* of the earth's atmosphere [4] (a bit less reaches the ground). We also know that about 29.3% of this is reflected by clouds, snow, and to a lesser extent water and terrain. So the earth system *absorbs* about 960 W/m². It absorbs this energy onto the area facing the sun: a projected disk of area $A_{\text{proj}} = \pi R_{\oplus}^2$. But the total surface area of the earth is four times this, all of it participating in the radiation to space (Figure 1.4). Equating the input and output for equilibrium conditions:

This 1,360 W/m², known as the solar constant, is the *incident* energy rate (power), or the *flux*, of sunlight incident on Earth.

$$P_{\text{in}} = 0.707 \times 1360 \, \text{W/m}^2 \times \pi R_{\oplus}^2 = P_{\text{out}} = 4\pi R_{\oplus}^2 \sigma T^4, \tag{1.10}$$

The 0.707 factor represents absorbed fraction after 29.3% is reflected.

which we can rearrange to isolate temperature, satisfying

$$T^4 = \frac{0.707 \times 1360\,\text{W/m}^2}{4\sigma}. \qquad (1.11)$$

Solving for T yields $T \approx 255$ K, or $-18°$C (about $0°$F). This is cold—too cold. We observe the average temperature of Earth to be about 288 K, or $15°$C ($59°$F). The difference of $33°$C is due to greenhouse gases—mostly H_2O—impacting the thermal balance by preventing most radiation from escaping directly to space. We'll cover this more extensively in Chapter 9.

Armed with Eq. 1.11, we can now estimate the impact of waste heat on Earth's equilibrium temperature. Using the solar input as a baseline, we can add increasing input using the exponential scheme from the previous section: starting today at 18 TW and increasing at 2.3% per year (a factor of 10 each century). It is useful to express the human input in the same terms as the solar input so that we can just add to the numerator in Eq. 1.11. In this context, our current 18 TW into the projected area $\pi R_\oplus^2$ adds 0.14 W/m^2 to the solar input (a trivial amount, today), but then increases by a factor of ten each century. Taking this in one-century chunks, the resulting temperatures—adding in the 33 K from greenhouse gases—follow the evolution shown in Table 1.4. At first, the effect is unimportant, but in 300 years far outstrips global warming, and reaches boiling temperature in a little over 400 years! If we kept going (not possible), Earth's temperature would exceed the surface temperature of the sun inside of 1,000 years!

A potential inconsistency in our treatment is that we based our exploration of energy scale on solar energy as a prelude to stellar energy capture. But in the thermodynamic treatment, we implicitly *added* our power source to the existing solar input. If the sun *is* the source, we should not double-count its contribution. Nonetheless, continued, relentless growth would eventually demand a departure from solar capture on Earth and drive the same thermodynamic challenges regardless. Synthesizing the messages: we can't continue 2.3% growth for more than a few centuries using sunlight on Earth. And if we invent something new and different to replace the fully-tapped solar potential, it too will reach thermodynamic limits within a few centuries.

Years	Power Density (W/m^2)	T (K)	ΔT (C)
100	1.4	288.1	0.1
200	14	288.9	~1
300	140	296.9	~9
400	1,400	344	56
417	2,070	373	100
1,000	1.4×10^9	8,600	8,300

Table 1.4: At a constant energy growth rate of 2.3% per year, the temperature climb from waste heat (not CO_2 emissions) is slow at first, but becomes preposterous within a few-hundred years. Water boils in just over 400 years, and by 900 years Earth is hotter than the sun! The scenario of continued growth is obviously absurd.

Connecting some ideas, we found in the previous section that we would be consuming the sun's entire output in 1,400 years at the 2.3% growth rate. It stands to reason that if we used a sun's worth of energy confined to the surface of the earth, the (smaller) surface would necessarily be hotter than the sun (in 1,400 years), just like a light bulb filament is hotter than human skin despite putting out the same power—owing to the difference in area.[19]

19: This can be gleaned from Eq. 1.8 or Eq. 1.9.

One key aspect of this thermal radiation scenario is that it *does not depend on the form of power source*. It could in principle be fossil fuels, nuclear fission, nuclear fusion, or some form of energy we have not yet realized and may not even have named! Whatever it is, it will have to obey thermodynamics. Thus, thermodynamics puts a time limit on energy growth on this planet.

1.4 Upshot: Physics Limits Physical Growth

We saw in this chapter that unabated growth leads to absurd results. First, we calibrated our intuition in the context of bacteria in jars. The key point is that the jar is half full one doubling time before it is full. While this seems obvious, it delays the drama to the very end, acting fast to impose hard limits and catch the inhabitants by surprise. The conditions that persisted for many generations—thus taken for granted—suddenly change completely.

Next, we found that continuing a modest growth rate in energy becomes hopelessly absurd in a matter of centuries. Then we saw another side to this coin, in the context of thermal consequences on the surface of the earth if energy growth continues.

In the end, physics puts a timeline on expectations with respect to growth in energy on Earth. Maybe the ~300 year scale is not alarming enough. But it imposes a hard barrier against preserving our historical growth rate. In reality, other practicalities are likely to assert themselves before these hard limits are reached. We can therefore expect our growth phase to end well within a few hundred years. Given that the growth phase has lasted for far longer than that, we can say that we are closer to the end of the saga than to the beginning, yet the world is not collectively preparing for such a new reality. This seems unwise, and we will evaluate related concerns in subsequent chapters.

Many factors will intercede to limit growth in both population and resource use: resource scarcity, pollution, aquifer depletion and water availability, climate change, warfare, fisheries collapse, a limited amount of arable land (declining due to desertification), deforestation, disease, to name a few. The point is only reinforced. By some means or another, we should view the present period of physical growth as a temporary phase: a brief episode in the longer human saga.

> Was the exercise pointless, since the math leads to absurdity? Is the math wrong? No—it's immensely valuable to learn that our assumption of continued growth (and application of the corresponding correct math) fails to make sense, ultimately. The logical conclusion is that growth cannot continue indefinitely.
>
> Note that a deviation from the assumed steady 2.3% growth rate changes all the numbers, and therein may lie the solution: ramp down growth!
>
> A number of these issues will be addressed in subsequent chapters.

1.5 Problems

Hint: for problems that require solving temperature when it appears as T^4, you'll need to take the fourth root, which is the same as raising to the $\frac{1}{4}$ power. So use the y^x button (or equivalent) and raise to the 0.25 power. You can check this technique by comparing the square root of a number to the result of raising that number to the 0.5 power. Another technique for the fourth root is to take the square root twice in a row.

1. Verify the claim in the text that the town of 100 residents in 1900 reaches approximately 100,000 in the year 2000 if the doubling time is 10 years.

2. Fill out Table 1.1 for the missing decades between 1940 and 2000.

3. Our example town from the text (page 3) starting at 100 people in the year 1900 and doubling every 10 years was said to take about 260 years (26 doubling times) to reach world population. Verify that the population indeed would approach 7 billion in 260 years (when the year would be 2160), by any means you wish.[20]

4. Use Eq. 1.5 with $b = 2$ to figure out exactly how many years—via a computation of doubling times, which may not be an integer—our example town from the text (page 3) would take to reach 7 billion people.

Hint: ln is the natural log function found on scientific calculators (sometimes as LN).

Hint: M is the ratio of the final population to the initial population.

5. If our example town from page 3, doubling every 10 years, reaches a population of 7 billion in 260 years, how many years before it reaches 14 billion?

6. In a classic story, a king is asked to offer a payment as follows: place one grain of rice on one square of a chess board (64 squares), then two on the next square, four on the next, 8 on the next, and double the previous on each subsequent square. The king agrees, not comprehending exponential growth. But the final number (adding all the grains) is one less than 2^{64}. How many grains is this?

ⓘ To get 2,000 kcal of metabolic content per day, a person would need to eat 30,000 grains of rice each day. The amount of rice computed for this problem would feed the current world population for 240 years, which you are encouraged to check for yourself!

7. In the bacteria example of Section 1.1, how many "doubling times" are present in the 24 hour experiment (how many times did the population double)?

8. A one-liter jar would hold about 10^{16} bacteria. Based on the number of doubling times in our 24-hour experiment, show by calculation that our setup was woefully unrealistic: that even if we started with a single bacterium, we would have far more than 10^{16} bacteria after 24 hours if doubling every 10 minutes.

9. If a one-liter jar holds 10^{16} bacteria, how many bacteria would we start in the jar so that the jar reaches full capacity after 24 hours if we increase the doubling time to a more modest/realistic 30 minutes?

ⓘ Roughly 10 bacteria fit within a cubic micron (tiny), so you would not be able to see this tiny starting amount.

10. A more dramatic, if entirely unrealistic, version of the bacteria–jar story is having the population double every minute. Again, we start the jar with the right amount of bacteria so that the jar will be full 24 hours later, at midnight. At what time is the jar half full now?

11. In the more dramatic bacteria–jar scenario in which doubling happens every minute and reaches single-jar capacity at midnight, at what time will the colony have to cease expansion if an explorer finds three more equivalent jars in which they are allowed to expand without interruption/delay?

12. What is the doubling time associated with 3.5% annual growth?

13. Using Eq. 1.5 and showing work, what annual growth rate, in percent, leads to the mathematically convenient factor-of-ten growth every century?

Hint: the exponential, e^x, "undoes" the natural logarithm.

14. Use Eq. 1.5 with $b = 1 + p$ to figure out how long it takes to increase our energy by a factor of 10 if the growth rate is closer to the historical value of 2.9% ($p = 0.029$). Using 2.3% as we did in the examples (starting on page 7) puts this at 100 years.[21]

21: Hint: a good way to check your math. Note that if we were to use 2.9% instead of 2.3%, *all* of the time estimates in Section 1.2 are reduced by the ratio of this question's answer to 100 years.

15. Extrapolating a constant growth rate in energy is motivated by historical performance. During this period, population was also growing, albeit not as fast. If population were to double every 50 years,[22] how many people would Earth host when we hit the energy/thermodynamic limits in roughly 300 years?

22: This corresponds to a 1.4% growth rate, but you don't need to use this number in your calculation.

ⓘ We are unlikely to reach such a number for a host of other reasons.

16. In extrapolating a 2.3% growth rate in energy, we came to the absurd conclusion that we consume all the light from all the stars in the Milky Way galaxy within 2,500 years. How much longer would it take to energetically conquer 100 more "nearby" galaxies, assuming they are identical to our own?

ⓘ Ignoring the fact that it impossible to get to them fast enough, even at light speed.

17. In the spirit of outlandish extrapolations, if we carry forward a 2.3% growth rate (10× per century), how long would it take to go from our current 18 TW (18×10^{12} W) consumption to annihilating an entire earth-mass planet every year, converting its mass into pure energy using $E = mc^2$? Things to know: Earth's mass is 6×10^{24} kg; $c = 3 \times 10^8$ m/s; the result is in Joules, and one Watt is one Joule per second.

Hint: Dividing the number of Joules associated with Earth's mass by the number of seconds in a year gives the number of Watts being consumed. You may wish to compare the result to the timescale before we would use the power output of all stars in the Milky Way galaxy?

18. Taking cues from the discussion of waste heat channels on page 10, describe some of the ways that all your energy output turns to heat when you go on a bicycle ride.

19. Your skin temperature is about 308 K, and the walls in a typical room are about 295 K. If you have about 1 m² of outward-facing surface area, how much power do you radiate as infrared radiation, in Watts? Compare this to the typical metabolic rate of 100 W.

ⓘ Air convection also takes some heat away, but then clothing reduces both to bring us to equilibrium/comfort.

20. The moon absorbs 90% of the solar energy incident on it.[23] How hot would you expect the surface to get under full sun? You don't need the factor of four here[24] because the moon rotates very slowly under the sun and we're considering a patch experiencing overhead sunlight (rather than averaging over the sphere). Compare the result to boiling water temperature.

23: ... incident at the same rate/flux as at Earth

24: Referring to the 4 that shows up in Eqs. 1.10 and 1.11.

21. Venus is, ironically, colder than Earth as an infrared radiator. This is because Venus is covered in bright clouds, absorbing only 25% of the incident solar flux. Sunlight is more intense there due to it's being closer to the sun: it's almost double, at 2,620 W/m². Adapting Eq. 1.11, calculate the equilibrium temperature of Venus in the infrared and compare it to the Earth value of 255 K.

ⓘ The surface of Venus is *much* hotter than that of Earth owing to a runaway greenhouse condition. On Earth, the greenhouse boost is only 33 K, but on Venus it's hundreds of degrees.

22. Adapt Eq. 1.11 to Mars to find its equilibrium temperature. The solar flux averages $590 \, \text{W}/\text{m}^2$ there, and it absorbs 75% of incident sunlight. Express the answer in both Kelvin and Celsius, and put in context.

23. If a human body having an outward surface area of $1 \, \text{m}^2$ continued to put out 100 W of metabolic power in the form of infrared radiation in the cold of space (naked; no sun), what would the equilibrium temperature be? Would this be comfortable (put in context)?

24. Verify the total solar power output of 4×10^{26} W based on its surface temperature of 5,800 K and radius of 7×10^8 m, using Eq. 1.9.

25. Verify that Earth would reach a temperature far in excess of boiling point of water[25] after 500 years if today's power output (18 TW) increased by a factor of 10 each century.

25: Water boils at 100°C, or 373 K.

2 Economic Growth Limits

Chapter 1 demonstrated that the laws of physics and mathematical logic render a constant few-percent growth trajectory to be absurd and untenable even a few hundred years into the future. In this chapter, we develop implications for the less physics-bound concept of *economic* growth, which is a cornerstone of modern society. Investment, loans, retirement and social security schemes all assume the march of growth. The conclusion of this chapter is that economic growth will not be able to continue at any significant rate in the absence of physical growth—which we have seen cannot continue indefinitely. The suggestion usually evokes quick criticism from economists, but they can be talked down, with patience.[1] This is how it goes.

1: See, for instance [5].

2.1 Historical Coupling

In subsistence times, esthetics held little value compared to physical goods: you couldn't eat a sculpture, for instance—nor would it help keep you warm.[2] Food, tools, resources like wood, pack or draft animals carried primary value. When basic subsistence requirements were met, gold or jewelry may have warranted some expenditure—but even these were physical resources.

2: Life, it turns out, is a struggle against thermodynamics.

Agriculture freed some individuals in society to think and create. The economy found more room to value arts and performance: things that fueled the mind, if not the body. During the Renaissance, patrons would support artists and scientists whose output had few other channels of economic support. In today's world, a complex economy distributes financial assets to a wide variety of practitioners in general accordance with society's values.

Banner photo pokes fun at what physics (gravity) finds more valuable: a silver dollar (real silver) vs. a $20 bill; Credit: Tom Murphy

But resources are still paramount. The United States prospered largely because it possessed a frontier rich in natural resources. Mining and animal pelts dominated early on, as well as agriculture (tobacco, cotton, corn, wheat). In the middle of the 20th century, the United States was the dominant oil exporter worldwide (first developed in Pennsylvania, then California and Texas). Escaping the World Wars largely unscathed in terms of domestic infrastructure, the country had tooled-up a massive manufacturing capability. Together with a can-do attitude, Americans set out to rack up superlatives in virtually every category. As a manufacturing powerhouse during the middle of the 20th century, American raw materials joined a well-educated workforce to drive innovation and production. It is no accident that many in the U.S. yearn to return to these "glory days;" however we cannot possibly do so, having permanently depleted some of the original stocks.

One might say that the U.S. was the Saudi Arabia of the day.

It is important to recognize that the past was not "glorious" for all people.

What was true in the past is largely still true today: resources like oil, steel, metals, agricultural products, and heavy machinery continue to fetch a significant price on the market. Resources fuel prosperity. It is not the *only* source, but remains a reliable and bedrock component. Figure 2.1 shows the tight correlation between economic scale and energy use, which is often expressed by saying that the two tend to be coupled.

Figure 2.1: Per capita energy use as a function of GDP on a logarithmic scale. Per capita GDP is the sum total of a country's economy divided by population, effectively indicating average annual income. The rate at which an individual uses energy is expressed as a power, in Watts. A strong correlation exists here across many orders-of-magnitude: rich countries use more energy, per person [6–8]. A few instructive cases (red dots) are labeled. ■ The dot areas are scaled to population.

One way to capture the physical connection to economic activity is to represent the energy expenditure associated with each dollar[3] spent. This economic *energy intensity* (see Definition 2.1.1) of a country is just the energy expenditure of society divided by the gross domestic product (GDP).[4]

3: Or converted monetary equivalent.

4: GDP is a measure of total monetary value of goods and services produced in a country within a year.

Energy intensity therefore provides a measure of how resource-heavy a country is in relation to the size of its economy. For instance, the U.S. uses about 10^{20} Joules of energy per year and has a GDP of approximately 20 trillion dollars. Dividing these gives an intensity of 5×10^6 J/\$, or 5 MJ/\$ (many variants are possible in terms of units). The world as a whole uses about 4.5×10^{20} J in a year at an estimated \$90 trillion gross world product, also resulting in 5 MJ/\$. The variation among developed countries is not especially large—generally in the single-digit MJ/\$ range.

We will cover units of energy in Chapter 5. For now, it is sufficient to know that a Joule (J) is a unit of energy, and that MJ means megajoules, or 10^6 J.
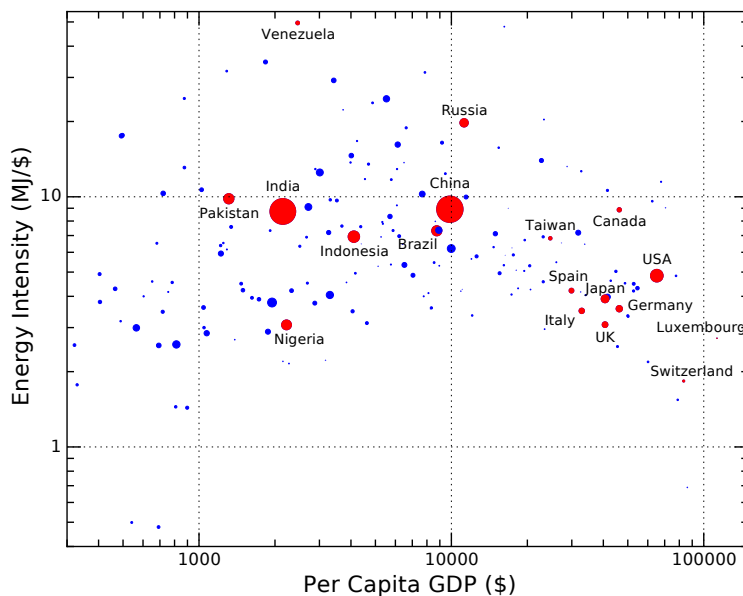
Figure 2.2: Energy intensity of countries, on a log–log plot. The vertical axis shows how energetically "hungry" each country is in relation to its economic output, while the horizontal axis sorts countries by economic output per person. A few instructive cases (red dots) are labeled. ■ The dot areas are scaled to population. Prosperous countries tend to have lower intensity than developing countries, but part of this may relate to moving manufacturing from the former to the latter [6–8].

Figure 2.2 illustrates the range of intensities for all the countries in the world. Among the factors driving energy use are geographical extent (large countries require more long-haul transportation), climate (cold countries require more heating), efficiency, and lifestyle. Russia, Canada, and the U.S. have large territories, and the former two require more heating than most. By contrast, Switzerland is geographically small

and outsources much of its heavy industry. Somebody should probably check on what's happening in Venezuela.[5]

2.2 Decoupling and Substitution

As economies expand beyond subsistence level, a larger fraction of the total activity can go to "frivolous" elements, such as art and entertainment. The intensity of such activities can be quite low. An art collector may pay $1 million for a coveted painting. Very little energy is required. The painting was produced long ago. It may even remain on display in the same location—only the name of the owner changing. Financial transactions that require no manufacture, transport, and negligible energy are said to be "decoupled" from physical resources. Plenty of examples exist in society, and are held up by economists as illustrating how we can continue to expand the economy without a commensurate expansion of resource needs.[6]

6: This is the hope, anyway.

> **Definition 2.2.1** *Decoupling is the notion that economic activities need not be strongly tied to physical (e.g., energy) requirements, so that energy intensity might become arbitrarily small. The degree to which some economic activity is decoupled forms a continuous scale, where intense utilization of energy and physical resources (e.g., steel production) are on one end and fine art dealing on the other.[7] The only way for an economy to maintain growth in the event that physical sector growth becomes limited is to increase the degree of decoupling in the society.*

7: Services, like plumbing, journalism, or marketing fall in between, using some physical resources, but not as much as heavy industry.

The dream is that as development progresses, economic energy intensity may decline (greater decoupling) so that more money is made per unit of energy expended. If the economy can decouple from energy needs, we are not constrained in our quest to continue economic growth, bringing smiles to the faces of investors and politicians. Such a transition would mean less emphasis on energy and resource-intensive industrial development/manufacturing, and more on abstract *services*,[8] broadly speaking.

8: Such services might include things like singing lessons, life coaching, psychotherapy, financial planning, and other activities that demand little physical input.

Because the world is a sort of "experiment," representing many countries having adopted many policies and in various states of development, Figure 2.2 can be viewed as a potential roadmap to decoupling.

The question is: as countries develop and become more prosperous, does intensity *decrease*, as we would want it to do as a signal of decoupling? On the large scale, any effect is modest. Going from India to the U.S. affords only a factor-of-two improvement in intensity, while spanning most of the horizontal extent in personal prosperity (a factor of 30 in per capita GDP).[9] That's pretty weak tea.

Part of the reason prosperous countries demonstrate a lower intensity is that manufacturing moves overseas. Driving the whole world toward lower intensity is a more difficult prospect, as the physical processes must still happen *somewhere*.

9: $65,000 vs. $2,100 for the U.S. and India, respectively.

At the upper end of personal income (right side of Figure 2.2), we might detect a downward slope. But we must be careful about cherry-picking in

the face of non-replicable circumstances. Not every country can assume the geography and financially-focused nature of Switzerland. And at the same time, if the U.S. imagines itself providing a model that other countries might emulate, the intensity of many European countries could actually increase if adopting U.S. habits. But more broadly, we don't have evidence that a country on the prosperous end of the distribution can operate at even a factor-of-four lower intensity than the 4 MJ/$ level typical of developed countries. In the present context of assessing the future of growth, in which we are concerned with order-of-magnitude scales and limits (as in Chapter 1), it does not appear that decoupling has very much to offer.[10]

10: That is, no orders-of-magnitude that will allow us to continue growth for centuries more after physical resources limit growth.

> **Definition 2.2.2** *Substitution refers to the ability to switch resources when one becomes scarce or a better/superior alternative is found. Substitution is often invoked to counter concerns about scarcity. A common and cute way to frame it is that the stone age did not end because we ran out of stones—we found bronze.*

The past is full of examples of substitution (Definition 2.2.2). Consider the progression in lighting technology from open fires to beeswax candles to whale oil lanterns to piped gas lanterns to incandescent electric bulbs to fluorescent lights to LED (light emitting diode) technology. Every step seems to be an improvement, and it is very natural to assume the story will continue developing along these lines.

Through this example, we can see how substitution and decoupling might be connected: efficiency improves through substitution, requiring less energy for the same lighting value.

Box 2.1: A Story of Lighting Efficiency

One way to quantify lighting progress is in the *luminous efficacy* of light, in units of lumens per Watt. In the 20th century, incandescent bulbs were so ubiquitous for so long that we fell into the bad habit of characterizing brightness in terms of the electrical power consumed by the bulb, in Watts. Thus we have generations of people accustomed to how bright a "100 W" or "60 W" bulb is. As technology changes, we should migrate to "lumens," which accurately captures how bright a source is perceived by the human eye.

Bulb packaging still refers to the "equivalent wattage" of a bulb, even though a "60 W equivalent" bulb may only consume 12 W of power.

11: We will see this in Sec. 5.10 (p. 79).

Table 2.1 and Figure 2.3 present the evolution of luminous efficacy as sources improved. Can this trend continue indefinitely? No. Every photon of light carries a minimum energy[11] requirement based on its wavelength. For photons spread across the visible spectrum (creating light we perceive as white), the theoretical limit is about 300 lm/W [9]. At this extreme, no energy is wasted in the production of light, putting 100% of the energy into the light itself. Engineering rarely reaches theoretical limits, due to a host of practical challenges. It would therefore not be surprising if lighting efficiency does not improve over where it is today by another factor of two, ending yet another centuries-long trend.

Table 2.1: Luminous efficacies [10, 11].

Light Source	lm/W
Candles	~0.3
Gas Lamp	1–2
Incandescent	8–15
Halogen	15–25
CFL	45–75
LED	75–120
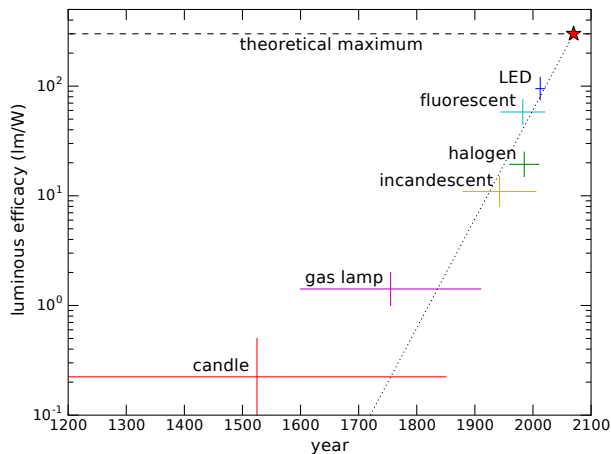
Figure 2.3: Historical progress of lighting efficiency on a logarithmic plot, using bars to indicate the approximate range of time and performance. The dashed line at top represents the maximum theoretical luminous efficacy for white light (no waste heat). The dotted line rises by our customary factor of ten per century (2.3% annual rate). Note that the guiding line reaches the theoretical maximum mid-century (red star), indicating that this centuries-long ride cannot continue much longer [10, 11].

The historical progress can fool us into thinking that we can expect a continued march to better substitutes. Having witnessed a half-dozen rabbits come out of the hat[12] in the example of lighting technology (Box 2.1), we are conditioned to believe more are forthcoming. It will be true until it isn't any more (e.g., see Figure 2.3) One way to put it is that 6 rabbits does not imply an infinite number. We should welcome each new rabbit, but not hinge our future on a continual stream of new rabbits.

For financially secure individuals at the top end of the wealth distribution, it is easier to buy into the allure of substitution as a way forward. Many have achieved wealth from humble beginnings, and have therefore lived a life of continual upgrades in terms of housing, transportation, clothing, food, travel, etc. Even those who have been surrounded by wealth their whole lives have been in a position to afford new upgrades as they become available. Yet, it is not always possible to export the capabilities of those at the top to a significant sector of the population. Not everything can scale.

12: . . . magician reference

We will return to this theme in the context of fossil fuels, which might be termed the *mother of all rabbits*, in this context. Having pulled such a stupendous rabbit out of the hat once, many assume we're set from now on. In this case, equating one to infinity is even more dubious.

An electric car having hundreds of kilometers of range seems like an obvious path forward beyond fossil fuels. But at a price tag above $40,000, it does not look like much of a solution to most people, and we can't be sure prices will fall steeply. Section D.3 covers electrified transportation in more detail.

> **Box 2.2: The Fate of the Concorde**
>
> The fate of the Concorde—which offered supersonic transatlantic passenger service between 1976–2003—may offer a useful lesson here: just because it is *possible* to construct a supersonic passenger airplane does not mean that enough people can afford it to result in an economically *viable* reduction in trans-oceanic travel time for all. Consumers no longer have the option for supersonic flight, even though 50 years ago it was assumed that this was the future. Sometimes we go backwards, when our dreams don't line up to practical reality.

More generally, sometimes the best possible solution and "peak" technology arrives at some early point in history. As much as we mess around with elements on the Periodic Table, we are never going to beat H_2O

as a vital substance.[13] Marketers might sell H_2O_2 as superior, having one more beneficial oxygen atom, but *please don't drink hydrogen peroxide*! Some technologies in use today would be recognized by pre-industrial people: wheels, string, bowls, glass, clothing. We won't always find better things, though we may make a series of incremental improvements over time. Not everything will experience game-changing developments.

In summary, decoupling and substitution are touted as mechanisms by which economic growth need not slow down as energy and other resources become constrained. We can make money using less of the resource (decoupling) or just find alternatives that are not constrained (substitution), the thinking goes. And yes, this is backed up by loads of examples where such things *have* happened. It would be foolish to claim that we have reached the end of the line and can expect *no* more gains from decoupling or substitution. But it would be equally foolish to imagine that they can produce dividends eternally so that economic growth is a permanent condition.

> **Box 2.3: Efficiency Limits**
>
> Efficiency improvements would seem to offer a way to tolerate a stagnation or decline in available energy resources. Getting more from less is very appealing. Yes, efficiency improvements are good and should be pursued. But they are no answer to growth limits, for the following reasons.
>
> 1. For the most part, realized efficiencies are already within a factor-of-two of theoretical limits.[14] A motor or generator operating at 90% efficiency has little room to improve. If efficiencies were typically far smaller than 1%, it would be reasonable to seek improvements as a "resource" for some time to come, but that is not the lay of the land.
> 2. Efficiency improvements in energy use tend to creep along at ~1% per year,[15] or sometimes 2%. Doubling times are therefore measured in decades, which combined with the previous point suggests an end to this train ride within the century.[16]
> 3. Efficiency improvements can backfire, in a process called the Jevons paradox or the rebound effect. Increased demand for the more efficient technology results in *greater* demand for the underlying resource. For example, improvements in refrigerator efficiency resulted in larger refrigerators and more of them,[17] for a net increase in energy devoted to refrigeration. Consider that per-capita global energy and material resource use has climbed inexorably amidst a backdrop of substantial efficiency improvements over the last century [12].
>
> Efficiency improvements are not capable of resolving resource demand.

13: Relatedly, consider that the Periodic Table is finite and fits easily on a single sheet of paper (Fig. B.1; p. 375). We don't have an unlimited set of substitute elements/compounds available. Astrophysical measurements validate that the whole universe is limited to the same set of elements.

14: Chapter 6 covers theoretical efficiency limits for thermal sources like fossil fuels.

15: ...meaning 30% one year might be 30.3% next year (not 31%, which would be a ~3% improvement)

16: ...similar to lighting technology, as per Box 2.1 and Figure 2.3

17: ...e.g., in basements or garages or offices

[12]: Garret (2014), *Rebound, Backfire, and the Jevons Paradox*

2.3 Physically Forced Economic Limits

Let us now consider a thought experiment. We will use Figure 2.4 as a guide as we go along. Colored numbers in the following text point to similarly-colored labels in the figure. We start by positing a constant growth rate for the entire economy (point 1; red curve in Figure 2.4) following the familiar 2.3% annual growth rate, picked for its convenient factor of 10 each century. Meanwhile, the scale of physical resources (energy, materials) in the economy also climbs at the same rate, starting at point 2. The vertical gap between the curves at the left-hand edge conveys that the economy is not 100% physical in the beginning: the total economy is larger than the physical piece.[18]

Verify this for yourself in Figure 2.4!

18: ...thus some room for services.
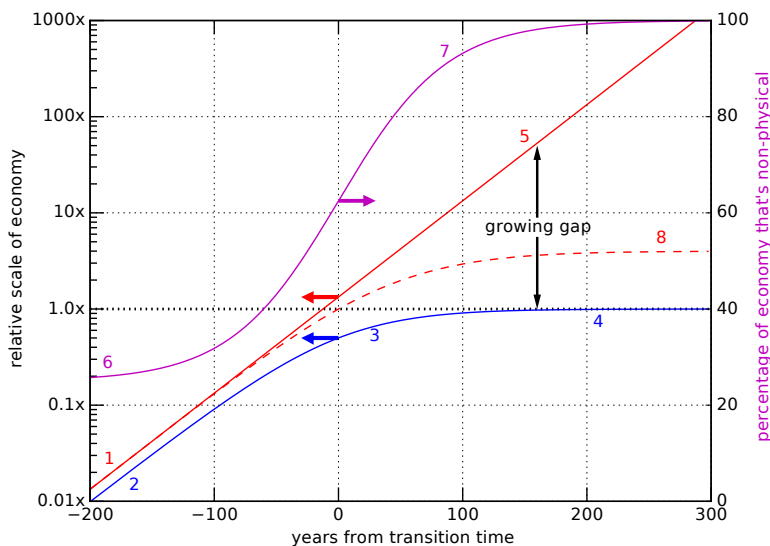
Figure 2.4: Model evolution of the economy after physical resources saturate. The blue curve is the scale of the physical economy (leveling out, or saturating). The solid red curve is the total economic scale, which we force to adhere to a constant growth rate (10× per century, or 2.3% annual rate). The magenta curve is the percentage of the economy in non-physical sectors, and the red dashed curve is a more realistic reaction of the economy to a saturating physical sector. Colored arrows point to the scale that each curve should use—logarithmic on the left for economic scales and linear on the right for the percentage curve. This model is constructed simply to illustrate the overall behavior: time scales and other quantitative details should not be taken literally.

Fast-forward to a time when physical resources have stopped growing, starting at point 3. Chapter 1—using energy and thermodynamics as the basis—made the case that we cannot expect physical growth to continue indefinitely, ending on a few-century timescale at the longest.[19] In this scenario, the scale of energy in our society flat-lines at a steady scale (point 4).

If we demand continued economic growth in the context of fixed energy, decoupling becomes increasingly necessary, shown as a growing gap in Figure 2.4. In other words, if the gross domestic product (GDP; as an indicator of economic activity) is to continue rising[20] (point 5), then overall intensity (energy per dollar) must continually decrease. For this to happen, less-energetic activities must assume increasing importance in the economy. So far, economists are on board: this is precisely what inspires an affinity for decoupling—a way forward in the face of physical limits. One might expect more abstract services, virtual experiences, art dealing, enhanced presentation: all requiring little or no additional

19: It is assumed here (optimistically) that we have managed to find a renewable alternative that can satisfy a constant demand effectively indefinitely. If not, the story is even worse and we are forced to *ramp down* the scale of the physical sector, which would force the blue curve in Figure 2.4 to descend in later years.

20: ...and not artificially via inflation, but in terms of real value

energy expenditure, or perhaps even less than before. In this way, the economic scale could keep rising while physical resources are held flat.

If the economy is to continue to expand on the basis of decoupled activities, a greater fraction of it must go toward these non-physical sectors. This means more monetary flow is associated with low-impact activities. In practical terms, then, a greater fraction of one's income is directed toward experiences not tied to energy or other physical demands. In Figure 2.4, we see, at point 6, the percentage of the economy in the non-physical sector starting at 25%: not dominant, but not negligible. The magenta curve must rise as the red and blue lines separate, until at point 7 it approaches 100% non-physical and continues to drive arbitrarily close to 100%.

During this process, the obvious converse consequence is that the energetically or physically costly activities—like transportation, food, heating, cooking, manufactured items—become an ever-smaller fraction of the economy, or an ever smaller fraction of monthly expenses, to put it more personally. In other words, they become cheap.

⚠ This, we will argue, is unrealistic.

Now, in our imagined scenario of continued economic growth, the ruthlessness of the exponential grabs the reins and drives the gulf ever wider, so that physical goods become arbitrarily cheap and demand an ever-smaller fraction of income. By the time we reach the right side of Figure 2.4, the economic scale is over 1,000 times as large as the physical scale, meaning that the physical component is less than 0.1% of the total economy. Table 2.2 illustrates the progression under the foregoing growth rate of 2.3%. If in the year 2000, 50% of one's income (and thus about half of one's work hours) goes toward physically intense products, this becomes ever smaller until by the end of the table it only takes 6 minutes of your annual work to earn enough for the physically intense goods: all your food, clothing, transportation, heating, cooking, manufactured goods.

⚠ Again, seems unrealistic.

⚠ Clearly absurd result.

If this is starting to feel like unrealistic fantasy, then good: your intuition is serving you well. How can essential, non-negotiable, life-sustaining commodities *that are in finite supply* become essentially free? The idea goes against another, more fundamental economic principle of **supply and demand**. A limited life-essential resource will always carry a moderately high value. Limited supply and inflexible demand dictate a floor to the price.

Table 2.2: Cost of physical goods.

Year	% income	hours
2000	50%	1,000
2100	5%	100
2200	0.5%	10
2300	0.05%	1
2400	0.005%	0.1

Box 2.4: Monopoly Made Easy

One way to highlight the absurdity of the scenario is that if the physically-limited but essential (life sustaining) resources became arbitrarily cheap in the fullness of time, a single person could buy them *all* for a pittance, and then charge a hefty price for anyone

who wants to keep living. We simply will not find ourselves in the situation where precious and limited resources become arbitrarily cheap. Alternatively, if people only needed to work an hour per year to accommodate basic needs, expect a lot less work to be done, acting as a drag on economic productivity and thus preventing inexorable growth—one way or another.

Once the price floor is reached, the cost of physical resources will not be able to fall any further. This happens pretty soon after physical resources cease to grow in scale. Indeed, it seems unlikely (to the author) that limited resources essential for survival would fall much below 10% of the total economic scale, which happens within a century of physical saturation in our 2.3% growth scenario. Point 8 in Figure 2.4 depicts a more realistic trajectory for the economy (red dashed line) in reaction to a saturated physical scale. In this case, the economy keeps growing a bit more than the physical sector, but eventually settles down itself into a non-growth phase.

> What seems like a reasonable lower limit to you? How economically insignificant can essentials be and still make sense?

We therefore have a logical sequence providing a few-century timescale for an end to economic growth. Thermodynamics limits us to at most a few centuries of energy growth on Earth, and economic growth will cease within a century or so thereafter, assuming a target rate of a few percent per year. In practice, growth may come to an end well before theoretical extremes are reached.

2.4 No-Growth World

The foregoing arguments spell out why economic growth cannot be expected to continue indefinitely—contrary to prevalent assumptions. When a mathematically-framed model delivers nonsense results—like the one we used to extrapolate energy use to absurd extremes—it does not mean the math itself is wrong, just that it has been misapplied or layered onto faulty assumptions. In this case, the breakdown indicates that the assumption of indefinite growth is untenable.

The growth regime is woven deeply into our current global society. And why wouldn't it be? We've enjoyed its benefits for many generations. We celebrate the myriad advantages it has brought, and therefore align our political and economic institutions toward its robust preservation. Community planning, interest rates, investment, loans, the very role of banks, social safety net systems,[21] and retirement plans all hinge on the assumption of growth.[22] Shock waves of panic reverberate at signs of (even temporary) recession, given the importance of growth to our institutions. Yet the message here is that we cannot expect its unfaltering continuance—implying that many things will have to change.

21: In the U.S., Social Security and Medicare are examples.

22: Growth in both workforce and investments are essential ingredients of these schemes that pay out more than an individual's past contributions to the program.

Returning to the roots of economic theory, the earliest thinkers—Adam Smith, David Ricardo, Thomas Malthus, John Stuart Mill—had foundations in natural philosophy[23] and saw growth as a temporary phase, ultimately limited by a prime physical resource: **land**. In that time, land held the key to outputs from farming, timber, mining, and game—thereby dictating economic development. What these pioneering economic thinkers did not foresee was the arrival of fossil fuels, and the technological developments that accompanied this energy explosion.

Now, we have fallen into something of a lulled complacency: having rescued ourselves so far from the end-of-growth predictions of the early economists, the temptation is to conclude that they were just wrong,[24] and we have outsmarted natural limits. This is dangerous thinking. In the end, nature is indifferent to how smart we imagined ourselves to be. If we were truly clever, we would *start thinking about a world that does not depend on growth, and how to live compatibly within planetary limits*. Chapter 19 touches on this theme, after intervening chapters paint a more complete picture of energy constraints.

2.5 Upshot: Economic Growth Will End

It is worth re-iterating the recipe for an end to economic growth in summary form, as spelled out in Box 2.5. Make sure you can trace the logic and connections from one point to the next—not to be memorized as disconnected facts.

Box 2.5: Economic Growth Limit
1. Physical resources (energy in our example) ultimately stabilize to a fixed annual amount.
2. Non-physical sectors of the economy must assume responsibility for continued economic growth, *if* growth is to continue.
3. The economy comes to be dominated by non-physical sectors.
4. Physical sectors are relegated to an ever-smaller fraction of the economy, ultimately vanishing if exponential growth is to hold.
5. In this scenario,[25] physical goods (energy among them) become arbitrarily cheap, requiring only one week's worth of earnings, then a day's worth, then an hour, a minute, a second.
6. This situation is impossible and does not respect common-sense supply/demand notions: a finite, limited but absolutely vital resource will never become arbitrarily cheap in a market system.
7. At some point, physical resources will "saturate" to a minimum fraction of the economy, at which point overall growth in non-physical sectors must also cease.

23: . . . closer to modern-day physics than to modern-day economics, rooted in the natural world

24: The classic example is Thomas Malthus, who warned of limits over 200 years ago based on finite resource limits before fossil fuels ripped the narrative apart. The lasting association is that "Malthus equals wrong," leading to the dangerous takeaway that all warnings in this vein are discredited and can be ignored. Note that the most consequential and overlooked lesson from the story about "the boy who cried wolf" is that a *real* wolf *did* appear.

As was stated before, experts frequently read complex sections more than once to fully absorb the arguments; feel free to do so here.

25: . . . which, let's be clear, we're arguing is ultimately not at all viable. . .

Just because we can point to some completely legitimate *examples* of decoupled activities and many impressive substitution stories does not mean that an entire economy can be based on indefinite continuance of such things. We are physical beings in a physical world and have non-negotiable minimum requirements for life. The activities and commodities that support critical functions cannot continue to expand indefinitely, and will not become arbitrarily cheap once their expansion hits physical limits. The finite nature of our world guarantees that such limits will be asserted, committing economic growth to stall in turn. Nothing, in the end, escapes physics.

So, while acknowledging that growth in the past has brought uncountable benefits to the human endeavor, we have to ask ourselves: If the end of growth is inevitable, why does it remain our prevailing plan?

2.6 Problems

1. At a 3.5% growth (interest) rate, how much would $1,000 invested at the time Columbus sailed to America be worth today (hint: use the rule of 70)? Put this in context (compare to richest individuals or find a similar GDP for some country).

   Retention of Chapter 1 material is assumed. The real world is not partitioned into chapters, and neither should your brain be.

2. As an indication of how sensitive the accumulation is to interest rate, compare the result from Problem 1 to what would happen for interest rates of 4% and 5%—again putting into context.

3. Find the energy intensity for at least four countries spanning a range of development levels. For each country, look up the GDP, and find energy consumption at: Wikipedia page on Primary Energy Consumption. [13] In order to compare to Figure 2.2, multiply the number in quadrillion Btu (qBtu) by 1.055×10^{18} J/qBtu. Also note that a trillion is 10^{12}.

   Look at the column for total energy consumption in units of quadrillion Btu.

4. Estimate the energy intensity of the UCSD campus, based on an annual electricity expenditure around 10^{15} J.[26] For the financial side, assume that student payments (tuition, fees, room and board) accounts for 40% of the total budget.[27] Use your knowledge of typical tuition/fees and enrollment to come up with a number. Compare your result to global figures for energy intensity.

   26: Based on a 30 MW electrical load times the number of seconds in a year; this won't account for all energy expenditures, missing transportation, for instance.

   27: Federal grants comprise most of the rest, and a small amount from state taxes

   Don't get hung up on in-state fraction; just make a crude guess (maybe guess an average) and clearly state assumptions.

5. Typical energy costs are in the neighborhood of $0.10 per kilowatt-hour (kWh), and 1 kWh is 3.6 MJ (megajoules). Take the ratio of these two figures to form an economic energy intensity *of energy itself*, in units of MJ/$.

   The result should be larger than the typical energy intensity for all economic activities within a country, since not all monetary expenditure goes toward energy.

6. If a country clocks in at 5 MJ/$ for its *overall* energy intensity, and its energy costs work out to an energy intensity of 30 MJ/$,[28] what percentage of the economy do we infer constitutes the energy sector?

   28: In the same sense as was calculated for Problem 5

   The answer should be well less than 100%.

7. Come up with some of your own examples (at least three; not listed in the text) of economic activities[29] that have little resource footprint and are therefore fairly decoupled. These are transactions for which the intensity (energy or resource expenditure) is very low compared to the dollar amount.

29: ... things that cost money

8. If a candle has a luminous efficacy of 0.3 lm/W and a modern LED light bulb achieves 100 lm/W, by what factor[30] have we improved lighting efficiency? If the theoretical limit is around 300 lm/W, what factor do we still have to go?

30: A factor is just a multiplicative scale: e.g., 24 is a factor of 6 larger than 4.

9. In going from 0.3 lm/W candle technology to the theoretical maximum luminous efficacy, we see a factor of 1,000 increase. Taking about 300 years to do this, we might recognize that we are following along our familiar factor-of-ten each century trajectory. Approximately how long[31] might we expect it to take to achieve the final factor of three to go from our current technology to the theoretical limit, at this same rate? Is it within your lifetime that we hit the limit?

31: You can just estimate, use appropriate math, or refer to the guiding line in Figure 2.3 to arrive at a rough number.

10. Provide your own example of a sequence of substitutions comprised of at least two qualitatively superior[32] substitutions over time (thus three steps: original, first replacement, second replacement).

32: ... perhaps defined by widespread or universal adoption or replacement.

11. List three substances or critical concepts we rely on that have no superior substitutes in the universe.

12. Based on your present state of knowledge, detail what you think an optimist might say about the superiority of post-fossil energy substitutes?

You might pick any subset of solar, wind, hydroelectric, geothermal, nuclear, etc. to guide your thinking.

13. Based on your present state of knowledge, detail what you think a pessimist might say about the lack of superiority of post-fossil energy substitutes?

14. Justify what, In your mind, is a reasonable lower limit to the percentage of the economy that could be based on decoupled (not energy or resource heavy) activities? Make an argument for what leads you to this "floor."

15. One form of decoupled activity that some will bring up is virtual reality: you can travel the world (or solar system?) without resource-hogging transportation and other material costs. Do you see this as a viable alternative that is likely to largely supplant physical travel? Why or why not?

16. Are you sold on the argument that the physics-imposed limit to resource/energy growth demands an ultimate cessation of economic growth as well? If so, highlight the persuasive elements. If not, why not?

3 Population

Underlying virtually every concern relating to our experience on this planet is the story of human population. The discussion of continued energy growth in Chapter 1 was based on the historical growth rate of energy, which is partly due to growing population and partly due to increased use per capita. But the notion that population will continue an exponential climb, as is implicit in the Chapter 1 scenario, is impractical—one of many factors that will render the "predictions" of Chapter 1 invalid and prohibit "growth forever."

So let's add a dose of reality and examine a more practical scenario. Americans' per-capita use of energy is roughly *five times* the global average rate. If global population eventually doubles, and the average global citizen advances to use energy at the rate Americans currently do,[1] then the total scale of energy use would go up by a factor of 10, which would take 100 years at our mathematically convenient 2.3% annual rate (see Eq. 1.5; p. 5). This puts a more realistic—and proximate—timescale on the end of energy growth than the fantastical extrapolations of Chapter 1.

Although the focus of this chapter will be on the alarming rate of population growth, we should keep the energy and resource context in mind in light of the overall theme of this book. To this end, Figure 3.1 shows the degree to which energy demand has outpaced population growth, when scaled vertically to overlap in the nineteenth century. From 1900 to 1950, per-capita energy consumption increased modestly, but then ballooned dramatically after 1950, so that today we have the equivalent of 25 billion people on the planet operating at nineteenth century energy levels.

Since population plays a giant role in our future trajectory, we need to better understand its past. We can also gain some sense for theoretical

---

Photo Credit: Tom Murphy

1: . . . so that global average energy use per capita increases by a factor of five from where it is today
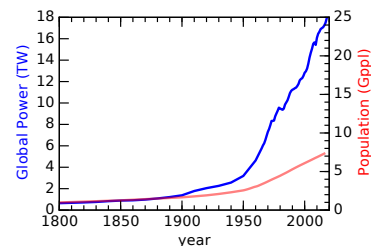
Figure 3.1: Population (red) and energy demand (blue) on the same plot, showing how much faster energy demand (power) has risen compared to population, which translates to increasing per-capita usage. The vertical axes are scaled so that the curves overlap in the nineteenth century. [14–16].

expectations, then discuss the heralded "demographic transition" and its implications.

3.1 Population History

Figure 3.2 shows a history of global population for the last 12,000 years. Notice that for most of this time, the level is so far down as to be essentially invisible. It is natural to be alarmed by the sharp rise in recent times, which makes the current era seem wholly unusual: an aberration. But wait—maybe it's just a plain exponential function. All exponential functions—ruthless as they are—would show this alarming rise at some point, sometimes called a "hockey stick" plot. In order to peer deeper, we plot population on a logarithmic vertical axis (Figure 3.3). Now we bring the past into view, and can see whether a single exponential function (which would have a constant slope in a logarithmic plot) captures the story.

Wait, what? It still looks somewhat like a hockey stick (even more literally so)! How can that be?! This can't be good news. Peering more closely, we can crudely break the history into two eras, each following exponential growth (straight lines on the plot), but at different rates. The early phase had a modest 0.044% growth rate. By the "rule of 70," the corresponding doubling time is about 1,600 years. In more recent times, a 1% rate is more characteristic (70 year doubling). Indeed, we would be justified in saying that recent centuries are anomalous compared to the first 10,000 years of the plot. If we extend the the 0.04% line and the 1% line, we find that they intersect around the year 1700, which helps identify the era of marked transition.

The recent rapid rise is a fascinating development, and begs for a closer look. Figure 3.4 shows the last ~1,000 years, for which we see several exponential-looking segments at ever-increasing rates. The doubling times associated with the four rates shown on the plot are presented in Table 3.1.

An interpretation of the population history might go as follows. Not much changed during the period following the Dark Ages.[2] The Renaissance (~1700) introduced scientific thinking so that we began to conquer diseases, allowing an uptick in population growth. In the mid-19th century (~1870), the explosive expansion of fossil fuel usage permitted industrialization at a large scale, and mechanized farming practices. More people could be fed and supported, while our mastery over human health continued to improve. In the mid-20th century (~1950), the Green Revolution [17] introduced a fossil-fuel-heavy diet of fertilizer and large-scale mechanization of agriculture, turning food production into an industry. The combination of a qualitative change in the availability of cheap nutrition and the march of progress on disease control cranked the population rate even higher.
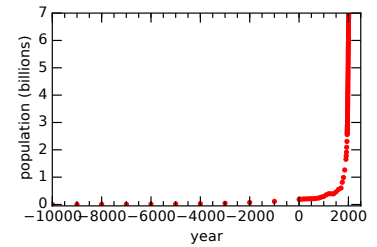
Figure 3.2: Global population estimate, over the modern human era, on a linear scale. Figure 3.1 offers a recent close-up. [14, 15].
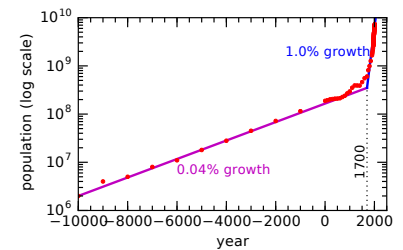
Figure 3.3: Global population estimate, over the modern human era, on a logarithmic scale. [14, 15].
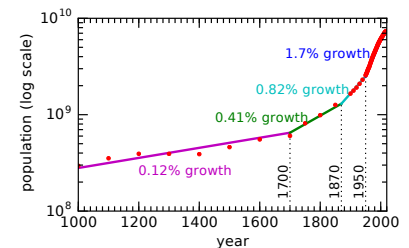
Figure 3.4: Global population estimate, over recent centuries. On the logarithmic plot, lines of constant slope are exponential in behavior. Four such exponential segments can be broken out in the plot, having increasing growth rates. [14, 15].

2: . . . except that famine and plague took a toll in the 14th century

Table 3.1: Doubling times for Fig. 3.4.

Years	% growth	t_2
1000–1700	0.12%	600 yr
1700–1870	0.41%	170 yr
1870–1950	0.82%	85 yr
1950–2020	1.70%	40 yr

In more recent years, the rate has fallen somewhat from the 1.7% fit of the last segment in Figure 3.4, to around 1.1%. Rounding down for convenience, continuation at a 1% rate would increase population from 7 billion to 8 billion people in less than 14 years. The math is the same as in Chapter 1, re-expressed here as

$$P = P_0 e^{\ln(1+p)(t-t_0)}, \tag{3.1}$$

where P_0 is the population at time t_0, and P is the population at time t if the growth rate is steady at p. Inverting this equation,[3] we have

$$t - t_0 = \frac{\ln\left(\frac{P}{P_0}\right)}{\ln(1 + p)}. \tag{3.2}$$

3: ... recalling that that the natural log and exponential functions "undo" each other (as inverse functions)

Example 3.1.1 We can use Eq. 3.1 to determine how many people we will have in the year 2100 if we continue growing at a 1% rate, starting from 7 billion in the year 2010. We set $P_0 = 7$ Gppl,[4] $t_0 = 2010$, $p = 0.01$, then compute the population in 2100 to be $P = 7e^{\ln 1.01 \cdot 90} = 17$ Gppl.

Eq. 3.2 is the form that was used to conclude that increasing from 7 to 8 Gppl takes less than 14 years at a 1% rate. The computation looks like: $\ln(8/7)/\ln 1.01 = 13.4$. Note that we need not include the factors of a billion in the numerator and denominator, since they cancel in the ratio.

4: Gppl is giga-people, or billion people

The actual time for adding one billion people has lately been 12 years, as we have been growing at a rate slightly higher than 1%.

Year	Population	Time	Rate	Doubling
1804	1 Gppl	—	0.4%	170
1927	2 Gppl	123	0.8%	85
1960	3 Gppl	33	1.9%	37
1974	4 Gppl	14	1.9%	37
1987	5 Gppl	13	1.8%	39
1999	6 Gppl	12	1.3%	54
2011	7 Gppl	12	1.2%	59
2023	8 Gppl	12	1.1%	66

Table 3.2: Population milestones: dates at which we added another one billion living people to the planet. The Time and Doubling columns are expressed in years. Around 1965, the growth rate got up to 2%, for a 35 year doubling time.

Table 3.2 and Figure 3.5 illustrate how long it has taken to add each billion people, extrapolating to the 8 billion mark (as of writing in 2020). The first billion people obviously took tens of thousands of years, each new billion people taking less time ever since. Growth rate peaked in the 1960s at 2% and a doubling time of 35 years. The exponential rate is moderating now, but even 1% growth continues to add a billion people every 13 years, at this stage. A famous book by Paul Ehrlich called *The Population Bomb* [18], first published in 1968, expressed understandable alarm at the 2% rate that had only *increased* to that point. The moderation to 1% since that period is reassuring, but we are not at all out of the woods yet. The next section addresses natural mechanisms for curbing growth.

Figure 3.5: Graphical representation of Table 3.2, showing the time between each billion people added [14, 15].

3.2 Logistic Model

Absent human influence, the population of a particular animal species on the planet might fluctuate on short timescales (year by year) and experience large changes on very long timescales (centuries or longer). But by-and-large nature finds a rough equilibrium. Overpopulation proves to be temporary, as exhaustion of food resources, increased predation, and in some cases disease (another form of predation, really) knock back the population.[5] On the other hand, a small population finds it easy to expand into abundant food opportunities, and predators reliant on the species have also scaled back due to lack of prey.

We have just described a form of negative feedback: corrective action to remedy a maladjusted system back toward equilibrium.

Definition 3.2.1 *Negative feedback simply means that a correction is applied in a direction* opposite *the recent motion. If a pendulum moves to the right, a restoring force pushes it back to the left, while moving too far to the left results in a rightward push. A mass oscillating on a spring demonstrates similar characteristics, as must all equilibrium phenomena.*

We can make a simple model for how a population might evolve in an environment hosting negative feedback. When a population is small and resources are abundant, the birth rate is proportional to the population.

Example 3.2.1 If a forest has 100 breeding-aged deer, or 50 couples, we can expect 50 fawns in a year (under the simplifying and unimportant assumption of one fawn per female per year). If the forest has 200 deer, we can expect 100 fawns. The birth rate is simply *proportional* to the population capable of giving birth.[6]

If the setup in Example 3.2.1 were the only element to the story, we would find exponential growth: more offspring means a larger population, which ultimately reaches breeding age to produce an even larger population.[7] But as the population grows, negative feedback will begin to play a role. We will denote the population as P, and its rate of change as $\dot{P}$.[8] We might say that the growth rate, or $\dot{P}$, is

$$\dot{P} = rP, \tag{3.3}$$

where r represents the birth rate in proportion to the population (e.g., 0.04 if 4% of the population will give birth in a year).[9] This equation just re-iterates the simple idea that the rate of population growth is dependent on (proportional to) the *present* population. The solution to this differential equation is an exponential:

$$P = P_0 e^{r(t-t_0)}, \tag{3.4}$$

5: For reference, the SARS-CoV2 pandemic of 2020 barely impacted global population growth rates. When population grows by more than 80 million each year, a disease killing even a few million people barely registers as a hit to the broader trend.

The word *negative* may sound like something we would not want, but its cousin— positive feedback—leads to disastrous runaway conditions. An example of positive feedback is the bacteria example from Chapter 1: having more bacteria only increases the rate of growth. Exponentials are the hallmark of positive feedback, while equilibrium signals negative feedback.

6: ...no negative feedback yet

7: We have just described a state of positive feedback: more begets more.

8: $\dot{P}$ is a time derivative (note the dot on top), defined as $\dot{P} = dP/dt$. But don't panic if calculus is not your thing: what we describe here is still totally understandable.

9: In terms of the growth rate we used before, p, as in Eq. 3.1, $r = \ln(1 + p)$. So for instance, if growing at 2%, $p = 0.02$ and r also is 0.02 ($r \approx p$ for small values of p).

which is really just a repeat of Eq. 3.1, where r takes the place of $\ln(1 + p)$.

> **Example 3.2.2** Paralleling the deer population scenario from Example 3.2.1, if we set $r = 0.5$, and have a population of $P = 100$ adult deer (half female), Eq. 3.3 says that $\dot{P} = 50$, meaning the population will change by 50 units.[10]
>
> We could then use Eq. 3.4 to determine the population after 4 years: $P = 100e^{0.5 \cdot 4} \approx 739$.

10: A more adorable term for "units" is fawns, in this case.

We ignore death rate here, but it effectively reduces r in ways that we will encounter later.

Let's say that a given forest can support an ultimate number of deer, labeled Q, in steady state, while the current population is labeled P. The difference, $Q - P$ is the "room" available for growth, which we might think of as being tied to available resources. Once $P = Q$, no more resources are available to support growth.

> **Definition 3.2.2** *The term "**carrying capacity**" is often used to describe Q: the population supportable by the environment. The carrying capacity (Q) for human population on Earth is not an agreed-upon number, and in any case it is a strong function of lifestyle choices and resource dependence.*

$Q - P$ quantifies a growth-limiting mechanism by representing available room. One way to incorporate this feature into our growth rate equation is to make the rate of growth look like

$$\dot{P} = \frac{Q - P}{Q} rP. \tag{3.5}$$

Figure 3.6: The rate of growth in the logistic model decreases as population increases, starting out at r when $P = 0$ and reaching zero as $P \rightarrow Q$ (see Eq. 3.5).

We have multiplied the original rate of rP by a term that changes the effective growth rate $r \rightarrow r(Q - P)/Q$. When P is small relative to Q, the effective rate is essentially the original r. But the effective growth rate approaches zero as P approaches Q. In other words, growth slows down and hits zero when the population reaches its final saturation point, as $P \rightarrow Q$ (see Figure 3.6).

> Try it yourself: pick a value for Q (1,000, maybe) and then various values of P to see how the effective growth rate will be modified.

The mathematical solution to this modified differential equation (whose solution technique is beyond the scope of this course) is called a logistic curve, plotted in Figure 3.7 and having the form

$$P(t) = \frac{Q}{1 + e^{-r(t-t_0)}}. \tag{3.6}$$

The first part of the curve in Figure 3.7, for very negative values[11] of $t - t_0$, is exponential but still small. At $t = t_0$ (time of inflection), the population is $Q/2$. As time marches forward into positive territory, P approaches Q. As it does so, negative feedback mechanisms (limits to resource/food availability, predation, disease) become more assertive

11: The parameter t_0 is the time when the logistic curve hits its halfway point. Times before this have negative values of $t - t_0$.
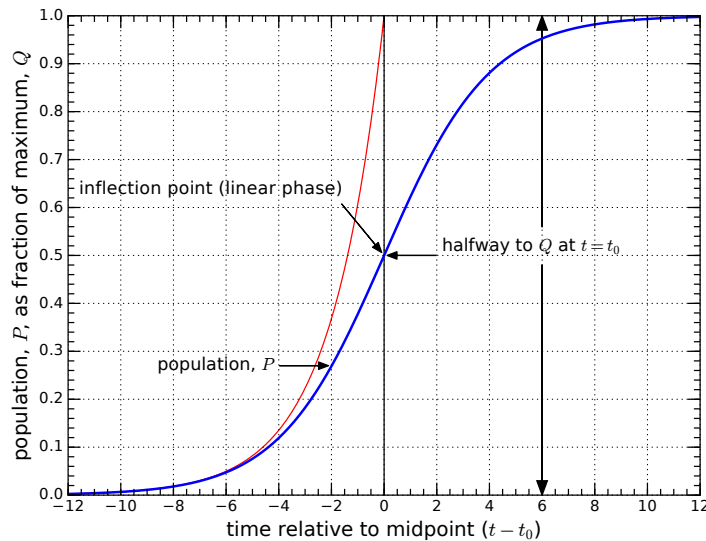
Figure 3.7: Logistic population curve (blue), sometimes called an S-curve, as given in Eq. 3.6, in this case plotting for $r = 0.5$ to match examples in the text. The red curve is the exponential that would result without any negative feedback.

and suppress the rate of growth until it stops growing altogether when P reaches Q.

Example 3.2.3 Continuing the deer scenario, let's say the forest can ultimately support 840 adults,[12] and keep $r = 0.5$ as the uninhibited growth rate. Using these numbers, Eq. 3.6 yields 100 adults at $t = t_0 - 4$ years (effectively the initial state in Example 3.2.1). One year later, at $t = t_0 - 3$, Eq. 3.6 yields 153—very close to the nominal addition of 50 members. But now four years in ($t = t_0$), we have 420 instead of the 739 we got under unrestricted exponential growth in Example 3.2.2.[13]

12: ... tuned for a convenient match to the numbers we have used in the foregoing examples

13: Not coincidentally, $P = Q/2$ at the halfway point, $t = t_0$.

The logistic curve is the *dream scenario*: no drama. The population simply approaches its ultimate value smoothly, in a tidy manner. We might imagine or hope that human population follows a similar path. Maybe the fact that we've hit a linear phase—consistently adding one billion people every 12 years, lately—is a sign that we are at the inflection, and will start rolling over toward a stable endpoint. If so, we know from the logistic curve that the linear part is halfway to the final population.

Three consecutive 12-year intervals appear in Table 3.2. If the middle one is the midpoint of a logistic linear phase—in 2011 at 7 billion people—it would suggest an ultimate population of 14 billion.

3.2.1 Overshoot

But not so fast. We left out a crucial piece: feedback delay. The math that leads to the logistic curve assumes that the negative feedback[14] acts instantaneously in determining population rates.

14: ... based on remaining resources, $Q - P$, at the moment in Eq. 3.5

Consider that human decisions to procreate are based on present conditions: food, opportunities, stability, etc. But humans live for many decades, and do not impose their full toll on the system until many years after birth, effectively delaying the negative feedback. The logistic curve and equation that guided it had no delay built in.

Definition 3.2.3 *Overshoot is a generic consequence of delaying negative feedback. Since negative feedback is a "corrective," stabilizing influence, delaying its application allows the system to "get away" from the control, thereby exceeding the target equilibrium state.*

By "generic consequence," we just mean an outcome that is characteristic of the situation, independent of details.

This is a pretty easy concept to understand. The logistic curve of Figure 3.7 first accelerates, then briefly coasts before decelerating to arrive smoothly at a target. Following an example from [1], it is much like a car starting from rest by accelerating before applying the brakes to gently come to a stop when the bumper barely kisses a brick wall. The driver is operating a negative feedback loop: seeing/sensing the proximity to the wall and slowing down accordingly. The closer to the wall, the slower the driver goes until lightly touching the wall. Now imagine delaying the feedback to the driver by applying a blindfold and giving voice descriptions of the proximity to the wall, so that decisions about how much to brake are based on conditions from a delayed communication process. Obviously, the driver will crash into the wall if the feedback is delayed, unless slowing down the whole process dramatically. Likewise, if the negative consequences—signals that we need to slow down population growth—arrive decades after the act of producing more humans, we can expect to exceed the "natural" limit, Q—a condition called overshoot.

[1]: Meadows et al. (1972), *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind*

Another example of feedback delay leading to overshoot: let's say you are holding down the space bar and trying to position the cursor in the middle of the screen. But your connection is lagging and even though you release the space bar when you *see* the cursor reach the middle, it keeps sailing past due to the delay: overshooting.

Example 3.2.4 We did not detail the mechanisms of negative feedback operating on the deer population in Example 3.2.3 that act to stabilize the population at Q, but to illustrate how delayed negative feedback produces overshoot, consider predation as one of the operating forces. To put some simple numbers on it, let's say that steady state can support one adult (hunting) mountain lion for every 50 deer. Initially, when the population was 100 deer, this means two predators. When the deer population reaches $Q = 840$, we might have ~17 predators. But it takes time for the predators to react to the growing number of prey, perhaps taking a few years to produce the requisite number of hunting adults. Lacking the full complement of predators, the deer population will sail past the 840 mark until the predator population rises to establish the ultimate balance. In fact, the predators will likely also exceed their steady population in a game of catch-up that leads to oscillations like those seen in Figure 3.8.

We can explore what happens to our logistic curve if the negative feedback is delayed by various amounts. Figure 3.8 gives a few examples of overshoot as the delay increases. To avoid significant overshoot, the delay (τ) needs to be smaller than the natural timescale governing the problem: $1/r$, where r is the rate in Eqs. 3.5 and 3.6. In our deer example using $r = 0.5$, any delay longer than about 2 years causes overshoot. For more modest growth rates (human populations), relevant delays are in decades (see Box 3.1).

Figure 3.8: Feedback delay generally results in overshoot and oscillation, shown for various delay values, τ. The black curve ($\tau = 0$) is the nominal no-delay logistic curve. As the delay increases, the severity of overshoot increases. Delays are explored in increments of 0.5 times the characteristic timescale of $1/r$ (using $r = 0.5$ here to match previous examples, so that a delay of $\tau = 1.5/r$ equates to 3 time units on the graph, for instance). The delay durations are also indicated by bar lengths in the legend.

Eventually all the curves in Figure 3.8 converge to the steady state value of 1.0,[15] but human population involves complexities not captured in this bare-bones mathematical model.[16] All the same, the generic phenomenon of overshooting when negative feedback is delayed is a robust attribute, even if the oscillation and eventual settling does not capture the future of human population well.

15: ... meaning that population P arrives at Q

16: For instance, a dramatic overshoot and collapse could be disruptive enough to take out our current infrastructure for fossil-fuel-aided agriculture so that the Q value essentially resets to some lower value.
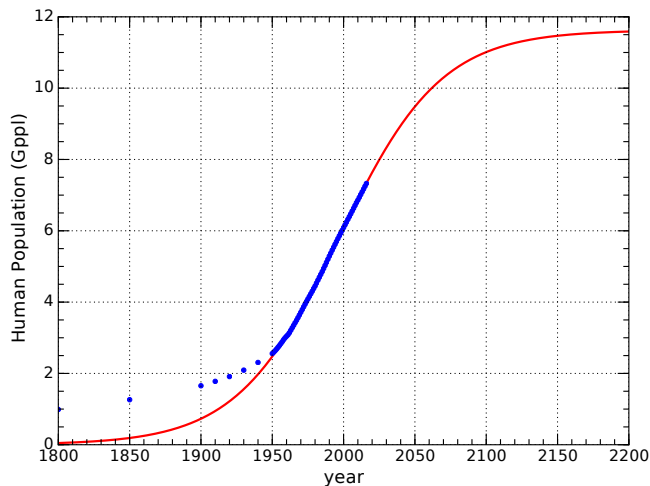
Figure 3.9: Human population data points (blue) and a logistic curve (red) that represents the best fit to data points from 1950 onward. The resulting logistic function has $Q \approx 12$ Gppl, $r = 0.028$, and a midpoint at the year 1997. The actual data sequence has a sudden bend at 1950 (Green Revolution?) that prevents a suitable fit to a larger span of data. In other words, the actual data do not follow a single logistic function very well, which is to be expected when conditions change suddenly (energy and technology, in this case) [14, 15].

Box 3.1: Will Human Population Overshoot?

Are humans in danger of population overshoot? What is our r value? It is tempting to take $r = 0.01$ corresponding to the present 1% growth rate. This would imply that any delay shorter than 100 years will not produce significant overshoot, which seems reassuring. But if human population is following a logistic curve rather than an exponential, resource availability is already exerting a moderating influence, now appearing to be in the linear "cruise" phase roughly

halfway to the limiting value. A fit to the data (Figure 3.9) suggests that $r \approx 0.028$, corresponding to a timescale of 36 years ($1/r$). This puts the overshoot-prone delay squarely into relevant timescales for human lifetimes, generations, and societal change—thus leaving the door open for an overshoot scenario.

3.2.2 Logistic Projection

As suggested by Figure 3.9, Human population is *not* following a strict logistic curve. If it were, the early period would look exponential at the ~2.8% rate corresponding to the best-fit logistic matching our recent trajectory, but growth was substantially slower than 2.8% in the past. Technology and fossil fuels have boosted our recent growth well beyond the sub-percent rates typical before ~1950. The point is that while reference to mathematical models can be extremely helpful in framing our thinking and exposing *robust, generic modes* of interest, *we should seldom take any mathematical model **literally***, as it likely does not capture the full complexity of the system it is trying to model. In the present case, it is enough to note that:

1. exponentials relentlessly drive toward infinity (ultimately unrealistic);
2. logistic curves add a sensible layer of reality, capping growth in some steady-state outcome;
3. other dynamical factors such as delays can prevent a smooth logistic function, possibly leading to overshoot; and
4. many other factors (medical and technological breakthroughs, war, famine, climate change) can muddy the waters in ways that could make the situation better or worse than simple projections.

3.3 Demographic Transition

Perhaps not surprisingly, the rate of a country's population growth is correlated to its wealth, as seen in Figure 3.10. An attractive path to reducing population growth would be to have poor countries slide down this curve to the right: becoming more affluent and transforming societal values and pressures accordingly to produce a lower net population growth rate.

Population growth happens when the birth rate exceeds the death rate.

Definition 3.3.1 *Birth rate, typically expressed in births per 1,000 people per year, minus **death rate** (also in deaths per 1,000 people per year) is the net population rate.[17] If the difference is positive, the population grows, and it shrinks if the difference is negative.*

17: This ignores immigration, which just shifts living persons around.
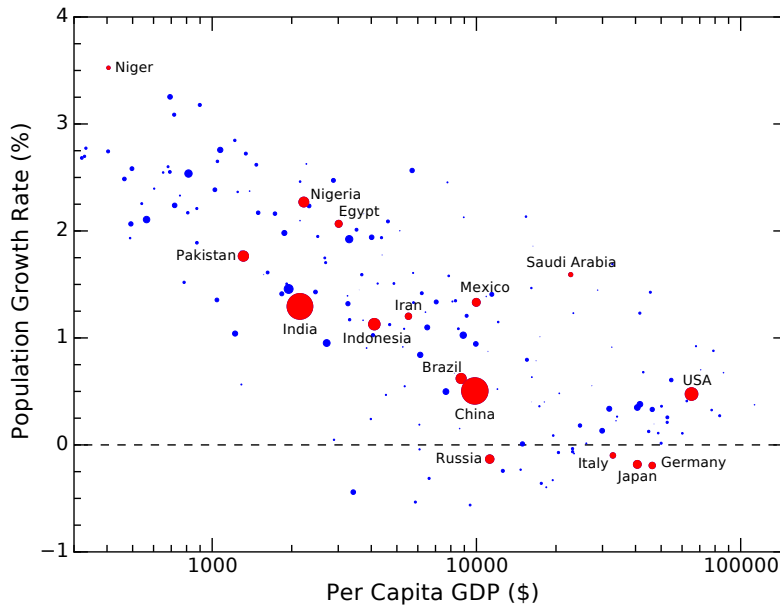
Figure 3.10: Net population rate, in percent, as a function of per-capita GDP. A clear trend shows wealthier countries having lower growth rates. A win–win solution would seem to present itself, in which everyone arrives at the lower right-hand side of this graph: more money for all and a stable population! Dot size (area) is proportional to population [6, 8, 19, 20].

Example 3.3.1 The U.S. has a birth rate of about 12 people per 1,000 per year, and a death rate of 8.1 people per 1,000 per year. The net rate is then roughly +4 per 1,000 per year, translating to 0.4% net growth.[18]

Niger has a birth rate of 46 per 1,000 per year and a death rate of 11, resulting in a net of positive 35, or 3.5%.

18: 4 per 1,000 is 0.4 per 100, which is another way to say 0.4 percent.

As conditions change, birth and death rates need not change in lock-step. Developed countries tend to have low birth rates *and* low death rates, balancing to a relatively low net population growth rate. Developing countries tend to have high death rates and even higher birth rates, leading to large net growth rates. Figure 3.11 depicts both birth rates and death rates for the countries of the world. A few countries (mostly in Europe) have slipped below the replacement line, indicating declining population.[19]

19: Note that immigration is not considered here: just birth rate and death rate within the country.

The general sense is that developed countries have "made it" to a responsible low-growth condition, and that population growth is driven by poorer countries. An attractive solution to many[20] is to bring developing countries up to developed-country standards so that they, too, can settle into a low growth rate. This evolution from a fast-growing poor country to a slow (or zero) growth well-off country is called the demographic transition.

20: . . . but unsolicited "preaching" to others

Definition 3.3.2 *The **demographic transition** refers to the process by which developing countries having high death rates and high birth rates adopt technologies, education, and higher standards of living that result in low death rates and low birth rates, more like advanced countries.*

In order to accomplish this goal, reduced death rates are facilitated by
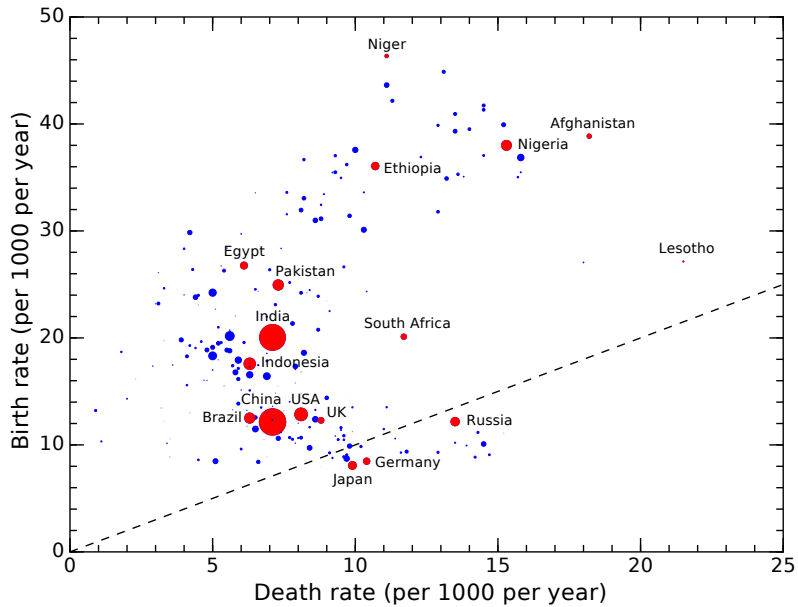
Figure 3.11: Birth rates and death rates for countries, where dot size is proportional to population. The diagonal line indicates parity between birth and death rates, resulting in no population growth. Countries above the line are growing population, while countries below are shrinking. A few countries fall a bit below this line [8, 19, 20].

introducing modern medicine and health services to the population. Reduced birth rates are partly in response to reduced infant mortality—eventually leading to fewer children as survival is more guaranteed. But also important is better education—especially among women in the society who are more likely to have jobs and be empowered to exercise control of their reproduction (e.g., more say in relationships and/or use of contraception). All of these developments take time and substantial financial investment.[21] Also, the economy in general must be able to support a larger and better-educated workforce. The demographic transition is envisioned as a transformation or complete overhaul, resulting in a country more in the mold of a "first-world" country.[22]

21: Better hospitals and schools are not free.

22: One may justifiably question whether this is the "correct" goal.

Figure 3.11 hints at the narrative. Countries are spread into an arc, one segment occupying a band between 5–10 deaths per 1,000 people per year and birth rates lower than 20 per 1,000 people per year. Another set of countries (many of which are in Africa) have birth rates above 20 per 1,000 per year, but also show higher death rates. The narrative arc is that a country may start near Lesotho, at high death and birth rates, then migrate over toward Nigeria as death rates fall (and birth rates experience a temporary surge). Next both death and birth rates fall and run through a progression toward Pakistan, India, the U.S., and finally the European steady state. Figure 3.12 schematically illustrates the typical journey.

The demographic transition receives widespread advocacy among Western intellectuals for its adoption, often coupled with the sentiment that it can't come soon enough. Indeed, the humanitarian consequences appear to be positive and substantial: fewer people living in poverty and hunger;

Figure 3.12: Schematic of how the demographic transition may play out in the space plotted in Figure 3.11. At points A and D, birth rates and death rates are equal, resulting in no population growth. Typically, death rates decline while birth rates increase (point B), and eventually death rates reach a floor while birth rates begin to fall (at C).

empowered women; better education; more advanced jobs; and greater tolerance in the society. It might even seem condemnable *not* to wish for these things for all people on Earth.

However, we need to understand the consequences. Just because we *want* something does not mean nature will comply. Do we have the resources to accomplish this goal? If we fail in pursuit of a global demographic transition, have we unwittingly unleashed even *greater* suffering on humanity by increasing the total number of people who can no longer be supported? It is possible that well-intentioned actions produce catastrophic results, so let us at least understand what is at stake. It may be condemnable *not to wish* for a global demographic transition, but failing to explore potential downsides may be equally ignoble.

3.3.1 Geographic Considerations

net population rate (annual births minus deaths per 1000 people)
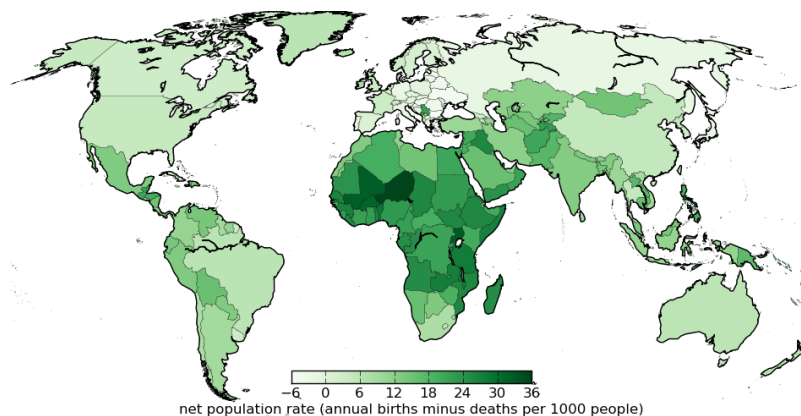
Figure 3.13: Net population growth rate by country: birth rate minus death rate per 1,000 people per year. The highest net growth (darkest shading) is Niger, in Saharan Africa [19, 20].

Figure 3.13 shows the net population rate (birth minus death rate) on a world map. Africa stands out as the continent having the largest net population growth rate, and has been the focus of much attention when discussing population dynamics.

But let us cast population rates in different countries in a new light. Referring to Figure 3.13, it is too easy to look at Niger's net population rate—which is about ten times higher than that of the U.S. (see Example 3.3.1)—and conclude that countries similar to Niger present a greater risk to the planet in terms of population growth. However, our perspective changes when we consider absolute population levels. Who cares if a country's growth rate is an explosive 10% if the population is only 73 people?[23]

23: But check back in 100 years!

Figure 3.14 multiplies the net rate by population to see which countries contribute the most *net new people* to the planet each year, and Table 3.3 lists the top ten. Africa no longer appears to be the most worrisome region in this light.[24] India is the largest people-producing country at present, adding almost 18 million per year. Far behind is China, in second

24: Although, the continent as a whole accounts for 35% of the total added population each year.

place. The U.S. adds about 1.6 million per year, a little beyond the top ten. This exercise goes to show that context is important in evaluating data.

Country	Population (millions)	Birth Rate	Death Rate	Annual Millions Added
India	1,366	20.0	7.1	17.7
China	1,434	12.1	7.1	7.2
Nigeria	201	38.0	15.3	4.6
Pakistan	216	24.9	7.3	3.8
Indonesia	271	17.6	6.3	3.1
Ethiopia	112	36.1	10.7	2.8
Bangladesh	163	20.2	5.6	2.3
Philippines	108	24.2	5.0	2.1
Egypt	100	26.8	6.1	2.1
DR Congo	87	36.9	15.8	1.8
Whole World	7,711	19.1	8.1	86

Table 3.3: Top ten populators [8, 19, 20], in terms of absolute number of people added to each country. Birth rates and death rates are presented as number per 1,000 people per year. These ten countries account for 55% of population growth worldwide.

Adding another relevant perspective, when one considers that the per-capita energy consumption in the United States is more than 200 times that of Niger,[25] together with the larger U.S. population, we find that the *resource impact* from births is almost 400 times higher for the U.S. than for Niger.[26] On a per capita basis, a citizen of the U.S. places claims on future resources at a rate 28 times higher than a citizen of Niger via population growth.[27] On a finite planet, the main reason we *care* about population growth is in relation to limited resources. Thus from the resource point of view, the problem is not at all confined to the developing world. Table 3.4 indicates how rapidly the top ten countries are creating energy demand (as a proxy to resource demands in general) based on population growth alone. Figure 3.15 provides a graphical perspective of the same (for all countries). For reference, one gigawatt (GW) is the equivalent of a large-scale nuclear or coal-fired power plant. So China, the U.S., and India each add the equivalent of 10–20 such plants per year just to satisfy the demand created by population growth.[28]

25: The average American rate of energy use is 10,000 W vs. 50 W for Niger.

26: In other words, for every additional kilogram of coal, steel, or whatever required by Niger's added population, the U.S. will require 400 kg of the same to satisfy its population growth.

27: 28 is smaller than 400 by the ratio of populations in the two countries.

28: This does not even consider rising standards placing additional burdens.

Country	Population (×10⁶)	Annual Growth (×10⁶)	Per Capita Power (W)	Power Added Annually (GW)	Power Added Per Citizen (W)
China	1,434	7.2	2,800	20.2	14
United States	329	1.6	10,000	15.6	48
India	1,366	17.7	600	10.5	8
Saudi Arabia	34	0.54	10,100	5.5	160
Iran	83	1.0	4,300	4.3	52
Mexico	128	1.7	2,000	3.3	26
Indonesia	271	3.1	900	2.8	10
Brazil	211	1.3	2,000	2.7	13
Egypt	100	2.1	1,200	2.5	25
Turkey	83	0.85	2,100	1.8	21
Whole World	7,711	86	2,300	143	18.4

Table 3.4: Top ten countries for growth in energy demand. Populations are in millions. Power is in Watts or 10^9 W (GW). The power added annually is the absolute increase in demand *due to population growth*, and is a proxy for resource demands in general. The last column provides some measure of an individual citizen's share of the responsibility in terms of increasing pressure on resources. The top three contributors to new power demand via population growth alone (China, the U.S., and India) account for about a third of the global total. [7, 8, 19, 20]
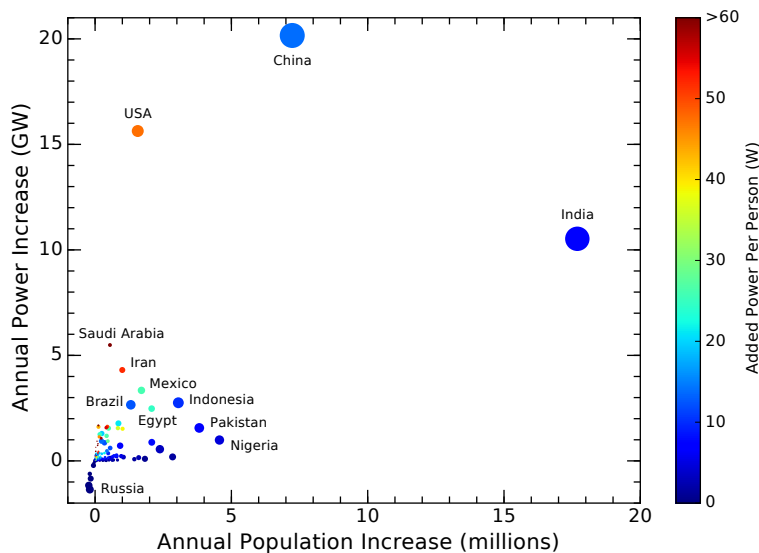
Figure 3.15: Graphical representation of Table 3.4, for all countries. Dots, whose size is proportional to population, indicate how many people are added per year, and how much additional energy demand is created as a consequence. Color indicates the added population-growth-driven power demand an individual citizen is responsible for generating each year as a member of the society. Negative cases (contracting) include Russia, Japan, Germany, and Ukraine [7, 8, 19, 20].

The last column in Table 3.4 is the per-citizen cost, meaning, for instance that each person in the U.S. adds about 50 Watts per year of energy demand via the country's net population growth rate.[29] In this sense, the last column is a sort of "personal contribution" an individual makes to the world's resource demands via net population rates and consumption rates in their society. Those having high scores should think twice about assigning blame externally, and should perhaps tend to their own house, as the saying goes.

29: A citizen of Niger, by comparison, only adds 1.7 W of demand per year on energy resources via population growth.

Before departing this section, let us look at continent-scale regions rather than individual countries in terms of adding people and resource demands. Table 3.5 echoes similar information to that in Table 3.4, in modified form. What we learn from this table is that Asia's demands are commensurate with their already-dominant population; North America creates the next largest pressure despite a much smaller population;

Country	Population (%)	Annual Growth (%)	Per Capita Power (W)	Power Added Annually (%)	Power Added Per Citizen (W)
Asia	59.7	55.1	1,800	60.5	18.9
N. America	7.6	5.5	7,100	23.0	56.1
Africa	16.9	34.7	500	9.9	10.8
S. America	5.5	4.4	2,000	5.4	18.1
Oceania	0.5	0.5	5,400	1.5	49.5
Europe	9.7	−0.1	4,900	−0.3	−0.6
Whole World	7,711 M	86 M	2,300	143 GW	18.4

Table 3.5: Population pressures from regions of the world, ranked by added power demand. Some of the columns are expressed as percentages of the total. The bottom row has totals in millions of people or total GW in place of percentages. [7, 8, 19, 20]

Africa is significant in terms of population growth, but constitutes only 10% of resource pressure at present. Finally, Europe holds 10% of the globe's people but lays no claim on added resources via population growth, resembling the target end-state of the demographic transition.[30]

30: Note that European countries are nervous about their decline in a growing, competitive world.

3.3.2 Cost of the Demographic Transition

A final point relates to the trajectory depicted in Figure 3.12 for demographic transitions: death rate decreases first while birth rates remain high—or rise even higher—before starting to come down. An example sequence is illustrated in Figure 3.16: initially the rates are high (at r_1), and the same (resulting in steady population); then the death rate transitions to a new low rate (r_2) over a time T; and the birth rate begins to fall some time τ later before matching the death rate and stabilizing population again. The yellow-shaded area between the curves indicates the region where birth rate exceeds death rate, leading to a net population growth (a surge in population).

The amount of growth in the surge turns out to be proportional to the exponential of the area between the curves. For this trapezoid cartoon, the area is just the base (τ) times the height (rate difference), so that the population increase looks like $e^{(r_1-r_2)\tau}$, where r_1 is the initial rate per year and r_2 is the final rate. The actual curves may take any number of forms, but the key point is that delayed onset of birth rate decrease introduces a population surge, and that magnitude of the surge grows as the area between the curves increases.

Figure 3.16: Schematic demographic transition time sequence.

Note in the cartoon example of Figure 3.16, the area between the curves is only dependent on the rate difference (height) and the delay, τ. The time it takes to complete the transition, T, is irrelevant, as the area of the parallelogram is just the base times the height. Thus the population surge associated with a demographic transition is primarily sensitive to the rate difference and the delay until birth rate begins to decline.

Example 3.3.2 If we start at a birth/death rate of 25 per 1,000 per year ($r_1 = 0.025$), end up at 8 ($r_2 = 0.008$; verify that these numbers are reasonable according to Figure 3.11), and have a delay of $\tau = 50$ years for the birth rate to start decreasing, we see the population increasing by a factor of

$$e^{(r_1-r_2)\tau} = e^{(0.025-0.008)\cdot50} = e^{0.85} = 2.34.$$

> This means that the population more than doubles, or increases by 134%.

So to effect a demographic transition means to increase the population burden substantially. Meanwhile, the transitioned population consumes resources at a greater rate—a natural byproduct of running a more advanced society having better medical care, education, and employment opportunities. Transportation, manufacturing, and consumer activity all increase. The net effect is a double-whammy: the combined impact of a *greater* population using *more* resources per capita. The resource impact on the planet soars.

The pertinent question is whether the Earth is prepared to host a dramatic increase in resource usage. Just because we might find appealing the idea that all countries on Earth could make it through the demographic transition and live at a first-world standard *does not mean nature has the capacity to comply*. The U.S. per-capita energy usage is roughly five times the current global average. To bring 7 billion people to the same standard would require five times the current scale. Completion of a global demographic transition would roughly double the current world population so that the total increase in energy would be a factor of ten. The blue-dashed projection in Figure 3.17 looks rather absurd as an extension of the more modest—but still rather remarkable—energy climb to date. As we are straining to satisfy current energy demand, the "amazing dream" scenario seems unlikely to materialize.

Energy in this context is a proxy for other material resources. Consider the global-scale challenges we have introduced today: deforestation, fisheries collapse, water pressures, soil degradation, pollution, climate change, and species loss, for instance. What makes us think we can survive a global demographic transition leading to a consumption rate many times higher than that of today? Does it not seem that we are already approaching a breaking point?

If nature won't let us realize a particular dream, then is it morally responsible to pursue it? This question becomes particularly acute if the very act of *pursuing* the dream increases the pressure on the system and *makes failure even more likely*. Total suffering might be maximized if the population builds to a point of collapse. In this sense, we cleverly stack the most possible people into the stadium to witness a most spectacular event: the stadium's collapse—which only happened because we packed the stadium. You see the irony, right?

The drive to realize a global demographic transition is strong, for the obvious set of reasons discussed above (improved quality of life, educational opportunity, greater tolerance, dignity, and fulfillment). Challenging the vision may be an uphill battle, since awareness about resource limits is not prevalent. This may be an example of the natural human tendency to extrapolate: we have seen the benefits of the demographic transition in many countries over the last century, and may expect this trend to
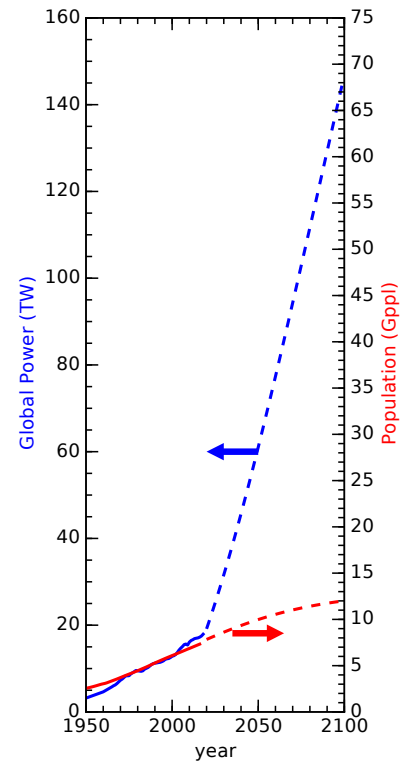
Figure 3.17: What our energy demand would have to do (blue-dashed line) if the growing global population (here projected as a red-dashed logistic curve) grew its per-capita energy consumption to current U.S. standards by the year 2100 (a factor-of-five increase). Historical energy and population are represented as solid curves. The departure from past reality would have to be staggering [15, 16].

continue until all countries have completed the journey. But bear in mind that earlier successes transpired during times in which global resource availability was not a major limitation. If conditions change, and we reach a "full" earth, past examples may offer little relevant guidance.

3.4 Touchy Aspects

3.4.1 Population Discussions Quickly Get Personal

Some of the decisions we make that translate into impact on our physical world are deeply personal and very difficult to address. No one wants to be told what they should eat, how often they should shower, or what temperature they should keep their dwelling. But the touchiest of all can be reproduction. It can be tricky to discuss population concerns with someone who has kids. Even if not intentional, it is too easy for the topic to be perceived as a personal attack on their own choices. And we're not talking about choices like what color socks to wear. Children are beloved by (most) parents, so the insinuation that having children is bad or damaging quickly gets tangled into a sense that their "angel" is being attacked—as is their "selfish" decision to have kids (see Box 3.2). The disconnect can be worse the larger the number of kids someone has. Couples having two kids take some solace in that they are exercising net-zero "replacement."

Having two kids is not a *strict* replacement, in that parents and children overlap (double-occupancy) on Earth. But the practice is at least consistent with a steady state.

One common side-step is to focus attention on the high birthrates in other countries, so that the perceived fault lies elsewhere. As pointed out above, if stress on the planet—and living within our means—is what concerns us, undeveloped countries are not putting as much pressure on global resources as many of the more affluent countries are. So while pointing elsewhere offers a bit of a relief, and is a very natural tendency, it does not get the whole picture.

The overall point is to be aware of the sensitive nature of this topic when discussing with others. Making someone feel bad about their choices—even if unintentionally—might *in rare cases* result in an appreciation and greater awareness. But it is *more likely* to alienate a person from an otherwise valuable perspective on the challenges we face.

> **Box 3.2: Which is More Selfish?**
>
> Parents, many of whom sacrifice dearly in raising kids—financially, emotionally, and in terms of time investment—understandably view their tireless commitment as being selfless: they often give up their own time, comfort, and freedom in the process. It is understandable, then, that they may view those not having kids as being selfish: the opposite of selfless. But this can be turned on its head. Why,

exactly, did they decide to have kids and contribute to the toll on our planet? It was their choice (or inattention) that placed them in parental roles, and the entire planet—not just humans—pays a price for their decision, making it seem a bit selfish.[31] In the end, almost any decision we make can be called selfish, since we usually have our own interests at least partly in mind. So it is pointless to try assigning more or less selfishness to the decision to have kids or not to have them. But consider this: if the rest of the Earth—all its plants and creatures—had a say, do you think they would vote for adding another human to the planet? Humans have the *capacity*, at least, to consider a greater picture than their own self interest, and provide representation to those sectors that otherwise have no rights or voice in our highly human-centric system.

31: Reasons for having children are numerous: genetic drive; family name/tradition; labor source; care in old age; companionship and love (projected onto not-yet-existing person). Note that adoption can also satisfy many of these aims without contributing additional population.

3.4.2 Population Policy

What could governments and other organizations do to manage population? Again, this is touchy territory, inviting collision between deeply personal or religious views and the state. China initiated a one-child policy in 1979 that persisted until 2015 (exceptions were granted depending on location and gender). The population in China never stopped climbing during this period, as children born during prior periods of higher birth rates matured and began having children of their own—even if restricted in number. The population curve in China is not expected to flatten out until sometime in the 2030–2040 period.[32] Such top-down policies can only be enacted in strong authoritarian regimes, and would be seen as a severe infringement on personal liberties in many countries. Religious belief systems can also run counter to deliberate efforts to limit population growth. In addition, shrinking countries are at a competitive disadvantage in global markets, often leading to policies that incentivize having children.

The net effect of the various exceptions meant that for most of this period half of Chinese parents could have a second child.

32: This is another case of delay in negative feedback resulting in overshoot.

One striking example of rarely-achieved sustainable population control comes from the South Pacific island of Tikopia [21]. Maintaining a stable population for a few thousand years on this small island involved not only adopting food practices as close to the island's natural plants as possible, but also invoking strict population controls. The chiefs in this egalitarian society routinely preached zero population growth, and also prevented overfishing. Strict limits were placed on family size, and cultural taboos kept this small island at a population around 1,200.[33] Population control methods included circumventing insemination, abortion, infanticide, suicide, or "virtual suicide," via embarking on dangerous sea voyages unlikely to succeed. In this way, the harshness of nature was replaced by harsh societal norms that may seem egregious to us. When Christian missionaries converted inhabitants in the twentieth century, the practices of abortion, infanticide and suicide were quenched and the population

[21]: Diamond (2005), *Collapse: How Societies Choose to Fail or Succeed*

33: A group size of 1,200 is small enough to prevent hiding irresponsible actions behind anonymity.

began to climb, leading to famine and driving the population excess off the island.

In the end, personal choice will be important, if we are to tame the population predicament. Either conditions will be too uncertain to justify raising children, or we adopt values that place short term personal and human needs into a larger context concerning ecosystems and long-term human happiness.

Nature, it turns out, is indifferent to our belief systems.

3.5 Upshot: Everything Depends on Us

We would likely not be discussing a finite planet or limits to growth or climate change if only one million humans inhabited the planet, even living at United States standards. We would perceive no meaningful limit to natural resources and ecosystem services. Conversely, it is not difficult to imagine that 100 billion people on Earth would place a *severe* strain on the planet's ability to support us—especially if trying to live like Americans—to the point of likely being impossible. If we had to pick a single parameter to dial in order to ease our global challenges, it would be hard to find a more effective one than population.

A common knee-jerk reaction to a statement that we would be better off with a smaller population is to demand an answer to who, exactly, we propose eliminating. Ideally, we should be able to discuss an important topic like population without resorting to accusations of advocating genocide. Of course we need to take care of those already alive, and address the problem via future reproductive choices.

Maybe we need not take any action. Negative feedback *will* assert itself strongly once we have gone too far—either leading to a steady approach to equilibrium or producing an overshoot/collapse outcome. Nature *will* regulate human population one way or another. It just may not be in a manner to our liking, and we have the opportunity to do better via awareness and choice.

Very few scholars are unconcerned about population pressure. Yet the issue is consistently thorny due to both its bearing on personal choice and a justified reluctance to "boss" developing nations to stop growing prior to having an opportunity to naturally undergo a demographic transition for themselves. Conventional thinking suggests that undergoing the demographic transition ultimately is the best solution to the population problem. The question too few ask is whether the planet can support this path for all, given the associated population surge and concomitant demand on resources. If not, pursuit of the transition for the world may end up causing more damage and suffering than would otherwise happen due to increased populations competing for dwindling resources.

3.6 Problems

1. The text accompanying Figure 3.1 says that Earth currently hosts the equivalent of 25 billion nineteenth-century-level energy consumers. If we had maintained our nineteenth-century energy appetite but

followed the same population curve, what would our global power demand be today, in TW? How does this compare to the actual 18 TW we use today?

2. Notice that on logarithmic plots,[34] factors of ten on the logarithmic axis span the *same distance*. This applies for *any* numerical factor—not just ten.[35] Shorter (minor) tick marks between labeled (major) ticks multiply the preceding tick label by 2, 3, 4, 5, 6, 7, 8, 9. The graphic below illustrates the constant distance property for a factor of two.[36] Now try a different multiplier (not 2 or 10), measuring the distance between tick marks, and report/draw how you graphically verified that your numerical factor spans the same distance no matter where you "slide" it on the axis.
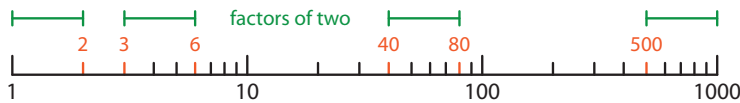
3. Looking at Figure 3.3, if humans had continued the slow growth phase characteristic of the period until about 1700, what does the *graph* suggest world population would be today, approximately, if the magenta line were extended to "now?"[37] Put the answer in familiar terms, measured in millions or billions, depending on what is most natural.[38]

4. Looking at Figure 3.4, if humans had continued the moderate growth phase characteristic of the period from the year 1000 to 1700, what does the *graph* suggest world population would be today, approximately, if the magenta line were extended to "now?"

5. If we were to continue a 1% population growth trajectory into the future, work out how many years it would take to go from 7 billion people to 8 billion, and then from 8 billion to 9 billion.

6. At present, a billion people are added to the planet every 12 years. If we maintain a 1% growth rate in population, what will global population be in 2100 (use numbers in Table 3.2 as a starting point), and how quickly will we add each new billion at that point?

7. A decent approximation to recent global population numbers using a logistic function is[39]

$$P = \frac{14}{1 + \exp[-0.025 \times (year - 2011)]}$$

in billions of people. First verify that inserting the year 2011 results in 7 (billion), and then add a column to Table 3.2 for the "prediction" resulting from this function. Working back into the past, when does it really start to deviate from the truth, and why do you

---

Hint: It is perfectly acceptable to hold a (preferably transparent) straight-edge up to a graph!

34: See, for example, Figures 3.3 and 3.4.

35: This is due to the property of logarithms that $\log\left(\frac{a}{b}\right) = \log a - \log b$. The property applies for any base, so $\log_{10}$ and ln behave the same way.

36: The green bars indicate that the same distance from 1 to 2 applies to 3–6, 40–80, and 500–1,000.

37: Determine graphically (may need to zoom in). See Problem 2 and the associated graphic to better understand how the tick marks work.

38: I.e., don't say 0.01 billion if 10 million is more natural, or 8,000 million when 8 billion would do.

See margin notes for Problem 3.

39: See Eq. 3.6.

think that is (hint: what changed so that we invalidated a single, continuous mathematical function)?

8. Using the logistic model presented in Problem 7, what would the population be in the year 2100? How does this compare to the exponential result at 1% growth as in Problem 6?

9. Which of the following are examples of positive feedback, and which are examples of negative feedback?

    a) a warming arctic melts ice, making it darker, absorbing more solar energy
    b) if the earth's temperature rises, its infrared radiation to space increases, providing additional cooling
    c) a car sits in a dip; pushing it forward results in a backward force, while pushing it backward results in a forward force
    d) a car sits on a hill; pushing it either way results in an acceleration (more force, thanks to gravity) in that direction
    e) a child wails loudly and throws a tantrum; to calm the child, parents give it some candy: will this encourage or discourage similar behaviors going forward?

10. Think up an example from daily life (different from examples in the text) for how a delay in negative feedback can produce overshoot, and describe the scenario.

11. Pick five countries of interest to you not represented in any of the tables in this chapter and look up their birth rate and death rate [19, 20], then find the corresponding dot on Figure 3.11, if possible.[40] At the very least, identify the corresponding region on the plot.

12. A country in the early stages of a demographic transition may have trimmed its death rate to 15 per 1,000 people per year, but still have a birth rate of 35 per 1,000 per year. What does this amount do in terms of net people added to the population each year, per 1,000 people? What rate of growth is this, in percent?

13. If the population of the country in Problem 12 is 20 million this year, how many people would we expect it to have next year? How many were born, and how many died during the year?

14. Figure 3.11 shows Egypt standing well above China in terms of excess birth rate compared to death rate.[41] Yet Table 3.3 indicates that China contributes a much larger annual addition to global population than does Egypt. Explain why. Then, using the first four columns in Table 3.3, replicate the math that produced the final column's entries for these two countries to reinforce your understanding of the interaction between birth and death rates and population in terms of absolute effect.

15. In a few clear sentences, explain why the maps in Figure 3.13

Comparison of this problem and Problem 6 highlights the difference the choice of mathematical model can make.

[19]: (2016), *List of Sovereign States and Dependent Territories by Birth Rate*
[20]: (2011), *List of Sovereign States and Dependent Territories by Mortality Rate*

40: Numbers may change from when the plot was made; population can help settle based on dot size.

41: ... much farther from dashed line

Show work and add one more decimal place to the answer as a way to validate that you did more than copy the table result.

and Figure 3.14 look so different, in terms of which countries are shaded most darkly?

16. Table 3.4 indicates which countries place the highest population-driven new demand on global resources using energy as a proxy. Which countries can American citizens regard as contributing more total resource demand? At the individual citizen-contribution level, what other citizens can Americans identify as being responsible for a greater demand on resources via population growth?

The point is that the U.S. is a major contributor to increased resource demand via population growth.

17. The last two columns in Table 3.4 were computed for this book from available information on population, birth and death rates, and annual energy usage for each country (as represented in the first four columns; references in the caption). Use logical reasoning to replicate the calculation that produces the last two columns from the others and report how the computation goes, using an example from the table.

Careful about 10^6 factors and GW = 10^9 W.

18. The bottom row of Table 3.4 is important enough to warrant having students pull out and interpret its content. What is world population, in billions? How many people are added to the world each year? What is the typical power demand for a global citizen (and how does it compare to the U.S.)? If a typical coal or nuclear plant puts out 1 GW of power, how many power-plant-equivalents must we add each year to keep up with population increase? And finally, how much power (in W) is added per global citizen each year due to population growth (and it is worth reflecting on which countries contribute more than this average)?

Some students may see this as free/easy points, but consider the value in internalizing the associated information.

19. If you were part of a global task force given the authority to make binding recommendations to address pressures on resources due to population growth, which three countries stand out as having the largest impact at present? Would the recommendations be the same for all three? If not, how might they differ?

20. Table 3.5 helps differentiate concerns over which region contributes pressures in raw population versus population-driven resource demand. By taking the ratio of population growth (in %) to population (as % of world population), we get a measure of whether a region is "underperforming" or "overperforming" relative to its population. Likewise, by taking the ratio of the added power (in %) to population, we get a similar measure of performance in resource demand. In this context, which region has the highest ratio for population pressure, and which region has the highest ratio for population-induced pressure on energy resources?

For instance, Oceania has a ratio of 1.0 for population growth (0.5% of population growth and 0.5% of global population), meaning it is not over- or under-producing relative to global norms. But in terms of power, it is 3 times the global expectation (1.5 divided by 0.5).

21. If a country starting out at 30 million people undergoes the demographic transition, starting at birth/death rates of 35 per 1,000 per year and ending up at 10 per 1,000 per year, what will the final population be if the delay, τ, is 40 years?

22. The set of diagrams below show five different time sequences on the left akin to Figure 3.16, labeled 1–5. The first four on the left have increasing τ (delay until birth rate begins falling), and the last increases birth rate before falling again. On the right are five trajectories in the birth/death rate space (like Figure 3.12), scrambled into a different order and labeled A–E.[42] Deduce how the corresponding trajectory for each time sequence would appear in the birth/death rate plot on the right, matching letters to numbers for all five.

42: Note that figures A. and E. differ only by whether the transition pauses (dwells) at the corner for some time.
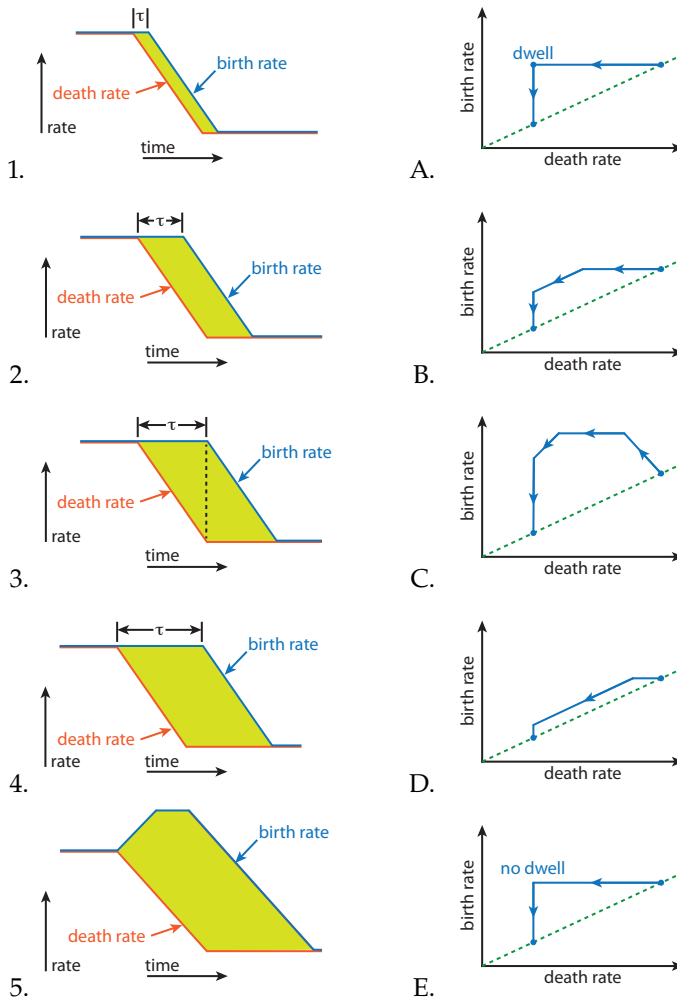
23. Referring to the figures for Problem 22 (and described within the same problem), which pair[43] corresponds to the largest population surge, and which pair produces the smallest? Explain your reasoning, consistent with the presentation in the text.

43: ...number and associated letter; not necessarily arranged next to each other

24. Referring to the figures for Problem 22 (and described within the same problem), which pair[44] is most similar to the actual trajectory we witness (i.e., Figure 3.11), and what does this say about the

44: ...number and associated letter; not necessarily arranged next to each other

population cost of the demographic transition in the context of Problem 23?

25. Considering Figure 3.11 in the context of a trajectory (as in Figure 3.12), would it appear that most countries in the world have begun the demographic transition? Have very few of them started? Is it about half-and-half? Justify your answer.

26. Express your view about what you learn from Figure 3.17. Do you sense that the prescribed trajectory is realistic? If so, justify. If not, what about it bothers you? What does this mean about the goal of bringing the (growing) world to "advanced" status by the end of this century? Are we likely to see this happen?

27. Make as compelling an argument as you can for why we should promote the demographic transition worldwide for those countries who have not yet "arrived" at the lower-right corner of Figure 3.10. What are the positive rewards?

28. Make as compelling an argument as you can for why pursuit of the demographic transition may be ill-advised and potentially create rather than alleviate hardship. What are the downsides?

29. List the pros and cons a young person without children might face around the decision to have a biological child of their own[45] Consider not only personal contexts, but external, global ones as well, and thoughts about the future as you perceive it. It does not matter which list is longer or more compelling, but it is an exercise many will go through at some point in life—although maybe not explicitly on paper.

45: Assume for the purpose of the question that it is biologically possible.

30. Do you think governments and/or tribal laws have any business setting policy around child birth policies? If so, what would you consider to be an acceptable form of control? If not, what other mechanisms might you propose for limiting population growth (or do you even consider that to be a priority or at all appropriate)?

4 Space Exploration vs. Colonization

This textbook assesses the challenges and limitations imposed upon us by living on a finite planet having finite resources. If harboring expectations that we will break out into a space-faring existence as a way to mitigate our earthly challenges, then it becomes harder for us to respond earnestly to information about where things are headed on Earth. This chapter is placed where it is to "close the exit" so that the content in the rest of the book might become more relevant and worth the investment to learn. Some of the sections in this chapter offer more of an author's perspective than might be typical for a textbook. Some may disagree with the case that is made, but consider that the burden of proof for a way of life unfathomably beyond our current means should perhaps fall to the enthusiasts.[1]

1: To quote Carl Sagan, extraordinary claims require extraordinary evidence.

4.1 Scale of Space

In the span of two hours, we can sit through a movie and "participate" in interstellar travel without getting tired. Let's step out of the entertainment (fiction) industry and come to terms with the physical scale of the *real* space environment.

Describing an analogous scale model of the solar system, galaxy, and universe—as we will do momentarily—is a fraught exercise, because in order to arrive at physical scales for which we have solid intuition (driving distance in a day?) we end up with inconceivably small (invisible) specks representing familiar objects like the earth. By the time we make Earth the size of something we can hold and admire, the scales become too big for easy comprehension. Figures 4.1 and 4.2 demonstrate how awkward or impossible correctly-scaled graphics are in a textbook.

The convention is to capitalize Earth when it is used as a proper *name*, and refer to the earth when it is an *object*. Similar rules apply to Moon and Sun.

Photo credit: NASA/Bill Anders from Apollo 8 [22].

Figure 4.1: Earth and Moon (far right) to scale. On this scale, the sun would be larger than the page and about 400 pages away. Mars would be 160 to 1,100 pages away. Since 1972, humans have not traveled beyond the black outline of the earth in this figure (600 km).

Sun   Earth (invisible)

Figure 4.2: Proving the point that textbooks are not conducive to correctly-scaled graphics of objects in space, by the time the Earth-Sun distance spans the page, Earth (on far right) is too small to be visible in print, at less than 1% the diameter of the orange sun at far left. The Earth–Moon distance is about the width of the arrow shaft pointing to Earth. Humans have *never* traveled more than the arrow shaft's width from Earth, and have not even gone 0.2% *that far* in about 50 years! Mars, on average, is farther from Earth than is the sun.

Let us first lay out some basic ratios that can help build suitable mental models at whatever scale we choose.

Table 4.1: Progression of scale factors.

Step	Factor
Earth diameter	(start)
Moon distance	30×
Sun distance	400×
Neptune distance	30×
Nearest Star	9,000×
Milky Way Center	6,000×
Andromeda Galaxy	100×
Universe Edge	6,000×

> **Definition 4.1.1** *Scale models of the universe can be built based on these approximate relations, some of which appear in Table 4.1 and Table 4.2:*
>
> 1. *The moon's diameter is one-quarter that of Earth, and located 30 earth-diameters (60 Earth-radii) away from Earth, on average (see Figure 4.1).*
> 2. *The sun's diameter is about 100 times that of Earth, and 400 times as far as the moon from Earth (see Figure 4.2).*
> 3. *Mars' diameter is about half that of Earth, and the distance from Earth ranges from 0.4 to 2.7 times the Earth–Sun distance.*
> 4. *Jupiter's diameter is about 10 times larger than Earth's and 10 times smaller than the sun's; it is about 5 times farther from the sun than is the earth.*
> 5. *Neptune orbits the sun 30 times farther than does Earth.*
> 6. *The Oort cloud[2] of comets ranges from about 2,000 to 100,000 times the Earth–Sun distance from the sun.*
> 7. *The nearest star[3] is 4.2 light years from us, compared to 500 light-seconds from Earth to the sun—a ratio of 270,000.*
> 8. *The Milky Way galaxy has its center about 25,000 light years away,[4] and is a disk about four times that size in diameter.*
> 9. *The next large galaxy[5] is 2.5 million light years away, or about 25 Milky Way diameters away.*
> 10. *The edge of the visible universe[6] is 13.8 billion light years away, or about 6,000 times the distance to the Andromeda galaxy.*

2: The Oort cloud marks the outer influence of the sun, gravitationally.

3: . . . Proxima Centauri

A light year is the *distance* light travels in a year.

4: That's 6,000 times the distance to the closest star.

5: . . . the Andromeda galaxy

6: The "edge" is limited by light travel time since the Big Bang (13.8 billion years ago), and is called our cosmic horizon. See Sec. D.1 (p. 392) for more.

We will construct a model using the set of scale relations in Definition 4.1.1, starting local on a comfortable scale.

We'll make Earth the size of a grain of sand (about 1 mm diameter). The moon is a smaller speck (dust?) and the diameter of its orbit would span the separation of your eyes. On this scale, the sun is 100 mm in diameter (a grapefruit) and about 12 meters away (40 feet). Mars could be anywhere from 4.5 meters (15 feet) to 30 meters (100 feet) away. Reflect

> As we build up our model, pause on each step to lock in a sense of the model: visualize it or even recreate it using objects around you!

Body	Symbol	Approx. Radius	Distance (AU)	Alt. Distance
Earth	$\oplus$	$R_\oplus \approx 6,400$ km	—	
Moon	$\mathbb{C}$	$\frac{1}{4}R_\oplus$	$\frac{1}{400}$	$60R_\oplus \approx 240R_{\mathbb{C}}$
Sun	$\odot$	$100R_\oplus$	1	$240R_\odot$
Mars	$\male$	$\frac{1}{2}R_\oplus$	0.4–2.7	
Jupiter	$\jupiter$	$10R_\oplus \approx \frac{1}{10}R_\odot$	4–6	
Neptune	$\neptune$	$4R_\oplus$	~30	
Proxima Centauri	—	$0.15R_\odot$	270,000	4.2 light years

Table 4.2: Symbols, relative sizes, and distances in the solar system and to the nearest star. An AU is an Astronomical Unit, which is the average Earth–Sun distance of about 150 million kilometers. The fact that both the sun and moon are 240 of their radii away from Earth is why they appear to be a similar size on the sky, leading to "just so" eclipses.

7: For this, picture a grain of sand sitting on the bridge of your nose representing the earth, and a speck of dust in front of one eye as the moon.

for a second that humans have never ventured farther from Earth than the moon, at 3 cm (just over an inch) in this scale.[7] Mars is outlandishly farther. Neptune is about four-tenths of a kilometer away (on campus at this scale), and the next star is over 3,000 km (roughly San Diego to Atlanta). So we've already busted our easy intuitive reckoning and we haven't even gotten past the first star. Furthermore, this was starting with the earth as a tiny grain of sand. We've only ever traveled two-finger-widths away from Earth on this scale,[8] and the next star is like going on a giant trip across the country. For apples-to-apples, compare how long it takes to walk a distance of two-finger-widths (3 cm) to the time it would take to walk across the U.S. The former feat of traveling to the moon was super-hard; the latter is comparatively impossible.

> Glance over to where Mars would be if the earth is a grain of sand on your nose.

8: The last time we went this far was 1972.

Box 4.1: When Will We Get There?

It took 12 years for Voyager 2 to get to Neptune, which is "in our back yard." The only spacecraft to date traveling fast enough to leave the solar system are the two Voyagers, the two Pioneers, and the New Horizons probe [23]. The farthest and fastest of this set is Voyager 1 at about 150 times the Earth–Sun distance after 43 years. The closest star is about 2,000 times farther. At its present speed of 17 km/s, it would reach the *distance* to the nearest star[9] in another 75,000 years.

The fastest spacecraft on record as yet is the Parker Solar Probe, which got up to a screaming 68.6 km/s, but only because it was plunging (falling) around the sun. Because it was so close to the sun, even this amount of speed was not enough to allow it climb out of the sun's gravitational grip and escape, as the five aforementioned probes managed to do. Even if Voyager 1 ended up with 70 km/s left over after breaking free of the solar system,[10] it would still take 20,000 years to reach the distance to the nearest star. Note that human lifetimes are about 200 times shorter.

Pushing a human-habitable spacecraft up to high speed is *immensely* harder than accelerating these scrappy little probes, so the challenges are varied and extreme. For reference, the Apollo missions to the very nearby Moon carried almost 3,000 tons of fuel [24], or about 80,000 times the typical car's gasoline tank capacity. It would take

[23]: (2020), *List of artificial objects leaving the Solar System*

9: It does not happen to be aimed toward the nearest star, however.

10: It only had 17 km/s left.

[24]: (2020), *Saturn V*

a typical car 2,000 years to spend this much fuel. Do you think the astronauts argued about who should pay for the gas?

Let's relax the scale slightly, making the sun a chickpea (garbanzo bean). Earth is now the diameter of a human hair (easy to lose), and one meter from the sun. The moon is essentially invisible and a freckle's-width away from the earth. The next star is now 300 km away (a 3-hour drive at freeway speed), while the Milky Way center is 1.5 million kilometers away. Oops. This is more than four times the *actual* Earth–Moon distance. We busted our scale *again* without even getting out of the galaxy.

So we reset and make the sun a grain of sand. Now the earth is 10 cm away and the next star is 30 km.[11] Think about space this way: the swarm of stars within a galaxy are like grains of sand tens of kilometers apart. On this scale, solar systems are bedroom-sized, composed of a brightly growing grain of sand in the middle and a few specks of dust (planets) sprinkled about the room.[12] It gets even emptier in the vast tracts between the stars. The Milky Way extent on this scale is still much larger than the actual Earth, comparable to the size of the lunar orbit.

In fairness, fuel requirements don't simply scale with distance for space travel, unlike travel in a car. Still, just getting away from Earth requires a hefty fuel load.

11: . . . a long day's walk

12: Even a solar system, which is a sort of local oasis within the galaxy, is mostly empty space.

Box 4.2: Cosmic Scales

It is not necessary to harp further on the vastness of space, but having come this far some students may be interested in completing the visualization journey.

As mind-bogglingly large as the solar system is, not to mention that it itself is dwarfed by interstellar distances, which in turn are minuscule compared to the scale of the galaxy, how can we possibly appreciate the largest scales in the universe? Let's start by making galaxies manageable. If galaxies are like coins (say a U.S. dime at approximately 1 cm diameter), they are typically separated by meter-like scales. The edge of the visible universe (see Sec. D.1; p. 392) would be only 1.5 km away. Finally, the picture is easy to visualize: coins as galaxies separated by something like arm's length and extending over an area like the center of a moderately-sized town. We can even imagine the frothy, filamentary arrangement of these galaxies, containing house-sized (5–50 m) voids empty of coins (galaxies). See Figure 4.3 for a visual explanation.

But penetrating the nature of the individual galaxies (coins, in the previous example scale) is extremely daunting: they are mostly empty space, and by the time we reduce the galaxy to a manageable scale (say 10 km, so that we can picture the whole thing as city-sized), individual stars are a few tenths of a meter apart and only about 50 atoms across (roughly 10 nm). Cells and bacteria are about 100–1,000 times larger than this. So it's nearly impossible to conceive of the scale of the galaxy while simultaneously appreciating the sizes of the stars and just how much space lies between.
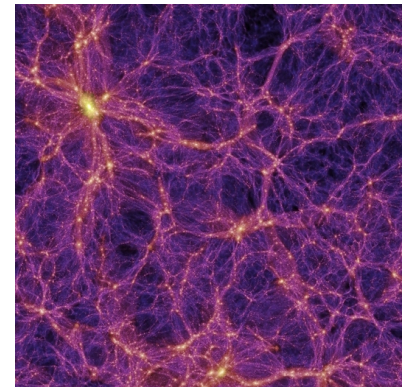
Figure 4.3: Galaxies are actually distributed in a frothy foam-like pattern crudely lining the edges of vast bubbles (voids; appearing as dark regions in the image). This structure forms as a natural consequence of gravity as galaxies pull on each other and coalesce into groups, leaving emptiness between. This graphic shows the bubble edges and filaments where galaxies collect. The larger galaxies are bright dots in this view—almost like cities along a 3-dimensional web of highways through the vast emptiness. From the Millennium Simulation [25].

Given the vastness of space, it is negligent to think of space travel as a "solution" to our present set of challenges on Earth—challenges that operate on a much shorter timescale than it would take to muster any meaningful space presence. Moreover, space travel is enormously expensive energetically and economically (see Table 4.3). As we find ourselves competing for dwindling one-time resources later this century, space travel will have a hard time getting priority, except in the context of escapist entertainment.[13]

Table 4.3: Approximate/estimated costs, adjusted for inflation (M = million; B = billion). [26–29]

Effort	Cost
Apollo Program	$288B
Space Shuttle Launch	$450M
Single Seat to ISS	$90M
Human Mars Mission	$500B

13: . . . which is great stuff as long as it does not dangerously distort our perceptions of reality

4.2 The Wrong Narrative

Humans are not shy about congratulating themselves on accomplishments, and yes, we have done rather remarkable things. An attractive and common sentiment casts our narrative in evolutionary terms: fish crawled out of the ocean, birds took to the air, and humans are making the next logical step to space—continuing the legacy of escaping the bondage of water, land, and finally Earth. It is a compelling tale, and we have indeed learned to escape Earth's gravitational pull and set foot on another body.

But let's not get ahead of ourselves. Just because we can point to a few special *example* accomplishments does not mean that such examples presage a new normal. A person can climb Mt. Everest, but it is not ever likely to become a commonplace activity. We can build a supersonic passenger airplane for trans-atlantic flight, but it does not mean it will be viable to sustain.[14] One can set up a backyard obstacle course for squirrels and generate viral videos, but the amusing demonstration does not signal a "new normal" in backyard design. We need to separate the *possible* from the *practical*. The moon landings might then be viewed as a nifty stunt—a demonstration of capability—rather than a path to our future. We encountered similar arguments in Chapter 2 in relation to decoupling: just because it *can* happen in certain domains of the economy does not mean that the entire economy can decouple and "defy gravity."

14: . . . or even still available today (see the story of the Concorde; Box 2.2; p. 22)

The attractive evolutionary argument misses two critical facets of reality. When fish crawled out of the sea, they escaped predation (as the first animals on land) and found new food sources free of competition. That's a win-win: less dangerous, more sustenance.[15] Likewise, when birds took flight (or we could discuss insects, which beat the birds to it), it was a similar story: evade ground-based predators who could not fly, and access a whole new menu of food—another win-win.

15: Evolution works on exploiting advantages, favoring wins and letting the "lose" situations be out-competed.

Going to space could easily be cast as a lose-lose. It's an extremely hostile environment offering no protection or safe haven,[16] and there's nothing to eat.[17] Think about it: where would you go to grab a bite in our solar system at present, outside of Earth? And a solar system is an absolute *oasis* compared to the vast interstellar void. The two factors that jointly

16: Earth **is** the safe haven.

17: Amusingly, consider that no cheeseburgers have ever smacked into a space capsule.

promoted evolution onto land and into the air *will not operate* to "evolve" us into space. It's a much tougher prospect. Yes, it could be possible to grow food on a spacecraft or in a pressurized habitat, but then we are no longer following the evolutionary meme of stumbling onto a good deal.

One "win" some imagine from space is access to materials. Yet Earth is already well-stocked with elements from the Periodic Table, and the economics of retrieval from space are prohibitive in any case.

Box 4.3: Accomplishments in Space

Before turning attention to what we have not yet done in space, students may appreciate a recap of progress to date. The list is by no means exhaustive, but geared to set straight common misconceptions.

- ► 1957: Sputnik (Soviet) is the first satellite to orbit Earth.
- ► 1959: Luna 3 (Soviet; unmanned) reaches the moon in a fly-by.
- ► 1961: Yuri Gagarin (Soviet), first in space, orbits Earth once.
- ► 1965: Alexei Leonov (Soviet) performs the first "space walk."
- ► 1965: Mariner 4 (U.S.; unmanned) reaches Mars.
- ► 1968: Apollo 8 (U.S.) puts humans in lunar orbit for the first time.
- ► 1969: Apollo 11 (U.S.) puts the first humans onto the lunar surface.
- ► Pause here to appreciate how fast all this happened. It is easy to see why people would assume that Mars would be colonized within 50 years. Attractive narratives are hard to retire, even when wrong.
- ► 1972: Apollo 17 (U.S.) is the last human mission to the moon; only 12 people have walked on another solar system body, the last about 50 years ago.
- ► 1973–now: as of this writing (2020), humans have not ventured farther than about 600 km from Earth's surface (called low earth orbit, or LEO; see Figure 4.4) since the end of the Apollo missions.
- ► 1981–2011: U.S. operates the Space Shuttle, envisioned to make space travel routine. After 135 launches (two ending in catastrophe), the shuttle was retired, leaving the U.S. with no human space launch capacity.
- ► 1998–now: The International Space Station (ISS) [30] provides an experimental platform and maintains a presence in space. It is only 400 km from Earth's surface (4-hour driving distance), and—despite its misleading name—is not used as a space-port hub for space travel. It *is* the destination.
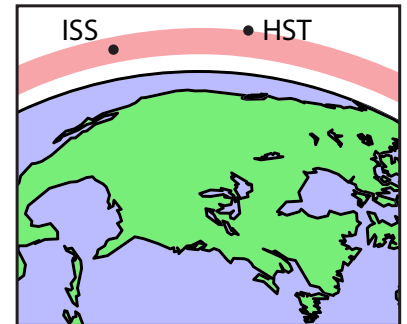
Figure 4.4: The pink band indicates the farthest humans have been from the surface of the earth for the last ∼ 50 years. The Hubble Space Telescope (HST) orbits at the top of this band at 600 km altitude, and the International Space Station in the middle at 400 km. Beyond the thin black line outlining the globe, Earth's atmosphere is too tenuous to support life.

[30]: (2020), *International Space Station*

4.3 A Host of Difficulties

If undeterred by the vast emptiness, hostile conditions, or lack of human-supporting resources in space, then maybe it's because you believe

human ingenuity can overcome these challenges. And this is correct to a degree. We *have* walked on one other solar system body.[18] We *have* had individuals spend a year or so in earth orbit. Either these represent first baby steps to a space future, or just rare feats that we can pull off at great effort/expense. How can we tell the difference?

Box 4.4: Comparison to Backpacking

The way most people experience backpacking is similar to how we go about space exploration: carry on your back all the food, clothing, shelter, and utility devices that will be needed for a finite trip duration. Only air and water are acquired in the wild. For space travel, even the air and water must be launched from Earth. So space travel is like a glorified and hyper-expensive form of backpacking—albeit offering breathtaking views!.

One way to probe the demonstration vs. way-of-the-future question is to list capabilities we have not yet demonstrated in space that would be important for a space livelihood, including:

1. Growing food used for sustenance;
2. Surviving long periods outside of Earth's magnetic protection from cosmic rays;[19]
3. Generating or collecting propulsive fuel away from Earth's surface;
4. Long-term health of muscles and bones for periods longer than a year in low gravity environments;
5. Resource extraction for in-situ construction materials;
6. Closed-system sustainable ecosystem maintenance;
7. Anything close to terraforming (see below).

19: The ISS (space station) remains within Earth's protection.

It would be easier to believe in the possibility of space colonization if we first saw examples of colonization of the ocean floor.[20] Such an environment carries many similar challenges: native environment unbreathable; large pressure differential; sealed-off self-sustaining environment. But an ocean dwelling has several *major* advantages over space, in that food is scuttling/swimming just outside the habitat; safety/air is a short distance away (meters); ease of access (swim/scuba vs. rocket); and all the resources on Earth to facilitate the construction/operation (e.g., Home Depot not far away).

20: Even just 10 meters under the surface!

⚠ This is not to advocate ocean floor habitation as a *good* idea; it is merely used to illustrate that space habitation is an even *less* practical idea, by far.

Building a habitat on the ocean floor would be vastly easier than trying to do so in space. It would be *even easier* on land, of course. But we have not yet successfully built and operated a closed ecosystem on land! A few artificial "biosphere" efforts have been attempted, but met with failure [31]. If it is not easy to succeed on the surface of the earth, how can we fantasize about getting it right in the remote hostility of space, lacking easy access to manufactured resources?

[31]: (2020), *Biosphere 2*

On the subject of terraforming, consider this perspective. Earth right now has a problem of excess CO_2 as a result of fossil fuel combustion (the subject of Chapter 9). The problem has flummoxed our economic and political systems, so that not only do we seem to be powerless to revert to pre-industrial CO_2 levels, but even arresting the annual increase in emissions appears to be beyond our means. Pre-industrial levels of CO_2 measured 280 parts per million (ppm) of the atmosphere, which we will treat as the *normal* level. Today's levels exceed 400 ppm, so that the modification is a little more than 100 ppm, or 0.01% of our atmosphere.[21] Meanwhile, Mars' atmosphere is 95% CO_2. So we might say that Earth has a 100 ppm problem, but Mars has essentially a *million* part-per-million problem. On Earth, we are completely stymied by a 100 ppm CO_2 increase while enjoying access to all the resources available to us on the planet. Look at all the infrastructure available on this developed world and still we have not been able to reverse or even stop the CO_2 increase. How could we possibly see transformation of Mars' atmosphere into habitable form as realistic, when Mars has zero infrastructure to support such an undertaking? We must be careful about proclaiming notions to be *impossible*, but we can be justified in labeling them as *outrageously impractical*, to the point of becoming a distraction to discuss. Figure 4.5 further illustrates the giant gap between tolerable conditions and actual atmospheres on offer in the solar system.

We also should recall the lesson from Chapter 1 about exponential growth, and how the addition of another habitat had essentially no effect on the overall outcome, aside from delaying by one short doubling time. Therefore, even if it is somehow misguided to discount colonization of another solar system body, who cares? We still do not avoid the primary challenge facing humanity as growth slams into limitations in a finite world (or even finite solar system, if it comes to that).

4.4 Exploration's Role

It is easy to understand why people might latch onto the idea that we will likely leverage our exploration of space into ultimate colonization. Much as early explorers of our planet opened pathways for colonization of "new worlds," the parallels in exploring literal new worlds like planets are obvious.[22] In short, it is a familiar story, and therefore an easy "sell" to primed, undoubting minds. Plus, we're captivated by the novelty and challenge space colonization represents—as attested by a vibrant entertainment industry devoted to stories of eventual life in space. But not all exploration leads to settlement, and entertainment is not truth.

Humans have explored (a small portion of) the crushing deep ocean, scaled Earth's highest and wholly inhospitable peaks, and visited the harsh ice cap at the north pole. In such instances, we had zero intention of establishing permanent residence in those locations. They represented

Terraforming is the speculative idea of transforming a planet so its atmosphere resembles that of Earth (chemical makeup, temperature, pressure) and can support human life.

21: While the increase from 280 to 400 is about 50%, as a fraction of Earth's total atmosphere, the ~ 100 ppm change is 100 divided by one million (from definition of ppm), or 0.01%.
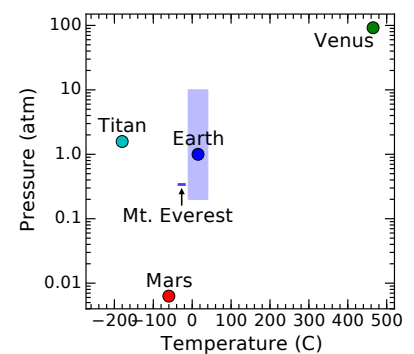
Figure 4.5: Rocky-body atmospheres in the solar system, showing *average* temperature (Celsius) and pressure (atmospheres). The range of "comfort" for Earth is shown as a blue rectangle going from $-10°C$ to $40°C$ and 0.2 atm (where the atmosphere would need to be 100% oxygen) to (arbitrarily) 10 atm. Not only are the other bodies *far* outside our comfort range, the compositions are noxious, and lack oxygen. Bear in mind that a change of even a few degrees—as in climate change—is a big deal. Even Mt. Everest, where humans can survive for only a few hours with supplemental oxygen is *substantially* more hospitable than Mars. ∎

22: Reaching the Americas involved a leap across a span of (life-supporting) ocean about twice the size of Europe. Reaching Mars involves a leap across inhospitable space 5,000 times the diameter of Earth—not very similar at all.

places to test our toughness and also learn about new environments. We do not view these sorts of explorations as *mistakes* just because they did not pave the way for inhabitation. Rather, we speak fondly of such excursions as feathers in our collective cap: feats that make us proud as a species. Space might be viewed in a similar way: superlative in terms of challenge and wonderment, reflecting positively on our curiosity, drive, ingenuity, and teamwork. We also derive benefits[23] in the way of technological advancement propelled by our quest to explore, and in furthering our scientific understanding of nature.

So even if space does not fulfill the fantasy of continued human expansion across the cosmos, it is in our nature to at least *explore* it. We would do well to put space exploration in the category of conquering Mt. Everest rather than that of Europeans stumbling upon the West Indies (one is as imminently uninhabitable as the other is inhabitable). Let us not make the mistake of applying the wrong narrative to space.

Many positive things might be said about space exploration, and hopefully we continue poking into our outer environment indefinitely. Yet hoping that such exploration is a pathway to human colonization of space is probably wrong and almost certainly counterproductive at present, given the short timescale on which human expansion is likely to collide with Earth's limits.

If, in the fullness of time, we do see a path toward practical space colonization, then fine. But given the extreme challenge and cost—both energetically and economically, and for what could only be a tiny footprint in the near term—it seems vastly more prudent to take care of our relationship with Planet Earth first, and *then* think about space colonization in due time, if it ever makes sense. Otherwise, not only do we spend precious resources unwisely, but (even worse) our mindset is tainted by unrealistic dreams that diminish the importance of confronting the real challenge right here on the ground. We need to have our heads in the real game. Perhaps twenty øne piløts said it best in the song *Stressed Out*:

> We used to play pretend, give each other different names
> We would build a rocket ship and then we'd fly it far away
> Used to dream of outer space but now they're laughing at our face
> Saying, "Wake up, you need to make money."
> Yeah.

Space colonization might be treated as a pretend fantasy for the moment. We would be better off waking up to face real here-and-now challenges. In some sense, perhaps the only way to achieve the dream of migration to space—should that be in the cards at all—is to first pretend that it is impossible and turn attention to the pressing matters on Earth. Otherwise we risk failing at *both* efforts.

23: . . . among them a deeper appreciation for the rare and precious Earth

Despite the pessimistic tone of this chapter, the author is himself captivated by space, and has built a life around it: *Star Wars* was a transformative influence as a kid, and later *Star Trek*. The movie *The Right Stuff* is still a favorite. He has peered to the edge of the universe—first through a 10-inch telescope he built in high school, and later using the largest telescopes in the world. He has worked on a Space Shuttle experiment, met astronauts, knew Sally Ride, and spent much of his career building and operating a laser system to bounce and detect individual photons off the reflectors placed on the lunar surface by the Apollo astronauts (as a test of the fundamental nature of gravity), which directly inspired part of a *Big Bang Theory* episode via personal interactions with the show's writers. So a deep fondness for space? Yes. Would volunteer to go to the moon or Mars? Yes. Believes it holds the key to humanity's future? No.

Box 4.5: Q&A on State of Exploration

After reading the first draft of this chapter, students had a number of remaining questions. Here are some of them, along with the author's responses.

1. How long before we live on other planets?

   Maybe never.[24] The staggering distances involved mean that our own solar system is effectively the only option. Within the solar system, Mars is the most hospitable body—meaning we might live as long as two minutes without life support. By comparison, Antarctica and the ocean floor are millions of times more practical, yet we do not see permanent settlements there.[25]

2. What is the status of searches for other planets to colonize?

   We understand our solar system pretty well. No second homes stand out. We have detected evidence for thousands of planets around other stars [32], but do not yet have the sensitivity to detect the presence of earth-like rocks around most stars. It is conceivable that we will have identified Earth analogs in the coming decades, but they will give new meaning to the words "utterly" and "inaccessible."

3. Haven't we benefited from space exploration in the technology spin-offs, like wireless headsets and artificial limbs?

   No doubt! The benefits have been numerous, and I would never characterize our space efforts to date as wasted effort. It's just that what we have done so far in space does not mean that colonization is in any way an obvious or practical next step. Actually, the banner image for this chapter from the Apollo 8 mission captivated the world and made our fragile shell of life seem all the more precious. So perhaps the biggest benefit to our space exploration will turn out to be a profound appreciation for and attachment to Earth!

24: True, never is a long time. The notion that we may *never* colonize space may seem preposterous to you now. Check back at the end of the book. The odds favor a more boring slog, grappling with our place in nature.

25: Staffed research stations are not the same as human settlements, in the case of Antarctica.

[32]: IPAC/NASA (2020), *NASA Exoplanet Archive*

4.5 Upshot: Putting Earth First

The author might even go so far as to label a focus on space colonization in the face of more pressing challenges as disgracefully irresponsible. Diverting attention in this probably-futile[26] effort could lead to *greater total suffering* if it means not only mis-allocation of resources but perhaps

26: . . . at least on relevant time scales

more importantly lulling people into a sense that space represents a viable escape hatch. Let's not get distracted!

The fact that we do not have a collective global agreement on priorities or the role that space will (or will not) play in our future only highlights the fact that humanity is not operating from a master plan[27] that has been well thought out. We're simply "winging it," and as a result potentially wasting our efforts on dead-end ambitions. Just because some people are enthusiastic about a space future does not mean that it can or will happen.

27: Prospects for a plan are discussed in Chapter 19.

It is true that we cannot know for sure what the future holds, but perhaps that is *all the more reason* to play it safe and not foolishly pursue a high-risk fantasy.[28] From this point on, the book will turn to issues more tangibly relevant to life and success on Planet Earth.

28: Tempted as we may be by the e-mail offer from the displaced Nigerian prince to help move his millions of dollars to a safe account, most of us know better than to bite. The promise of wealth can lead the gullible to ruin.

Box 4.6: Survey Says?

It would be fascinating to do a survey to find out how many people think that we will have substantial populations living off Earth 500 years from now. It is the author's sense that a majority of Americans believe this to be likely. Yet, if such a future is not to be—for a host of practical reasons, including the possibility that we falter badly and are no longer in a position to pursue space flight—we would find ourselves in a situation where most people may be completely wrong about the imagined future. That would be a remarkable state of affairs in which to find ourselves—though not entirely surprising.

4.6 Problems

1. If the sun were the size of a basketball, how large would Earth be, and how far away? How large would the moon be, and how far from Earth, at this scale?

2. Find objects whose sizes *approximately* match the scales found in Problem 1 and place them in your environment at the scaled/appropriate distances. Submit a personalized/unique picture of your arrangement.

   ⓘ This kind of exercise might seem like a hassle, but it can really help internalize the scales in a way words never will.

3. How far would the nearest star be at the scale from Problem 1, and how big is this in relation to familiar objects?

4. Find an Earth globe and an object about one-fourth its size to represent the moon, then place at the appropriate distance apart. Report on how far this is. Take a personalized/unique picture to document, and take some time appreciating how big Earth would look from the moon.

   ⓘ By doing this, you can get maybe 2% of the enjoyment of a trip to the moon for less than one-billionth the cost: a real bargain! Can you make out Florida? Japan?

5. Highway 6563 in New Mexico has signs along a roughly 30 km stretch of road corresponding to the solar system scale from the Sun to Neptune. On this scale, how large would Earth, Sun, and Jupiter be, in diameter? Express in convenient units appropriate to the scale.

6. Using the setup in Problem 5, how fast would you have to travel on the road to match the speed of light, for which it takes 500 seconds to go from Earth to the sun? Express in familiar/convenient units.

7. Note that the size of the moon in Figure 4.1 is about the same size as the sun in Figure 4.2. Explain how this is related to the fact that they appear to be about the same size in our sky. Hint: imagine putting your eye at the earth location in each figure and looking at the other body.

ⓘ This is why eclipses are special on Earth.

8. Use Table 4.1 to accumulate (multiplicatively combine) scale factors and ask: which is a bigger ratio: the distance to the nearest star compared to the diameter of Earth, or the distance to the edge of the universe compared to that to the nearest star? Compared to the large numbers we are dealing with, is one much smaller or much bigger than the other, or are they roughly the same?

9. It may be tempting to compare Earth to a life-sustaining oasis in the desert—maybe spanning 100 m. But this is a pretty misleading view. One way to demonstrate this is to consider that in a real desert, the next oasis might be a perilous 100 km journey away. Using the ratio of distance to size (diameter of planet or oasis), how close would another Earth have to be (and compare your answer to other solar system scales) to hold the analogy?[29] How long would it take to drive this distance? Do we have another oasis or potential oasis within this distance?

29: In other words, oasis size is to distance between oases as Earth's diameter is to *how far*?

10. Another way to cast Problem 9 is to imagine that the actual distance between Earth and a comparable oasis is more like the distance between stars.[30] In this case, how far would the next oasis be in the desert if we again compare the 100 m scale of the oasis to the diameter of the Earth?[31] How long would it take to drive between oases at freeway speeds (cast in the most informative/intuitive units)?

30: . . . since Earth is the only livable "oasis" in our own solar system

31: In other words, Earth diameter is to interstellar distances as a 100 m oasis is to *how far*?

ⓘ The insight you develop will not depend on exact choices for distance and speed, as long as they are reasonable.

11. On the eighth bullet of Box 4.3 (the one that asks you to pause), imagine someone from the year 2020 traveling back 50 years and explaining that we have not been to the moon since 1972, and that Americans get to space on Russian rockets. How believable do you think they would be, and what assumptions might be made to reconcile the shock?

12. Prior to exposure to this material, what would you honestly have said in response to: How far have humans been from the planet in

the last 45 years;

a) 600 km (about $\frac{1}{10}$ Earth radius; low earth orbit)
b) 6,000 km (roughly Earth radius)
c) 36,000 km (about 6 $R_\oplus$; around geosynchronous orbit)
d) 385,000 km (approximate distance to the moon)
e) beyond the distance to the moon

Explain what led you to think so (whether correct or not).

13. List at least three space achievements that impress you personally, even if they do not bear directly on colonization aims.

14. In the enumerated list beginning on page 60, which item is most surprising to you as not-yet accomplished, and why?

15. List three substantive challenges that prevented successful long-term operation of the artificial biosphere project [31].

16. Since Figure 4.5 spans the range of atmospheres found in our solar system, we can imagine how likely it would be that a random planet might happen to be livable for humans.[32] Imagine throwing a dart at the diagram to get a random instance. How likely are we to hit the comfort zone of Earth, by your estimation?

32: ...just in terms of temperature and pressure; ignoring composition and a host of other considerations!

17. Come up with three examples (not repeating items in the text) of feats that are technically possible, but not common or practical.

18. For some perspective, imagine you were able to drive your car up a ramp to an altitude characteristic of low-earth orbit (about 320 km, or 200 miles). It takes about 5×10^{10} J of energy[33] to win the fight against gravity. Meanwhile, each gallon of gasoline can do about 25×10^6 J of useful work. How many gallons would it take to climb to orbital height in a car? Roughly how many miles per gallon is this (just counting vertical miles)?

33: We'll encounter gravitational potential energy later, but this quantity is computed as mgh with $m \approx$ 1,500 kg.

34: The same amount of energy to climb against gravity must go into accelerating to speed (kinetic energy).

19. In Problem 18, we ignored the energy required to provide the substantial orbital speed ($\sim$ 8 km/s, but will not need), which essentially *doubles* the total energy.[34] How much gasoline will it now take, and how massive is the fuel if gasoline is 3 kg per gallon, compared to the 1,500 kg mass of the car?

ⓘ Given this mass of fuel, and its significance compared to the car's mass, we can see that we've underestimated the fuel required since we'll have to put substantial additional energy into lifting the fuel away from the earth.

Part II

ENERGY AND FOSSIL FUELS

What worked for us in the past
cannot work for us in the future.
We must learn the language of our old friends
in order to say a proper goodbye.

5   Energy and Power Units

This chapter provides a baseline for understanding the rest of the content in this book, so that students may learn to interpret and convert units, while building a useful intuition in the process. Sec. A.10 (p. 370) in the Appendices offers some tips on manipulating units and performing unit conversions.

Unlike most chapters, this one does not tell a single story or advance our perspective on the world. But it builds a foundation, putting us in a position to start looking at consequential matters of energy use in our society in chapters to come. Hopefully, patience will be rewarded.

5.1 Energy (J)

First, what is energy?

> **Definition 5.1.1** *Energy is defined as the* capacity to do work. *Work is well-defined in physics as the **application of force through a distance**.*[1] *The colloquial use of the word "work" matches relatively well, in that pushing a large couch across the floor (applying force through a distance) or lifting a heavy box up to a shelf feels like work and can tire you out.*

1: This definition applies to the common circumstance when the motion is aligned with the direction of force, like pushing a box across a level floor, propelling a car along the road, or lifting a weight.

The SI unit of force is the Newton (N), breaking down more fundamentally to $kg \cdot m/s^2$. The best way to remember this is via Newton's Second Law: $F = ma$ (force equals mass times acceleration). Mass has units of kg, and acceleration[2] is measured in meters per second squared.

Since work is force times distance, the unit for work (and thus energy) is Newtons times distance, or $N \cdot m$. We give this unit its own name: the

2: Acceleration is the rate of change of velocity. Since velocity is measured in meters per second, the rate at which it changes will be meters per second *per second*, or m/s/s, or m/s^2. Some students may know that gravitational acceleration on Earth's surface is 9.8 m/s^2, which is another way to remember.

Energy units from everyday life. Clockwise from upper left: a utility bill (kWh and Therms); a hot water heater label (Btu/hr); EnergyGuide for same hot water heater (Therms); U.S. nutrition label for peanut butter (Calories; should be kcal); a German nutrition label for Nutella (kJ, kcal); and rechargeable AA batteries (2200 mAh, 1.2 V).

Joule (J). Thus, the application of 1 N of force across a distance of 1 m constitutes 1 J of work, requiring 1 J of energy to perform. Table 5.1 offers contextual examples (unit prefixes are on page 420).

> **Example 5.1.1** Several examples[3] illustrate force times distance, the first two amounting to one Joule of energy:
>
> ▶ Pushing a book across a table, applying 2 N of force and sliding it 0.5 m amounts to 1 J of work.
> ▶ Pushing a matchbox toy car across the floor might require only 0.1 N of force. One would have to push it through a distance of 10 m to make up one Joule of energy.
> ▶ A car on level ground may require 150 N of force to roll against friction. Pushing a car 5 m would then require 750 J of work.

Writing out Newtons as $kg \cdot m/s^2$, we find that the unit of energy amounts to $J = N \cdot m = kg \cdot m^2/s^2$. Notice that this looks like mass times velocity-squared. Box 5.1 explores how this makes a lot of sense.

> **Box 5.1: The Units Make Sense!**
>
> Think about the famous equation $E = mc^2$. Energy is mass times the speed of light squared. The units work!
>
> Also, kinetic energy is K.E. $= \frac{1}{2}mv^2$, telling a similar story in terms of units: mass times velocity-squared.
>
> Gravitational potential energy is just the weight of an object times the height it is lifted through.[4] The weight (force) is mass (m) times the acceleration due to gravity[5] (g), so that lifting (applying a force equal to the weight) through a height (h) results in a potential energy gain of P.E. $= mgh$. The units again check out as
>
> $$mgh \rightarrow kg \cdot \frac{m}{s^2} \cdot m = \frac{kg \cdot m \cdot m}{s^2} = \frac{kg \cdot m^2}{s^2} = J.$$

We'll encounter other ways to describe energy in this book, but *any energy unit can always be cast into units of Joules*, if desired. Later sections in this chapter detail alternative units whose acquaintance we must make in order to interpret energy information in our lives.

5.2 Energy Forms and Conservation

Energy manifests in a variety of forms, which we will treat in greater detail in application-specific chapters in Part III of this text. For now we just want to name them and point to related chapters and applications, as is done in Table 5.2.

Table 5.1: Approximate energy for familiar activities. The first freeway example is just kinetic energy; the second is the energy cost of a whole trip.

Action	Energy
Nerf football toss	15 J
Lift loaded bookbag	100 J
Fast-pitch baseball	120 J
Speeding bullet	5 kJ
Charge cell phone	30 kJ
Car on freeway (K.E.)	675 kJ
human daily diet	8 MJ
1 hour freeway drive	250 MJ

3: For examples like these, framed as statements and not questions, you can practice solving several types of problems by covering up one number and then solving for it using still-available information. So each statement can be seen as several examples in one!

More on mass-energy in Chapter 15.

More on kinetic energy in Chapter 12.

More on gravitational potential energy in Chapter 11.

4: Another example of work (energy) being force times distance.

5: The force needed to hold against gravity is just $F = ma = mg$

Sec. A.10 (p. 370) in the Appendices provides additional guidance on manipulating units.

Energy Form	Formula	Chapter(s)	Applications
gravitational potential	mgh	11, 16	hydroelectric, tidal
kinetic	$\frac{1}{2}mv^2$	12, 16	wind, ocean current
photon/light	$h\nu$	13	solar
chemical	$H - TS$	8, 14	fossil fuels, biomass
thermal	$c_{\mathrm{P}}m\Delta T$	6, 16	geothermal, heat engines
electric potential	qV	15	batteries, nuclear role
mass (nuclear)	mc^2	15	fission and fusion

Table 5.2: Energy forms. Exchange is possible between all forms. Chemical energy is represented here by Gibbs free energy.

A bedrock principle of physics is conservation of energy, which we take to *never* be violated in any system, ever.[6] What this means is that energy can *flow* from one form to another, but it is *never created or destroyed*.

6: The only exception is on cosmological scales and times. But across scales even as large as the Milky Way galaxy and over millions of years, we are on solid footing to consider conservation of energy to be inviolate. It is fascinating to note that conservation of energy stems from a symmetry in time itself: if the laws and constants of the Universe are the same across some span of time, then energy is conserved during such time—a concept we trace to Emmy Noether. See Sec. D.2 (p. 393) for more.

Box 5.2: Energy: The Money of Physics

A decent way to conceptualize energy conservation is to think of it as the *money* of physics. It may change hands, but is not created or destroyed in the exchange. A large balance in a bank account is like a potential energy: available to spend. Converting to another form—like heat or kinetic energy—is like the act of spending money. The *rate* of spending energy is called *power*.

Example 5.2.1 traces a few familiar energy conversions, and Figure 5.1 provides an example illustration. A more encompassing narrative connecting cosmic sources to daily use is provided in Sec. D.2.2 (p. 395).

Example 5.2.1 Various illustrative examples:

▶ A rock perched on the edge of a cliff has gravitational potential energy. When it is pushed off, it trades its potential energy for kinetic energy (speed) as it races toward the ground.

▶ A pendulum continually exchanges kinetic and potential energy, which can last some time in the absence of frictional influences.

▶ A stick of dynamite has energy stored in chemical bonds (a form of potential energy). When ignited, the explosive material becomes very hot in a small fraction of a second, converting chemical energy into thermal energy.

▶ The fireball of hot material from the exploding dynamite expands rapidly, pushing air and nearby objects out of the way at high speed, thus converting thermal energy into kinetic energy.

▶ Light from the sun (photons) hits a black parking lot surface, heating it up as light energy is converted to thermal energy.

▶ A uranium nucleus splits apart, releasing nuclear (potential) energy, sending the particles flying off at high speed (kinetic energy). These particles bump into surrounding particles transferring kinetic energy into thermal energy.

▶ Thermal energy from burning a fossil fuel or from nuclear fission

Figure 5.1: Example exchange of potential energy (P.E.) into kinetic energy (K.E.) as an apple drops from a tree. The total energy always adds to the same amount (here 7 J). The apple speeds up as it gains kinetic energy (losing potential energy). When it comes to rest on the ground, the energy will have gone into 7 J of heat (the associated temperature rise is too small to notice).

P.E. = 7 J
K.E. = 0 J

P.E. = 5 J
K.E. = 2 J

P.E. = 3 J
K.E. = 4 J

P.E. = 1 J
K.E. = 6 J

can be used to make steam that drives a turbine (kinetic energy) that in turn generates electrical energy (voltage, current).

Any of the forms of energy (e.g., in Table 5.2) can convert into the other, directly or indirectly. In each conversion, 100% of the energy is accounted for. In the general case, the energy branches into multiple paths, so we do not get 100% efficiency into the channel we want. For instance, the pendulum example above will eventually bleed its energy into stirring the air (kinetic energy) and friction (heat) at the pivot point. The stirring air eventually turns to heat via internal (viscous) friction of the air.

One useful clarification is that thermal energy is really just random motions—kinetic energy—of individual atoms and molecules. So in the case of nuclear fission in Example 5.2.1, the initial kinetic energy of the nuclear fragments is already thermal in nature, but at a higher temperature (faster speeds) than the surrounding material. By bumping into surrounding atoms, the excess speed is diffused into the medium, raising its temperature while "cooling" the fragments themselves as they are slowed down.

> The differences between kinetic and thermal energy is about *coherence*, in that we characterize the kinetic energy of a raindrop by its bulk motion or velocity. Meanwhile, water molecules *within* the drop are zipping about in *random* directions and at very high speeds exceeding 1,000 meters per second.

If accounting for all the possible paths[7] of energy, we are confident that they always add up. Nothing is lost.[8] Energy is never created or destroyed in any process we study. It just sloshes from one form to another, often branching into multiple parallel avenues. The sum total will always add up to the starting amount. Sec. D.2.3 (p. 396) provides a supplement for those interested in better understanding where energy ultimately goes, and why "losing energy to heat" is not actually a loss but just another reservoir for energy.

> 7: ...sometimes called channels
>
> 8: Actually, the principle is so well established that new particles (like the neutrino) have been discovered by otherwise unaccounted energy in nuclear processes.

5.3 Power (W)

Before getting to the various common units for energy, we should absorb the very important concept and units of power.

> **Definition 5.3.1** *Power is simply defined as energy per time: how much energy is expended in how much time. The SI unit is therefore J/s, which we rename Watts (W).*

> One Watt is simply one Joule per second.

While energy is the capacity to do work, it says nothing about how quickly that work might be accomplished. Power addresses the *rate* at which energy is expended. Figure 5.2 provides a sense of typical power levels of familiar animals and appliances.

> **Example 5.3.1** Lifting a 10 kg box, whose weight is therefore about 100 N, through a vertical distance of 2 m requires about 200 J of energy. If performed in one second, the task requires 200 W (200 Joules in one second). Stretching the same task out over four seconds requires only 50 W.

> Weight is mg. In this case, m is 10 kg. If we're being sticklers, $g = 9.8$ m/s^2, but for convenience we can typically use $g \approx 10$ m/s^2 without significant loss of precision.
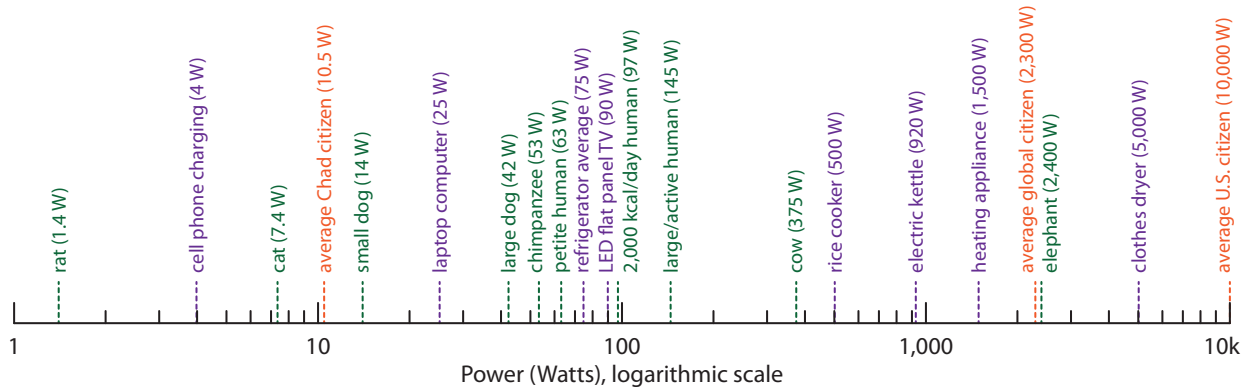
Figure 5.2: Various power levels for comparison and intuition-building. Green entries correspond to metabolic power [33]. Purple entries are devices and appliances. Orange entries are per-capita totals for societal (non-metabolic) energy use. Note that appliances whose job it is to create heat demand the greatest power. The "heating appliance" entry stands for things like microwave ovens, toaster ovens, space heaters, or hair dryers plugged into electrical outlets. Do not take the numbers provided as definitive or exact, as almost everything in the figure will vary somewhat from one instance to another.

Of course, we commonly apply the usual multipliers of factors of 10^3 to the unit to make it more useful. Thus we have the progression W, kW, MW, GW, TW, etc. For reference, a large college campus will require several tens of MW (megawatts) for electricity. A large power plant is typically in the 1–4 GW range. See Table 5.3 for scales at which we are likely to use the various multiplying factors, and a more complete set of multipliers on page 420.

Although it won't come up too often in this course, it is worth mentioning that the common unit of horsepower equates to 745.7 W. Thus a 100 hp car is capable of delivering about 75 kW of power.

Table 5.3: Power multipliers and contexts

Factor	Unit	Context
1	W	phones; computers
10^3	kW	microwave oven
10^6	MW	campus; community
10^9	GW	power plant; city
10^{12}	TW	societal scale

It is usually sufficient to remember that 1 hp is about 750 W.

5.4 Kilowatt-hour (kWh)

Definition 5.4.1 *The **kilowatt-hour** is an amount of energy (not a power) resulting from an expenditure of energy at a* rate *of 1 kW for a* duration *of one hour, and is the unit of choice for residential electricity usage.*

This unit causes no end of confusion, but it's really pretty straightforward. The kilowatt-hour is a kilowatt *times* an hour. Thus it is power multiplied by time, which is energy (since power is energy over time).

It may help to think of the sequence: kilowatt × hour; kW×h; kW-h; kWh.

Example 5.4.1 Let's say you plug in a space heater rated at 1,000 W (1 kW) and run it for one hour. Congratulations—you've just spent 1 kWh.

Or maybe you turn on a 100 W incandescent light bulb (0.1 kW) and leave it on for 10 hours: also 1 kWh!

What if you run a 500 W rice cooker (0.5 kW) for half an hour? That's 0.25 kWh.

It is straightforward to convert back to Joules, because 1 kW is 1,000 J/s and one hour is 3,600 s. So 1 kWh is 1 kW times 1 hr, which is 1,000 J/s times 3,600 s, and is therefore equal to 3,600,000 J, or 3.6 MJ. A related measure sometimes comes up: the watt-hour (Wh). In much the same vein, this is equivalent to 1 J/s for 3,600 seconds, or 3,600 J.[9]

9: A Wh is one-thousandth of a kWh, not surprisingly.

Box 5.3: Don't be one of those people...

If you ever hear someone say "kilowatts per hour," it's likely a mistake,[10] and has the side effect of leading people to erroneously think that kilowatts is a unit of energy, not a power. Kilowatts is already a *rate* (speed) of energy use: 1,000 Joules per second.

10: Literally, kW/hr would be a sort of *acceleration* through energy. It's a real thing that can happen, but it's usually not what people mean.

One tendency some people have is to mix up kW and kWh.[11] Kilowatts is a unit of *power*, or how fast energy is being used. Think of it like a speedometer: how fast are you moving (through space or energy)? Kilowatt-hours is a multiplication of power times time, becoming an *energy*. It's more like the odometer: how much have you accumulated (distance or energy)? Just like distance is rate (speed) times time, energy is rate (power) times time.

11: ⚠ Perhaps related to Box 5.3.

Example 5.4.2 We will explore kWh using a light bulb for an example. Let's say the light bulb is labeled as 100 W.[12] How much energy does it use?

12: ...an incandescent, for instance

Well, it depends on how long it's on. If it is never turned on, it uses *no* energy. If it is on for 10 seconds, it uses far less than if it's on for a day.

The characteristic quality of the light bulb is the power it expends when it's on—in this case 100 W. It only has one speed. In analogy to a car and speedometer, it's similar to saying that a car travels at a constant speed,[13] and asking how far it travels. Well, it depends on how much *time* it spends traveling at speed.

13: ...maybe 30 m/s; 67 m.p.h.; 108 k.p.h.

So view kWh (energy) as an *accumulated* amount that increases with time. On the other hand, kW is a *rate* of energy expenditure.

5.5 Calories (kcal)

A common unit for describing chemical and thermal processes is the calorie and its siblings.

Definition 5.5.1 *A calorie is defined as the amount of energy it takes to heat one gram of water (thus also 1 mL, or 1 cm³, or 1 cc) by one degree Celsius (Figure 5.3). One calorie (note the small "c") is 4.184 J of energy.*

One Calorie (note the capital "C")[14] is 1,000 calories, or 1 kilocalorie

14: This might win the prize for the dumbest convention in science: never define a unit as case-sensitive, as it cannot be differentiated in spoken language!

(1 kcal), equating to 4,184 J. Most memorably, it is the amount of energy it takes to heat one kilogram *(or one liter; 1 L) of water by 1°C. Due to the tragic convention of Calorie, we will opt for* **kcal** *whenever possible.*

Food labels in the U.S. are in Calories, describing the energy *content of the food we eat.[15] We would all do ourselves a favor by calling these kcal instead of Calories (same thing). Many other countries sensibly use either kJ or kcal for quantifying food energy.*

15: Human metabolism is not the same as heating water, but the energy involved can still be counted in an energy unit that is defined in terms of heating water. It's still just energy.

20°C ☐ + 50 calories (209 J) ⟶ ☐ 70°C

1 cm³ = 1 mL = 1 g

Figure 5.3: Following the definition of a calorie, adding 50 cal to one gram of water raises its temperature by 50°C.
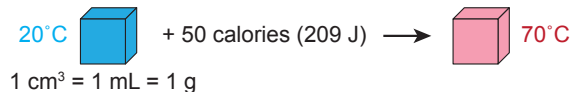
Example 5.5.1 To change 30 mL (30 g) of water by 5°C requires 150 cal, or a little over 600 J.

Injecting 40 kcal of energy into a 2 L (2 kg) bottle of water will heat it by 20 degrees.

Drinking 250 mL of ice-cold water and heating it up to body temperature (thus raising its temperature by approximately 35 degrees) will take about 8,750 cal, or 8.75 kcal, or a bit over 36 kJ of energy.

It is usually sufficient to remember that the conversion factor between calories and Joules is about 4.2—or just 4 if performing a crude calculation.

No deep significance attaches to the fact that 1 cal happens to equate to 4.184 J, other than to say this describes a property of water (called specific heat capacity).

$$
\begin{array}{lcccccc}
1\text{ cal} & = & 4.184\text{ J} & \approx & 4.2\text{ J} & \sim & 4\text{ J} \\
1\text{ kcal} & = & 4{,}184\text{ J} & \approx & 4.2\text{ kJ} & \sim & 4\text{ kJ}
\end{array}
$$

Two examples will help cement use of the kcal (a more useful scale in this class than the much smaller calorie).

Example 5.5.2 A typical diet amounts to a daily intake of about 2,000 kcal of food energy. If you think about it, 2,000 kcal/day is a *power* (energy per time). We can convert to Watts by changing kcal to J and one day to seconds. 2,000 kcal is 8.368 MJ. One day has 86,400 seconds. The division of the two is very close to 100 W.[16]

16: It would serve little purpose to perform exact math here—producing 96.85 W in this case—since the idea that someone's daily diet is exactly 2,000.00 kcal is pretty preposterous. It will likely vary by at least 10% from day to day, and by even larger amounts from individual to individual, so that 100 W is a convenient and approximate representation.

A second example hews closely to the definition of the kcal: heating water.

Example 5.5.3 Let's say you want to heat a half-liter (0.5 kg) of water from room temperature (20°C) to boiling (100°C). Since each kcal can heat 1 kg by 1°C, that same energy will raise our half-kg by 2°C.[17] So raising the temperature by 80°C will require 40 kcal, or 167 kJ.

If the water is heated at a rate of 1,000 W (1,000 J/s), it would take 167 seconds for the water to reach boiling temperature.

17: Make sure this is clear to you; by understanding, we are installing concepts instead of formulas, which are more powerful and lasting.

Notice that we did not apply an explicit formula in Example 5.5.3. By proceeding stepwise, we attempt to keep it intuitive. We *could* write a

Appendix Sec. A.8 (p. 368) addresses this philosophy in a bit more detail.

formula, but we implicitly create the formula on the fly by recognizing that the amount of energy required should scale with the mass of water and with the amount of temperature increase. Hopefully, this approach leads to a deeper understanding of the concept, while printing a formula on the page might short-circuit comprehension.

> This would be an excellent opportunity to *create your own* formula to capture the idea, like an expert!

5.6 British Thermal Unit (Btu)

Why would we waste our time talking about the arcane British thermal unit (Btu)? It's because data provided by the U.S. Energy Information Administration on global energy use is based on the Btu. More specifically, country-scale annual energy expenditures are measured in units of quadrillion (10^{15}) Btu (see Box 5.4). Also, heating appliances in the U.S.[18] are rated in Btu/hour—a unit of power that can be converted to Watts.

We need to cover the unit in this chapter in order to be energy-literate in the U.S., and because it will come up later in this book.

18: ...hot water heaters, furnaces, air conditioners, ovens and stoves

> **Definition 5.6.1** *The **Btu** is the Imperial analog to the kcal.[19] One Btu is the energy required to heat one pound of water one degree Fahrenheit.*
>
> *In terms of Joules, 1 Btu is about 1,055 J, or not far from 1 kJ.*

19: Recall that 1 kcal is the energy it takes to heat one kilogram of water by 1°C.

We can make sense of the conversion to Joules in the following way: a pound is *roughly* half a kilogram and one degree Fahrenheit is *approximately* half a degree Celsius. So a Btu should be roughly a quarter of a kcal. Indeed, 1,055 J is close to one quarter of 4,184 J.

> **Box 5.4: Quads: qBtu**
>
> The U.S. uses quadrillion Btu to represent country-scale annual energy expenditures. It is denoted as qBtu, or informally "quads." One qBtu is approximately 10^{18} J.[20]
>
> The U.S. uses about 100 quads per year. Since a year is about 3.16×10^7 seconds,[21] dividing energy in Joules by time in seconds tells us that the U.S. *power* is about 3×10^{12} W (3 TW), working out to about 10,000 W per person as a per-capita *rate* of energy use.

20: 1.055×10^{18} J, more precisely.

21: A cute and convenient way to remember this, approximately, is $\pi \times 10^7$ seconds per year.

> **Example 5.6.1** For appliances characterized by Btu/hr, we can relate to power in Watts via 1 Btu/hr as 1,055 J per 3,600 s, working out to 0.293 W.
>
> Thus, a hot water heater rated at 30,000 Btu/hr is effectively 8,800 W.
>
> Let's also pause to understand how long it will take to heat a shower's worth of hot water at this rate. We'll do it two ways:
>
> 1. Heating 15 gallons[22] (125 pounds) from a cool 68°F to a hot 131°F at 30,000 Btu/hr will take how long? We must put in

22: Typical shower flow is about 2 gallons, or ~8 L, per minute.

> $125 \times 63 = 7,900$ Btu of energy at a rate of 30,000 Btu/hr, so it will take 7,900/30,000 of an hour, or just over 15 minutes.
> 2. In metric terms, the equivalent to 15 gallons is 57 L (57 kg), and we heat from 20°C to 55°C at 8,800 W.[23] Since one kcal heats one kilogram of water 1°C, heating 57 kg by 35°C will require 57×35 kcal, or $57 \times 35 \times 4,184$ J = 8.35 MJ, which at 8,800 W will take 950 seconds, also just over 15 minutes (reassuringly, the same answer).

5.7 Therms

We will rarely encounter this unit, but include it here because natural gas utility bills[24] in the U.S. often employ Therms. Since part of the goal of this book is to empower a personal understanding of energy and how to compare different measures of energy (e.g., on a utility bill), conventions in the U.S. demand that we cover the unit here.[25]

> **Definition 5.7.1** *One **Therm** is 100,000 Btu, or 1.055×10^8 J, or 29.3 kWh.*

> **Box 5.5: Why Therms?**
>
> The Therm is partly adopted for the near-convenience that 100 cubic feet of natural gas (CCF or 100 CF), which meters measure directly, equates to 1.036 Therms. Relatedly, one gallon (3.785 L) of liquid propane gas[26] contains 91,500 Btu, which is 0.915 Therms. Thus the Therm very closely matches convenient measures of natural gas (100 cubic feet) or liquid propane (a gallon).

> **Example 5.7.1** It might take approximately 10,000 kcal of energy[27] to heat a fresh infusion of cold water into a hot water heater tank. How many Therms is this?
>
> We do a two-step conversion: first, 10,000 kcal is 41.84 MJ, which at 1,055 J per Btu computes to about 40,000 Btu, which is the same as 0.4 Therm, requiring approximately 40 cubic feet of natural gas, or a little less than half-a-gallon (about 2 L) of liquid propane.
>
> If we learn that the hot water heater is rated at 30,000 Btu per hour, it will take an hour and 20 minutes to complete the job.

27: Based on a capacity of 200 L, pulling in chilly water at 5°C and heating it to 55°C, thereby requiring 200 kg $\times$ 50 C kcal.

It is interesting to reflect on the notion that 200 L of water can be heated by 50°C for only 2 L of liquid fuel: 1% of the water volume in fuel. If heating to boiling, it would take twice as much fuel, so 2% of the water volume. Seems like a good bargain—especially for backpackers who want to boil water and have to lug the fuel around to do so. Inefficiencies in getting heat into the water might require more like 10% fuel volume.

5.8 Electrical Power

Electronic interactions are governed by charges pushing on each other. For the purposes of this course, we need only understand a few concepts. The first is voltage.

Voltage is a measure of *electric potential*, in Volts, and can be thought of as analogous to how high something is lifted.[28] A higher voltage is like sitting higher on the shelf, and can do more work if allowed to be released.

Charge is moved around by electrical forces, and the amount of charge moved plays a role similar to that of mass in gravitational settings. The unit of charge is the Coulomb (C), and the smallest unit of charge we encounter in normal situations is from the proton ($+1.6 \times 10^{-19}$ C) or the electron (-1.6×10^{-19} C).

> **Definition 5.8.1** *The amount of energy in a charge, q, at a voltage, V, is*
>
> $$E = qV. \tag{5.1}$$
>
> *One Coulomb of charge at a potential of 1 V has an energy of 1 J.*

Current is the rate at which charge flows, and is usually symbolized by the letter I. Imagine setting up a toll booth in a conducting wire and counting how many charges (or how much cumulative charge) pass the gate per unit time. This gives rise to the Definition 5.8.2.

> **Definition 5.8.2** *Current is measured in Amps,*[29] *which is defined as one Coulomb per second.*

Moving one Coulomb through one Volt every second would constitute one Joule of energy every second, which is the definition of one Watt. Putting the concepts of Definition 5.8.1 and Definition 5.8.2 together, we find ourselves able to define electrical power.

> **Definition 5.8.3** *Electrical power is simply current multiplied by voltage:*
>
> $$P = IV. \tag{5.2}$$
>
> *Current, I, is in Amps, and voltage, V is in Volts.*

> **Example 5.8.1** Households in the U.S. often have circuit breakers allowing maximum currents of 15 or 20 Amps for regular power outlets. At a voltage of 120 V,[30] this corresponds to a maximum power of 1,800 W or 2,400 W, respectively.[31]

Finally, we are in a position to understand how much energy a battery will hold. Batteries are rated by two numbers: voltage, and charge capacity. Since current is charge per time, multiplying current and time results in just charge.[32] Therefore, charge capacity in batteries is characterized as Amp-hours (Ah) or milli-amp-hours (mAh). Since Amps times Volts is Watts (Eq. 5.2), Amp-hours times Volts is Watt-hours, a familiar unit of energy from Section 5.4.

28: …making electric potential a lot like gravitational potential energy in flavor

29: Amperes, formally

30: The alternating current nature is already accommodated in this measure of voltage.

31: Safety regulations limit continuous use to 80% of the breaker current capacity, so that realistically the limits are 1,400 W and 1,920 W, respectively. This is why "heating appliances" in Figure 5.2 top out around 1,500 W: circuit/safety limits.

32: For example, 0.1 Amps (0.1 Coulombs per second) of current sustained for a duration of 100 seconds results in 10 Coulombs of charge flow.

Example 5.8.2 A typical 9-volt battery has a capacity of 500 mAh. How much energy is this?

500 mAh is 0.5 Ah. Multiplying by 9 V produces 4.5 Wh. Recall that 1 Wh is 1 J/s times 3,600 s (one hour), or 3,600 J. So 4.5 Wh is 16.2 kJ.

How long can we power a 1 W LED array from this battery? We can go the long way (16.2 kJ divided by 1 J/s) and say 16,200 seconds, or recognize that a 4.5 Wh battery can dispense 1 W for 4.5 hours. It's the same either way.[33]

33: Approaching a problem from multiple directions provides validation and also promotes greater flexibility.

5.9 Electron Volt (eV)

The electron-volt (eV) is the unit of choice for energy at the atomic scale. This makes it ideal for discussing individual chemical bond strengths, the energy of individual photons of light emitted from atoms, and thermal energy per atom or molecule.[34] We also use the eV for nuclear physics, but must increase the scale one million-fold and therefore speak of the mega-electron-volt, or MeV.

34: Really, this is just the kinetic energy of the particle.

We have already hit all the relevant concepts for understanding the eV in Section 5.8. The main reason to have its own section is so that it appears separately in the table of contents, making it easier to find and reference. The definition follows Definition 5.8.1 closely.

Definition 5.9.1 *One **electron-volt** is the energy associated with pushing one fundamental unit of electron charge, $|e| = 1.6 \times 10^{-19}$ Coulombs, through an electric potential of 1 V:*

$$1 \, eV = 1.6 \times 10^{-19} \, C \cdot 1 \, V = 1.6 \times 10^{-19} \, J \qquad (5.3)$$

The electron-volt, at 1.6×10^{-19} J, is a tiny amount of energy. But it's just the right level for describing energetic processes for individual atoms.

Example 5.9.1 When 12 grams of carbon (one mole, or 6×10^{23} atoms[35]) reacts with oxygen to form CO_2, about 394 kJ of energy is released.[36] How much energy is this *per carbon atom* in electron-volts?

35: See Appendix B for a primer/refresher on chemistry.

36: Tables in chemistry books contain this type of information.

Since we have one mole, or 6×10^{23} carbon atoms, we divide our total energy (3.94×10^5 J) by the number of atoms to get 6.5×10^{-19} J per atom. This is just a bit larger than 1 eV (1.6×10^{-19} J), and the division leads to something very close to 4 eV per atom.

Because CO_2 has a total of four bonds between the carbon atom and the two oxygen atoms,[37] we see that each bond accounts for about 1 eV. Chemical bonds are often in this range, highlighting the usefulness of the eV unit at the atomic level.

37: Each carbon-to-oxygen link is a *double* bond, meaning that two electrons participate in the link, for a total of four.

5.10 Light Energy

Light energy and its spectrum will be explored more extensively in Chapter 13, but the main concepts are covered here for completeness.

Light can be used to describe any part of the electromagnetic spectrum, from radio waves and microwaves, through infrared, visible, ultraviolet, and on to X-rays and gamma rays. Like atoms, light is "quantized" into smallest indivisible units—in this case particles called photons. An individual photon's energy is characteristic of its wavelength, λ (Greek lambda), or frequency, ν (Greek nu).[38]

38: The two are related by the speed of light, c, via $\lambda \nu = c$.

Definition 5.10.1 *The energy of a photon is given by*

$$E = h\nu = \frac{hc}{\lambda},\tag{5.4}$$

where $h = 6.626 \times 10^{-34}$ J · s is Planck's constant and $c \approx 3.0 \times 10^8$ m/s is the speed of light.

Example 5.10.1 Visible light has a wavelength of 0.4–0.7 μm,[39] corresponding to 2.8–5.0×10^{-19} J for each photon.

39: A micron (μm, or micrometer) is another way to say 10^{-6} m.

We also routinely express photon energy in electron-volts (eV) according to Definition 5.10.2.

Definition 5.10.2 *Given the wavelength in microns (μm), the energy of a photon in eV units is*

$$E_{\text{eV}} = \frac{1.24}{\lambda(\mu\text{m})}\,\text{eV}.\tag{5.5}$$

Example 5.10.2 The red-end of the visible spectrum, around 0.7 μm, corresponds to photon energies around 1.8 eV, while the blue-end, around 0.4 μm, corresponds to 3.1 eV.

5.11 Upshot on Units

Every chapter has an upshot, usually distilling key lessons from the chapter or offering final thoughts. Such a treatment is not necessary here, although we could reinforce the idea that energy can always be expressed in Joules, or converted into any of the units described in the chapter. Also critical is the notion that energy is conserved—only exchanging from one form to another but never truly disappearing or coming from nowhere.

Students may wish to see a master table of conversions between all the units discussed—and what a glorious table this would be! But it is intentionally left out for three reasons:

1. It could short-circuit your effort to learn the material;
2. Problems will ask you to do some of this;
3. This would be a fantastic opportunity for you to design and populate your *own* master conversion table. Then you'll really own it.

Great idea! Go for it!

5.12 Problems

1. A typical textbook may have a mass of 1 kg, and thus a weight of about 10 N. How high could the textbook be lifted (against the force of gravity) by supplying one Joule of energy?

   Weight is mg, where $g \approx 10\,\text{m/s}^2$.

2. If you look in your "energy wallet" and only have 24 J of energy available to spend, how far can you expect to slide an empty box across the floor if it takes 6 N of force to move it along?

3. A 50 kg crate might require 200 N to slide across a concrete floor. If we must slide it 10 m along the floor and then lift it 2 m into a truck, how much energy goes into each action, and what fraction of the total energy expenditure is each?

4. Come up with your own scenario (a force and a distance) that would result in 100 J of energy expenditure.

5. The numbers in Table 5.1 are *reasonable* but should not be thought of as *right*.[40] You can make your own table by using mgh for lifting and $\frac{1}{2}mv^2$ for kinetic energy. For this exercise, pick three familiar activities or situations that allow you to estimate an energy scale in Joules and compute/estimate the results.

   40: Every nerf toss is not 15 J; the bookbag lift depends on how heavy and how high the lift; every example would have a range of reasonable numbers.

6. Just for fun, compute the energy associated with the mass of a tiny bit of shaving stubble having a mass of 0.01 mg[41] using $E = mc^2$. Make sure you use the correct units to put the result in Joules. The speed of light, c, is approximately 3×10^8 m/s.

   41: ...based on 0.1 mm diameter and 1 mm long

7. What exchanges of energy (between what forms) happens when a hand grenade explodes and sends pieces of its casing flying away from the explosion at high velocity? You may wish to describe more than one step/exchange.

8. Follow the evolution of energy exchanges for a wad of clay that you throw high into the air. Describe what is happening as the clay moves upward, as it reaches its apex, as it falls back down, and finally hits the ground with a thud. Where does the initial energy you put into the clay end up?

9. A couch might take 100 N to slide across a floor. If someone slides the couch 4 meters and does it in 8 seconds, how much power did they expend?

   ⓘ The equivalent force to lifting 10 kg, or 22 lb of pushing force.

10. If a 70 kg person (weight: 700 N) is capable of putting out energy at a rate of 500 W in short bursts, how long will it take the person to race up a flight of stairs 4 m high, considering only the vertical energy[42] required?

11. If asked to compute the power associated with performing a pull-up,[43] what specific information would you need to solve the problem (and what are the units of each)? Write out the math that would give the final answer.

12. How many kcal will it take to heat 1 liter of water (e.g., in a pot) from room temperature (20°C) to boiling (100°C)? How many Joules is this?

13. If a microwave operates at a power of 1,600 W (1,600 J/s), how long will it take to heat 0.25 L of water from room temperature to boiling (changing temperature by 80°C) if 50% of the microwave energy is absorbed by the water?

14. A smaller or less active person may require only 1,300 kcal per day of food intake, while a larger or more active person might demand 3,000 kcal per day. Approximately what range of power does this spread translate to, in Watts?

15. If a typical metabolic intake is 2,000 kcal each day, approximately how much energy does this translate to for one day, in units of kWh? Compare this to a typical American household's electricity usage of 30 kWh in a day.

16. The chapter banner image (page 68) shows food labels for peanut butter and Nutella. The former indicates 188 Calories in a 32 g serving, while Nutella is 539 kcal in 100 g. To compare, we must adjust to the same serving size. Using 100 g as a sensible reference, which of the two is more energetic for the same serving size, and by how much (as a percentage)?

17. Based on the peanut butter label in the chapter banner image (page 68), showing 188 Cal per 32 g serving, how much mass of peanut butter would need to be consumed daily to constitute a 2,000 kcal/day diet? If a baseball has a mass of 145 g, how many baseballs of peanut butter would need to be consumed each day?

18. A generic $10 pizza might contain about 2,500 kcal. What is this in kWh? Electricity typically costs $0.15 per kWh,[44] so how much would a pizza's amount of energy cost in electrical terms? Which of the two is a cheaper form of energy?

19. A refrigerator cycles on and off. Let's say it consumes electrical power at a rate of 150 W *when it's on*, and (essentially) 0 W *when it's off*. If it spends half of its time in the on-state, what is its *average* power? How much energy does it consume in a 24-hour day, in

42: Ignoring inefficiencies of moving legs, rounding flights, etc.

43: ... or chin-up, lifting your entire body up to a bar using your arms

ⓘ 50% is typical for microwave efficiency.

ⓘ The result can help inform your sense for the typical range of human metabolic power.

Hint: it may be convenient to first get power in Watts and round to a nice number before proceeding.

ⓘ Comparable to a full day's intake.

44: ... regionally variable

kWh? At a typical electricity cost of $0.15 per kWh, about how much does it cost per *year* to run the refrigerator?

20. The chapter banner image (page 68) shows data from the author's utility bill, indicating 230 kWh of electrical usage for a 30-day period in 2020. What does this rate of energy usage translate to, in Watts?

21. Heating a typical house might require something like 200 W of power for every degree Celsius difference between inside and outside temperatures. If the inside temperature is kept at 20°C and the outside temperature holds steady all day and night at 0°C, how much power is required to maintain the temperature?

(i) For instance, if the temperature difference is 10°C, the house will require 2,000 W of steady input to maintain temperature.

22. If Problem 21 had resulted in 5,000 W,[45] how much energy is used in a 24-hour day, in Joules? Express in the most natural/convenient multiplier (i.e., J, kJ, MJ, GJ, etc.) depending on the scale.

45: ⚠ it does not, exactly

23. If Problem 21 had resulted in 5,000 W,[46] how many kilowatt-hours (kWh) are expended in a 24-hour period? At an electricity cost of around $0.15 per kWh,[47] about how much will it cost, per day, to maintain heat?

46: ⚠ it does not, exactly

47: ...regionally variable

24. If Problem 21 had resulted in 5,000 W,[48] how many Btu are required in a day to maintain temperature? How many Therms is this? At a typical cost of around $1.25 per Therm, about how much does it cost per day to heat the home?

48: ⚠ it does not, exactly

25. If Problem 21 had resulted in 5,000 W,[49] how many gallons of liquid propane[50] would be consumed in heating the home for a day? At a cost of around $2.50 per gallon, about how much does it cost per day to heat the home?

49: ⚠ it does not, exactly

50: Hint: related to Problem 24 and the fact that a gallon of propane contains 0.915 Therms of energy.

26. The chapter banner image (page 68) shows data from the author's utility bill, reflecting 230 kWh of electricity and 4 Therms of gas usage. Annoyingly, the units are different. How do the actual energies compare, if expressed in the same units?[51] How would you capture in a simple sentence the approximate comparison of energy use for each?

51: ...recommend kWh as common basis

27. The chapter banner image (page 68) shows part of the hot water heater label in the author's home, showing a rating of 40,000 Btu/hr. How much power is it capable of putting out, in Watts?

28. The chapter banner image (page 68) shows the energy label associated with the author's hot water heater, estimating that it will use 242 Therms per year. If the estimated energy cost is distributed evenly across 12 months, what would the utility bill be expected to report? Based on an actual utility bill in the same image, the usage for one billing period was 4 Therms. How does actual usage compare to estimated usage, as an approximate percentage?

29. The chapter banner image (page 68) has two panels relating to the same hot water heater. One indicates the rate of gas usage when the heater is on (ignited, heating water) as 40,000 Btu/hr, and the other anticipates 242 Therms per year will be used. How many hours per day is the heater expected to be on (heating water) based on these numbers?

Hint: useful to convert to Therms/hr

30. Gather up or compute conversion factors from the chapter to start your own conversion table (empty version below). Express kWh, cal, kcal, Btu, and Therms in terms of Joules.

From →	kWh	cal	kcal	Btu	Therms
To: J					

31. Perhaps assisted by Problem 30, create a table for conversions between kWh, kcal, Btu, and Therms in terms of one another. The table is started out below, populating the diagonal (no conversion necessary) and also providing a start that 1 Therm is 29.3 kWh.

Not all these conversions are likely to be useful, but a few will come up in practice.

From →	kWh	kcal	Btu	Therms
To: kWh	1			29.3
To: kcal		1		
To: Btu			1	
To: Therms				1

32. A car headlight using light emitting diodes (LEDs) operates at about 15 W. If drawing from the car's 12 V battery, how much current, in Amps, flows to the headlight?

33. Houses in the U.S. are equipped with circuit protection rated to 100 or 200 Amps, typically. If a 100 A house is operating at 80% of its rated capacity,[52] how much power is it consuming (at 120 V)? If sustained for a month, how many kWh will show up on the bill? At $0.15/kWh, what is the cost?

52: All circuits blazing at the safety limit!

34. The chapter banner image (page 68) shows a rechargeable AA battery, operating at 1.2 V and holding 2,200 mAh of charge. How many Joules is this, and how long could it power a 1 W LED array?

35. If we have 6×10^{23} molecules,[53] and each molecule releases 1 eV in a chemical reaction, how many kJ (per mole, as it turns out) is this reaction?

53: This is one mole, as covered in Sec. B.1 (p. 375).

36. Considering the typical wavelength of light to be 0.55 μm, what is a typical photon energy, in Joules, and how many photons per second emerge from a 1 W light source?[54]

54: Assuming 100% efficiency

37. At what wavelength, in microns (μm), is the corresponding photon energy in eV the same number? A deliberately wrong example to illustrate would be if a 2.6μm wavelength corresponded to 2.6 eV (it doesn't').

6 Putting Thermal Energy to Work

We have already encountered thermal energy in two contexts. The first was infrared radiation (Eq. 1.8; p. 10), and the second was in the definition of the kilocalorie (Sec. 5.5; p. 73). Otherwise, heat has often been treated as a form of "waste" in a chain of energy conversion: friction, air resistance, etc. The insinuation was that heat is an unwanted byproduct of no value.

Yet 94% of the energy we use today is thermal in nature [34]: we *burn* a lot of stuff for energy![1] Sometimes heat is what we're after, but how can we use it to fly airplanes, propel cars, and light up our screens? This chapter aims to clarify how heat is used, and explore limits to the efficiency at which heat can perform non-thermal work.

Like the previous chapter, this topic represents a slight detour from the book's overall trajectory, which otherwise aims to build a steady narrative of what we *can't* expect to continue doing, what options we *might* use to change course, and finally how to bring about such change. Nonetheless, the way we utilize thermal energy is a key piece of the story, and relates to both current and future pathways to satisfying our energy demands.

[34]: U.S. Energy Inform. Administration (2011), *Annual Energy Review*

1: The exceptions are wind, hydroelectricity, and solar.

6.1 Generating Heat

Before diving in to thermal issues, let's do a quick run-down of the various ways we can generate heat.

> **Example 6.1.1 Ways to Generate Heat**: Roughly arranged according to degree of sophistication:

A locomotive engine as an example heat engine. Photo credit: South Australian Government Photographer.

1. Rub your hands together (or other forms of friction).
2. Harvest sunlight, possibly concentrating it, for heat; drying clothes outside and letting sunlight warm a room through a window are examples.
3. Access geothermal heat in select locations.
4. Burn wood in a fireplace or stove.
5. Burn a fossil fuel for direct heat; gas is often used in homes for space heating, as well as for heating water and cooking.
6. Run electrical current through a coil of wire that glows orange; seen in toaster ovens, hair dryers, space heaters.
7. Use electricity to run a heat pump (Section 6.5).
8. Allow nuclear material to undergo fission in a controlled chain reaction.
9. Contrive a plasma hot enough to sustain nuclear fusion—as the sun has done for billions of years.

6.2 Heat Capacity

First, we'll connect a basic thermal concept to something we already covered in Sec. 5.5 (p. 73) in the context of the calorie. The statement that it takes 1 kcal to heat 1 kilogram of H_2O by 1°C is in effect defining the *specific heat capacity* of water. In SI units, we would say that H_2O has a specific heat capacity of 4,184 J/kg/°C.[2] Very few substances top water's specific heat capacity. Most liquids, like alcohols, tend to be in the range of 2,000 J/kg/°C. Most non-metallic solids (and even air) come in around 1,000 J/kg/°C. Metals are in the 130–900 J/kg/°C range—lighter metals at the top, and heavier ones at the lower end.[3] Table 6.1 provides a sample of specific heat capacities for common substances.

Knowing the specific heat capacity of a substance allows us to compute how much energy it will take to raise its temperature. A useful and approximate guideline is to treat water as 4,000 J/kg/°C and *all other stuff* (air, furniture, walls) as 1,000 J/kg/°C. Mixtures, like food, might be somewhere between, at 2,000–3,500, due to high water content. If in doubt, 1,000 J/kg/°C is never going to be *too* far off. For estimation purposes, deviate from this only for high water-content[4] or for metals.[5]

Example 6.2.1 A 2,000 kg pick-up truck is transporting a one-cubic-meter container of water. How much energy will it take to raise the temperature of the whole ensemble by 5°C?

A cubic meter of water (1,000 L) is 1,000 kg and has a heat capacity around 4,000 J/kg/°C; the truck is mostly steel, so we might guess 500 J/kg/°C. Multiply each specific heat capacity by the respective mass and the 5 degree change to get 20 MJ to heat the water and 5 MJ to heat the truck for a total of 25 MJ.[6]

Table 6.1: Specific heat capacities of common materials.

Substance	J/kg/°C
steel	490
rock, concrete	750–950
glass	840
aluminum	870
air	1,005
plastic	1,100–1,700
wood	1,300–2,000
alcohol	2,400
flesh	3,500
water	4,184

2: For temperature changes, it is always possible to interchange per-degrees-Celsius and per-Kelvin because the two are only different by a constant offset, so that any *change* in temperature is the same measure in both.

3: The pattern here is that substances like water or alcohols containing light atoms like hydrogen have higher heat capacities than substances like metals containing heavier atoms.

4: ...go as high as 4,000 J/kg/°C in this case

5: ...500 for heavier metals like steel; although light metals like aluminum are not far from 1,000 J/kg/°C

6: Notice that the water demands far more energy to heat, even though it is half the mass.

To perform computations using specific heat capacity, try an intuitive approach rather than some algorithmic formula.[7] The following should just make a lot of sense to you, and can guide how to put the pieces together: it takes *more energy* to heat a *larger* mass **or** to raise the temperature by a *larger* amount. It's all proportional. The units also offer a hint. To go from specific heat capacity in J/kg/°C to energy in J, we need to multiply by a mass and by a temperature change.

7: Although, this would be a good opportunity for a student to make up their *own* formula, driving home the concept and the fact that equations simply capture a concept. Also, the choice of symbols is arbitrary, which the experience would reinforce.

Example 6.2.2 To compute the amount of energy it will take to heat a 30 kg piece of furniture[8] by 8°C, we will multiply the specific heat capacity by the mass—to capture the "more mass" quality—and then multiply by the temperature change—to reflect the "more temperature change" element. In this case, we get 240 kJ.

8: . . . assuming 1,000 J/kg/°C

6.3 Home Heating/Cooling

Our personal experience with thermal energy is usually most connected to heating a living space and heating water or food. Indeed, about two-thirds of the energy used in residential and commercial spaces[9] relate to thermal tasks, like heating or cooling the spaces, heating water, refrigeration, drying clothes, and cooking.

9: . . . in the form of natural gas, electricity, and fuel oil

When it comes to heating (or cooling) a home, we might care about two things:

▶ how long will it take to change its temperature by some certain amount; and
▶ how much energy it will take to keep it at the desired temperature.

The first depends on how much stuff is in the house,[10] how much ΔT you want to impart, and how much power is available to create[11] the heat. The energy required is mass times ΔT times the catch-all 1,000 J/kg/°C specific heat capacity. The time it takes is then the energy divided by the available power.

10: . . . including walls, furniture, air

11: . . . or to remove, if cooling

Example 6.3.1 How long will it take to heat up the interior of a mobile home from 0°C to 20°C using two 1,500 W space heaters? We'll assume that we must heat up about 6,000 kg of mass.[12]

The first job is to find the energy required and then divide by power to get a time. We'll use the good-for-most-things specific heat capacity of 1,000 J/kg/°C.

Multiplying the specific heat capacity by mass and temperature change results in 120 MJ of energy. At a rate of 3,000 W, it will take 40,000 s to inject this much energy, which is about 11 hours.

12: Only 300 kg is in the form of air: most of the mass to be heated is in the walls, floor, and ceiling.

How much it takes to *maintain* temperature depends on how heat flows out of (or into) the house through the windows, walls, ceiling, floor,

and air gaps. But it also depends *linearly* on ΔT—the difference between inside and outside temperatures—that is being maintained. A house can be characterized by its heat loss rate in units of Watts per degree Celsius.[13] This single number then indicates how much power is needed to maintain a certain ΔT between inside and outside. Box 6.1 explores an example of how to compute the heat loss rate for a house, and Example 6.3.2 applies the result to practical situations.
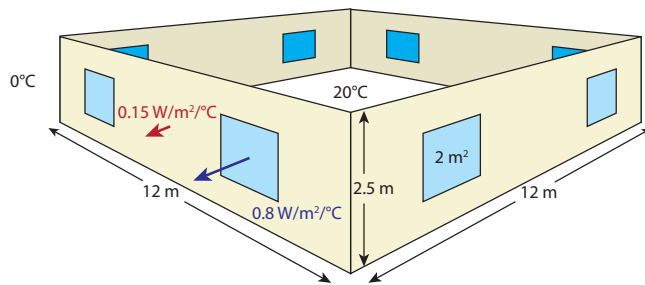
13: ...or equivalently, Watts per degree Kelvin

Figure 6.1: External walls and windows for the house modeled in Box 6.1. The floor and ceiling are not shown. The numbers in $W/m^2/°C$ are U-values, and in this case represent the very best engineering practices. Most houses will have larger values by factors as high as 2–6. Don't forget the door in a real house!

Box 6.1: House Construction

The very best practices result in a snugly-built house qualified as a "Passive House," achieving 0.15 W/°C *for each square meter* of external-interfacing surface[14] and 0.8 W/°C per square meter of windows.

Let's imagine a house having a square footprint 12 m by 12 m, walls 2.5 m high, each of the four walls hosting two windows, and each window having an area of 2 m² (Figure 6.1). The floor and the ceiling are both 144 m², and the wall measures (perimeter times height) $48 \times 2.5 = 120\,m^2$. But we deduct 16 m² for the eight windows, leaving 104 m² for the walls. The resulting heat loss measure for the house is 13 W/°C for the windows (0.8 W/m²/°C $\times$ 16 m²), plus 59 W/°C for the walls/floor/ceiling for a total of 72 W/°C.

The loss rate for a *decently*-constructed house might be about twice this, while a *typically*-constructed house (little attention to efficiency) might be 3–6 times this—several hundred W/°C. Of course, smaller houses have smaller areas for heat flow and will have smaller loss rates.

14: ...outer walls, ceiling-to-unconditioned attic, floor-to-crawl-space

The numbers used to characterize heat loss properties of walls and windows are called U-values, in units of $W/m^2/°C$, where low numbers represent better insulators. In the U.S., building materials are described by an inverse measure, called the R-value, in ugly units of $°F \cdot ft^2 \cdot hr/Btu$. The two are numerically related as $R = 5.7/U$, so that our Passive House wall has $R \approx 38$ and the windows have $R \approx 7$—both rather impressive and hard to achieve.

Example 6.3.2 Let's compare the requirements to keep three different houses at 20°C while the temperature outside is 0°C (freezing point). The first is a snugly-built house as described in Box 6.1, where we round the heat loss rate to a more convenient 75 W/°C. We'll then imagine a decently built house at 150 W/°C, and a more typical[15] house at 300 W/°C.

15: ...not "thermally woke"

The temperature difference, ΔT, is 20°C, so that our super-snug house

will require 75 W/°C times 20°C, or 1,500 W[16] to keep it warm, while the decent house needs 3,000 W and the shoddy house needs 6,000 W.

Once we understand how much power it takes to maintain a certain temperature (ΔT) in a house, we can anticipate the behavior of the house's heater. Heaters are typically either on full-blast or off. Regulation is achieved by turning the heat on and off—usually controlled by a thermostat. Given the *rating* of a heater,[17] it is then straightforward to anticipate the duty cycle: the percentage of time it has to be on to produce an *average* output meeting the power requirement for some particular ΔT.

In a sensible world, heaters are characterized by W (or kW). In the U.S., the measure for many appliances is Btu/hr. Since 1 Btu is 1,055 J and one hour is 3,600 s, one Btu/hr equates to 0.293 W.[18] A whole house heater—sometimes in the form of a furnace—might be rated at 30,000 Btu/hr (about 10 kW), in which case the three outcomes in Example 6.3.2 would require the heater to be on about 15%, 30%, or 60% of the time[19] to maintain $\Delta T = 20°C$ in the three houses.

It is also possible to assess how much ΔT the foregoing heater could maintain in the three houses. It should stand to reason that a house requiring 100 W/°C and having a 10,000 W heater can support a ΔT as high as 100°C.[20] Thus, the three houses from Example 6.3.2 could support ΔT values of 133°C, 67°C, and 33°C if equipped with a 10 kW (~30,000 Btu/hr) heater. The snug house does not need such a powerful heater installed. The poorly built house can maintain a $\Delta T = 33°C$ differential at full-blast, which means that if the temperature drops below −13°C (8.6°F) outside, it will not be able to keep the inside as high as 20°C. So a house in a cold climate should either be built to better thermal standards, or will require a bigger heater—costing more to heat the home.[21]

Cooling a home (or refrigerator interior, or whatever) is also a thermal process, but in this case involves *removing* thermal energy from the cooler environment. Removing heat is harder to do, as witnessed by the length of human history that has utilized heating sources—starting with fire—compared to the relatively short amount of time when we have been able to produce cooling on demand.[22] Section 6.5 will get into how this is even possible, in principle. For now, just be aware that the rating on air conditioners uses the same units as heaters: how much thermal energy can be moved (out of the cooler environment) per unit time. In SI units, we know this as the Watt. In the U.S., it's Btu/hr.

6.4 Heat Engines

Now we get to the part where thermal energy can be used to do something other than just provide direct heat to a home. It may seem odd to always

characterize burning fuel as a purely *thermal* action, since what transpires within the cylinder of a gasoline-burning internal combustion engine seems like more of a little *explosion* than just the generation of heat. This is not wrong, but neither is it the whole story. The process still begins as a fundamentally thermal event. When the fuel-air mixture ignites, the temperature in the cylinder increases dramatically. To appreciate what happens as an immediate consequence, we turn to the ideal gas law:

$$PV = Nk_{\mathrm{B}}T. \tag{6.1}$$

This is the physicist's version, which looks a little different than the chemist's $PV = nRT$. For a comparison, see Sec. B.4 (p. 381).

P, V and T are pressure, volume, and temperature (in $\mathrm{N/m^2}$, $\mathrm{m^3}$, and Kelvin). N is the number of atoms or molecules, and $k_{\mathrm{B}} = 1.38{\times}10^{-23}$ J/K is the Boltzmann constant, which we will see again in Sec. 13.2 (p. 199). The temperature rise upon ignition is fast enough that the cylinder volume does not have time to change.[23] Eq. 6.1 then tells us that the pressure must also spike when temperature does, all else being held constant. The increase in pressure then pushes the piston away, increasing the cylinder volume and performing work.[24] But it all starts thermally, via a sharp increase in temperature.

23: The moving piston allows the volume to change, but on slower timescales.

24: Work is measured as pressure times the change in volume. Pressure is force per unit area, so the units work out to force times distance, as they should given the definition of work.

In the most general terms, thermal energy tries to flow from hot to cold—out of a pot of hot soup; or into a cold drink from the surrounding air; or into your feet from hot sand. Some part of this flow can manifest as physical work, at which point the system can be said to be acting as a heat engine.

Definition 6.4.1 *A heat engine is loosely defined as any system that turns heat, or thermal energy into mechanical energy: moving stuff.*

Example 6.4.1 Example heat engines: when heat drives motion.

1. Hot air over a car's roof rises, gaining both kinetic energy and gravitational potential energy;
2. Wind is very similar, in that air in contact with the sun-heated ground rises and gains kinetic energy on an atmospheric scale;
3. The abrupt temperature increase in an internal combustion cylinder drives a rapid expansion of gas within the cylinder;
4. Steam in a power plant races though the turbine because it is flowing to the cold condenser.

Table 6.2: Schemes for electricity generation. Most are thermal in nature, and nearly all employ a turbine and generator. Data for 2018 from Table 8.2a of [34].

Source	% elec. in U.S.	therm.	turb./ gen.
Nat. Gas	35.3	✓	✓
Coal	27.3	✓	✓
Nuclear	19.2	✓	✓
Hydroelec.	7.0		✓
Wind	6.6		✓
Solar PV	2.2		
Biomass	1.5	✓	✓
Oil	0.6	✓	✓
Geotherm.	0.4	✓	✓
Sol. Therm.	0.09	✓	✓

The last example deserves its own graphic, as important as this process is in our lives: almost all of our electricity generation—from all the fossil fuels and even from nuclear fission—follows this arrangement. Figure 6.2 illustrates the basic scheme. Table 6.2 indicates that 98% of our electricity involves turning a turbine on a shaft connected to a generator, and 84% involves a thermal process as the motive agent for the turbine—most often in the form of steam.

Figure 6.2: Generic power plant scheme, in which some source of heat at T_h generates steam that flows toward the condenser—where the steam cools and reverts to liquid water, by virtue of thermal contact to a cool source at T_c provided by a body of water or evaporative cooling towers. Along the way, the rushing steam turns a turbine connected to a generator, exporting electricity. This basic arrangement is employed for most power plants using fossil fuels, nuclear, solar thermal, or geothermal sources of heat.

6.4.1 Entropy and Efficiency Limits

A deep and powerful piece of physics intervenes to limit how much useful work may be extracted out of a flow of heat from a hot source at temperature T_h to a cold source at temperature T_c. That piece is entropy. You don't need to fully grasp the deep and subtle concept of entropy in order to follow the development in this chapter and understand the role entropy plays in limiting heat engine efficiency. All the same, it is a stimulating topic that we'll dip a toe into for some appreciation.

> **Definition 6.4.2** *Entropy is a measure of how many ways a system might be organized at the microscopic level while preserving the same internal energy.*[25]

25: E.g., at constant temperature, pressure, volume.

This definition may be an obscure disappointment to those expecting entropy to be defined as a measure of *disorder*.[26] Consider a gas maintained at constant pressure, volume, and temperature—thus fixing the total energy in the gas. The atoms/molecules comprising the gas can arrange into a staggeringly large number of configurations: any number of positions, velocities, rotational speeds and axis orientations, or vibrational states of each molecule, for instance—all while keeping the same overall energy.

26: Entropy is indeed *related* to disorder, in that there are many more ways to configure matches in a mess than there are ways to neatly stack them.

> **Example 6.4.2** To illustrate, consider a tiny system containing 3 molecules labeled A, B, and C, having a total energy of 6 units split

between them in some way. They can all have exactly 2.0 units of energy apiece, or can have individual energies of 1.2, 1.8, and 3.0 units; or 3.2, 0.4, and 2.4; or any other of myriad combinations adding to the same thing. Entropy provides a measure of how many combinations[27] are possible.

27: It is far beyond the scope of this book to detail the counting scheme, but it is perhaps important to appreciate that energy levels are discrete—or *quantized*—which prevents an infinite number of possible energy combinations.

0:4 (1)   1:3 (16)   2:2 (36)   3:1 (16)   4:0 (1)
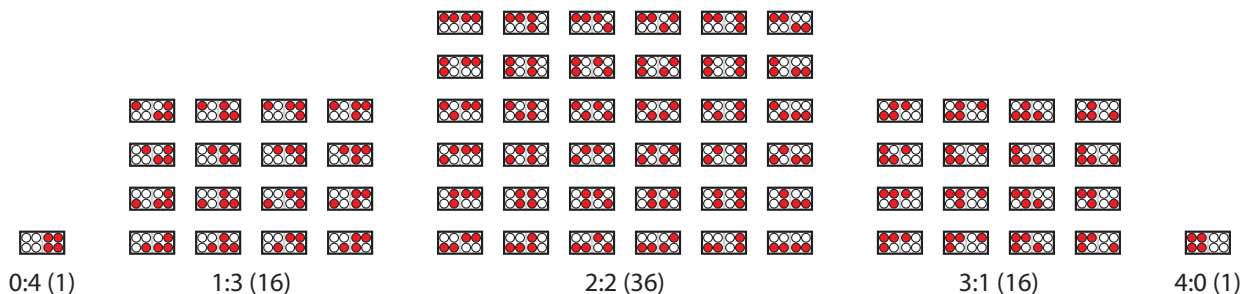
Figure 6.3: A box containing 4 atoms or molecules of one type (white) and 4 of another type (red) has many more configurations available (number in parentheses) when species are equally distributed so that left and right sides both have two of each. Entropy is related to the number of ways a system can distribute itself (at the same energy level), acting to favor disordered mixing over (improbable) orderly separation.

Example 6.4.3 To better elucidate the connection between entropy and disorder, imagine a box of air, containing both N_2 and O_2 molecules. As Figure 6.3 illustrates, a thoroughly-mixed arrangement has a larger number of possible configurations, thus the highest entropy. Nature does not give rise to spontaneous organization in a closed system.[28]

28: It is, however, possible to see lowered entropy in one place if balanced by an increase elsewhere: life organizes matter, but at the expense of increased entropy in the wider universe.

The First Law of Thermodynamics is one we already encountered as conservation of energy:

Definition 6.4.3 *First Law of Thermodynamics: the energy of a closed system is* conserved, *and cannot change if nothing—including energy—enters or leaves the system boundaries.*

Now we are ready for the Second Law.

Definition 6.4.4 *Second Law of Thermodynamics: the total entropy of a closed system* may never decrease.

It is entropy that governs *which way* heat flows (hot to cold, if left alone) and in a deep sense defines the "arrow of time."

Box 6.2: The Arrow of Time

Consider that if you were shown videos of a rock splashing into water, a coffee mug shattering on the floor, or an icicle melting, you would have no difficulty differentiating between the forward and reverse playbacks of the video.

The reverse action, you would conclude, is preposterous and can

never happen. Pieces of ceramic strewn about the floor will never spontaneously assemble into a mug and leap from the floor! Energy is not the barrier, because the total energy in all forms is the same[29] before and after. It's entropy: the more ordered states are less likely to spontaneously emerge. To appreciate how pervasive entropy is, imagine how easy it is to spot a "fake" video run backwards.

29: . . . provided that the system boundary is drawn large enough that no energy escapes

These two laws of thermodynamics, plus a way to quantify entropy changes that we will see shortly, are all we need to figure out the maximum efficiency a heat engine can achieve in delivering work. If we draw an amount of heat, ΔQ_h from a hot bath[30] at temperature T_h, and allow part of this energy to be "exported" as useful work, ΔW, then we must have the remainder flow as heat (ΔQ_c) into the cold bath at temperature T_c. Figure 6.4 offers a schematic of the process. The First Law of Thermodynamics[31] requires that $\Delta Q_h = \Delta Q_c + \Delta W$, or that all of the extracted heat from the hot bath is represented in the external work and flow to the cold bath: nothing is lost.

30: By "bath," we mean a large reservoir at a constant temperature that is large enough not to appreciably change its temperature upon extraction of some amount of thermal energy, ΔQ.

31: . . . conservation of energy

Figure 6.4: Heat engine energy balance. Heat flowing from the hot bath to the cold bath can perform useful work, ΔW, in the process—subject to conservation of energy ($\Delta Q_h = \Delta Q_c + \Delta W$), where ΔQ is a heat flow. Entropy constraints limit how large ΔW can be. Arrow widths are proportional to energy, and red numbers are example energy amounts, for use in the text.

So where does entropy come in? Extracting heat from the hot bath in the amount ΔQ_h results in an entropy change in the hot bath according to Definition 6.4.5.

Definition 6.4.5 *Entropy Change: when energy (heat, ΔQ, in J) is moved into or out of a thermal bath at temperature T, the accompanying change in the bath's entropy, ΔS, obeys the relation:*

$$\Delta Q = T\Delta S. \qquad (6.2)$$

When heat is removed, entropy is reduced. When heat is added, entropy increases. The temperature, T, must be in Kelvin, and entropy is measured in units of J/K.

So the extraction of energy from the hot bath results in a *decrease* of entropy in the hot bath of ΔS_h according to $\Delta Q_h = T_h\Delta S_h$. Meanwhile, ΔS_c of entropy is *added* to the cold bath according to $\Delta Q_c = T_c\Delta S_c$. The Second Law of Thermodynamics enforces that the *total* change in

Table 6.3: Thermodynamic symbols.

Symbol	Describes (units)
T	temperature (K)
ΔT	temp. change (K,°C)
ΔQ	thermal energy (J)
ΔW	mechanical work (J)
ΔS	entropy change (J/K)
ε	efficiency
η	entropy ratio

entropy may not be negative (it can't decrease). In equation form (symbol definitions in Table 6.3):[32]

$$\Delta S_{\text{tot}} = \Delta S_{\text{c}} - \Delta S_{\text{h}} \geq 0, \tag{6.3}$$

where we have subtracted ΔS_{h} since it was a *deduction* of entropy, while ΔS_{c} is an addition. We therefore require that

$$\Delta S_{\text{c}} \geq \Delta S_{\text{h}}. \tag{6.4}$$

Now we are in a position to ask what fraction of ΔQ_{h} can be diverted to useful work (ΔW) within the constraints of the Second Law. We express this as an efficiency,[33] denoted by the Greek epsilon:

$$\varepsilon = \frac{\Delta W}{\Delta Q_{\text{h}}} = \frac{\Delta Q_{\text{h}} - \Delta Q_{\text{c}}}{\Delta Q_{\text{h}}}. \tag{6.5}$$

The second step applies conservation of energy: $\Delta Q_{\text{h}} = \Delta Q_{\text{c}} + \Delta W$.

> **Example 6.4.4 Actual Efficiency**: If a heat engine is observed to remove 30 J from the hot bath and deposit 20 J into the cold bath, as in Figure 6.4, what is the efficiency of this heat engine in producing useful work?
>
> Whether we deduce that $\Delta W = 10$ J and use the first form in Eq. 6.5 or apply the second form using the given heat flows, the answer is 1/3, or 33%.

We can add a step to Eq. 6.5 to express it in terms of entropy changes:

$$\varepsilon = \frac{\Delta W}{\Delta Q_{\text{h}}} = \frac{\Delta Q_{\text{h}} - \Delta Q_{\text{c}}}{\Delta Q_{\text{h}}} = \frac{T_{\text{h}}\Delta S_{\text{h}} - T_{\text{c}}\Delta S_{\text{c}}}{T_{\text{h}}\Delta S_{\text{h}}}, \tag{6.6}$$

where we have re-expressed each ΔQ as an equivalent $T\Delta S$ withdrawal/deposit of entropy. Now we can divide both numerator and denominator by ΔS_{h} to be left with

$$\varepsilon = \frac{T_{\text{h}} - T_{\text{c}}\eta}{T_{\text{h}}}, \tag{6.7}$$

where we create η (eta) to represent the ratio of entropies: $\eta = \Delta S_{\text{c}}/\Delta S_{\text{h}}$, which we know from Eq. 6.4 cannot be smaller than one:[34]

$$\eta \geq 1. \tag{6.8}$$

Looking at Eq. 6.7, if we want the highest possible efficiency in extracting work from a flow of heat, we want the numerator to be as large as possible. To achieve this, we want to subtract as little as possible from T_{h}. If η were allowed to be very large, then the numerator would be reduced. So we want the *smallest possible* value for η, which we know from Eq.

32: Remember: treat equations as *sentences* expressing important concepts in precise ways—not merely as algorithmic machines to memorize for plugging in while solving problems. What does it *say*?

33: This definition of efficiency captures what we care about: what fraction of the extracted heat can be turned into useful work.

34: If $A \geq B$, then we know that $A/B \geq 1$.

6.8 happens when $\eta = 1$. We therefore derive the maximum physically allowable efficiency of a heat engine as

$$\varepsilon_{\max} = \frac{T_h - T_c}{T_h} = \frac{\Delta T}{T_h}, \qquad (6.9)$$

⚠ Temperature must be in Kelvin. Recall that $T(K) \approx T(°C) + 273$.

where we have designated $\Delta T = T_h - T_c$ as the temperature difference between hot and cold baths. A major takeaway is that efficiency improves as ΔT gets bigger, and becomes vanishingly small for small values of ΔT.

Example 6.4.5 If operating between a hot bath at 800 K and ambient temperature around 300 K,[35] a heat engine could produce a maximum efficiency of 62.5%.

35: 300 K is a convenient and reasonable temperature for "normal" environments, corresponding to 27°C or 80.6°F.

Example 6.4.6 A heat engine operating between boiling and freezing water has $T_h \approx 373$ K and $\Delta T = 100$ K, for a maximum possible efficiency of $\varepsilon_{\max} = 0.268$, or 26.8%.

Example 6.4.7 A heat engine operating between human skin temperature at 35°C and ambient temperature at 20°C has a maximum efficiency of $\varepsilon_{\max} = 15/308 \approx 0.05$, or 5%.

If the cold bath is fixed,[36] the maximum possible efficiency improves as the temperature of the hot source goes up. Conversely, for a given T_h, the efficiency improves as the cold temperature decreases and thus ΔT increases.

36: This is a common situation, as T_c is usually set by the ambient temperature of the air or of a body of water.

Box 6.3: At the Extreme Limit...

If T_c approaches 0 K[37], the maximum efficiency approaches 100%. We can trace this back to the relation $\Delta Q = T\Delta S$, which implies that when T is very small, it does not take much heat (ΔQ) to meet the requirement for the amount of entropy added to the cold bath (ΔS_c) to be large enough to satisfy the prohibition on net entropy decrease, so the arrow width in Figure 6.4 for ΔQ_c can be rather thin (small) allowing ΔW to be about as thick (large) as ΔQ_h, meaning that essentially all the energy is available to do work and the efficiency can be very high. In practice, Earth does not provide baths cold enough for this effect to kick in, but discussing it is a means to better understand how Eq. 6.9 works.

37: ...absolute zero temperature, −273°C

Real heat engines like power plants (Figure 6.2) or automobile engines tend to only get about halfway to the theoretical efficiency due to myriad practical challenges. A typical efficiency for an electrical power plant is 30–40%, while cars are typically in the 15–25% range. In contrast, combustion temperatures around 700–800°C suggest a maximum theoretical efficiency around 60%.

6.5 Heat Pumps

We can flip a heat engine around and call it a heat pump. In this case, we *apply* some external work to *drive* a heat flow opposite its natural direction—like pushing heat uphill. This is how a refrigerator[38] works, for instance. Figure 6.5 sets the stage.
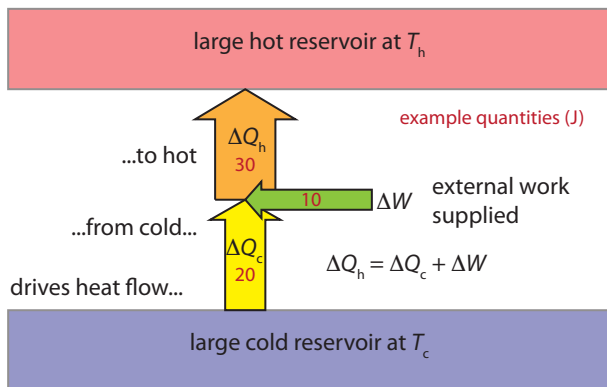
Figure 6.5: Heat pump energy balance. The application of work (ΔW; from an electrical source, for instance) can drive heat to flow—counterintuitively—from a cold reservoir (like the interior of a freezer) to a hotter environment. Example $T_c \rightarrow T_h$ pairs might include freezer-interior $\rightarrow$ room-air; cooled-inside $\rightarrow$ summer-outside; winter-outside $\rightarrow$ warmed-inside. We still must satisfy conservation of energy ($\Delta Q_h = \Delta Q_c + \Delta W$), where ΔQ is a heat flow. Entropy constraints limit how large ΔQ_c can be for a given ΔW input. Arrow widths are proportional to energy, and red numbers are example energy amounts, for use in the text.

A very similar chain of logic can be applied to this configuration, invoking the Second Law to guarantee no entropy decrease. We define the efficiency according to the application and what we care about, giving rise to two different figures of merit.

Definition 6.5.1 ε_{cool}: *For cooling applications,*[39] *we care about how much heat can be removed from the cooler environment (ΔQ_c) for a given input of work (ΔW). The efficiency is then characterized by the ratio $\varepsilon_{cool} = \Delta Q_c/\Delta W$.*

Definition 6.5.2 ε_{heat}: *For heating applications,*[40] *we care about the heat delivered to the hot bath (ΔQ_h) for a given amount of input work (ΔW). The efficiency is then characterized by the ratio $\varepsilon_{heat} = \Delta Q_h/\Delta W$.*

The derivation goes similarly to the one above, but now we require that the entropy *added* to the hot bath must not be smaller than the entropy *removed* from the cold bath so that the total change in entropy is not negative.[41] In this case, the maximum allowed efficiencies for cooling and heating via heat pumps are:

$$\varepsilon_{cool} \leq \frac{T_c}{T_h - T_c} = \frac{T_c}{\Delta T}, \tag{6.10}$$

and

$$\varepsilon_{heat} \leq \frac{T_h}{T_h - T_c} = \frac{T_h}{\Delta T}. \tag{6.11}$$

These look a lot like Eq. 6.9, but *turned upside down*. The maximum efficiencies can be larger than unity![42]

Example 6.5.1 What is the limit to efficiency of maintaining a freezer at −10°C in a room of 20°C?

First, we express the temperatures in Kelvin: $T_c \approx 263$ K and $\Delta T = 30$ K.[43] The maximum efficiency, by Eq. 6.10, computes to $\varepsilon_{\text{cool}} \leq 8.8$ (880%).

43: Note that $\Delta T = 30$ in either K or °C.

Example 6.5.2 What is the limit to efficiency of keeping a home interior at 20°C when it is −10°C outside?

First, we express the temperatures in Kelvin: $T_h \approx 293$ K and $\Delta T = 30$ K.[44] The maximum efficiency, by Eq. 6.11, computes to $\varepsilon_{\text{heat}} \leq 9.8$ (980%).

44: Note that $\Delta T = 30$ in either K or °C.

Box 6.4: Is >100% Really Possible?

At first, it seems to be spooky and impossible that efficiencies can be greater than 100%. Example 6.5.1 essentially says that as many as 8.8 J of thermal energy can be *moved* for a mere 1 J *input* of work! The situation bears analogy to the martial art of Jiu Jitsu, whereby the opponent's momentum is used to their detriment, requiring little work to direct its flow. In this case, we convince a bundle of thermal energy sitting in the freezer to move outside where it is hotter (uphill; against natural flow) and in the process use less energy than the amount of thermal energy residing in the bundle.

The fact that our "efficiency" metrics come out to be greater than 100% is an illusion: an artifact of how we defined $\varepsilon_{\text{cool}}$ and $\varepsilon_{\text{heat}}$. Conservation of energy is not violated; we're just putting the small piece (ΔW) in the denominator to form the efficiency metric.[45] In this sense, it's not the usual sort of efficiency measure, which puts the *largest* quantity (total budget) in the denominator.

Maybe the situation can be compared more understandably to money transfers, where one might pay a $20 fee to wire $1,000 from account A to account B. It doesn't mean that $1,000 was created out of $20—just that $20 was spent (like ΔW) to move a much larger sum into account B. But if account A belonged to somebody else, it would seem like you just turned $20 of your own money into $1,000 at a gain of 5,000%, even though it really came from elsewhere.

45: Following the example numbers in Figure 6.5, we would say that $\varepsilon_{\text{cool}}$, defined as $\Delta Q_c / \Delta W$, is 2.0, and $\varepsilon_{\text{heat}}$ is 3.0.

In the case of heating, it is worth comparing the output of a heat pump to the application of direct heat. Let's revisit the scenarios explored in Section 6.3.

Example 6.5.3 If a house's thermal performance is 150 W/°C and we want to maintain 20°C inside while the outside temperature is a frigid −20°C, we would need to supply 6,000 W of energy[46] to the home in the form of burning fuel (natural gas, propane, firewood) or electricity for direct-heating application.[47]

But according to Eq. 6.11, a heat pump could theoretically move 6,000 W of thermal energy by only applying 820 W without violating the Second Law, since $\varepsilon_{\text{heat}} \leq 293/40 = 7.3$ and 6,000 J (ΔQ_h) divided by 7.3 (to get ΔW) is 820 J.[48]

46: 150 W/°C times 40°C.

47: . . . e.g., four space heaters each expending 1,500 W

48: We are solving for $\Delta W = \Delta Q_h / \varepsilon_{\text{heat}}$, and consider the energy moved in one second in order to go from W to J.

Engineering realities will prevent operating right up to the thermody-

namic limit, but we might at least expect to be able to accomplish the 6,000 W goal of Example 6.5.3 for under 2,000 W. Thus the heat pump has shaved a factor of three (or more) off the energy required to provide heat inside. Heat pumps are very special.

As Eq. 6.10 and Eq. 6.11 imply, heat pumps are most efficient when ΔT is small. Thus a refrigerator in a hot garage must not only work harder to maintain a large ΔT, it does so less efficiently—making it a double-whammy. For home heating, heat pumps offer the most gain in milder climates where ΔT is not so brutal.

6.5.1 Consumer Metrics: COP, EER, HSPF

When shopping for heat pumps or air conditioners (or freezers/refrigerators), products are specified by the coefficient of performance (COP) or energy efficiency ratio (EER) or heating seasonal performance factor (HSPF), as in Figure 6.6. How do these relate to our $\varepsilon_{\text{heat}}$ and $\varepsilon_{\text{cool}}$ values? The first one is easy.

> **Definition 6.5.3** *COP: Heat pumps used for heating are specified by a coefficient of performance (COP), which turns out to be familiar already:*
>
> $$\text{COP} = \varepsilon_{\text{heat}}. \qquad (6.12)$$

> **Example 6.5.4 COP Example**: Using the red numbers in Figure 6.5, we can compute $\varepsilon_{\text{heat}}$, the COP, and then determine the minimum T_c theoretically permissible (resulting in maximum possible efficiency) if $T_h = 300$ K.[49]
>
> We go back to the original definition of $\varepsilon_{\text{heat}}$ as $\Delta Q_h/\Delta W$, which for our numbers works out to 30/10, or 3.0 The COP is then simply 3.0.
>
> Setting $\varepsilon_{\text{heat,max}} = T_h/\Delta T$ equal to 3.0, we find that ΔT is 100 K, so that the minimum permissible $T_c = 200$ K in this case.

The EER is different, and perhaps a little odd. EER is defined as the amount of heat moved (ΔQ_c), in Btu, per work input (ΔW), in watt-hours (Wh). What?! Sometimes the world is just loopy. But we can manage this. If handed an EER (Btu/Wh), we can convert it to our same/same numerator/denominator units by converting both numerator and denominator to the same units. We could convert Btu to Wh in the numerator and be done, or convert Wh to Btu in the denominator and be done, or we could convert both numerator *and* denominator to Joules[50] to get there. For illustrative purposes, we'll pick the last approach. To get from Btu to Joules, we multiply (the numerator) by 1,055. To get from Wh to Joules, we multiply the denominator (or divide the EER construct) by 3,600.[51] The net effect is highlighted in the following definition.
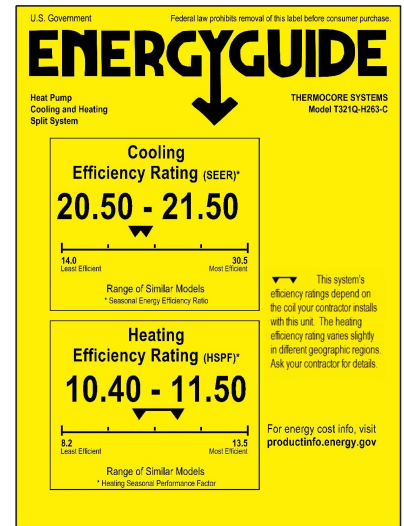
Figure 6.6: Typical heat pump energy label in the U.S., showing an EER around 21 and a HSPF around 11. From U.S. DoE.

49: This corresponds to maintaining the hotter environment at 27°C, for instance in the context of heating a house.

50: . . . or any other energy unit of choice

51: 1 watt-hour (Wh) is 1 J/s times 3,600 s.

Definition 6.5.4 **EER***: Heat pumps used for cooling are specified by the* *energy efficiency ratio* *(EER), which modifies Eq. 6.10 as follows.*

$$\varepsilon_{\text{cool}} = \text{EER} \left(\frac{\text{Btu}}{\text{Wh}}\right) \frac{1055\,\text{J/Btu}}{3600\,\text{J/Wh}} = \text{EER} \cdot 0.293, \qquad (6.13)$$

or the converse

$$\text{EER} = \frac{\varepsilon_{\text{cool}}}{0.293} \approx 3.41 \times \varepsilon_{\text{cool}}. \qquad (6.14)$$

Example 6.5.5 **EER Example**: Using the red numbers in Figure 6.5, we can compute $\varepsilon_{\text{cool}}$, the EER, and then determine the maximum T_{h} theoretically permissible (resulting in maximum possible efficiency) given a target T_{c} of 260 K, as we might find in a freezer.

We go back to the original definition of $\varepsilon_{\text{cool}}$ as $\Delta Q_{\text{c}}/\Delta W$, which for our numbers works out to 20/10, or 2.0. The EER is then 3.41 times this amount, or 6.8.

Setting $\varepsilon_{\text{cool,max}} = T_{\text{c}}/\Delta T$ equal to 2.0, we find that ΔT is 130 K, so that the maximum permissible $T_{\text{h}} = 390$ K in this case.

Because the theoretical maximum efficiency depends on ΔT—according to Eq. 6.10 and Eq. 6.11—and therefore can fluctuate as outdoor temperatures change, a seasonal average is often employed, called the SEER (seasonal EER). In a similar vein, the HSPF measures the same thing as the COP, but in units of EER[52] and averaged over the heating season.

52: . . . Btu/Wh

Definition 6.5.5 **HSPF***: Heat pumps used for heating are sometimes specified by the* *heating seasonal performance factor* *(HSPF), which modifies Eq. 6.11 as follows.*

$$\varepsilon_{\text{heat}} = \text{HSPF} \left(\frac{\text{Btu}}{\text{Wh}}\right) \frac{1055\,\text{J/Btu}}{3600\,\text{J/Wh}} = \text{HSPF} \cdot 0.293, \qquad (6.15)$$

or the converse

$$\text{HSPF} = \frac{\varepsilon_{\text{heat}}}{0.293} \approx 3.41 \times \varepsilon_{\text{heat}} = 3.41 \times \text{COP}. \qquad (6.16)$$

Example 6.5.6 **HSPF Example**: Using the red numbers in Figure 6.5, we can compute $\varepsilon_{\text{heat}}$ and the HSPF.

We go back to the original definition of $\varepsilon_{\text{heat}}$ as $\Delta Q_{\text{h}}/\Delta W$, which for our numbers works out to 30/10, or 3.0. The COP is then 3.0, and the HSPF is 3.41 times this, or 10.2.

Typical COP values for heat pumps range from about 2.5–4.5.[53] This means an energy savings by a factor of 2.5 to 4.5 for heating a house via heat pump vs. direct electrical heating. Quite a bargain. An air conditioner EER rating is typically in the range 10–20, corresponding to 3–6 in terms of $\varepsilon_{\text{cool}}$—similar to the range for heat pumps in heating

53: . . . mapping to HSPF from ~8–15

mode.[54] Houses equipped with electric heat pumps can typically be run for both cooling and heating applications, making them a versatile and efficient solution for moving thermal energy in or out of a house.

Heat pumps leveraging the moderate-temperature ground just below the surface as the external thermal bath are called "geothermal" heat pumps, but have nothing to do with geothermal energy (as a source). Compared to heat pumps accessing more extreme outside air temperatures, geothermal heat pumps benefit from a smaller ΔT, and thus operate at higher efficiency.

6.6 Upshot on Thermal Energy

Sometimes we just want heat. Cooking, home heating, and materials processing all need direct heat. Burning fossil fuels, firewood, biofuels, extracting geothermal energy, or simply letting the sun warm our houses all directly utilize thermal energy. Specific heat capacity tells us how much thermal energy is needed to change something's temperature, using 1,000 J/kg/°C as a rough guess if lacking more specific information.[55] We also saw how to estimate home heating demand using a metric of heat loss rate, such as 200 W/°C.

But it turns out that we use heat for much more than this. 84% of our electricity is produced by heat engines, using heat flow to drive a turbine to turn a generator. The maximum efficiency a heat engine can achieve is set by limits on entropy and amounts to $\varepsilon_{\max} < \Delta T / T_{\mathrm{h}}$, although in practice we tend to be a factor of two or more short of the thermodynamic limit.[56] In any case, thermal energy plays a giant role in how we run our society.

Heat pumps are like heat engines in reverse: driving a flow of thermal energy against the natural hot-to-cold direction by putting in work. Any refrigeration or cooling system is likely to use this approach.[57] Because heat pumps only need to *move* thermal energy, each Joule they move can require a small fraction of a Joule to accomplish, making them extremely clever and efficient devices.

6.7 Problems

1. How many Joules does it take to heat your body up by 1°C if your (water-dominated) mass has a specific heat capacity of 3,500 J/kg/°C?

2. How long will it take a space heater to heat the air[58] in an empty room by 10°C if the room has a floor area of 10 m² and a height of 2.5 m and the space heater is rated at 1,500 W? Air has a density[59]

of 1.25 kg/m^3. Express your answer as an approximate number in minutes.

3. When you put clothes on in the morning in a cool house at 15°C, you warm them up to something intermediate between your skin temperature (35°C) and the ambient environment.[60] If your clothes have a mass of 2 kg, how much energy must be deposited into the clothes? If you are emitting power at 100 W, how long will this take?

   60: The inside surface of the clothing will be near skin temperature, and the outside will be near ambient temperature.

4. You score this massive 1 kg burrito but decide to put it in the refrigerator to eat later. It comes out at 5°C, and you want to heat it in the microwave up to 75°C before eating it. If the microwave puts energy into the burrito at a rate of 700 W.[61] How long should you run the microwave for a high-water-content burrito having an effective specific heat capacity of 3,000 J/kg/°C?

   61: (i) Note that a microwave oven might be rated for 1,500 W, but not all the energy ends up in the burrito, so we pick 700 W to be realistic.

5. Let's say you come home from a winter vacation to find your house at 5°C and you want to heat it to 20°C. Let's say the house contains: 500 kg of air;[62] 1,000 kg of furniture, books, and other possessions; plus walls and ceiling and floor that amount to 6,000 kg of effective[63] mass. Using the catch-all specific heat capacity for all of this stuff, how much energy will it take, and how long to heat it up at a rate of 10 kW? Express in useful, intuitive units, and feel free to round, since it's an estimate, anyway.

   62: ...appropriate for a 150 m^2 footprint

   63: (i) We only count half-thickness of exterior walls since they are not heated to the interior temperature all the way to the outside.

6. In a house achieving a heat loss rate of 200 W/°C equipped only with two 1,500 W space heaters, what is the coldest it can get outside if the house is to maintain an internal temperature of 20°C?

7. In a house achieving a heat loss rate of 200 W/°C equipped a 5,000 W heater, what will the internal temperature be if the outside temperature is −10°C and the heater is running 100% of the time?

8. In a super-tight house achieving 100 W/°C equipped with a 5,000 W heater, what percentage of the time will the heater need to run in order to keep the internal temperature at 20°C if the temperature outside is at the freezing point?[64]

   64: Hint: compute the average power that would be needed in this case.

9. How much will it cost per day to keep a house at 20°C inside when the external temperature is steady at −5°C using direct electric heating[65] if the house is rated at 150 W/°C and electricity costs $0.15/kWh?

   65: ...no heat pump: just straight energy deposition at 100% efficiency

10. Provide at least one example not listed in the text in which heat flows into some other form of energy.[66] In the text, we mentioned hot air over a car, wind, internal combustion, and a steam turbine plant.

    66: Think about motion deriving from or caused by heat or thermal release.

11. What is the only form of significant electricity production in the

U.S. that does not involve a spinning shaft?

12. If a can of soda (350 mL; treat as water) cools from 20°C to 0°C, how much energy is extracted, and how much is the entropy (in J/K) in the can reduced using the average temperature and the relation that $\Delta Q = T\Delta S$?

13. What would the maximum thermodynamic efficiency be of some heat engine operating between your skin temperature and the ambient environment 20°C cooler than your skin?

14. We can think of wind in the atmosphere as a giant heat engine[67] operating between the 288 K surface and the top of the troposphere[68] at 230 K. What is the maximum efficiency this heat engine could achieve in converting solar heating into airflow?

15. Since the sun drives energy processes on Earth, we could explore the maximum possible thermodynamic efficiency of a process operating between the surface temperature of the sun (5,800 K) and Earth's surface temperature (288 K). What is this maximum efficiency?[69]

16. A heat engine pulls 100 J out of a hot bath at 800 K, and transfers 80 J of heat into the cold bath at 300 K. What efficiency does this heat engine achieve in producing useful work, and how does it compare to the theoretical maximum?

17. Human efficiency[70] is in the neighborhood of 25%, meaning that in order to do 100 J of external work, we need to eat 400 J of energy content. To investigate whether human energy is working as a heat engine, figure out what the cold temperature, T_c, would have to be to achieve this efficiency, thermodynamically.[71] Do you conclude that our biochemistry operates as a heat engine, or no?[72]

18. A 350 mL can of soda[73] at 20°C is placed into a refrigerator having an EER rating of 10.0. How much energy will you have to spend (ΔW) to remove the thermal energy from the soda and bring it to a frosty 0°C?

19. If a refrigerator works at *half* of its theoretical ε_{cool} limit, how much more energy does it take to maintain an internal temperature of 0°C in a 40°C garage vs. a 20°C house interior? Two things are going on here: even at the same efficiency, the cooling energy scales as ΔT, but the efficiency also changes for a double-whammy.

20. Changing from direct electrical heating to a heat pump operating with a COP of 3 means spending one-third the energy for a certain thermal benefit. If a house averages 30 kWh/day in heating cost through the year using direct electrical heating at a cost of $0.15/kWh, how long will it take to recuperate a $5,000 installation cost of a new heat pump?

67: And it really is!

68: ⓘ Atmospheric wind and weather are confined to the lowest layer of the atmosphere, called the troposphere, extending to about 12 km high.

69: We would not expect any solar-derived process to exceed this limit in the Earth environment.

70: . . . in terms of converting food energy into useful work

71: ⓘ The hot temperature, T_h, would be internal body temperature of 37°C.

72: Hint: do our bodies have regular access to temperatures this cold?

73: Treat as water, and recall that the density of water is one gram per milliliter.

7 The Energy Landscape

Now that we have a handle on common energy units and thermal processes, we can take a look at various sources of energy data and make sense of the information, allowing meaningful cross-comparisons. In this chapter, we will do exactly that, gaining in the process a perspective on the past and present roles different energy sources play at a national and global level.

Most of the information in this chapter comes from the U.S. Energy Information Administration's (EIA's) *Annual Energy Review* [34], and from a compilation of global data owing to Vaclav Smil and the British Petroleum *Statistical Review of World Energy* [16]. Rather than laboriously citing each instance, it is sufficient to assume for this chapter that numbers for the U.S. come from the former and global numbers come from the latter, unless stated otherwise.

7.1 The Annual Energy Review

Until 2011, the Annual Energy Review (AER) was compiled for the U.S. as an annual report. Since then, a web interface provides access to many of the same products, but not as a single document. An impressive amount of detail is available in the AER products, and we will only scratch the surface in this book, looking at high-level overviews. Later chapters will sometimes rely on deeper information to provide state-by-state use of hydroelectric, solar, wind, etc. But for now, we stick mostly to section 1 of the Annual Energy Review (AER), labeled *Energy Overview*.

An oil pump dominates the foreground, while wind makes a visible presence in the background. Photo credit: Tom Murphy

7.1.1 Energy Flow

Section 1.0 of the AER is a one-page PDF graphic that conveys at a glance the flow of energy into and out of the U.S. Figure 7.1 shows the 2018 version.

U.S. energy flow, 2018
quadrillion Btu

Figure 7.1: The flow of energy in the U.S. for 2018, as presented in [34]. Units are quadrillions of Btu (qBtu), unfortunately. From U.S. EIA.

From past experience, many students dislike this graphic. Firstly, it's a product of the EIA, and not a creation of this book. Secondly, it is actually not so bad, once you get the hang of it.

Resources come in from the left. Expenditures or exports go off to the right. The format guarantees that all inputs must match all outputs.[1] We also see at a glance the big players vs. small players.

To understand, let's start in the middle section. To the left of center, we see that the total supply sums to 122.44 qBtu. Of this, we consume 101.25 qBtu (right of center) and export the remaining 21.19 qBtu. Now we focus on the central column to get a powerful visual and quantitative snapshot of how our energy is partitioned.[2] From this, we see that 13% is coal, 31% is natural gas, 36.5% is petroleum (oil), 8% is nuclear energy, and 11.5% is renewable energy.[3]

1: That is, no significant amount of energy is stored or drawn from a stockpile.

2: By luck, total consumption is very nearly 100 qBtu, so the amount of each source in qBtu is already approximately a percentage!

See how they add up?

3: Think of the three forms of fossil fuels as solid (coal), liquid (petroleum/oil) and gas (natural gas; not the same as liquid gasoline, which is a petroleum product).

Now the right-hand side shows the sectors into which the energy flows, finding *roughly* equal distribution between residential (homes), commercial (businesses), industrial (manufacture), and transportation (both personal and commercial/shipping). In this graphic, we lose entirely any sense for how much of each energy source contributes to each sector,[4] but that is coming in the next section.

4: For instance, we cannot tell how much coal is used in the industrial sector.

Finally, the left-hand side indicates the inputs, grouped as domestic fossil fuel supply at top (out of our own ground), nuclear energy and renewable in the middle, and imports at bottom. From this, we can learn that we export some coal,[5] that almost all of our natural gas and 100% of our nuclear is domestic, and that 62% of our petroleum comes from domestic crude oil production.

5: For instance, the supply of 15.33 qBtu that is mined is larger than the 13.24 qBtu we consume.

Other insights are present in the graphic as well. Don't be afraid to subtract or divide numbers to aid new discoveries.

> Ask yourself what else you can learn from the numbers!

Box 7.1: 100 quads? So what?

To put the scale into a bit of perspective, 100 qBtu in a year for the U.S. is about 10^{20} J in a year. A year is 3.156×10^7 seconds long,[6] meaning that the U.S. power budget is just over 3 TW (3×10^{12} W). Distributed among a little over 300 million people, the average contribution per person is about 10,000 W.[7] That's a lot. As we have seen in Sec. 5.5 (p. 73), human metabolism is about 100 W. So Americans have approximately 100 times as much energy available as their personal metabolism. The situation has been compared to each person *having 100 energy servants*! No wonder we live better than royalty of ages past. Even though the U.S. uses about 4.5 times the global energy per capita (about 20% of the world's energy and 5% of population[8]), the average citizen of Earth still has over 20 energy servants available, on average, thanks almost entirely to fossil fuels. They have been an unqualified game changer.

6: Neat trick: roughly $\pi \times 10^7$ seconds in a year.

7: This showed up in Table 3.4 (p. 43) and also in Box 5.4 (p. 75).

8: American energy usage is much higher than average because of consumerism, diet, comfort standards, prevalence of detached housing, and transportation.

7.1.2 Source and Sector

Figure 7.2 provides a more detailed breakdown of how energy flows from source to usage sectors.[9] In other words out of the 101.25 qBtu consumed in 2018, we see how much comes from each source, and within each source can track how much goes to each end-use category. For example, we learn that 91% of coal and 100% of nuclear go to electricity, and that 92% of transportation is based on petroleum.

9: Notice that the source and end-use numbers in the boxes match the numbers in Figure 7.1 within rounding error.

Notice the black and gray block at lower center, representing electricity. We derive electricity from all the sources on the left, and electricity is consumed in all sectors. Also, of the 38.3 qBtu going *into* making electricity, only 13.0 qBtu (34%) makes it out the door as electricity, due to thermodynamic losses that were covered in Chapter 6.

U.S. energy consumption by source and sector, 2018
(Quadrillion Btu)
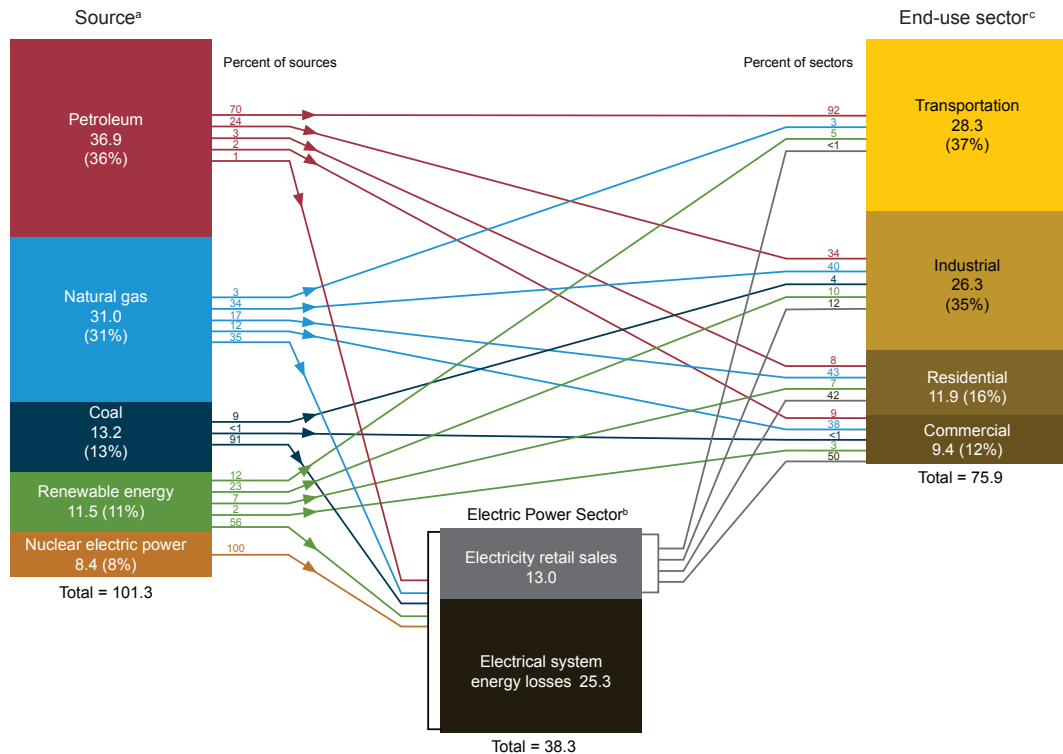
Figure 7.2: Tracking of energy sources and end-use in the U.S. for 2018, from section 2.0 of the AER. Small numbers beside the blocks represent percentages. Numbers that are not percentages are qBtu (quads). From U.S. EIA.

In principle, it is possible (and would be nice) to put percentages where the arrows enter and exit the electricity sector, but enough numbers are present to work this out, as Example 7.1.1 demonstrates. Without these numbers, the story is a little misleading. For instance, only 17% of natural gas goes *directly* to residential use, but some natural gas produces electricity, which then flows to residences. It is therefore not immediately obvious what percentage of residential energy ultimately comes from natural gas, but it's more than the 43% indicated in the figure.

A similar graphic combining some elements of both Figure 7.1 and Figure 7.2 is provided by Lawrence Livermore National Lab [35].[10]

[35]: LLNL (2019), *Energy Flow Charts*

10: See also [36] for a fascinating animated version.

Example 7.1.1 Let's work through the numbers in Figure 7.2 to elucidate what percentage of residential energy ultimately derives from natural gas. The same technique can be pursued to ask similar questions about any source-to-sector pathway, by incorporating the electricity contribution.

We start simply, by noting that 43% of the 11.9 qBtu residential energy

budget comes directly from natural gas. So that's 5.1 qBtu.[11]

Now, 35% of natural gas goes toward electricity, which we can compute to be 10.9 qBtu.[12]

So of the 38.3 qBtu total energy coming into the electricity block, 10.9 qBtu (28%) is from natural gas.[13]

Assuming the 34% efficiency[14] of electricity production applies equally across all sources (close to the truth), we can say that 28% of the electricity output comes from gas: 28% of 13.0 qBtu (electricity output) is 3.7 qBtu.

But not all of this goes into homes. The home gets 42% of its 11.9 qBtu from electricity, or 5.0 qBtu. We can assume that 28% of the 5 qBtu of electricity flowing into the home derives from natural gas, as decided above. So that's 1.4 qBtu of gas-derived electricity flowing into the home.

We can add this 1.4 qBtu of gas-derived electricity to the 5.2 qBtu[15] of direct gas-to-home to learn that 6.6 qBtu of residential input is sourced from natural gas—either directly or via electricity. Compared to the 11.9 qBtu total for residences, natural gas therefore contributes 55% of the energy used in homes, not just 43% as listed. Now we know.

7.1.3 Detailed Mix

Delving a bit further into the AER, Section 1.3 provides a more detailed breakdown of consumption, now separating out the "renewable" category into its constituent parts, as seen in Table 7.1 and Figure 7.3.

In sum, 80% of the U.S. energy in 2018 came from fossil fuels. Less than 2.5% came from wind, and less than 1% was solar in origin—the other 16% mainly in the form of nuclear, biomass, and hydroelectricity. Most of the renewable energy is from biomass—like burning wood. The wider world is pretty similar, in that about 80% of energy is from fossil fuels. It's still our main squeeze. Table 7.2 breaks out electricity sources separately.

Box 7.2: Thermal Equivalent

Note that the EIA—and thus Table 7.1—habitually applies a thermal conversion factor to some energy sources in order to more meaningfully compare one source to another. Fossil fuel energy is characterized by its *thermal* content, which makes sense as they are burned for thermal energy. Often—but not always—the thermal energy is turned into electrical energy. Meanwhile, some sources, like solar, hydroelectric, wind, nuclear, and geothermal are almost exclusively used for electricity production and are most easily mea-

---

11: As a check, we note that the other side of the blue arrow has 17% of 31.0 qBtu, or 5.3 qBtu leaves the gas block for homes: close enough to the 5.1 qBtu we got on the other side (essentially the same, to rounding error).

12: 35% of 31.0 qBtu

13: Wouldn't it be nice if Figure 7.2 printed a blue 28 where the blue arrow comes into the electricity block?

14: 13.0 qBtu of electricity is produced from 38.3 qBtu energy input.

15: ...averaging the two estimates from before

Table 7.1: U.S. energy consumption for 2018 in thermal equivalent terms.

Resource	qBtu
Petroleum	36.88
Natural Gas	31.09
Coal	13.25
Nuclear	8.44
Biomass	4.98
Hydroelectric	2.77
Wind	2.48
Solar	0.92
Geothermal	0.21
Total	101.0
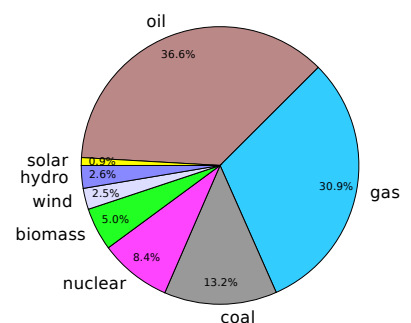
Figure 7.3: 2018 Energy sources for the U.S. Figure 7.6 shows the global distribution.

sured by electrical *output*, not thermal input (which is meaningless for solar, wind, and hydro).

Multiplying the electrical output by a factor of about 3 recovers the thermal equivalent.[16] The interpretation is: how much fossil fuel (thermally) would have been necessary to achieve the same result? As a consequence, when Table 7.1 says the solar contribution is 0.92 qBtu, and therefore about 1% of the total, the *actual* solar energy was smaller by a factor of three, but the practice is fair because now we can directly compare solar to the fossil fuels. Reporting electrical output alongside thermal inputs would make the renewables appear to have a smaller contribution than they effectively do, against fossil fuels.

16: The actual factor is just the inverse of the electrical conversion efficiency discussed above (34%, so 1/0.34). The conversion efficiency adopted by the EIA has slowly increased over time, and is tracked in Appendix A6 of the AER—now at 37.5%, leading to a conversion factor of 2.67.

Region	Coal	Gas	Oil	Nuclear	Hydro	Wind	Solar	Bio	Geo
U.S.	27.3	34.9	0.6	19.2	7.0	6.5	2.3	1.5	0.4
World	38.0	23.0	2.9	10.1	16.2	4.8	2.1	2.4	0.5

Table 7.2: Percentages of **electricity** derived from various sources in the U.S. and globally in 2018. Bio includes burning wood and waste, and Geo means geothermal. Data are from Table 7.2a of [34] and from [37].
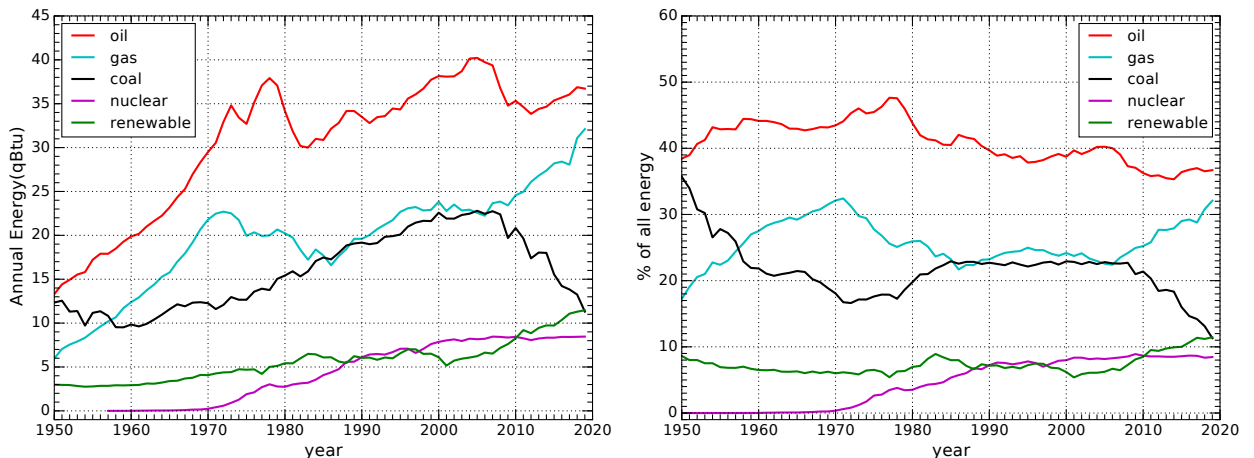
Figure 7.4: Recent history of primary energy consumption in the U.S. The three fossil fuels and nuclear are shown separately, and then all renewable sources are grouped together. Note that at the end of the plot, coal has sunk into a tie with renewable resources. The plot on the right shows percentages of total energy. Most of the lines are fairly flat, although in recent years the main story is gas replacing coal.

7.1.4 Energy Trends

It is worth looking at trends to understand not only the state of affairs today, but what happened over past decades and trends that may carry into the near future. Section 1.3 of the AER includes data going back to 1950 on the categories in Table 7.1.

Figure 7.4 shows the trends for the fossil fuels over the last 70 years, along with the slow rise of the sub-dominant non-fossil sources. Recent news touted the fact that the renewable sources[17] surpassed coal as an energy source in the U.S. Indeed, the lines basically meet on the

17: The term "renewable," will be more fully explained in Chapter 10.

right-hand side of the plot, and the trends suggest a clear reversal of rank going forward. Note, however, that this result is largely due to natural gas replacing coal at electrical power plants. The sharp rise in natural gas nearly mirrors the decline in coal, while the rise in renewable resources is more modest. So this is really more a story of trading gas for coal than renewables replacing coal. Figure 7.4 also shows each source as a percentage of all energy. For a few decades (1980–2010), coal and gas were essentially tied, while oil sat at almost double these two. Lately, gas is approaching oil while coal plummets.
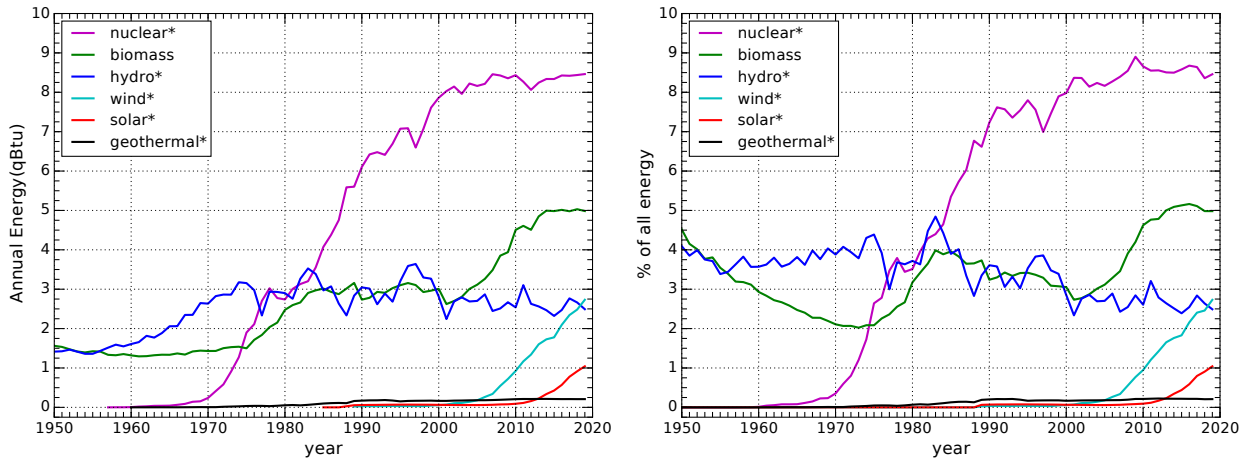
Figure 7.5: Recent history of non-fossil energy consumption in the U.S. Nuclear, hydroelectric, and biomass have dominated, while wind and solar are rising to join as players. Asterisks indicate thermal equivalents, as described in Box 7.2. The same data are plotted at right as a percentage of total energy. Aside from the rapid rise of nuclear in the middle years of the plot, the recent entry of wind and solar (though still only a few percent) are the most interesting developments.

The non-fossil consumption in Figure 7.5 clarifies the breakdown of the "renewables" curve in Figure 7.4, alongside nuclear. From this, we see that nuclear dominates non-fossil energy, rising quickly from 1970 to 2000 and holding steady since then. Hydroelectric has been pretty stable over the last 50 years as other sources surpass it and lower its rank. The surge in biofuels around 1980 appears to be largely driven by increased burning of wood, while the next surge (2000–2010) was due to biofuels—mostly ethanol. Wind is approaching a 3% contribution to our total ~100 qBtu consumption budget, edging up about 0.2% per year. Solar is also on the move, reaching the 1% level recently and rising more slowly than wind. Geothermal is and will continue to be a paltry contributor.

7.2 Global Energy

Not surprisingly, the global story is not dramatically different from the story in the U.S., as Figure 7.6 and Figure 7.7 show. Fossil fuels dominate, with oil at the top. Coal has held a lead over natural gas in the wider world, unlike the U.S. Also, while nuclear and renewables are
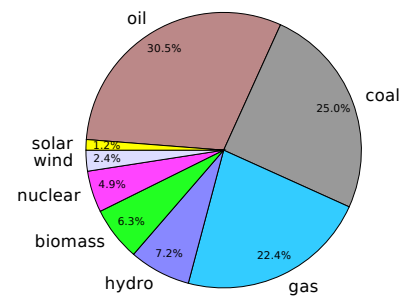
Figure 7.6: 2018 Energy sources for the world. Figure 7.3 shows the U.S.

comparable in the U.S., this is not true globally, for reasons discussed shortly. Note that different assessments of global energy may report different percentage contributions depending on whether or not thermal equivalents are used (see Box 7.2).
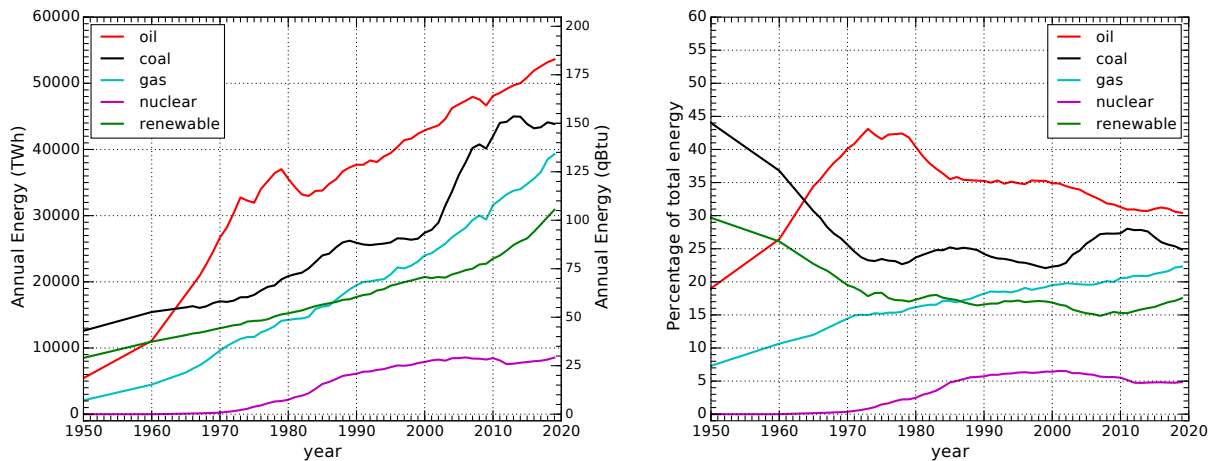
Figure 7.7: Recent history of primary energy consumption in the world. The three fossil fuels and nuclear are shown separately, while renewable sources are grouped together. The plot on the right shows the same data as a percentage of the whole.

For non-fossil contributions, Figure 7.8 shows the evolution of recent decades. Here, we see that a large part of the reason why renewables exceed nuclear energy globally is because of biomass. This makes sense, as countries having a lower standard of living are more likely to burn wood and less likely to have nuclear power.
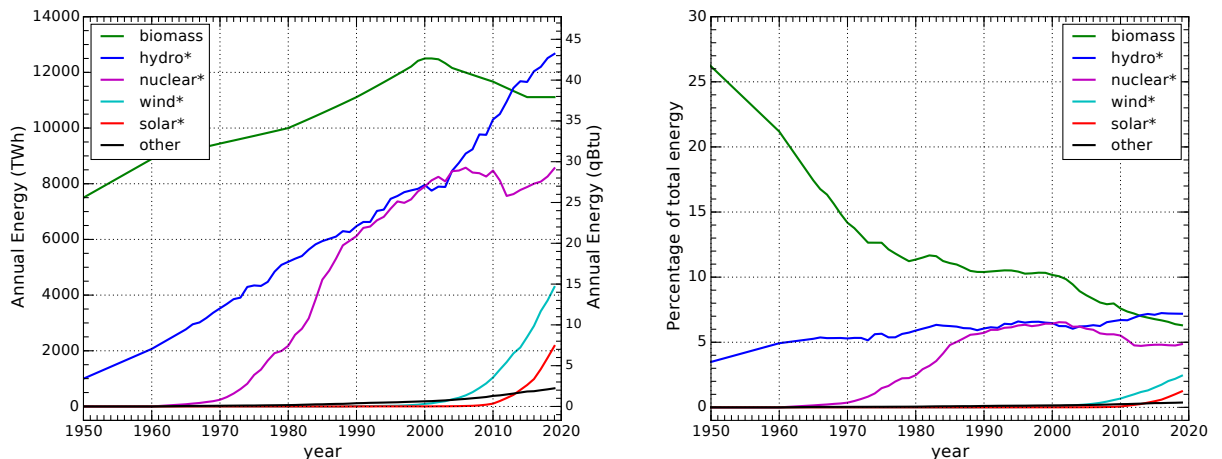
Figure 7.8: Recent history of non-fossil global energy consumption. Asterisks indicate thermal equivalents, as described in Box 7.2. The plot at right shows each source as a percentage of the total energy. Biomass accounted for a quarter of global energy in 1950.

Box 7.3: TWh vs. qBtu

You may have noticed that as soon as we departed from the AER

---

data, which expressed energy in qBtu, the units on the plot (Fig. 7.7) changed to terawatt-hours (TWh). It means what it sounds like: tera is 10^{12}, so this is 10^{12} watt-hours (Wh). We use kWh more often than Wh, so a TWh is the same as a giga-kWh, or GkWh (can you do that?). One kWh is 3.6×10^6 J, so 1 TWh is 3.6×10^{15} J. Meanwhile, 1 qBtu is 1.055×10^{18} J, facilitating a conversion. The figures for global power also put qBtu on the right side for easier comparison between plots.

The source of numbers for this section [16] mix thermal and electrical output, so the plots have multiplied some entries (asterisks in plot legends) by 3.06 for reasons described in Box 7.2.

[16]: Smil (2017), *Energy Transitions: Global and National Perspectives*

7.2.1 U.S. Global Share

A final overview to help frame a number of discussions in this textbook looks at the U.S. share of consumption of various energy resources compared to the global total. The evolution seen on the left side of Figure 7.9 contains a crucial insight into geopolitics. In 1950, the U.S. used an astounding 84% of global natural gas and 72% of petroleum. At only 6% of the world's population at the time, Americans used more than ten times the global average oil and gas, and substantially more than the rest of the world combined. Since energy per year is the definition of power, we can understand how the U.S. was a literal *superpower* during this era. Parroting Bill Clinton: It's the resources, stupid.

This may be a factor in nostalgia for what some Americans see as the "glory days" of the 1950s. To the extent that U.S. energy share played a role, longing for a return to that era is not likely to materialize.
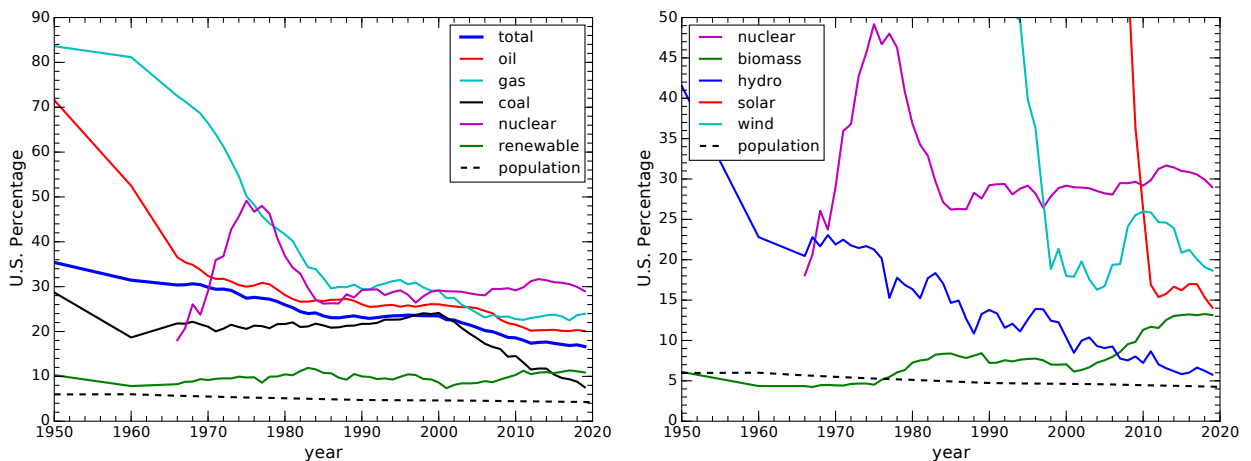
Figure 7.9: The left figure combines Figure 7.4 and Figure 7.7 to show the percentage of energy resources consumed by the U.S. over time. The overall picture is of a world catching up to an early leader. The U.S. was a literal "superpower" in the middle of the twentieth century. The dashed line at bottom represents the fraction of U.S. population in the world, so that energy use above this line means a greater-than-average share, which is true for all sources. The plot at right combines Figure 7.5 and Figure 7.8 to show the percentage of renewable and nuclear energy resources consumed by the U.S. over time. Solar and wind are characteristic of a nation known for innovation: first on the scene.

The thicker dark blue line in the left panel of Figure 7.9 represents all sources of energy, combined. Around 1950, Americans used a third of all the global energy, corresponding to almost 8 times[0mm] the global

0mm: The math is 35% over 6% of population compared to 65% over 94% of population: $(35/6)/(65/94) \approx 8.4$.

average per non-American. Today, the ratio is closer to 4.

The right side of Figure 7.9 similarly explores U.S. share of renewables. The only up-trending resource is biomass, due to mandates for ethanol usage.[18] But it is a minor player in the scheme of things. Solar and wind are interesting, in that the U.S. initially held a large global share as pioneers of the technology before the rest of the world joined in.

18: More on biofuels in Chapter 14.

7.3 Upshot: Go to the Source

The purpose of this chapter was twofold: first to introduce students to sources of reliable information on national and global energy production; and second to communicate the landscape of energy use. What emerges is a picture of a world still firmly in the grip of fossil fuels, whose annual usage continues to increase. Wind and solar are making inroads, but only at the few-percent levels thus far. The U.S. has played an outsized role in global energy relative to its population, especially in the mid-twentieth century.

7.4 Problems

1. Referring to Figure 7.1 and Figure 7.2, figure out the following measures:

   a) What percentage of energy consumption in the U.S. is from petroleum?
   b) What percentage of transportation is powered by petroleum?
   c) What percentage of petroleum goes directly to transportation?
   d) What percentage of petroleum goes directly to industrial processes (ignoring via electricity)?

   ⚠ Not the same as previous question.

2. The electricity block at the bottom center of Figure 7.2 is said to be 38.3 qBtu in size. Using the qBtu numbers in the sources at left, and the percentages of each going to electricity, figure out how many qBtu each line connecting to the left side of the electricity block represents.[19] What is the total, and does it match the 38.3 qBtu expectation, within reasonable rounding errors?

   19: Example 7.1.1 may offer guidance.

3. Building off the result in Problem 2, calculate the percentages[20] of contributions coming into the left side of the electricity block in Figure 7.2? Which is the dominant input?

   20: Verifying that they add to 100% is a good check.

4. Following a similar approach as for Problem 2,[21] concentrate on the output side of electricity production[22] and figure out how many qBtu are delivered to each sector on the right-hand side of the figure, based on input percentages to each of the four sectors

   21: See also Example 7.1.1.

   22: 13.0 qBtu delivered; implying 34% conversion efficiency from primary sources to delivered electricity

and their total qBtu amounts. Treat "< 1%" as 0.5%. Do these add to 13 qBtu, as they should, within rounding error?[23]

23: This is a great way to check the correctness of your answers.

5. Figure 7.2 hides contributions of sources to end sectors behind the "electric black box."[24] Following similar logic to that in the margin, and using results from Problem 4, figure out "corrected" values for what percentage of coal provides energy to each of the four end-sectors (re-distributing the 91% going to electricity into end-sectors).[25]

24: For example, the figure indicates that 17% of natural gas goes directly to residential end-users. But a substantial fraction of natural gas (35%) also goes to electricity, and 38.5% of electrical output goes toward residential use—a result of Problem 4. So the fraction of natural gas ending up satisfying residential demands is the direct 17% *plus* 38.5% of 35%, adding to 30.5%.

25: The four numbers you get should add to 100%, within rounding error.

6. Figure 7.2 makes it look as if residential demand is satisfied without coal or nuclear, but 42% of residential demand comes from electricity, which *does* depend in part on coal and nuclear. Using numbers derived in Problem 3, and following a logic similar to that in Problem 5 and Example 7.1.1, redistribute this 42% residential contribution from electricity into its primary sources to ascertain what fraction of residential demand comes from each of the five source categories. For instance, petroleum would be the direct 8% plus 42% times the fraction (or percentage) of electricity coming from petroleum.[26]

26: Make sure your five numbers add to 100%, within rounding error.

7. While no energy source is free of environmental harm, arguably the last four entries in Table 7.1 are the cleanest, requiring no burning and no evidently problematic "waste." What percentage of the total U.S. energy is in this "clean" form, at present?

8. Let's say that in the course of one year a county in Texas produces 5 million kWh of electrical output from wind, and also pumps 100,000 barrels of oil from the ground containing a (thermal) energy content of about 6 GJ per barrel. What percentage of total energy production came from wind, if scaling wind in terms of thermal equivalent, as explained in Box 7.2?

9. Referring to Figure 7.4, what is the fastest-growing energy source in the U.S., and is it one of the fossil fuels?

10. If the approximately linear trends for recent increases in solar and wind seen in Figure 7.5 were to continue at the current (linear) pace, approximately how long would it take for the pair of them to cover our current ~ 100 qBtu per year demand?[27]

ⓘ Keep it simple, as there is no single correct way to extrapolate this far into the future; just explain your approach.

27: We can hope to see faster-than-linear expansion in renewables, but this question asks what *would* happen without dramatic changes to the recent trends.

11. If the downward trend in U.S. coal use continues at its current pace, approximately what year would we hit zero?

12. Globally, do any of the resources appear to be phasing out, as coal is in the U.S. (as in Problem 11)? If so, how long before we would expect to reach zero usage, globally, based on simple extrapolation?

13. Globally, would you say that renewable energy sources are climbing faster than the combined fossil fuels, or more slowly? Can we therefore confidently project a time when renewables will overtake

fossil fuels, based on trends to date?

14. As explored in Problem 11, the U.S. usage of coal is falling precipi-
    tously. According to the left plot in Figure 7.9, is the U.S. usage of
    coal greater or less than the global per-capita average?

15. Is the U.S. per-capita usage of *any* energy source lower than the
    global average, according to Figure 7.9?

8 Fossil Fuels

We are now ready to dive into the core content of the book: assessing global energy demands and prospects. For most of human history, we derived energy from food-supplied muscle power of people and work animals, burning firewood, and harnessing wind and water flow (all deriving from solar energy). Then a most remarkable thing happened: the discovery and widespread utilization of fossil fuels. The abundance of energy delivered by fossil fuels profoundly changed the human condition, such that many elements of our modern world would seem like magic to someone living 200 or even 100 years ago.

Fossil fuels still completely dominate our energy usage. Every country is reliant on some amount of fossil fuels—especially for transportation. Even though fossil fuels cannot be our future—due to finite resource depletion and climate change concerns—it is critical that we look at these pillars of modern life, assessing what makes them both amazing and terrible, and what we might expect going forward. Facing the stark and underappreciated reality of fossil fuels will sharpen our desire to learn more about what might come after, as subsequent chapters address.

8.1 The Most Important Plot Ever

We have so far gained a few big-picture perspectives on the human endeavor. First, we illustrated the absurdity of constant growth in both physical and economic terms, concluding that growth must be confined to a temporary phase and will not be physically allowed to continue indefinitely. Next, we looked at population realities to understand how that story might develop. Then we looked at the scale of the universe, how minuscule Earth is in the vast emptiness, and explored the extreme

---

Oil pipelines and gas flaring on the Alaskan tundra at Prudhoe Bay. A drill rig fades into the fog at top center. Note the optical illusion that makes the photo's bottom border look crooked! Photo credit: Tom Murphy

difficulties of colonization—putting the emphasis on managing our challenges right here on Earth.

In order to frame just how important fossil fuels are and have been, we again take a broad view to put our energy trajectory in perspective before getting into the nuts and bolts of fossil fuels. The picture that emerges has the potential to reframe personal perspectives on our future.

The result may have greater impact if you are an active participant in its creation. So get some paper, the back of an envelope, or something. Draw a horizontal axis as a timeline. Label the left edge as –10,000 years (past). The right edge is +10,000 years (future). The middle is 0 (now; see the example in the margin). The vertical axis represents global energy production, on a linear scale. For ages, this was too tiny to see poking up above the floor. Only about 200 years ago did it become visible. So for the first 98% of the way from -10,000 to 0, draw a line hugging the floor. In the last 200 years, energy usage has increased exponentially.[1] So draw a smooth curve connecting the previous line into a steep rise at present (middle of the plot), using much or all of the available vertical space.

What emerges is the classic "hockey stick" plot that applies to many physical attributes of our world: population, carbon dioxide, temperature, and—in the present case—energy use. In the long flat portion of the plot, our energy came from firewood and muscle (both animal and human labor). But the sudden transformative rise is really a story of fossil fuels. Even today, having added hydroelectric, nuclear, solar, wind, geothermal, and tidal power to the mix, fossil fuels still account for over 80% of the total.[2]

Let us then continue the plot *in the context of fossil fuels*. Being a finite resource, we know in broad terms what the curve *must* look like. It must drop back down to zero and ride into the future looking much as it did in the past: at zero. One may debate the exact timing of the peak of fossil fuel use, but for a variety of reasons we would be well justified in placing it sometime this century. We'll leave it to individual preference if you want to allow the curve to climb a bit more before turning down, but don't stray too far. This century ends only 1% of the way from 0 to +10,000, so don't let the peak get very far at all from the middle of the plot. Once turning down, the curve is likely to look reasonably symmetric, returning to zero in short order and staying there.

Independent of individual choices, if keeping within reason we're all looking at the same basic plot (as in Figure 8.1): fossil fuels are a blip on the time scales we associate with history. We live in a most abnormal time.[3] Because the upswing has lasted for generations, *it seems entirely normal* to most people: it's the only reality we or any person we've ever met has known. Lacking perspective, a child will view their life circumstances as *normal*, no matter how impoverished or privileged: it's the only world they've ever known or seen. Likewise, we accept and define our current world as normal—even if historical perspective

-10,000   0   10,000

-10,000   0   10,000
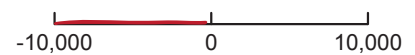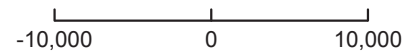
1: . . . a smooth curve peeling up off the floor and rocketing to an essentially vertical recent trajectory

-10,000   0   10,000
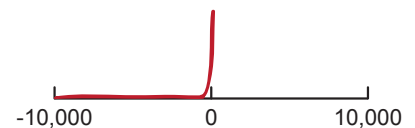
2: It's even worse than it sounds, since 10% is still in the form of biomass, much of which is old-technology firewood, leaving only 10% in the more modern forms of hydro, nuclear, solar, wind, geothermal, and tidal.

3: Social scientists are trained to not label their own time as *abnormal*, as such thinking may reflect a sloppy bias that all people through history might be tempted to adopt. Yet, neither should we declare that abnormal times can *never* happen. Any quantitative assessment of the current human scale and planetary resource impact argues that we are justified in allowing ourselves an exception for the present age.

ultimately considers the last century or two to be the most insanely unusual period of the human experience—like a fireworks show.
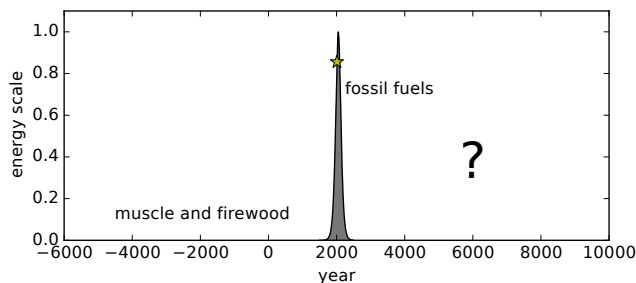
Energy over the ages, in the form of fossil fuels. Up until the present, fossil fuels capture the bulk of the human energy story. We know what it must look like in the long term as well. The huge question is how the second half of human history looks, after fossil fuels are depleted or abandoned. The yellow star is a guess as to our current position, based on evidence addressed later in the chapter suggesting that the resources are nearly halfway depleted.

Figure 8.1 should stimulate a swarm of questions. Where are we on the curve? When is the peak? Is the decline phase marked by escalating energy scarcity, or the advent of a renewable energy future? Might the far future look more like the past (muscle and firewood) than the present? Will this plot change how we interpret the world and our own plans for the future? The only fair conclusion is that we really do not know how the future will unfold.[4] We can label the left side as "muscle and firewood," and the spike as fossil fuels, but the only credible occupant of the right-hand side is a gigantic question mark.

4: We *can* rule some things out, though, like unending growth and fossil fuels lasting centuries more.

The idea of Figure 8.1 is not original to this textbook, having been portrayed in various incarnations over the last half-century or so [38]. When anyone makes a claim about what they think will happen by late-century, think about this plot. So many of our assumptions are based on the recent but abnormal past. All bets are off in defining the future. In one sense, those who rightly point out that we can't expect to be clever enough to foresee the future are correct—but perhaps in an unintentionally symmetric way. The future could be far more dismal than our dreams currently project. That would also be a surprise to many. We need to approach the future with humility, and set aside preconceived notions of where things are heading so that we can make choices now that will help define what comes next. Taking it for granted is a risky move.[5] Only by acknowledging the potential for a disastrous outcome can we take steps to mitigate that possibility. Waving it off is the most dangerous move we could make.

[38]: Hubbert (1962), "Energy resources: a report to the Committee on Natural Resources of the National Academy of Sciences; National Research Council"

5: In this sense, taking the risk seriously fits the definition of the word "conservative," even if present political alignments are mislabeled in this regard.

> **Box 8.1: Will Renewables Save Us?**
>
> Just because fossil fuel energy must return to pre-industrial levels in Figure 8.1 does not dictate that human society must return to pre-industrial energy levels. After all, solar, wind, nuclear, hydroelectricity are available to us now. Yet we will struggle to match today's energy levels on these resources alone. More disturbing is the notion that we may not be able to maintain high-technology approaches in a world devoid of fossil fuels. No one has demonstrated how, yet.

Also, the very disruption of losing such a critical resource without adequate advanced preparation may damage our capabilities. The short answer is: we simply do not know. The question mark in Figure 8.1 is the most fair statement we can make.

Note that Figure 8.1 is not intended to predict a particular future path. But it can serve to to counterbalance the prevailing optimism about a technologically marvelous future by providing a sanity check so that we might acknowledge that *we really do not know*. How can it be wrong to say that we do not know what the future holds? Yet, accompanying this uncertainty is a glimmer of hope: if the future is so uncertain and unscripted, then perhaps we have the power to write the script and set ourselves onto a viable and pleasant future path. If we elect to do so, it is of paramount importance that we do not ignore limitations imposed by nature in the process.

8.2 Overview: Coal, Oil, and Gas

Fossil fuels are found in three principal forms: coal, oil (petroleum), and natural gas.[6] They are essentially a form of ancient solar energy that plants once captured and stored as chemical energy to be locked away underground for many millions of years.[7] Sporadic, low-level use of fossil fuels dates back millennia, but modern use began in earnest in the eighteenth century with coal in Britain. Figure 8.2 makes clear that the use of coal did not *really* gather steam until the mid-nineteenth century, when industrialization took off. One may suspect that much of the rise in the use of fossil fuels is simply a reflection of population growth, but this turns out to be wrong. The right-hand side of Figure 8.2 divides the amount of fossil fuel use by global population to show that energy use per capita has *also* risen steeply over this time period, so that the exponential-looking phenomenon in the left panel is a *combination* of more people *and* more use per person. Today, the global average rate of use of fossil fuel use is a little over 2,000 W per person.[8] From Figure 8.2, we may say that coal really ramped up starting around 1850, oil around 1915, and natural gas around 1970.[9]

8.2.1 Coal

Coal—which looks like black rock—is the remnant of plant matter deposited, turned to peat, and heated/compressed by burial to form a mostly-carbon substance that can be combusted with oxygen to generate heat. The heat can be used to make steam, which can then power machinery or turbines for producing electricity.[10] Or the heat may be used directly for materials processing, like creating molten steel in blast furnaces.

6: Think of the three forms of fossil fuels as solid (coal), liquid (petroleum) and gas (natural gas).

7: It is in this sense that the word "fossil" is appropriate: ancient remnants of life buried underground.

8: 15 TW of fossil fuel use divided by nearly 8 billion people is about 2,000 W per person. Compare to the U.S. total energy appetite of 10,000 W per person.

9: All of these sources were *first* used much earlier, but at insignificant levels. Natural gas makes a meaningful appearance starting around 1920, but heavy use began 50 years later after pipeline infrastructures were in place.

10: . . . covered in Chapter 6
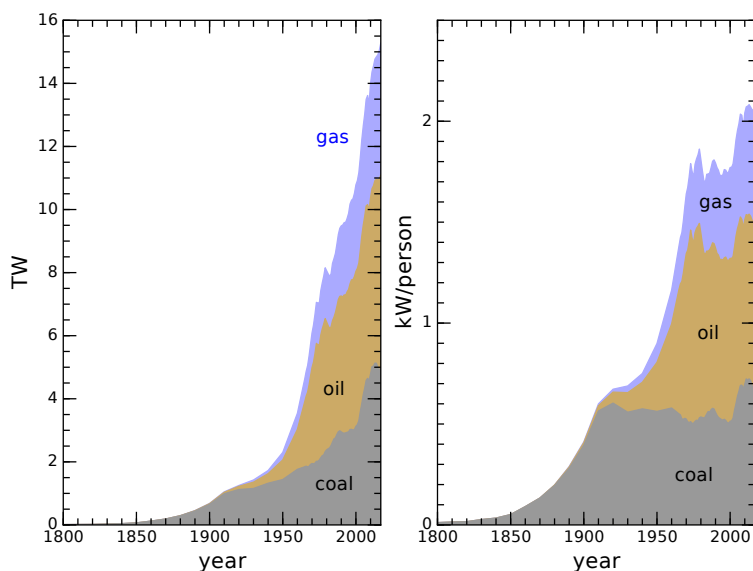
Figure 8.2: Historical use of fossil fuels worldwide, which may be viewed as a zoom-in of the left-hand side of the peak in Figure 8.1. The three types are stacked on top of one another, so that gas makes the smallest contribution, not the biggest. On the left is the raw usage rate expressed in terawatts, while the right is a per-capita measure showing that the left-hand rise is much more than just a reflection of population growth [16].

Coal opened the door on the Industrial Revolution[11] in the late eighteenth century, allowing locomotion (trains), mechanized manufacturing, large-scale materials processing, and heating applications. Somewhat circularly, the first major use of the steam engine[12] was to pump water out of coal mines to accelerate the extraction of. . . coal. This fact further highlights that from the very start, the Industrial Revolution was focused on the fossil fuel resource that enabled it.

Today in the U.S., coal accounts for 13% of total energy consumption[13]—down considerably from 23% in 2000.[14] For the world at large, coal still accounts for 25% of primary energy use.[15] The vast majority of coal (91%) in the U.S. goes to electricity production, the remainder fueling industrial processes requiring lots of heat. The quality of coal varies greatly. Table 8.1 presents properties of the four main coal categories. Anthracite is the king of coals, but has been largely consumed at this stage. Coal grades having lower energy content contain more non-combustible materials[16] like SiO_2, Al_2O_3, Fe_2O_3, and water.

11: . . . which history may rename the Fossil Fuel Revolution

12: The first widely adopted steam engine design is credited to James Watt, from whom we get the name of our unit for power.

13: Fig. 7.4 (p. 107)

14: As discussed in Chapter 7, coal's decline in the U.S. is largely due to increased reliance on natural gas for electricity.

15: Fig. 7.7 (p. 109)

16: . . . sometimes called "ash" content and volatiles

Table 8.1: Four classes for grades of coal, in order of decreasing energy content and value. Anthracite has been largely depleted and is a rare find today. [39, 40]

Grade	Carbon Content (%)	Energy Density (kcal/g)
Anthracite	86–97	6–8
Bituminous	45–86	5.5–8
Sub-bituminous	35–45	4.5–6.5
Lignite	25–35	2.5–5

8.2.2 Petroleum (Oil)

Petroleum—also called oil—is ubiquitous in our world as the source for gasoline, diesel, kerosene, lubricating oils, tar/asphalt, and even most

plastics. Virtually all[17] transportation: planes, trains, automobiles, and ships run on petroleum-based energy.

Petroleum first entered the modern scene around 1850, and the first drilled well[18] was in 1858 in Pennsylvania. Early uses were for kerosene lamps.[19] The first commercial internal combustion engine closely followed in 1859, arriving at an essentially modern form in 1876 at the hands of Nikolaus Otto.[20] The first production automobile using a gasoline-powered internal combustion engine was developed by Karl Benz in 1885 and Henry Ford's Model T began mass-production in 1913. In the intervening years, electric cars surprisingly were more popular, but quickly gave way to the gasoline[21] car due to superior range, quick refueling, and cost.

Today, petroleum supplies 37% of energy consumption in the U.S.[22] 70% of petroleum goes to transportation (92% of transportation energy is in the form of petroleum), while another 24% goes to industrial processes.[23]. Globally, petroleum usage represents a slightly smaller fraction of total energy than in the U.S., at 31% of total energy consumption.[24]

The petroleum extracted from the ground is often called crude oil, and consists primarily of hydrocarbon chains of various lengths. The lighter molecules (shorter chains)—typified by octane (Figure 8.3)—are useful for gasoline, while the much heavier (longer) molecules are found in tar/asphalt, lubricants, or used as "petrochemical feedstock" for plastics. The process of refinement separates constituents by chain length, producing gasoline, kerosene, diesel, heating oil, lubricants, tar, etc. 92% of crude oil goes to energy production of some form (burned), while 8% is used to create petrochemical products, as depicted in Figure 8.4.
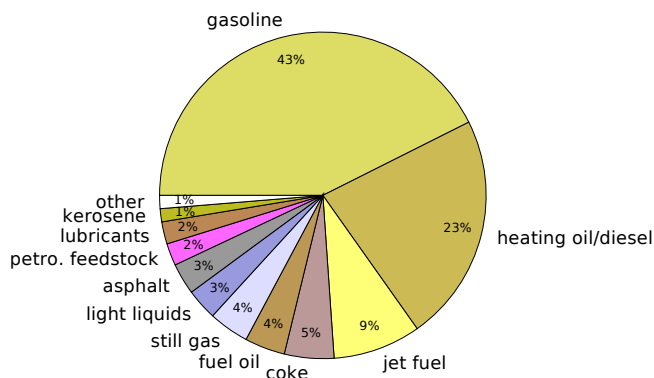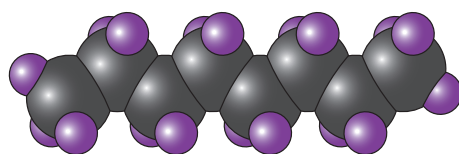
17: Even electric cars may depend on fossil fuels, since > 60% of electricity in the U.S. is fossil-generated.

18: . . . using a steam engine powered by coal

19: . . . a relief from expensive and declining whale oil resources

20: Why isn't it Otto-mobile, then?

21: For clarity, *gasoline* is a liquid that derives from petroleum. Natural gas is in gaseous form, not directly related to gasoline.

22: Recall that Chapter 7 presented these breakdowns in graphical form.

23: Fig. 7.2 (p. 105)

24: Fig. 7.4 (p. 107) and Fig. 7.7 (p. 109)

Figure 8.3: Octane (C_8H_{18}, containing 8 carbon atoms and 18 hydrogens) is among the shorter/lighter hydrocarbon chains found in oil, and is typical of gasoline. Longer chains of the same basic design are found in lubricants, tar, and as feedstock for plastics.

Figure 8.4: Fractional use of a barrel of petroleum, from [41]. All but asphalt, petrochemical feedstock, lubricants, and "other" are burned for energy, amounting to 92% burned. Still gases include methane, ethane, propane and butane in gaseous form, while the light liquids are also mostly propane and butane in liquid form. Coke is not the soft drink.

Petroleum is measured in barrels (bbl), equating to 159 L (42 gal). Each barrel of crude oil contains about 6.1 GJ of energy (1,700 kWh; 5.8 MBtu). For reference, the world consumes about 30 billion barrels a year (the U.S. is about 7 billion barrels per year, or 20 million barrels per day). No single country produces oil at a rate greater than about 12 million barrels per day.[25]

To provide some perspective on how special/rare oil is, the chances of finding any by drilling a random spot on the planet is about 0.01%.[26] This is because many geological conditions must be met to make oil:

1. Organic material must be deposited in an oxygen-poor environment to inhibit decomposition, like dead animal and plant remnants settling to the bottom of a still, shallow sea;
2. The material must be buried and spend time under at least 2 km of rock, to "crack" large organic molecules into the appropriate size, like octane, for instance (Figure 8.3);
3. The material must not go below about 4 km of rock, or the pressure will "overcrack" the molecules to form natural gas (still useful, if trapped underground);
4. An impermeable caprock structure must sit atop the permeable and porous rock (Figure 8.5) that holds the high-pressure oil to keep it from simply escaping.[27]

Oil deposits are rare and tend to be clustered in certain regions of the world where ancient shallow seabeds and geological activity have conspired to sequester organic material and transform it appropriately. The process takes millions of years to complete, and we are depleting the resource about 100,000 times faster than it is being replenished.[28]

Many early oil wells were "gushers"—under enough pressure to push up to the surface under no effort. Modern extraction is not so lucky, having depleted the easy oil already. A combination of techniques is used to push or pull the oil out of its porous rock, including pumps, injecting water under high pressure, bending the drill path to travel horizontally through the deposit, or fracturing[29] the underground rock via pressurized fluids. More work is required to coax the oil out of the ground as time moves forward.

8.2.3 Natural Gas

Natural gas is familiar to many as a source of heat in homes (stoves, hot water, furnace), but is also a major contributor to electricity production and industrial processes (usually for direct heat in furnaces/ovens). It is also used extensively in the production of fertilizer via the Haber process.[30]

Natural gas is primarily methane (CH_4). Its formation process is similar to that of oil, but deeper underground where the pressure is higher and

25: As a consequence, the U.S. is presently unable to support its petroleum needs from domestic resources alone.

26: ... based on a crude calculation of the total resource and assuming a typical deposit thickness of 10 m
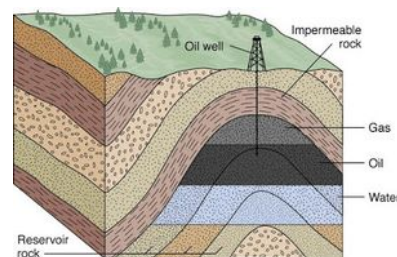
Figure 8.5: Oil and gas embedded in porous rock, under an impermeable caprock [42]. From U. Calgary.

27: Losing even a drop per second adds up to 20 million barrels over one million years, which is short on these geological timescales.

28: A simple way to see this is that it took tens of millions years to create the resource that we are consuming over the course of a few centuries: a ratio of at least 100,000 (see Box 10.2; p. 169). This is like charging a phone for 3 hours and then discharging it in 0.1 seconds! Viva Las Vegas! Fireworks!

29: ... colloquially called fracking

30: The Haber process uses the energetically cheap hydrogen in methane (CH_4) to produce ammonia (NH_3) as a chief ingredient in nitrogen-rich fertilizers.

longer-chain hydrocarbons are broken down to single-carbon methane molecules. We find natural gas trapped in underground reservoirs, often on top of oil deposits (Figure 8.5). Thus petroleum drilling operations typically also produce natural gas output.[31] The gas itself tends to flow out freely once a well is drilled, since it is under great pressure and not viscous like oil. The first commercial use of natural gas started with a well in New York in 1821, leading to a pipeline distribution for street lighting in Philadelphia in 1836. Because of its low density[32] compared to coal or petroleum, it is often impractical to collect, store and transport the gas, strongly favoring a pipeline infrastructure for its delivery. Lack of pipeline infrastructure delayed widespread use of natural gas until about 1970. It is also possible to liquefy natural gas (called LNG) by cooling to $-160°C$ and then storing/transporting in cryogenic vessels.

Natural gas constitutes 31% of energy consumption in the U.S., and 22% globally.[33] Because of the need for pipeline infrastructure in order to deliver gas to consumers, remote areas are typically unable to take advantage of the resource. The uses for natural gas in the U.S. are more diverse than for coal or oil: 35% goes to electricity production, 34% for industrial purposes, and 29% for residential and commercial heating.[34]

31: Unless a gas pipeline is in place at the drill site, the natural gas is too voluminous to be contained/stored, so is often flared (burned; wasted) at the well-head.

32: ...on account of its being gaseous

33: Fig. 7.4 (p. 107) and Fig. 7.7 (p. 109)

34: Fig. 7.2 (p. 105)

8.3 Chemical Energy

Chemical energy is released as heat when combustible materials are ignited in the presence of oxygen. Sec. B.3 (p. 379) in Appendix B provides some background.

Fossil fuels all work the same way, chemically. The three key reactions for coal, methane, and octane[35] are:

35: Gasoline—the main product extracted from petroleum—is a blend of medium-sized hydrocarbon chains, and we use octane as a decent representative for oil.

$$
\begin{aligned}
\text{coal}: \quad & C + O_2 \rightarrow CO_2 + 32.8\,\text{kJ/g} \\
\text{gas}: \quad & CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O + 55.6\,\text{kJ/g} \\
\text{oil}: \quad & C_8H_{18} + \tfrac{25}{2}O_2 \rightarrow 8CO_2 + 9H_2O + 48.0\,\text{kJ/g}
\end{aligned}
\tag{8.1}
$$
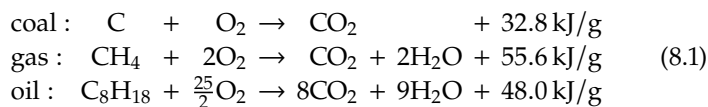
The energy amounts above represent the total energy available per gram of input fuel.[36] Table 8.2 provides several key attributes of fossil fuel combustion. Energy density, in kJ per gram or often kcal/g, is a fundamentally important measure of the fuel's potency. By expressing in kcal/g, we can compare to food labels in the U.S., for which fats are around 9 kcal/g, while carbohydrates and proteins clock in around 4 kcal/g.

36: ...not counting the oxygen; just the carbon-based fuel

Both fossil fuels and food are a type of chemical storage ultimately tracing back to photosynthesis in plants.

Table 8.2: Combustion properties of fossil fuels.

Fuel	Representative	molar mass	kJ/mol	kJ/g	kcal/g
coal	C	12	393.5	32.8	7.8
natural gas	CH_4	16	890.3	55.6	13.3
petroleum	C_8H_{18}	114	5,471	48.0	11.5

Note that fossil fuels are more like fat (near 10 kcal/g) than carbohydrates (at 4 kcal/g).[37]

Box 8.2: Superlative Energy Density

To put these energy densities into perspective and demonstrate how amazing fossil fuels are, consider that the explosive TNT[38] has an energy density of just 1.0 kcal/g. But comparing to TNT is somewhat unfair, as explosives must carry their oxygen with them.[39] Hydrogen gas tops the energy density charts, chemically, at 34 kcal/g, because hydrogen is such a light atom.[40] If having to carry the oxygen along, as rockets must, for instance, the hydrogen-plus-oxygen source is down to 3.8 kcal/g. Rocket fuels and explosives, in general, tend to be in this range of a few kcal/g for this reason. Aside from hydrogen, very few compounds outperform methane for energy density. So crudely, 15 kcal/g is about the top of the chemical scale.

8.4 Fossil Fuel Pros and Cons

8.4.1 What Makes Fossil Fuels Amazing

Energy Density: We have seen in Section 8.3 that the energy density of fossil fuels is quite respectable: about the best that chemistry delivers. Anything over 10 kcal/g is a "superfood" energetically. Table 8.3 compares to other substances, by which we see that fossil fuels are two orders-of-magnitude more energy-dense than battery storage.

Safety: Fossil fuels have greater energy density than explosives, without being particularly explosive! The safety aspect of fossil fuels is a big selling point. Sure, gasoline burns, but really it's the vapor mixed with oxygen that goes poof. If you (foolishly; please don't do this!) throw a match onto a bowl of gasoline, you'll certainly get some lively fire, but the thing won't *explode*. Only the vapor above the pool will be on fire. Think about how many cars you've seen in your life, and how many of those have exploded.[41] How many wrecked cars have you seen, and how many of those exploded? It is not impossible to have an explosive accident from gasoline, but it's pretty rare.

Cheap: Fossil fuels were bestowed upon us as a byproduct of biological and geological processes on our planet. They are essentially free—at least the way we have historically viewed natural resources as ours to grab. How cheap are they? Hiring a physical laborer to exert 100 W of mechanical power (digging, for instance) for 40 hours a week at $15/hr costs $600 for a week. For that price, we receive 4 kWh of work. In electricity terms, the same 4 kWh costs $0.60 at typical rates (1,000 times cheaper than human labor). Gasoline—for which one gallon contains

---

[37]: The simplest way to understand this is that carbohydrates (sugars, such as glucose: $C_6H_{12}O_6$) already have oxygen in the molecules, and are in some sense already half-reacted (combusted) with oxygen, as elaborated in Sec. B.3 (p. 379).

[38]: ... $C_6H_2(NO_2)_3CH_3$

[39]: An explosion is too fast—and violent—to get oxygen from the surrounding air.

[40]: But hydrogen is both bulky and so highly flammable as to be dangerous to store in gaseous form (look up Hindenburg), so don't get too excited.

Table 8.3: Energy densities of familiar energy substances. The Tesla Powerwall represents available lithium-ion capability. Alkaline batteries are familiar AA or AAA cells, and lead-acid batteries are the 12 V ones found in most cars. For hydroelectricity, a 50 m dam is assumed.

Substance	kcal/g
Gasoline	11
Fat (food)	9
Carbohydrates	4
Rocket Fuel	4
TNT explosive	1
Alkaline battery	0.11
Tesla Powerwall	0.10
Lead-acid battery	0.03
Hydroelectric (50 m)	0.0001

[41]: ... discounting dramatic events the entertainment industry prepares for us

---

37 kWh and costs \$4—would be just \$0.43. Efficiency differences, and the cost of the machine to perform the labor also factor in. But the point should be clear enough.

Perfect Storage: Effectively, fossil fuels represent a form of long-term storage of ancient sunlight, captured in plant matter and (sometimes via animal ingestion) ending up buried underground as chemical energy. Compared to other forms of storage, like rechargeable batteries, flywheels, or even hydroelectric reservoirs (pumped storage), fossil fuels are astoundingly superior. Fossil fuel deposits are tens or hundreds of millions of years old. Try finding a battery that will hold its charge that long! Seemingly permanent man-made dams/reservoirs are unlikely to last even one-thousandth as long. Combined with their superior energy density, fossil fuels are perhaps the best form of energy storage available to us, aside from nuclear materials.

Food Production: The Green Revolution in agriculture would not have been possible without fossil fuels. Not only did they provide the motive force for mechanized farming (plowing larger tracts of land, harvesting and processing crops quickly), but the all-important fertilizer is derived from natural gas.[42]

42: Methane (CH_4) provides an energetically favorable source of hydrogen to make ammonia (NH_3) as a way to deliver nitrogen to plants (called the Haber process). Water (H_2O) may seem like a more obvious and abundant source for hydrogen, but in this case substantial energy would have to be injected to extract hydrogen. Methane, by contrast, will give up its hydrogen more easily.

Technology Catalyst: Fossil fuels opened the door to widespread mechanization and electrification, completely transforming our way of life. As central as their role has been, it is difficult to claim that many of the benefits we enjoy today—whether health care, technology, scientific knowledge, or comfortable living standards—would have been possible without them. Much that we celebrate in this world rode on the back of fossil fuels.

8.4.2 What Makes Fossil Fuels Terrible

Climate Change: Nothing comes for free. Fossil fuels also bring many downsides. Chief on many peoples' minds today is climate change, via CO_2 emission—an unavoidable consequence of combustion (Eq. 8.1). Extracting energy from fossil fuels,[43] leaves no choice but to accept CO_2 as a byproduct, in large quantities. We will get to the details of climate change in Chapter 9, but for now will just say that increased CO_2 in the atmosphere changes the equilibrium temperature of Earth by altering how effectively the surface can radiate heat away to space through the atmosphere. The physical mechanism is *very* well understood, and the amount of CO_2 that fossil fuel combustion has produced is *more than enough* to account for the measured CO_2 increase in our atmosphere. What is less certain is how the complex, nonlinear, interconnected climate systems will react, and whether positive feedbacks that exacerbate the problem dominate[44] over negative feedbacks that act to tame the consequences. In the meantime, fossil fuels have handed us a global-scale

43: ...that's the whole point

44: All evidence says positive dominates.

problem of uncertain magnitude and may end up costing us—and other species—dearly.

Population Enabler: Human population pressures on our planet may also be traced to fossil fuels via agricultural mechanization and fertilizer feedstock (the Green Revolution). Since so many new global challenges—deforestation, fisheries collapse, species loss, climate change—scale with the population, perhaps all of these ills can be attributed to fossil fuels—in that it is doubtful these problems would exist at the present scale had we never discovered or utilized them.

Military Conflict: Fossil fuels represent such a prize that access and control of the resources has played a key role in many armed conflicts. Put another way, how many have lost their lives to fights over these precious resources? It is hard to view the complex and fraught relationships in the middle-east as being disconnected from the fact that it is the most oil-rich region in the world.[45]

45: Countries and regions lacking important resources receive far less attention from the developed world.

Environmental Toll: Environmental effects from the extraction of fossil fuels can be pretty destructive. We have seen oil tankers crash and coat beaches and wildlife in tarry sludge. The Deepwater Horizon drill platform failure in 2010 spewed vast amounts of oil into the ocean. Coal extraction can leave mountaintops bare and contaminate local water sources from the tailings. Hydraulic fracturing (fracking) can contaminate groundwater supplies. Natural gas wells—including fracking sites—often leak methane into the atmosphere, which is 80 times more potent than CO_2 as a greenhouse gas[46] on short timescales.

46: Although, methane does not last in the atmosphere as long as CO_2. Still, this is why gas is often flared (burned) at drill sites lacking pipeline infrastructure, rather than allowing it to escape as methane.

Substance Addiction: Finally, the very fact that fossil fuels are *finite* may be viewed as a serious negative. Granted, an effectively inexhaustible supply would be devastating for the climate change story. Setting that aside, the fossil fuel inheritance might be viewed as a sort of bait-and-switch trick. We have built up to our current state wholly in the context of cheap and available fossil fuels, and simply *do not know* if we can continue to live at a similar standard in a post-fossil world. Fossil fuels have lasted long enough (several generations) to seem normal. We take them for granted, and have not formulated a master plan for a viable world devoid of these critical resources. How will air travel, ships, trains, and long-haul trucking[47] be handled without petroleum? The current situation is precarious. Failure to plan wisely for a post-fossil world would not be the fault of fossil fuels by themselves. But the fossil fuel endowment that happened to grace our planet was large enough to harm the climate and to lull us into complacency. Had it been a much smaller amount, we would be less likely to fall into the trap.[48] This is the "rabbit out of a hat" referred to in Chapter 2: just getting one conditions us to expect an eternal state of rabbits.

47: All of these modes of transportation are difficult to accomplish via electric drive (Sec. D.3; p. 397), and critical to our global supply chains for manufacture of consumer goods.

48: By the same token, it is unlikely that we would be at a comparable technological level if our inheritance had been much smaller.

8.4.3 On Balance?

Deciding whether fossil fuels have had a net-positive or net-negative influence on humanity may not be answerable (Table 8.4 provides a summary of the previous two subsections). How many lives has it saved through better technology and health care? How many lives has it destroyed through conflict, pollution, and transportation accidents? How many lives has it created, through vast increases in agricultural productivity—as well as via better medical care? How many species has it destroyed, by promoting habitat loss both directly via extraction and indirectly as a catalyst to population growth via increased agricultural productivity? Sometimes it is even hard to decide which category to put these impacts into. For instance, in the fullness of time, will we see all the lives created on the back of fossil fuels as a good thing? If the result is overshoot, collapse, and the unprecedented suffering of billions of people, then perhaps not. It's a mess.

In essence, humanity is running this global-scale unauthorized experiment on the planet without a plan. Nothing like this has ever happened, so we don't know how it will turn out. We have plenty of evidence that past civilizations overextend and collapsed [43], but we can't identify a fitting analog to successful navigation of the fossil fuel phenomenon. Meanwhile, plenty of signs justify grave concern.

Table 8.4: Pros and cons of fossil fuels.

Pro	Con
energy dense	climate change
safe	overpopulation
inexpensive	agent of war
long storage	environ. damage
agriculture	overdependency
technology	so yesterday

[43]: Diamond (2005), *Collapse: How Societies Choose to Fail or Succeed*

8.5 The Future of Fossil Fuels

8.5.1 Scenarios

Figure 8.1 provocatively asserts that fossil fuel use must fall back to essentially zero in a relatively short time (within a century or two). This fact alone does not define our future on the spectrum of dismal to glorious, but it is one we need to consider carefully given the fundamentally important role fossil fuels have played in getting us to where we are today. The return to zero fossil fuels could take a variety of forms:

1. We discover a new form of cheap energy not yet known or appreciated that is a game changer, quickly abandoning the fossil fuels still left in the ground.
2. Known renewable energy sources (solar, wind) are developed to the point of being effectively superior to fossil fuels so that market forces naturally move us away from fossil fuels before actually running out.
3. Climate change concerns result in politically enforced financial dis-incentives to using fossil fuels, so that we migrate away—albeit likely at higher cost, politically controversial, and not globally adopted.

These are not strictly exclusive of each other, so some combinations are also possible.

4. Increased difficulty in extracting fossil fuels drives their price up so that the market is ultimately forced to accept less convenient and more expensive forms of energy.

5. We fail to find suitable substitutes to this precious and unique resource, so that global geopolitics increasingly center on competition for remaining fuel, likely touching off destructive resource wars.

6. Perhaps together with the previous point, society slowly grinds to a less energy-rich state, diminishing agricultural capacity and decreasing both the number and standard of living of people on the planet.

We cannot predict which of these paths might manifest, but it is not hard to find adherents to any of these narratives. Part III of this book covers alternatives to fossil fuels, and Chapter 17 summarizes practical challenges to the various alternatives. One lesson that emerges is that fossil fuels beat out alternatives on a host of considerations, leaving a gap between the two groups. If not for the finite supply and climate ills, we would have no incentive to adopt otherwise inferior sources of energy at higher cost. But first, we should briefly look into future prospects for extraction of fossil fuels. How limiting is the physical resource?

8.5.2 Timescales

The simplest approach to evaluating a timescale for resource availability is the R/P ratio: reserves to production.[49] The idea is very intuitive: if you have $10,000 in a bank account, and tend to spend $1,000 per month on living expenses, you can predict that—absent additional income—you will be able to go for ten months. So if we have an estimate for resource remaining in the ground, and the current rate of use, we simply divide to get a timescale.

Table 8.5 reports the proven reserves in the world and in the U.S. for the three fossil fuels, the estimated fraction used so far globally, the rate of consumption,[50] and the timescale given by the R/P ratio.

Region	Resource	Remaining	% Used	Annual Use	R/P (years)
World	oil	1,700 Gbbl	~45%	30 Gbbl	~60
	gas	200 Tcm	~33%	3.5 Tcm	~60
	coal	900 Gt	~30%	8 Gt	~110
U.S.	oil	35 Gbbl		7 Gbbl	~5
	gas	8.5 Tcm		0.85 Tcm	~10
	coal	250 Gt		0.7 Gt	~360

The world has already consumed 1.5 trillion barrels of oil, which is nearly the same amount as the 1.7 trillion barrels of proven reserves—indicating that we are roughly halfway through the resource.[51] Certainly, we can expect that additional resources will be discovered and added to the

49: Here, "production" means "obtaining from the ground," not fabricating artificially.

50: Consumption and production are essentially identical: no stockpiling.

Table 8.5: Summary of proven reserves, usage rates, and time remaining for the world and for the U.S. (if using *only* its domestic supply) [44–46]. Oil is measured in giga-barrels (Gbbl; 10^9 bbl), gas in tera-cubic-meters (Tcm; 10^{12} m^3), and coal in gigatons (Gt; 10^{12} kg; noting that 1 ton is 1,000 kg).

51: This fact is one justification for believing we may be near the top of the symmetric curve in Figure 8.1.

proven reserves,[52] but the globe is pretty well explored now, and we would not expect huge surprises like another hidden middle-east-size oil deposit. Note that for natural gas, the estimated total resource in the U.S. (what we *think* we may yet find beyond proven reserves) is about 55 Tcm, which would last just over 60 years.[53]

It is difficult to compare the remaining resource in the three forms directly, since different units are used for each. But we can cast each in terms of energy units for comparison. Doing so, the global reserves of oil, gas, and coal correspond to 10, 8, and 20 ZJ[54] remaining, respectively. We have so far consumed 8, 4, and 8 ZJ of oil, gas, and coal (Table 8.6). These form the basis of the estimated fraction consumed in Table 8.5. Note that the amount of oil and gas remaining are roughly comparable in energy, while coal is roughly twice as much.

Coal[55] therefore seems to be our most abundant fossil fuel, which prompts two comments. The first is that it is the worst offender in terms of CO_2 emission, emitting roughly twice as much CO_2 per unit of delivered energy as the other fossil fuels (covered in Chapter 9). The second is a caution in trusting the reserves estimates for coal, having often been vastly overestimated and then reduced significantly. For instance, Britain had to downward-revise their estimated coal reserves over the period from 1970–2000 to about 1% of their original because most of the estimated resource turned out to be in seams too thin and difficult to be commercially viable [47].

For some, the R/P numbers in Table 8.5 may seem alarmingly short, while for others they may signal a comfortable amount of time to devise alternative energy strategies. Either way, this century is critical. But it is also important to recognize that the story is not quite as simple as the R/P ratio. While it provides a useful *scale*,[56] we should consider these nuances:

1. The production (thus consumption) rate is not steady, but on the whole has grown over time (continued growth would shorten timescale).
2. New exploration and discovery adds to reserves (lengthening the timescale), but with diminishing success lately.
3. Advances in oil extraction technologies increase the amount of accessible oil (lengthening timescale).
4. Geological challenges limit the rate of production (lengthening timescale but also limiting resource availability).
5. Demand (thus production) could plummet if superior substitutes are found.

Point number 4 deserves some elaboration. We should not think of fossil fuel reserves as a bank account from which we may withdraw funds at an arbitrary rate, or as a cavernous underground lake just waiting to be slurped out by whatever straw we wish to shove in. Coal, firstly, does not flow, requiring substantial physical effort to expose and remove. The

52: Also, technological advances can make previously impractical resources available, adding to reserves.

53: Because gas is harder to transport, and typically delivered by pipelines, domestic supply is more relevant for gas than it is for the globally-traded oil resource.

54: ZJ is zetta-Joules, or 10^{21} J.

Table 8.6: Proven reserves and amount used, in energy terms.

Fuel	Proven 10^{21} J	Used 10^{21} J
Coal	20	8
Oil	10	8
Gas	8	4

55: Coal reserves estimates [46] are broken into higher-quality anthracite and bituminous (~7 kcal/g), then sub-bituminous and lignite (~4.5 kcal/g) varieties, totaling 480 Gt (gigatons) and 430 Gt, respectively. (see Table 8.1).

[47]: Rutledge (2011), "Estimating long-term world coal production with logit and probit transforms"

56: If the number worked out to 5 years, we would be in a panic. If it worked out to 5,000 years, climate change would loom as the chief concern.

A car's gas tank is another tempting, but flawed mental model. Getting water out of wet sand is closer to the truth for oil extraction.

rate at which it can be removed depends on the thickness of the seam, how deep it is located, and how hard it is to dig out surrounding rock. Even oil is not in some sloshing reservoir, but permeates porous rock, limiting how quickly the viscous fluid can be coaxed to flow out of the rock and into the pump tube. Gas is the quickest to escape its rocky tomb, but at this stage the U.S. has moved to "tight gas" that does not so easily break free—forcing a technique of fracking the rock to open channels for gas to flow. The same technique is being used to access "tight oil" that otherwise refuses to be pumped out of the ground by conventional means.

In all cases, it is obvious that we would pursue the easiest resources first: the low-hanging fruit. As time marches on, we are forced to the more difficult resources.[57] Adding to the geological factors is the simple fact that we do not possess unlimited extraction machinery, limiting the rate at which fossil fuels can be delivered from the ground. It is also worth pointing out that drilling deeper will not continue to pay dividends, as Section 8.2.2 points out that oil buried too deep will be "cracked" into gas.

57: ...deeper underground, under deep water, or in "tight" formations

Figure 8.6 illustrates three variants of possible trajectories for a finite resource. The left-most panel corresponds to the R/P ratio: how long can we go at *today's* rate of use, if we locked in consumption at a steady value? The second assumes we continue an upward trajectory, which shortens the time compared to the R/P ratio before the resource runs out (using it ever-faster). Both of these are unrealistic in their own ways—the second one because of the physical constraints on extraction listed above (not a free-flowing resource). The third case is more realistic: a peak and somewhat symmetric decline. This is how real fossil fuel resources behave in practice. All three scenarios could create shocks to the system, but note that the (realistic) peak scenario brings the trauma of declining supplies soonest—long before the R/P ratio would suggest.

Figure 8.6: Three scenarios for a finite resource playing out, all based on the same initial history (the red dot is "now") and the same remaining amount (blue-shaded region). The red bar over each represents the remaining time until resource decline. See text for details.

8.5.3 Clues in the Data

Despite the uncertainties listed above, we can say for sure that Earth is endowed with a finite supply of fossil fuels, and that in order to consume the resource, deposits must first be discovered via exploration and then developed into active wells. Even in areas known to have oil,[58] only about one in ten exploratory wells bears fruit. The chances of striking oil at a random location[59] on Earth is in the neighborhood of 0.01%. Section 8.2.2 indicated the chain of events that must transpire to produce oil.

58: ...also applies to gas

59: Think about throwing a dart at the globe.

A plot of the discovery history of conventional oil is revealing, seen in Figure 8.7. In it, we see that discovery peaked over 50 years ago. Since we can't extract oil we have not yet discovered—much like we can't possess an iPhone model that hasn't even been designed yet, the area under the consumption (red) curve must ultimately be *no larger* than
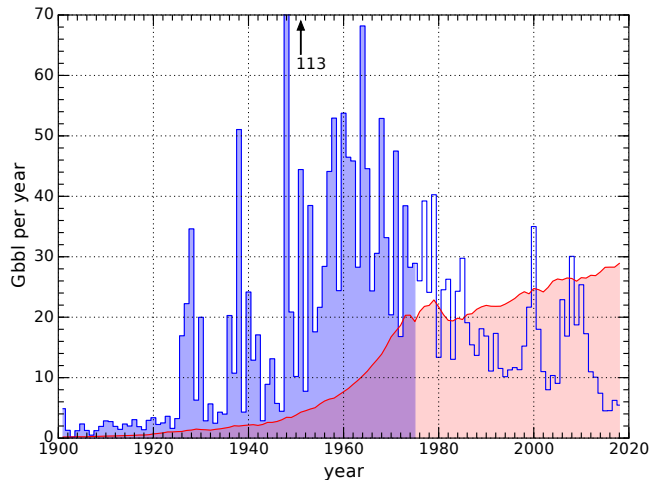
Figure 8.7: Historical discovery rate of conventional oil (blue), measured in billion barrels (Gbbl) discovered each year [48]. The red curve shows annual global consumption of conventional crude oil. Until about 1985, we tended to discover more oil than we used each year, but the rate of discovery peaked decades ago and is now in decline as we complete the job of exploring Earth's resources. The blue area is made equal to area under the red curve, which itself represents the amount of oil used to date. This effectively means that we have used all the oil discovered up to 1976, and are now left with a dwindling bank account (oil reserve)—our annual income (new discovery) being less than our spending (consumption).

the area under the discovery data (blue). It is therefore inevitable that consumption will peak and fall at some point, by whatever means. Note that a symmetric curve peaks when the resource is half-consumed.

The information in Figure 8.7 can also be re-cast to ask how many years remain in the resource. For any given year, the total remaining resource can be assessed as the cumulative amount discovered to date minus the cumulative amount consumed. Then dividing by that year's annual production (same as consumption) rate produces an estimate of remaining time (the R/P ratio again). Figure 8.8 shows the result.
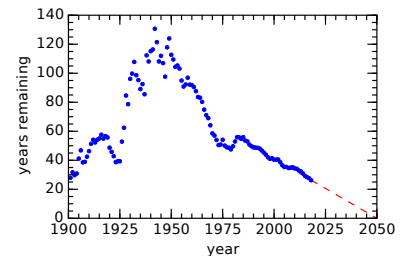
Figure 8.8: Years remaining in the global conventional oil resource as a function of time, extracted from the data in Figure 8.7. Since 1982, the world has been on a steady path toward depletion of the conventional oil resource by 2050.
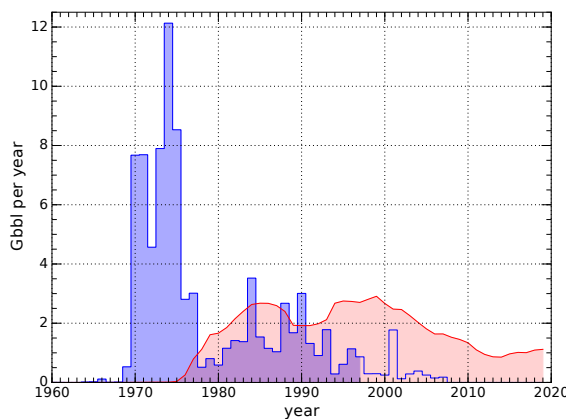
Figure 8.9: North Sea (U.K.) oil discoveries (blue, in giga-barrels per year) peaked in the 1970s and have basically ended. Production (red) lags discovery, and cannot carry on much longer as the last of the discovered oil (unshaded blue outline) is extracted. Plot conventions follow those in Figure 8.7.

We have seen this story play out numerous times within oil-producing regions. Discovery of oil in the North Sea put the U.K. into the oil business about 50 years ago (Figure 8.9). At first, the discovery rate was brisk, followed by 20 years of modest discovery. It appears that nothing is left to find, as discoveries have stopped. The production shows a double-peak structure—maybe echoing the discovery lull around 1980—but in any case is nearing the end of extraction. Only about 6% of the discovered oil (effectively that discovered after 1996; unshaded in Figure 8.9) is left: not much remains to pump out.

The U.S. experienced a similar history (Figure 8.10) in that discovery of conventional oil peaked around 1950, and production peaked two decades later, around 1970. Nobody wanted this to happen, although some oil geologists (notably M. King Hubbert) pointed out its inevitability based on the preceding discovery peak and simple logic.[60] The U.S. had been the largest oil producer since the dawn of the oil age, and was now slipping.[61] The peak and subsequent fall caused great anxiety and stimulated tremendous effort to find and develop additional oil resources, leading to the discovery of oil at Prudhoe Bay in Alaska—responsible for the second (lower) peak in the mid 80s. But then the decline resumed for another couple of decades, to the chagrin of many.[62]
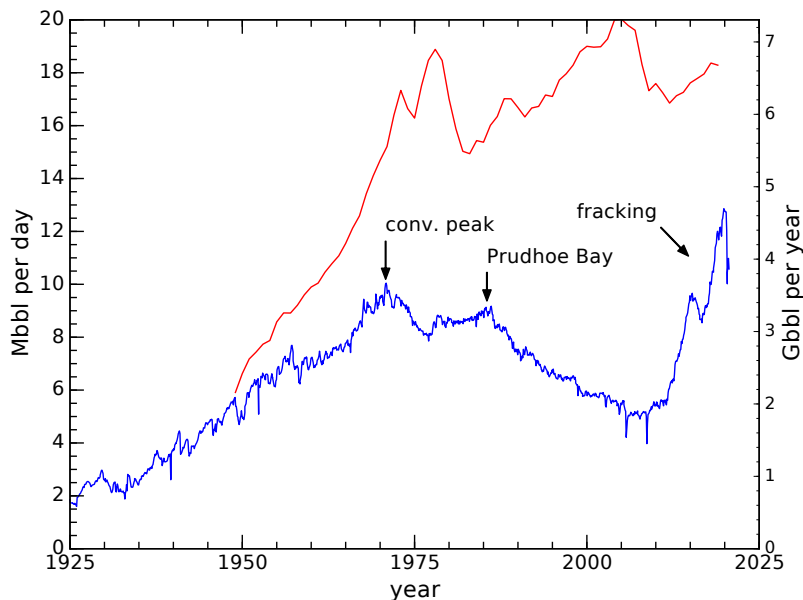
60: Still, the prevailing attitude was one of denial, until it actually happened.

61: This is a large factor in the prosperity of the U.S.: it was the "Saudi Arabia" of the first half of the 20th century, leading oil exports and expansion of automotive transportation.

62: To reiterate a key point: it wasn't for lack of will or effort.

Figure 8.10: U.S. oil production history (blue; from [49]) and consumption history (red; from [34]), in both million barrels per day (left axis) and billion barrels per year (right). The conventional production peak is visible around 1970, a second peak around 1985 from Prudhoe Bay in Alaska, and finally a dramatic upturn due to hydraulic fracturing practices in the last decade. The gap between blue and red curves is made up by imports. The downturn in fracking production in 2020 coincides with the COVID pandemic, so it is not clear whether U.S. oil production will resume the climb or if we are past the peak.

Something unexpected happened next, which may serve as a cautionary tale to those who might attempt confident predictions of the future. The "fracking" boom opened access to "tight" oil deposits that were previously untenable for conventional drilling. The history is shown in Figure 8.10.

How long will the fracking boom last? One aspect to appreciate is that conventional wells take something like a decade to fully "develop,"[63] and even after individually peaking continue to deliver at diminishing rates for many years. Notice the approximate symmetry of the curve[64] in Figure 8.10 and its slow decline phase prior to 2010. Fracking "plays"[65] are fast: once the small region has been fractured and pumped, the whole process can be over in a matter of a few years. Thus it is certainly possible that the fracking boom on the right-hand-side of Figure 8.10 will end as abruptly as it started—the easy plays being exploited first, leaving less productive fields to round out the declining phase of this boom. In any case, declaring the current state of oil production in the U.S. to represent a "new normal" seems premature.

63: By develop, we mean populate the deposit with multiple drill sites and pumps.

64: Note that the curve is an aggregation of many hundreds of individual wells whose individual production rates rise and fall on shorter time scales.

65: . . . the term for a field to be exploited

8.5.4 Geopolitics

Another wrinkle worth mentioning is the geopolitical angle. Much of the world's proven reserves are not owned by the countries having the highest oil consumption. Figure 8.11 shows which countries hold the largest stocks, with a caveat that the deposits in Venezuela and Canada are heavy oils,[66] which are harder to extract and refine into lighter forms like gasoline, making the middle-east (Saudi Arabia, Iran, Iraq, UAE, and Kuwait) the "real" leaders of light[67] crude oil dominated by the more useful shorter-chain hydrocarbon molecules like octane (Figure 8.3). One thing that should cause Americans alarm is to go around the circle looking for close allies. Aside from Canada, with its inconvenient heavy-oil, the picture is not terribly reassuring. Proven reserves of oil in the U.S. amount to 35 billion barrels. At a consumption rate of 20 million barrels per day, the math suggests only 5 years, if we only used our own supply. The proven reserve, however, is a conservative number, often short of estimated total resource: exploration can add to proven reserves. The estimated resource in the U.S. is closer to 200 Gbbl, which would last a little less than 30 years without imports at the present rate of consumption. These short timescales offer some relief for climate change concerns, but perhaps represent bad news for global economies utterly dependent on fossil fuels.

66: ... e.g., tar sands; long-chain hydrocarbons

67: ... sometimes called "sweet"
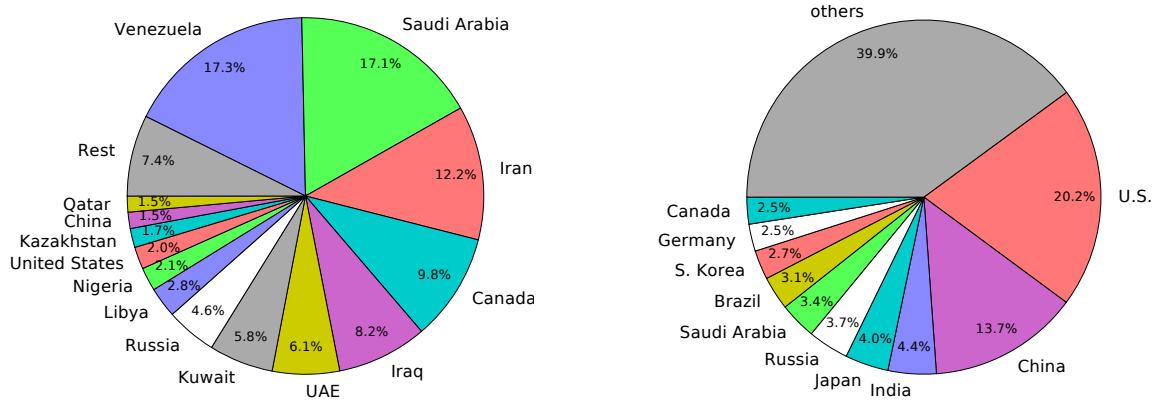
Figure 8.11: Distribution of proven oil reserves by country, on left, according to the U.S. Energy Information Administration. The oil in Venezuela and Canada is heavy oil, harder to extract and process than the light oil characteristic of the middle-east. At right is the oil consumption by country for the top ten consumers (U.S. EIA). Note that the U.S. possesses 2% of the oil, but consumes about 20% of annual production, and an overall lack of correlation between who *has* oil and who *needs* it.

Because the rate of extraction can be a limiting factor, it often happens that the rate of production begins to slow down (peaks) around the time half the resource has been exhausted,[68] producing a symmetric usage curve over time. This suggests that the peak can occur *well before* the timescales resulting from the R/P ratio, as depicted in Figure 8.6. Once the world passes the peak rate of oil production, a sequence of panic-driven damaging events could ensue, making it more difficult (less likely) for us to embark on a renewable-centered post-fossil world. Boxes Box 8.3 and Box 8.4 paint scenarios that cause concern.

68: ... about where we appear to be on oil

Box 8.3: Resource War

Imagine the scenario in which oil prices climb from their current $50/bbl to $100/bbl.[69] Some major oil-producing country—seeing the writing on the wall that this precious resource is only going to become more valuable as supplies inevitably diminish—will decide that its economy was doing just fine at $50/bbl, so can sell half as much at $100/bbl and have the same income. Removing that oil from the market pushes oil prices up further to $150/bbl, at which point other countries may begin playing the same game, but now selling a third as much oil for the same income. The resulting domino effect will cause international crisis, and some military power, acting as the world's police,[70] will step in to ensure continued flow of this vital global resource. Other countries possessing military strength will object to this one country's presumption and control of important segments of the global oil supply, and might potentially engage in a resource war. Sadly, this turn of events would consume massive amounts of energy and other resources to *destructive* ends, rather than channel these resources into *constructive* activities like building a post-fossil renewable energy infrastructure.

69: Oil has been as high as about $160/bbl, in June 2008 (inflation-adjusted for 2020). A compelling argument can be made that the stress of high oil prices on many sectors of our economy provided a trigger for the financial crisis—putting an end to the growth-fueled bubble in the sub-prime housing market.

70: This hypothetical country may also have built numerous military bases in the middle-east, in anticipation of this day.

Box 8.4: The Energy Trap

If we find ourselves in a state of annual decline in energy resources—having clung too tightly to fossil fuels as a cheap and largely superior energy resource—we will have a hard time politically pulling out of the dive, because to do so means transitioning away from fossil fuels via a renewable infrastructure. But such an enormous enterprise will require substantial energy investment. And energy *is the very thing in short supply*. To embark on this transition, the society would have to voluntarily sacrifice *even more* than they already are in the energy decline crisis by diverting energy toward the decades-long initiative. The temptation to vote for a politician who would end the program and bring instant energy relief in the short term may be overwhelming. In other words, we could find ourselves in an energy trap. Witness the difficulty the world is having weaning itself off of fossil fuels despite obvious perils in the form of climate change. If it were easy, cheap, and superior to move to renewables, it would have already happened in a heartbeat. Maybe we're stuck on the flypaper.

This notion is further explored in Sec. 18.3 (p. 310).

8.6 Upshot: Amazing, Terrible, and Limited

History may very well view this time period as the Fossil Fuel Age rather than the Industrial Age. Fossil fuels are a ubiquitous and defining characteristic of this unusual time. The current level of technology,

global population, or impressive state of knowledge would not have been possible without fossil fuels. We therefore owe a great debt of gratitude to these three amazing resources. Perhaps the first species on any planet to discover and use fossil fuels[71] will follow a similar madcap trajectory and even temporarily poke into space, as we have.

Yet fossil fuels bring a number of downsides, like climate change, potential population overshoot (and associated myriad pressures on the planet), pollution and environmental damage. More subtly, a near-complete dependence on fossil fuels has transformed human expectations in a way that could result in failure to adapt once they are no longer available. Superior substitutes are not guaranteed, and inferior replacements may not be gracefully adopted.

One thing we know for sure about fossil fuels is that the supply is finite. We are arguably approaching the halfway point[72] in extraction, and have naturally harvested the easiest deposits of the resources. As extraction gets harder, supply-rate (relative to demand) may become the limiting factor *well* before the R/P ratio says we will "run out" (see Box 8.5). Recall that fossil fuels are not situated in the equivalent of a single bank account permitting withdrawals of arbitrary size and speed.

71: It is plausible that fossil fuels would be a common result of billions of years of evolution, resulting from buried biological matter on planets supporting rich ecosystems—Earth being the only one we know about (see Sec. 18.4; p. 312).

72: ... especially in the hardest-to-replace oil resource

> **Box 8.5: Running Out One Day?**
>
> Fossil fuels will not abruptly run out one day, or even one year (see Figure 8.6). Production will taper off slowly over decades as ever-smaller deposits are harder to access and extract. In this sense, "running out" of fossil fuels will not be a sudden, jarring event in human history that sends us into a panicked chaos. Nonetheless, passing the peak and having less available with each passing year creates its own set of hardships. In the best scenario, alternatives ramp up fast enough to offset declining fossil fuel supplies. But the challenge is enormous, and success is far from guaranteed.

Given the important role the diminishing fossil resource plays in our world, today's insignificant contribution from renewable sources—as presented in Chapter 7—is all the more worrisome. This fate has been apparent to many for at least 50 years, but fossil fuel use has only continued to increase, while growth of alternatives has been lackluster. Part of the reason has to do with the low cost and amazing convenience of fossil fuels compared to alternatives.[73] Another part is lack of awareness. Sometimes old—yet no less important—stories have trouble maintaining currency in our news-oriented society.

73: One is justified in asking why prices are not raised to discourage fossil fuel use and catalyze development of alternatives. See Box 8.6.

> **Box 8.6: Why Not Raise the Price?**
>
> If continued reliance on fossil fuels is risky—both from resource

scarcity and climate change points of view—then why do prices remain low, serving to encourage continued use and hinder adoption of alternatives? Why doesn't the government raise the price?

The rookie mistake here is assuming that adults are in charge. Markets are in charge. Governments may impose taxes and tariffs, but cannot go overboard before voters[74] object. Global competition without global government penalizes those countries self-imposing additional costs on their citizens. And finally, short-term sacrifice for long-term benefit is not a human strong suit—especially in the face of uncertainty. Convincing people of a future problem that has never surfaced for generation after generation turns out to be hard.

74: ...in democracies, anyway

8.7 Problems

1. Make a zoom-in[75] of Figure 8.1 showing the central fossil fuel spike. You could have it "leave the floor" around 1850, reach a peak maybe at 2050 (fine for the purposes of this problem), and return to zero in symmetric fashion. Now draw—perhaps using a different color—the part of the curve you've lived through, and project out using a dotted line the part of the curve you think you'll live through (over the peak?). Now draw a segment representing your parents,[76] and do the same for your grandparents and great grandparents. You'll end up with overlapping lines.[77] Don't worry about exact dates; we're just looking for a visual impression. Has anybody you've ever met known any period but the rapid growth phase in energy you've experienced in your life?

75: Figure 8.2 is at least a good example of the left-hand side of the spike.

76: ...or something representative of their generation

77: Just stack them a bit so you can tell them apart.

2. If you had to fill in the big question mark in Figure 8.1 with a prediction of the scenario you think is most likely to result in a few thousand years, what would you say? How do you think humans will live?[78] This is really an exercise to make us think about possibilities: no one knows the "right" answer.

78: E.g., what energy sources, primitive vs. technological, dwelling style, etc.

3. If for some reason we are grossly mistaken about the amount of fossil fuels remaining, and have 1,000 years instead of ~100 left, how qualitatively different would Figure 8.1 look?

4. Today, 20% of energy comes from non-fossil resources. Redraw Figure 8.1 under the condition that we manage to hold on to this capability indefinitely, after fossil fuels are gone.

5. Guided by Figure 8.1, how do you think humans 200 years from now will view the period from 1900–2100?

6. It is fair to say that the scientific consensus has held for a while that curtailing our use of fossil fuels would be in the best interest of the planet. From Figure 8.2, report on what has happened to

global fossil fuel use (total, not per-capita) during the last 20 years.

7. Coal usage in the U.S. has declined dramatically in the last 20 years as natural gas has replaced much of the electricity production from coal. What does Figure 8.2 say about the *global* coal trend during this period?

8. As we exploit the best coal resources first, working our way from the premium Anthracite towards Lignite (Table 8.1), will we need to mine *more* coal, or *less*, to achieve the same energy output from coal, in terms of mass removed?

9. Referring back to Fig. 7.2 (p. 105), deduce what fraction of the 38.3 qBtu electricity budget derives from coal. How much would the U.S. need to reduce its electricity dependence if we suddenly stopped using coal?

ⓘ This problem has much in common with Prob. 3 (p. 111) in Chapter 7.

10. Octane is C_8H_{18}. On either side is heptane and nonane, containing 7 and 9 carbons, respectively. Referring only to Figure 8.3 and recognizing the pattern, what would the chemical formulas for heptane and nonane be, in the form of C_xH_y?

11. The U.S. uses approximately 20 million barrels of oil per day, and has a population of about 330 million people. On average, then, how many barrels *per year* is one person responsible for consuming?[79]

79: Even if not personally consuming this much, it is used on behalf of individuals to provide goods and services for them.

12. If an average American is responsible for consuming a barrel[80] of oil every 18 days, what power does this correspond to, in Watts?

80: A barrel contains about 6.1 GJ of energy.

13. Using the values in Table 8.2, compute the energy content of a gallon of gasoline assuming that octane (C_8H_{18}) is a good representative, energetically. Express your answer in both MJ and kWh. One gallon is 3.785 L and in the case of gasoline has a mass of 2.8 kg.[81]

81: The density of gasoline is about 0.75 times that of water.

14. Every day, Americans use about 9×10^8 J of energy per person. Since we know that 37%, 13%, and 31% of this comes from oil, coal, and gas, respectively, use Table 8.2 to figure out how much mass of each is used per day on American's behalfs, and take a moment to compare to equivalent-mass volumes of water to provide familiar context.

15. What if we could get our energy from drinking gasoline?[82] Referring to Table 8.2, how many grams of gasoline[83] would we have to drink daily to satisfy the typical 2,000 kcal/day diet? How much volume does this represent if gasoline is 0.75 g/mL? Relate this to a familiar container for holding liquids.[84]

82: ⚠ Don't do this! It won't work!

83: ... represented by octane, C_8H_{18}

84: For reference, 100 mL is 3.4 oz.

16. One liter of gasoline (1,000 mL) has a mass of about 750 g and contains about 9.7 kWh of energy. Meanwhile, a typical AA battery

occupies 7.4 mL of volume at a mass of 23 g, while holding about 0.003 kWh of energy. How much volume and how heavy would a collection of AA batteries be in order to match the energy in a liter of gasoline, and by what factors (in volume and mass) is gasoline superior?

17. A gallon of gasoline contains about 37 kWh of energy and costs about $4, while a typical AA battery holds about 0.003 kWh and costs about $0.50 each, in bulk. By what factor are batteries more expensive, for the same amount of energy?

18. Putting the cheapness of fossil fuels into perspective, a gallon of gasoline purchased for $4 might deliver 6 kWh of mechanical energy after accounting for efficiency of the associated engine. A laborer might be expected to export 100 W of mechanical power, on average, and be limited to 8 hours per day. How many hours would it take for the laborer to accomplish the equivalent output of a gallon of gasoline? At a rate of $15/hr, how much will this cost you?

19. A number of attempts to estimate the energy investment in our food arrive at the conclusion that every kcal of food we eat took 10 kcal[85] of fossil fuel input energy, so that we are effectively eating our fossil fuels! As a sanity check, what fraction of our fossil fuel energy would have to go into food production in the U.S. if diets are typically 2,000 kcal/day and we use fossil fuels at a rate of 8,000 W?[86] Does the answer seem plausible?

[85]: Before fossil fuels—when food-driven muscle power was used instead—if we put in more energy than we got out, we would have starved and died out.

[86]: 80% of the 10,000 W American energy rate per person

20. List at least five ways in which your life benefits from fossil fuels.

21. List at least three negative impacts of fossil fuels that most concern you (or explain why not, if they don't concern you).

22. Which of the possibilities from the list on page 125 (or combination thereof) seem most likely to play out, to your mind? Explain what makes you think so.

23. Let's say that Earth was originally endowed with one million flerbits,[87] and that we have already used up 400,000 of them. We currently extract 15,000 per year. How long does the R/P ratio suggest the resource will last?

[87]: ... a made-up thing that we presume is irreplaceable

24. Proven remaining reserves of oil, gas, and coal are 10, 8, and 20 ZJ,[88] while we have used 8, 4, and 8 ZJ of each. What fraction of the *original* total fossil fuel resource have we already used, then?

[88]: ZJ is zetta-Joules, or 10^{21} J.

25. Explain, both in practical and mathematical terms, why the R/P ratio overestimates the time remaining for a resource if the rate of production (use) of that resource is continually increasing.

26. It is hard for many people to appreciate that fossil fuels will not just

"run out one day," because they don't appreciate the substantial amount of work that must go into extracting the resource from a reluctant ground. What common, day-to-day personal experiences do you imagine contributes to this disconnect?[89]

89: In other words, what is your personal experience obtaining resources that can suddenly just run out?

27. Simultaneously considering Table 8.5 and the lessons from Figure 8.6, what sort of timescale might you guess for when the world might see a more-or-less permanent downturn in oil production? Say how many years you think we have until a downturn and explain your reasoning.

28. What analogy from everyday life can you think of that would help someone understand the idea that extraction of oil from the ground must be preceded by exploration and discovery of the resource, and that we can't produce more than we discover?

29. Explain why the area under the red curve in Figure 8.7 or Figure 8.9 cannot be larger than the total area under the blue curve.[90]

90: Note that the shaded blue area is not the entire area under the blue curve, but has been set to equal the red area.

30. If the inevitable decline in fossil fuel availability is a potentially important disrupter of the status quo in the decades to come, what are some reasons it gets little attention compared to, say, climate change? No right answer here, but what do you think contributes?

9 Climate Change

Climate change stands tall among the global scale problems created by our energy appetite—caused by the accumulation of carbon dioxide (CO_2) in our atmosphere from the burning of fossil fuels. This chapter aims to provide a no-nonsense account of the basis for climate change that leaves little room for the kind of uncertainty often injected by (alarmingly successful) disinformation campaigns. While the *response* of our complex climate system is more difficult to predict in detail, the core physics is unassailable. We will see that the rise in CO_2 is not at all mysterious, stemming from fossil fuels. We will also explore a few scenarios and connect the CO_2 rise to temperature consequences.

9.1 The Source of CO_2

The climate change forces at play today are primarily due to increased concentration of CO_2 in our atmosphere as a result of burning fossil fuels.

The chemistry is unambiguous (Eq. 8.1; p. 121): energy is released when fossil fuels are combusted with oxygen (O_2) to get CO_2 and H_2O. Table 9.1 extends properties of fossil fuels first presented in Chapter 8, adding CO_2 attributes.

Table 9.1: Combustion properties of fossil fuels, including CO_2 emission per gram of input and per MJ of energy out.

Fuel	Representative	molar mass	kJ/mol	kJ/g	kcal/g	CO_2 g/g	CO_2 g/MJ
coal	C	12	393.5	32.8	7.8	3.67	112
natural gas	CH_4	16	890.3	55.6	13.3	2.75	49
petroleum	C_8H_{18}	114	5,471	48.0	11.5	3.09	64

Of chief interest in Table 9.1 for this chapter are the last two columns:

Elephants doing their best. Photo courtesy of O'Connell & Rodwell.

1. **mass ratio**: how many grams of CO_2 are produced per gram[1] of input fuel;
2. **carbon intensity**: how many grams of CO_2 are produced per unit of energy delivered.

While all forms produce a mass ratio of *approximately* **3 units of** CO_2 **for every unit of fossil fuels**, the lower energy density of coal together with its slightly higher mass ratio[2] make it *more than twice* as carbon-intense as natural gas.

2: Coal produces more CO_2 per gram of fuel because the other fossil fuels also contain mass in the form of hydrogen, which not only adds to energy production but also does not end up in CO_2.

> **Example 9.1.1** Roughly how much CO_2 is produced from each full tank of gasoline in a car?
>
> A typical tank might hold about 50 L of gasoline (13 gallons). The density of gasoline is 0.75 kg/L, so that one tank has a gasoline mass of about 38 kg. Applying the simple and convenient factor-of-three ratio of CO_2 mass to input fuel mass, we see that one tank of gasoline will produce something like 110 kg of CO_2—not a small amount!

9.1.1 CO_2 Measurements

Beginning in 1958, Dave Keeling of the Scripps Institution of Oceanography began recording CO_2 concentration in the atmosphere from the top of Mauna Loa in the middle of the Pacific ocean.[3] In addition to seeing annual variation due to the seasonal cycle of photosynthesis,[4] he began to see a steady year-by-year increase in the level. The measurements have continued to the present, now known as the "Keeling Curve," shown in Figure 9.1.

3: …far from continental influences

4: Plants seasonally absorb and then release CO_2 as leaves grow and then die.
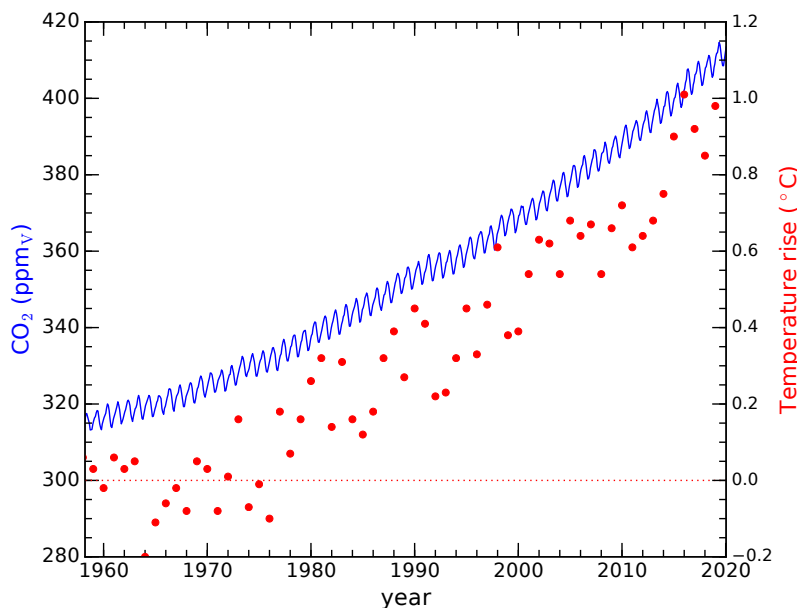
Figure 9.1: In blue (left axis), CO_2 measurements from Mauna Loa (Hawaii) for the last 60 years, showing a relentless and accelerating upward trend now at abut 2.6 ppm_v per year [50]. Seasonal variations due to photosynthesis are seen on top of this trend. Pre-industrial levels were around 280 ppm_v, so that we have added about 130 parts per million (ppm). Red dots (right axis) show global average mean temperature records over the same period [51]. Thus far, global average temperature has risen about 1°C. Note that the Kyoto Protocol in 1997 and Paris Agreement in 2015 (Box 19.4; p. 320) do not visibly curb the upward trajectory of CO_2 emissions.

When the measurements started, the atmosphere consisted of less than 320 parts per million by volume (ppm_v), or < 0.032%. By now, we are beyond 410 ppm_v.

Measurements of trapped air bubbles in the Greenland ice sheet going back about 100,000 years and the Antarctic ice sheet going back 800,000 years indicate that CO_2 has fluctuated between 180–280 ppm_v, reaching the higher end of the range during the warmer periods between ice ages (interglacial periods). For at least the last thousand years before the Fossil Fuel Age, CO_2 held steady around 280 ppm_v.

9.1.2 CO_2 Expectations
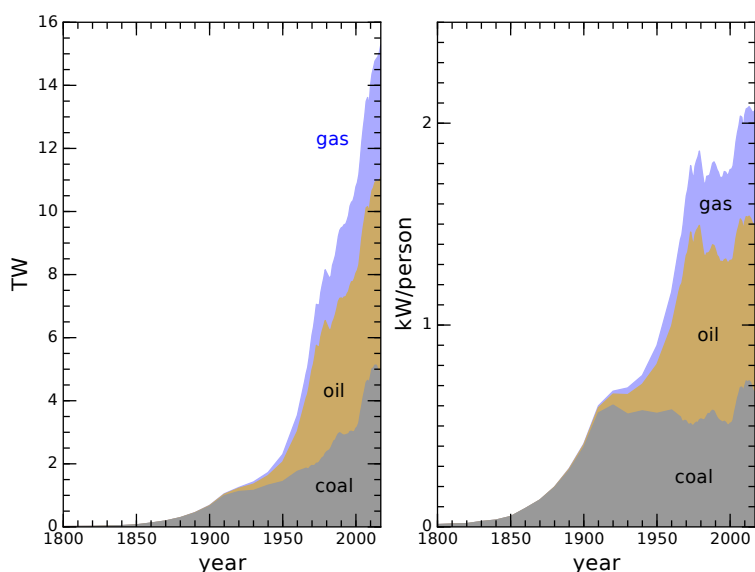
Figure 9.2: Historical use of fossil fuels worldwide, repeated from Figure 8.2 [16]. On the left is the raw usage rate expressed in terawatts, while the right is a per-capita measure. The three fossil fuel types are stacked on top of one another, so that gas makes the smallest contribution, not the largest. For instance, on the right-hand-edge of the left panel, coal goes from 0 to 5 TW, oil from 5 to 11 (thus 6 TW from oil), and gas from 11 to 15, indicating 4 TW from gas. The left-hand panel makes clear that fossil fuel use is still rising dramatically, and thus CO_2 emissions.

We saw in Figure 8.2—repeated as Figure 9.2—a history of fossil fuel usage for the world, in coal, oil, and natural gas. Meanwhile, Table 9.1 indicates how much CO_2 each fuel contributes per kilogram or Joule used. These two pieces can be combined to make an estimate of how much CO_2 is emitted globally each year, and to track total CO_2 emission over time. Table 9.2 and Box 9.1 elucidate how to go from the fossil fuel power figures (TW) in Figure 9.2 to CO_2 atmospheric concentrations.

Box 9.1: Computing CO_2 ppm_v from TW

We will use oil as an example. In Figure 9.2, we appear to get about 6 TW from oil (5 TW from coal, 4 TW from natural gas). Multiplying by 10^{12} puts this in Watts (J/s) and by 3.156×10^7 seconds per year results in the annual global energy from oil in Joules: 1.9×10^{20} J/yr. Table 9.1 indicates that oil contains about 11.5 kcal/g, so the number of grams of oil used per can be determined by first converting J to kcal

---

Table 9.2: Stepwise procedure to convert TW to ppm_v of CO_2. FF means fossil fuel, which can be coal, oil, or gas—each computed separately using the various values provided on the right.

Operation	Resulting Units	Coal	Oil	Gas
Starting value	TW			
$\times 10^{12}$ W/TW	W (J/s)			
$\times 3.16 \times 10^7$ s/yr	J/yr			
$\div 4{,}184$ J/kcal	kcal/yr			
$\div x$ kcal/g	FF g/yr	$x = 6.5$	$x = 11.5$	$x = 13.3$
$\div 1{,}000$ g/kg	FF kg/yr			
$\times y$ CO_2 kg/kg	CO_2 kg/yr	$y = 3.67$	$y = 3.09$	$y = 2.75$
$\div 5 \times 10^{18}$ kg	CO_2 frac/yr			
$\times 10^6$	CO_2 ppm_m/yr			
$\div 44/29$	CO_2 ppm_v/yr	total emissions		
$\div \sim 2$	CO_2 ppm_v/yr	stays in atmosphere		

(divide by 4,184), yielding 4.5×10^{16} kcal/yr, and thus 3.9×10^{15} g/yr of oil, or 3.9×10^{12} kg/yr.[5] Table 9.1 shows that each kilogram of oil combusted produces 3.09 kg of CO_2, translating to 1.2×10^{13} kg/yr of CO_2 from oil. Incidentally, we get the same answer using 64 g/MJ from Table 9.1 and 1.9×10^{14} MJ/yr from a few lines up. The remainder of the work, converting to parts per million, is covered in the text and the lower part of Table 9.2.

5: At 120 kg per barrel, this turns into the expected 30 billion barrels per year as a check to see that we're on the right track.

The mass of the atmosphere is about 5×10^{18} kg, obtained by multiplying 10,000 kg of air[6] sitting over every square meter by the $4\pi R_\oplus^2$ surface area of the earth. Dividing the 1.2×10^{13} kg[7] of oil-generated CO_2 by the mass of the atmosphere yields 2.4×10^{-6}, or 2.4 parts per million.[8] We are almost there. The quantity we have calculated is parts per million by mass (ppm_m), not the conventional parts per million by volume (ppm_v). Since air[9] averages 29 g/mol, and CO_2 is 44 g/mol, the mass concentration of CO_2 is higher than the volume occupation in air by a factor of 44/29, or 1.52. Thus we divide our 2.4 ppm_m result by 1.52 to get 1.6 ppm_v. A final correction is that only half of this stays in the atmosphere, so that today we are putting 0.8 ppm_v into the atmosphere each year from oil.

6: This is a close approximation to the actual value, obtained by dividing standard atmospheric pressure of 101,325 Pa by $g \approx 9.8$ m/s^2.

7: ... the result of Box 9.1

8: Just multiply by one million, or 10^6 to get ppm.

9: Air is about 75% N_2 at 28 g/mol plus 25% O_2 at 32 g/mol.

Can you validate this number in the left panel of Figure 9.3 for oil?

Figure 9.3 shows the result of this computation as a function of time in terms of annual rate and cumulative emission. We find that even though oil and coal deliver about the same global annual energy now, the carbon-intensity of coal is much higher so that its CO_2 emission dominates[10] over the other sources. In fact, coal *always* has been the dominant CO_2 producer, seen by the fact that the black curve in the left-hand panel of Figure 9.3 has always been higher than the other two CO_2 sources. In total, this estimate suggests that we should see CO_2 levels rising by 2.6 ppm_v per year, a little over half of which is due to coal (1.4 ppm_v/year).

10: Why do we keep using coal if it's the worst? Because replacement infrastructure is very expensive, and fossil fuel extraction does not work like a bank account allowing withdrawals at an arbitrary rate. We could not suddenly switch over and continue to satisfy demand, even if everyone wanted to—which they don't.

Summing up the rising contributions over time, this method of estimation suggests that we have increased the atmospheric CO_2 by 123 ppm_v,[11] and that 75 ppm_v of this (61%) is attributed to coal (right-hand panel of Figure 9.3).

11: ... very close to the ~130 ppm_v we observe!
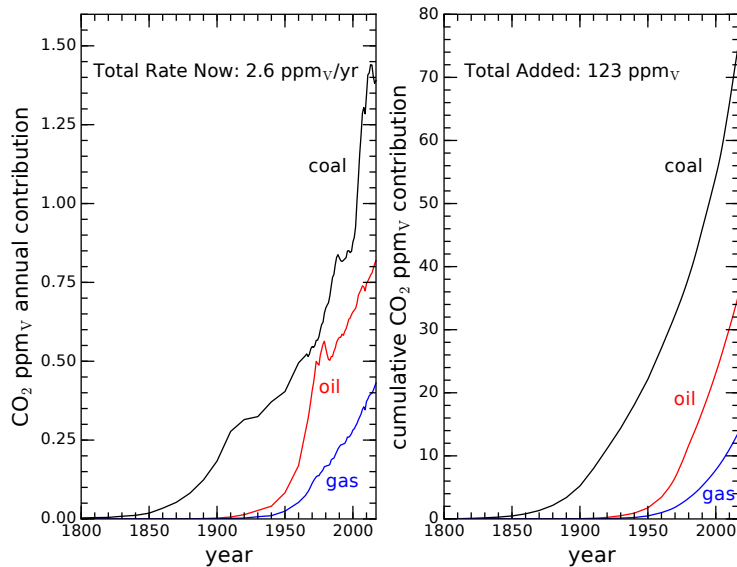
Figure 9.3: Estimated CO_2 contributions from known fossil fuel expenditures based on chemistry and the assumption that half of CO_2 stays in the atmosphere, while the rest is absorbed by the ocean and land. Units are parts per million by volume. The left-hand panel shows the annual addition, adding to 2.6 ppm_v per year and accounting for the slope in Figure 9.1. The right-hand panel is the cumulative emission to date as a function of time—essentially adding up all the annual emissions from the left-hand panel. These curves are not stacked as are the ones in Figure 9.2, so each can be read directly from the vertical axis. Note that oil and gas are still on the rise in the left-hand panel: we emit more CO_2 each year than we did the year before.

Adding all three contributions from the right-hand panel of Figure 9.3 and plotting the result on top of the Keeling Curve,[12] we find astounding overlap in the shape—as shown in Figure 9.4.

12: …actual CO_2 measurements
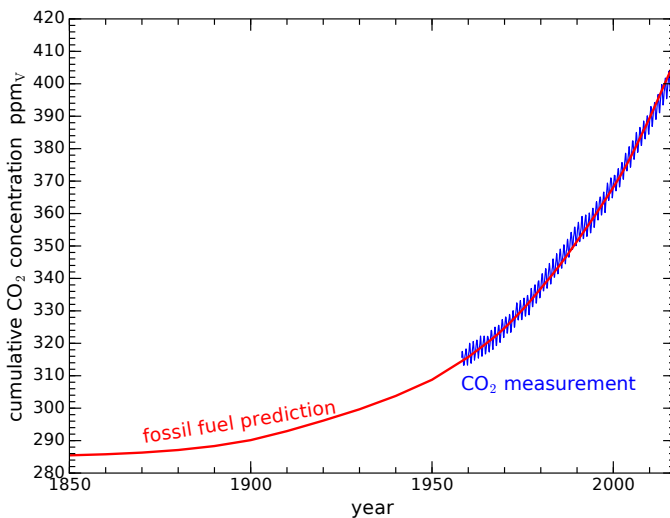
Figure 9.4: Fossil fuel contribution to CO_2 (red) on top of CO_2 measurements (blue). The red curve uses a starting point of 285 ppm_v and has 49% of CO_2 emissions staying in the atmosphere. The overlap is remarkably good and convincing.

The curve computed from fossil fuel use overlaps the Keeling Curve so faithfully that little mystery is left as to where the excess CO_2 in our atmosphere originates. The chemistry and historical use of fossil fuels are not in dispute. The only "fudge" is in what fraction of the CO_2 emitted from fossil fuel combustion remains in the atmosphere vs. being absorbed by the ocean and other "sinks." Empirically, about half stays in the atmosphere, while the rest disappears into the ocean,[13] and into plant matter that gets buried in the ground. If unaware of the oceanic and land absorption mechanisms, we would have *overestimated* the amount of CO_2 due to fossil fuels by a factor of two (see Box 9.2).

13: …acidifying the ocean's water

Box 9.2: If We Ignored Oceans

Reflect on what our reaction might have been had we not corrected for oceanic and land absorption: we would conclude that we have *no trouble* quantitatively accounting for the CO_2 rise based on fossil fuels, and would be left asking why we don't see an even *larger* rise. In other words, it is no stretch to account for the cause of excess CO_2—making it far from mysterious. If our CO_2 increase were *not* due to humans,[14] we would be left with a *real* mystery: then where is all of the *known* CO_2 emission from burning fossil fuels disappearing to?

14: . . . via fossil fuels

9.1.3 Chief Contributors

Climate change is a global phenomenon. Even if all emissions came from one country or region, atmospheric circulation would spread the result around the globe—albeit more slowly across the equator. It is, therefore, a global problem. All the same, it is interesting to look at chief emitters.

Table 9.3: Leading CO_2 emitters, 2018. Gt means gigatons, or 10^{12} kg, as 1 ton is 1,000 kg.

Country	Gt/yr	% share
China	9.43	27.8
U.S.	5.15	15.2
India	2.48	7.3
Russia	1.55	4.6
Japan	1.15	3.4
Germany	0.73	2.1
Top 6 Total	20.49	60.4
World	34	100

Figure 9.5: Cumulative contributions to CO_2 emissions to date, grouped by continent. Major contributing countries are called out explicitly, and the remaining contributions from each continent are grouped as a "rest of" category [52].

Figure 9.5 indicates that the U.S. is the single country bearing the largest responsibility for cumulative CO_2 emissions: roughly twice that of the second-largest (China) [52]. Today, China is the largest emitter, at 9.4 Gt per year, while the U.S. is in second place at 5.15 Gt/yr. Table 9.3 lists the top six emitters lately, accounting for about 60% of the 34 Gt per year [53].

[52]: Ritchie (2019), *Who has contributed most to global CO_2 emissions*

[53]: Rapier (2019), *The World's Top 10 Carbon Dioxide Emitters*

9.2 Warming Mechanism

The presence of excess CO_2 in our atmosphere is undeniably from fossil fuel combustion. But how does this alter our climate? How can such a minor constituent of the atmosphere (now 0.04%) cause so much trouble? The answer lies in infrared radiation [54]. Recall from Sec. 1.3 (p. 10) that

[54]: Pierrehumbert (2011), "Infrared radiation and planetary temperature"

this is the mechanism by which energy leaves the earth, the power of which is governed by the Stefan–Boltzmann law $P = A_{\text{surf}}\sigma T^4$, where the Stefan–Boltzmann constant,[15] $\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4, and T is the temperature of the radiating surface, in Kelvin.

15: ...easy as 5-6-7-8

Sun

infrared radiation from $4\pi R^2$

πR^2 projection intercepted sunlight

Sun's view
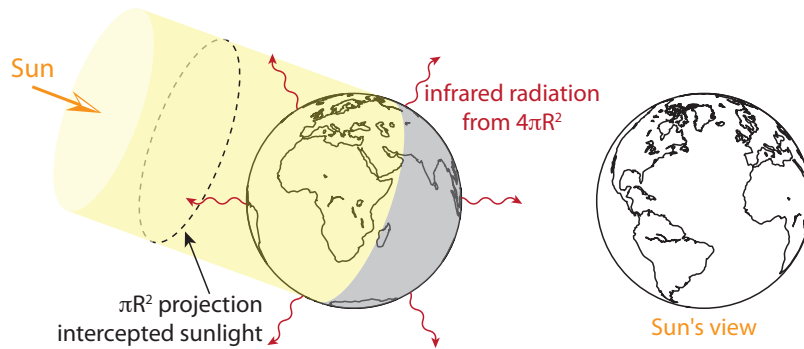
Figure 9.6: Earth intercepts sunlight across the projected area of the Earth's disk (πR^2), while radiating from the entire surface area, which is four times larger ($4\pi R^2$).

The sun delivers energy to the *top* of the earth's atmosphere at a rate[16] of 1,360 W/m^2. About 30% of this light—29.3%, to be precise—is immediately reflected by clouds, snow, and to a lesser extent water and terrain. The remaining 70.7% of the light intercepts the earth in a projected disk of area $A_{\text{proj}} = \pi R_{\oplus}^2$ (Figure 9.6). But the total surface area of the earth is four times this, all of it contributing to infrared radiation to space. In perfect balance,[17] energy absorbed equals energy radiated:

16: This is called the solar constant [4], and will appear again in Chapter 10 and Chapter 13.

17: An imbalance would mean energy is accumulating or being lost, leading to warming or cooling. Even under present conditions, the balance is good to within 1 W/m^2.

$$0.707 \times 1360\,\text{W/m}^2 \times \pi R_{\oplus}^2 = 4\pi R_{\oplus}^2 \sigma T^4. \tag{9.1}$$

The $\pi R_{\oplus}^2$ factors cancel, and we can rearrange to isolate temperature:

$$T^4 = \frac{0.707 \times 1360\,\text{W/m}^2}{4\sigma}, \tag{9.2}$$

solving to $T \approx 255$ K, or $-18°$C (about $0°$F). This is about $33°$C colder than the 288 K ($15°$C; $59°$F) we actually observe as the average temperature of Earth. The $33°$C difference[18] is due to greenhouse gases—mostly H$_2$O—impacting the thermal balance by preventing most radiation from escaping directly to space.

Verify for yourself. Two square roots in a row accomplish a fourth root.

18: Life on Earth is adapted to and reliant upon this $33°$C greenhouse effect. Abruptly *changing* it is what causes problems.

We understand this mechanism perfectly. Being at a temperature of 288 K, the surface emission peaks at a wavelength around 10 μm.[19] The atmosphere is not transparent at all wavelengths, its various absorption features depicted in Figure 9.7. The blue curve at upper right in this figure is the emission spectrum associated with infrared radiation.

19: We will see how/why in in Section 13.2 (Eq. 13.5; p. 199).

Of the greenhouse gases contributing to absorption as pictured in Figure 9.7, water vapor is the dominant player, followed by CO$_2$. Notice that the blue solid portion in the figure[20] is mirrored in white in the total absorption panel just below,[21] and that the window is mostly defined by water vapor. But the right-hand—longer wavelength—side of the water window is stepped on by the CO$_2$ absorption feature, seen more clearly in Figure 9.8. This CO$_2$ feature is responsible for the sharp cutoff on the

20: ...the infrared radiation that directly escapes to space

21: Thus the white portions indicate the open "windows."

Radiation Transmitted by the Atmosphere

right side of the solid blue[22] shape in Figure 9.7. As CO_2 concentration in the atmosphere increases, this absorption feature gets wider, cutting deeper into the right edge of the escaping radiation (solid blue feature), allowing less radiation to escape.

22: . . . escaped radiation
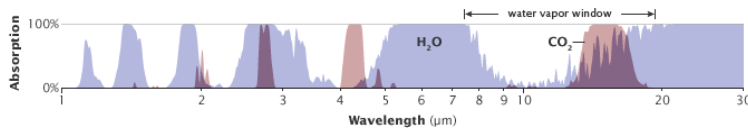
If some portion of the infrared radiation does not escape to space but is absorbed by the atmosphere, the planet does not cool as effectively, adding some offset to Eq. 9.2—in Earth's case 33°C. It is like the earth is wearing a blanket that raises its temperature by 33°C. Figure 9.9 illustrates the mechanism. Water vapor is responsible for ~20°C of this 33°C, and CO_2 is responsible for another ~8°C, leaving ~5°C for ozone, methane, and other minor contributors (Table 9.4). Incidentally, methane (CH_4) is about 80 times more potent than the same amount of CO_2 as a greenhouse gas, but is at a far lower concentration than CO_2, and also shorter-lived in the atmosphere before being chemically destroyed.[23] We focus on CO_2 because this is what human activity is changing rapidly by burning fossil fuels. The vast ocean–air interface means water concentration is impossible to control, and simply responds to temperature due to the fact that warmer air holds more moisture— becoming an important feedback agent. Water is not the *driver* of climate

Table 9.4: Greenhouse contributions [56].

Molecule	ΔT (°C)
H_2O	20
CO_2	8.6
O_3 (ozone)	2.6
CH_4 (methane)	1.5
N_2O (nitrous oxide)	0.5
Total	33

23: Methane emission becomes important via leaks from drill sites and also permafrost melt.

change, but a hefty *passenger*.
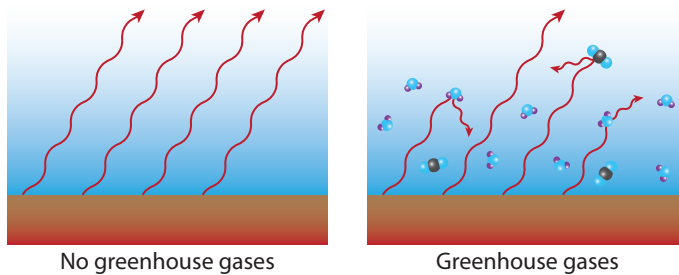
No greenhouse gases   Greenhouse gases

Naïvely speaking, going from 280 ppm$_v$ to 420 ppm$_v$ (an increase by a factor of 1.5), might be expected to turn the 8.6°C greenhouse effect from CO_2 into 12.9°C (1.5×) for a 4.3 degree human-caused warming.[24] But the CO_2 absorption feature at $\sim 15\,\mu m$ is *saturated*, so as CO_2 is added, it becomes *wider*, but logarithmically rather than linearly as a function of CO_2 concentration. Climate scientists often express the impact of various influences as a radiative forcing, measured in W/m^2.

24: ⚠ This is not how it works.

> **Definition 9.2.1** *Radiative forcing is used to describe the areal power (in W/m^2) of absorbed solar energy and infrared radiation to space. Various influences or constituents each contribute their own radiative forcing. In equilibrium, a balance exists so that the net[25] radiative forcing is zero.*

25: ... sum of all: positive and negative

The average solar forcing is

$$RF_\odot = 1360\,\text{W/m}^2 \times 0.707/4 \approx 240\,\text{W/m}^2, \tag{9.3}$$

sharing substantial overlap with Eq. 9.2—the factors having been explained in that context. Without adding to the pre-industrial set of greenhouse gases (GHGs), we would solve for temperature[26] as

26: ... adding the baseline 33°C GHG contribution

$$T = \left(\frac{RF_\odot}{\sigma}\right)^{0.25} + 33, \tag{9.4}$$

evaluating to 288 K, or about 15°C. If we add (or deduct) radiative forcing from another source, it would add (subtract) in the numerator of Eq. 9.4. The addition of CO_2 over the original amount ($CO_{2,\text{orig}}$; 280 ppm$_v$) generates a radiative forcing of

$$RF_{CO_2} = 5.35\ln\left(\frac{CO_2}{CO_{2,\text{orig}}}\right)\,\text{W/m}^2, \tag{9.5}$$

where the ln() function is the natural logarithm.[27] At our present ~420 ppm$_v$, the ratio of current CO_2 to $CO_{2,\text{orig}}$ is 1.5 and $RF_{CO_2} \approx 2.2\,\text{W/m}^2$, so that the new temperature (adding the new forcing to Eq. 9.4) is

27: ... capturing the logarithmic broadening of the width of the CO_2 absorption feature at 15 μm in Figure 9.7

$$T = \left(\frac{RF_\odot + RF_{CO_2}}{\sigma}\right)^{0.25} + 33 \approx 288.6, \tag{9.6}$$

which is about 0.6°C larger than it was before increasing the amount of CO_2. We can express this as a *sensitivity*: how much ΔT do we get for a given imbalance in radiative forcing? In this case, 0.6°C from a 2.2 W/m^2 increase is 0.27°C per W/m^2. But when the known feedback mechanisms are included—most of them in the positive direction—the temperature sensitivity becomes 0.8°C for every W/m^2 of additional forcing: called the climate sensitivity parameter.[28]

28: See [57] for a good synopsis and references to primary material within.

> **Definition 9.2.2** *The **climate sensitivity parameter** connects the amount of warming expected for a given amount of radiative forcing. Current understanding puts this at 0.8°C per W/m^2 of radiative forcing.*

Therefore, our current 2.2 W/m^2 of additional (fossil-fuel added) radiative forcing translates[29] to a 1.7°C temperature increase (Figure 9.10), which is about three times what the non-feedback calculation would provide.

29: Just multiply 2.2 W/m^2 by 0.8 °C per W/m^2.

> **Example 9.2.1** If we double our CO_2 concentration from pre-industrial levels, what would we expect the temperature increase to be?
>
> Pre-industrial CO_2 was 280 ppm$_v$, so doubling it adds 280 ppm$_v$ for a total of 560 ppm$_v$. The radiative forcing is then $5.35 \ln(2) \approx 3.7$ W/m^2. Multiply this by the climate sensitivity of 0.8°C per W/m^2 to get a temperature increase of about 3.0°C (would be 1°C without feedback).

The positive feedbacks are important, and include factors like:

1. A warmer planet means less ice (glaciers, Arctic cap), resulting in less reflected sunlight, increasing the 0.707 absorption factor in Eq. 9.3 to increase solar forcing.
2. Warmer air can hold more water vapor—the principal greenhouse gas, thus driving up the nominal 33°C greenhouse gas contribution.
3. A warmer environment leads to additional CO_2 loss from drying forests, desertification, and accelerated decomposition of plant matter and peat.

A few negative feedback mechanisms[30] exist as well, but are outweighed by the positive feedback terms.

Global temperature increase is already about 1.0°C [58]. Note that even if we never added another CO_2 molecule to the atmosphere, the temperature would continue to rise as the ocean[31] slowly catches up to the new equilibrium. We would expect the temperature to stabilize around 1.7°C higher for today's CO_2 excess, according to the calculation above. Thus the climb is about 55% done. Of course, more CO_2 will be added, so the eventual temperature rise is destined to be higher still.

Figure 9.10: As CO_2 concentration increases, the radiative forcing (left axis) increases, driving the temperature (right axis) up. We are now at 420 ppm$_v$, corresponding to a radiative forcing of 2.2 W/m^2 and 1.7°C of ultimate temperature increase (red star). Presently, the temperature has only increased by 1.0°C (red circle), but will catch up to a new equilibrium once oceans warm and ice melts.

30: ⚠ By far the most important negative feedback mechanism is the infrared radiation itself, increasing dramatically as temperature increases (as T^4), thus opposing the temperature rise by a cooling influence. Here, we mean negative feedback influences *in addition to* this main one.

[58]: National Oceanic and Atmospheric Administration (NOAA) (2019), *Global Climate Report*

31: . . . lots of thermal mass, or heat capacity

9.3 Possible Trajectories

Launching from the data used to generate Figure 9.3, we can now play a few games to understand what our future might hold in terms of total CO_2 rise and corresponding ΔT increases by the year 2100 under various contrived scenarios.[32]

First, let's imagine that we suddenly arrest the upward climb characteristic of fossil-fuel usage to date[33] and maintain present-day levels of fossil fuel use from now until 2100. Figure 9.11 shows what happens. The total added CO_2 rises to 2.75 times the current excess, to 339 ppm_v[34] above pre-industrial levels. The associated radiative forcing would be 4.25 W/m^2 and result in a 3.4°C temperature increase. Table 9.5 summarizes this scenario and the three to follow.
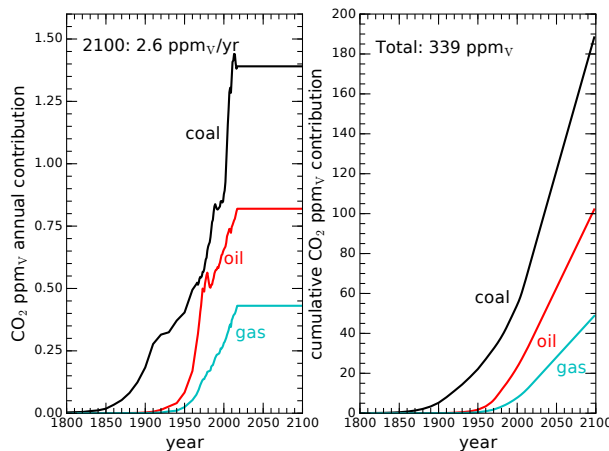
> Verify yourself following a procedure like Example 9.2.1 as good practice.

Figure 9.11: CO_2 rise if fixing fossil fuel use at today's levels for the rest of the century, following the conventions of Figure 9.3. We would still add 2.6 ppm_v per year from now until the end of the run in 2100, and would have accumulated a total rise of 339 ppm_v, or 2.75 times the problematic amount already accumulated to date. The associated temperature rise would be 3.4°C. For this and all subsequent scenarios, the plots show only the half of emitted CO_2 that remains in the atmosphere.

Table 9.5: Summary of scenario outcomes.

Scenario	ΔCO_2 (ppm$_v$)	vs. today	CO_2 (ppm$_v$)	RF_{CO_2} (W/m^2)	ΔT (°C)
Arrest FF climb	339	2.75 ×	620	4.25	3.4
Arrest; no coal	268	2.18 ×	548	3.6	2.9
Curtail by 2100	235	1.91 ×	515	3.3	2.6
Curtail by 2050	169	1.37 ×	450	2.5	2.0

We're already seeing serious problems emerging today, at about 1°C increase, so this 3.4°C scenario is not desirable.[35] And reflect for a moment how unlikely it is that we can even arrest the climb of fossil fuel use so suddenly. It would seem that the *rate* of CO_2 emission is destined to climb higher than it is today: we have not yet found the peak!

The second scenario focuses on eliminating coal, since it is the highest intensity CO_2 emitter,[36] as Figure 9.3 makes clear. What if natural gas—the fossil fuel having the lowest carbon intensity—could replace all coal applications? This is already happening—gradually—in the electricity generation sector. Countless advocates encourage such a transition as rapidly as can be accomplished. The pretend world of

simulation allows us to evaluate the best-case (and unrealistic) scenario of *instant*, complete replacement, to put a limit on how much benefit such a move brings. Figure 9.12 shows what happens. The rate of CO_2 emission would immediately drop to 1.8 ppm_v/year.[37] That definitely helps, but the total emission by 2100—if carrying on at today's energy demand via fossil fuels—would climb to 268 ppm_v. The effect would more than double the 123 ppm_v that we've *already* contributed to the atmosphere, and would approximately double the pre-industrial CO_2 level in the atmosphere, leading to a forcing of 3.6 W/m^2 and $\Delta T \approx 2.9°C$ (summarized in Table 9.5). So as beneficial as the termination of coal would be, any path that involves carrying our fossil fuel use forward at today's levels—even substituting the best form for the worst form—does not look promising.

See if you can replicate these numbers!
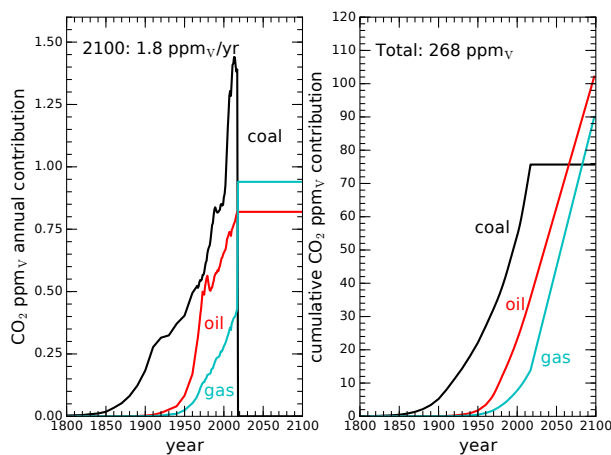
Figure 9.12: CO_2 rise if instantly substituting coal (worst CO_2 intensity) with natural gas (best CO_2 intensity) and then maintaining current levels for the rest of this century. Our annual contribution would drop from 2.6 ppm_v/yr to 1.8 ppm_v/yr based on this substitution, and the total accumulation would be 268 ppm_v by century's end (2.2 times the accumulation to date). The associated temperature rise would be 2.9°C.

The emphasis, then, should be to taper off fossil fuel use so that we wean ourselves of dependency. The transition could be fast or slow. A slower version might target the year 2100 for a full termination of fossil fuels. Figure 9.13 shows an idealization of what this might look like. Notice that the resulting curves are *roughly* symmetric, in that the downslope is not terribly different from the upslope. Let's pause to reflect on how incredible and fast the rise of fossil fuels has been. A descent as steep as the rise represents change at an astounding pace—which would be pretty disruptive in the best circumstances. In the absence of suitable substitutions, this would be a tremendously difficult journey, but one we may be forced to make by any number of paths.[38] In any case, the eventual added CO_2 would end up at 235 ppm_v—almost doubling what we have already emitted, and nearly doubling the pre-industrial CO_2 level in the atmosphere. The forcing in this case would be 3.3 W/m^2 and $\Delta T \approx 2.6°C$.

38: . . . driven by policies, markets or—more certainly—resource limits

Again, follow along for good practice.

Reducing fossil fuel use even more quickly, tapering to zero by 2050, results in Figure 9.14. The descent is alarmingly steep, and difficult to imagine happening in practice unless major disruptions[39] force this upon us. In any case, should we manage such a feat, our total CO_2

39: . . . resource wars, devastating climate change repercussions
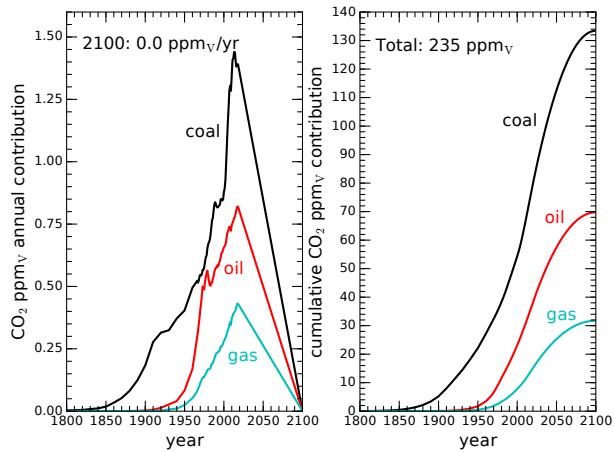
Figure 9.13: CO_2 rise if immediately reversing fossil fuel use in an ambitious decline reaching zero by the year 2100. The accumulation would come to 235 ppm_v at the end, which is almost twice the current level (1.9 times). The associated temperature rise would be 2.6°C.
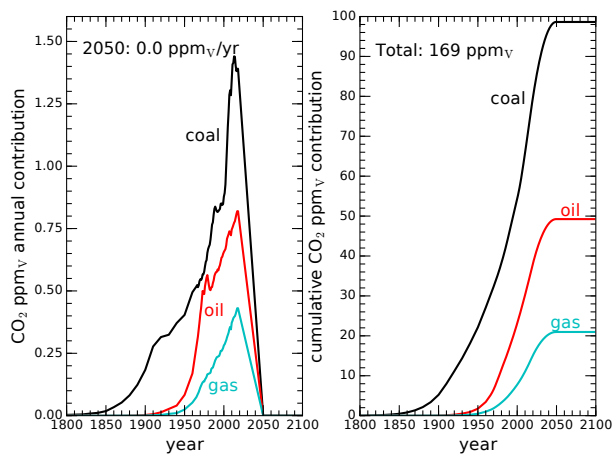
Figure 9.14: CO_2 rise if we mounted a super-aggressive weaning of fossil fuels, completed by the year 2050. In this extreme case, the total CO_2 emission would be 169 ppm_v, or 37% more than we have produced to date. The associated temperature rise would be 2°C.

contribution to the atmosphere would be an increase of 169 ppm_v, which is 37% more than we have emitted to date. Adding another $\sim 40\%$, then, seems like the best we could hope to see, but possibly accompanied by serious hardships in adapting. The radiative forcing associated with this scenario is 2.5 W/m² , corresponding to $\Delta T \approx 2°C$.

Try replicating!

9.4 Climate Change Consequences

Turning up the thermostat on the planet results in too many effects to chronicle here. Obviously, the climate is impacted—in terms of storm frequency and intensity, rainfall, snowfall and water supply, season durations, and the ability of plant and animal species to adapt to the changes. The timescale over which we are changing the climate is far faster than evolution can track, except for microbial life and maybe insects, whose shorter generational turnover permits a more dynamic response. Humans are late arrivals to a long evolutionary sequence, which laid a foundation to support our lives in complex interconnected

ways that we are not close to fully understanding. Climate change messes with the system in such a way to prevent accurate prediction of the long-term consequences of one species or another disappearing from the web of life.

The consequences of climate change are elaborated in many sources that are not difficult to find. Rather than try to add to the general awareness, this section—in the spirit of the book—aims to provide students with some tools[40] to be able to quantitatively understand how the physical world reacts to changes in radiative forcing. Specifically, we concentrate on the process of heating up[41] elements of the planet, and on sea level rise.

40: ...and make connections to earlier content

41: For amusement, try substituting the phrase "climate change" with "hotting up."

9.4.1 Heating Up

Recall that the radiative forcing of 2.2 W/m^2 arising from a 50% enhancement to the pre-industrial CO_2 concentration[42] is expected to result in 1.7°C of eventual warming. But measurements indicate only 1.0°C of warming so far. Is our understanding wrong?

42: ...from 280 ppm$_v$ to 420 ppm$_v$

As we saw in Sec. 6.2 (p. 85), it takes energy to change something's temperature. When the *rate* of energy input[43] is limited, it takes *time* to accomplish a temperature rise.[44]

43: ...which we know as power

44: A burrito in the microwave does not heat up instantly, for instance.

Earth is by-and-large in thermodynamic equilibrium. The sun deposits energy onto Earth at a rate of 240 W/m^2, when averaged over the surface (Eq. 9.3). Prior to the modern increase in CO_2 concentration, we had no additional radiative forcing from CO_2 and had an average surface temperature of 288 K (15°C), as in Eq. 9.4. Because it was in equilibrium, we know that the infrared radiation from Earth must have *also* totaled 240 W/m^2 to match the solar input.
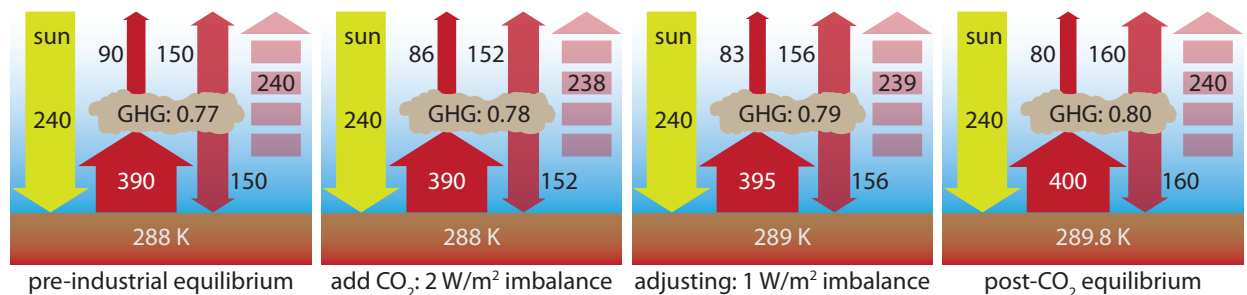
Figure 9.15: Four steps to illustrate (in a grossly simplified way) the process of Earth adapting to an increase in greenhouse gas (GHG). Starting from the left within each panel, solar input is held constant at 240 W/m^2. Most of the radiation leaving the ground—quantitatively adhering to σT^4—is absorbed by GHGs (fraction absorbed indicated in GHG "cloud"), the rest escaping directly to space. Half the absorbed energy is radiated up (escaping) and half back down. The dashed arrow at right is the net radiation escaping. Integer numbers are in W/m^2, and arrow widths are scaled accordingly. Ground temperature is indicated at bottom. See text for narrative sequence.

Figure 9.15 sums up the story.[45] The **first panel** shows the pre-industrial equilibrium condition, in which 77% of the infrared radiation from the ground was intercepted by the greenhouse gases, while 23% (90 W/m^2)

45: Note that in each panel, adding the two top arrows or subtracting the two bottom ones both yield the same number—matching the dashed arrow on the right.

directly escaped. The 77% absorbed (300 W/m^2) is re-radiated equally up and down (150 W/m^2 each). The net outgoing radiation matches the 240 W/m^2 solar input[46] exactly, signaling equilibrium.

Pretend that we suddenly increased the CO_2 concentration to 420 ppm$_v$ in one instant,[47] which takes us to the **second panel** of Figure 9.15. The ground has not had time to change temperature yet, but the added GHG absorbs more of the outgoing radiation (78%). Now the numbers are not balanced. Only 238 W/m^2 radiates away, leaving a 2 W/m^2 net influx of energy. This is what we have been calling the radiative forcing.

After some time (**third panel**; representing our current status, roughly), the extra input energy starts to heat up the earth environment to 289 K (16°C),[48] which cranks up the radiation leaving the ground according to the σT^4 radiation law. At the same time, the higher temperature drives some positive feedback effects, putting more GHG into the atmosphere[49] and raising the absorption fraction. Meanwhile, the imbalance has moderated to 1 W/m^2 as the system edges toward a new equilibrium.

Finally, equilibrium is re-established in the **last panel** of Figure 9.15, at which point outgoing energy and incoming energy are again matched at 240 W/m^2. The feedback mechanisms essentially tripled the change in GHG absorption: initially a 1% bump ultimately becoming 3%. Without this effect, the system would equilibrate at 288.6 K (15.6°C),[50] in which case 393 W/m^2 leaves the ground, and 153 W/m^2 is re-radiated from the atmosphere.

9.4.2 Heating Earth's Skin

Let's now understand a bit more about the process of heating up the Earth's air, water, land, and ice during the time when outgoing energy does not match incoming energy.[51] We know from Sec. 6.2 (p. 85) that it takes energy to change something's temperature. For instance, it takes 4,184 J to raise the temperature of a kilogram (liter) of water by 1°C, or about 1,000 J for many other substances like air and rock. Also relevant, and not introduced before, is that it takes a substantial amount of energy to melt ice.

> **Definition 9.4.1** *The heat of fusion of ice is 334 J per gram, meaning that every gram of ice that goes from just below freezing to just above[52] requires an input of 334 J.*

To put this in context, that same 334 J would heat a gram of liquid water by 80°C, but gets eaten up in the phase change without changing the temperature really at all.[53]

Using the properties of various constituents in Table 9.6, we can construct Table 9.7 to describe how much energy is needed to "charge up" different planetary components to a different temperature.[54] Note that the ocean

46: In reality, the solar absorption can also change as surface reflectivity changes—like when Arctic ice caps melt and expose dark water.

47: Obviously, it took time, but this approach is illustrative.

48: ...as a result of no longer being in temperature equilibrium

Try re-creating the upward radiation numbers using σT^4

49: H_2O, CH_4, for example.

50: ...just as in Eq. 9.6

51: ...middle two panels of Figure 9.15

52: ...e.g., from −0.001°C to +0.001°C

53: This is why ice in a glass of water does not all melt suddenly—keeping the water around it basically right at 0°C as the ice slowly melts, limited by the rate of heat transfer.

54: Two entries in the table are blank, because it does not make much sense to talk about heating just a layer of the atmosphere, or heating the "whole" ground: how deep makes sense?

Component	area (10^{12} m^2)	volume (10^{15} m^3)	mass (10^{18} kg)	density (kg/m^3)	thermal (J/kg)
Atmosphere	510	4,000	5.3	1.3	1,000/°C
Land/Rock	150	—	—	2,000	1,000/°C
Ocean	360	1,400	1,400	1,000	4,184/°C
Ice	16	29	2.6	917	334,000

Table 9.6: Properties of surface mass components of Earth. About 90% of the ice volume is in the Antarctic ice sheet, 10% in the Greenland ice sheet, and less than half-percent in glaciers [59, 60]. The volume of air corresponds to what it would be if compressed to uniform (sea-level) density. The last column captures specific heat capacity, or heat of fusion for ice.

takes about 1,000 times more energy than the atmosphere to heat it up by the same amount.

Component	charge per meter (10^{21} J/m)	total charge (10^{24} J)
Atmosphere	—	0.0053/°C
Land/Rock	0.3/°C	—
Ocean	1.5/°C	5.9/°C
Ice	4.9	8.8

Table 9.7: Energy requirements to heat up (thermally "charge") Earth components, derived from Table 9.6. The energy investment for the first three components depends on the temperature change sought, while melting ice is independent of temperature change.

Example 9.4.1 We will use the ocean as an example of how to interpret and use Table 9.7. The two numbers tell us what it takes to heat up the ocean per meter of depth and to heat the entire volume.

If we ask how much energy it would take to raise the temperature of the upper 10 m of the ocean by 2.5°C, we multiply 1.5×10^{21} J/m/°C by 10 m and 2.5°C to get 3.75×10^{22} J. Heating the entire volume of the ocean by 0.5°C would require 5.9×10^{24} J/°C times 0.5°C for 3×10^{24} J of energy.

Similarly—but without any temperature element—it would take $\sim 49 \times 10^{21}$ J to melt 10 m of Earth's ice, and would take 8.8×10^{24} J to melt it all.

We are now in a position to appreciate how long it can take to change temperatures on a planetary scale for a certain imbalance in radiative forcing. If, for instance, the imbalance is 1 W/m^2,[55] then Earth receives an extra 5.1×10^{14} J each second, or 1.6×10^{22} J in a year.[56] We can compare this annual surplus energy to the values in Table 9.7 to understand how deeply the components would be heated or melted per year for a 1 W/m^2 radiative forcing imbalance.

55: ...as it roughly is now; third panel in Figure 9.15

56: 1 W/m^2 times the 5.1×10^{14} m^2 area of Earth, then 3.16×10^7 seconds in a year.

Example 9.4.2 If we could direct *all* of the annual surplus 1.6×10^{22} J arising from a 1 W/m^2 imbalance into one component only,[57] we could ask: to what depth will each component be heated by 1°C or melt the ice?

57: ⚠ not how it really works

component	math	depth (m)
land	1.6×10^{22} J/$(0.3 \times 10^{21}$ J/m/°C · 1°C)	54
ocean	1.6×10^{22} J/$(1.5 \times 10^{21}$ J/m/°C · 1°C)	11
ice	1.6×10^{22} J/$(4.9 \times 10^{21}$ J/m)	3.3

From this, we see that land is more easily heated, and ice is most resistant—only shaving about 3 m per year if somehow the entire wrath of radiative imbalance were directed upon it.

Likewise, we can explore how long it would take to raise the temperature of *entire* components—or melt *all* of the ice—for an imbalance of $1\,\text{W/m}^2$.

Example 9.4.3 If we could direct *all* of the annual surplus 1.6×10^{22} J arising from a $1\,\text{W/m}^2$ imbalance into one component only,[58] we could ask: how much will the temperature rise of the entire body be per year, or how much ice would melt?

component	math	ΔT (°C)	years to 1°C
atmosphere	1.6×10^{22} J/$(5.3 \times 10^{21}$ J/°C)	3.0	0.33
ocean	1.6×10^{22} J/$(5.9 \times 10^{24}$ J/°C)	0.0027	367
ice	1.6×10^{22} J/$(8.8 \times 10^{24}$ J)	0.185%	545 to melt

The atmosphere is a wimp: it takes very little to change its temperature. The ocean, however, is very sluggish to change temperature. For ice, we look at fractional loss per year instead of temperature increase.

What we learn from the examples is that the ocean and ice are both substantial thermal brakes on fast heating. Even though ice has a much larger energy cost per kg,[59] its total mass is substantially smaller than that of the entire ocean—the two effects roughly balancing. In reality, we might expect oceanic circulation patterns to concentrate heating in the upper layers rather then distributing uniformly to the full depth. So the upper layer of the ocean—which then controls air temperature—can reach a 1°C increase well before 367 years elapse. Indeed, we are already seeing warming at this scale in less than a century.

The way things *really* work is that the excess 1.6×10^{22} J annually associated with a radiative excess of $1\,\text{W/m}^2$ gets distributed into lots of channels at once. If ice only gets 3% of the attention in proportion to its area, only 5×10^{20} J goes into the ice in a year. Dividing by 4.9×10^{21} J/m (from Table 9.7), we find that we might expect 0.1 m of ice to disappear each year. Since the ocean is roughly 25 times larger than the ice areas,[60] the associated sea level rise from redistributing the ice melt across the entire ocean surface would be about 25 times less, or about $4\,\text{mm/yr}$.[61]

Meanwhile, the constant, swirling contact between water and air, and between air and land, keeps all three in sync with each other: one will not race off to get hot without the others. And in this case, the ocean—with its giant thermal mass and extensive air-water interface—is the limiting

If the radiative forcing imbalance is larger or smaller than $1\,\text{W/m}^2$, the effect will be proportionally deeper or shallower. Likewise, if we ask about a temperature increase larger than 1°C, the depth would be smaller in proportion.

58: ⚠ not how it really works

As before, this pretends that the entire radiative imbalance is directed upon a single component, in turn. If the radiative imbalance is larger or smaller than $1\,\text{W/m}^2$, the effect will be proportionally more or less severe.

59: …334 kJ vs. ~4 kJ to heat water by 1°C

Tip: don't treat sections like this as the textbook "telling" you stuff. It is far more productive to follow the logic, reasoning, and numbers yourself as an active participant in the story. Otherwise, OMG is it boring! The value is empowerment: bestowing tools for transparent understanding of fundamental processes.

60: Ice covers 3% of the globe, while the ocean covers 71%.

61: This is not far from reality, as will be seen in Section 9.4.3.

factor on how fast things can heat up. If we confine ocean heating to the top 300 m[62] of water, the excess 1.6×10^{22} J per year leads to an annual temperature rise of about 0.035°C per year, or about 30 years to climb 1°C (at a 1 W/m^2 imbalance).

See if you can replicate this result.

Although this section may seem to be long, convoluted, and perhaps even boring, it accomplished a number of things for us:

1. it showed how a change in GHG absorption fraction leads to a radiative forcing imbalance;
2. it indicated how a radiative imbalance changes the surface temperature until the earth re-establishes a new equilibrium (balance) at a higher temperature, including feedback effects;
3. it assessed energy requirements for heating up relevant masses of material and melting ice via straightforward physics;
4. it showed that the two most important thermal masses on the planet are the ocean (first) and the ice sheets (second);
5. it established approximate timescales over which we might expect temperature to climb, and why the ocean in particular is important in slowing down the consequences.

An additional insight is that even if we stopped CO_2 emissions today, Earth's temperature will continue to climb as the oceans slowly adjust to the new radiative reality imposed by 420 ppm$_v$ of CO_2 in the atmosphere.

62: The ocean is such a dominant thermal player that the rate at which temperature rises depends critically on how well and deeply mixed the thermal influence is.

9.4.3 Sea Level Rise

The previous section covered the energetics of melting ice sheets. The resulting melt-water flows to the ocean[63] and contributes to sea level rise. Besides melting ice, sea level also rises as a consequence of thermal expansion of water as it warms up. Figure 9.16 shows the recent history.

63: Ice floating on the ocean is already displacing water, so its melting does not impact sea level.

Contributors to global sea sea level rise (1993-2018)
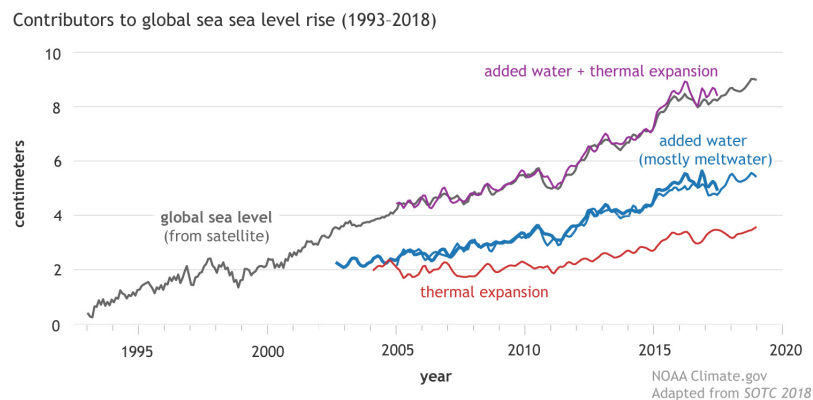
Figure 9.16: Satellite measurements of sea level since 1993, showing a rise of 3.6 mm per year. Melting ice is the largest contribution, although thermal expansion plays a role as well [61]. From NOAA.

Melting ice contributes about 2.4 mm/yr of rise, while thermal expansion accounts for about 1.2 mm/yr for a total rate of 3.6 mm/yr [61]. Since

[61]: Lindsey (2020), *Climate Change: Global Sea Level*

1880, sea level has risen about 230 mm At the *current* rate, we would expect a comparable addition by 2100 for a total of 0.5 m.

But the current rate is not likely to be the right measure, since warming air temperatures result in a faster rate of ice melting. Positive feedbacks also accelerate ice melt. For instance, melted pools of water on top of the ice are darker than ice, increasing the rate at which solar energy is absorbed.

We can get a quick handle on how much sea level rise might possibly be in store, based on the fact that the vast majority of "permanent" ice on the planet is in Antarctica and Greenland. These two ice sheets constitute 2.7% and 0.3% of the globe's surface area, respectively. From there, it is easy to estimate sea level rise, because the ocean (71% of the globe) has an area 26 times bigger than the Antarctic ice sheet and 210 times larger than the Greenland ice. What this means is that it takes 26 meters of ice melting from Antarctica[64] to raise sea level by 1 meter, and 210 meters off Greenland to do the same.[65] See Figure 9.17 to understand the logic here.
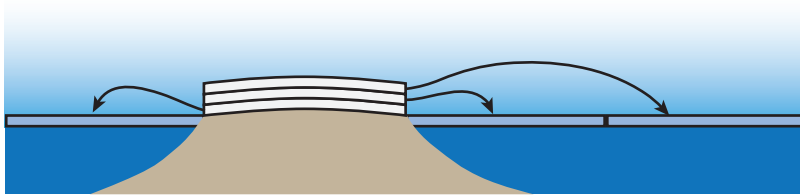
Now if we just knew the average thickness of each ice sheet, we could figure out how much sea level would rise if all the ice melted. The Greenland ice sheet is estimated to be 2.85 million cubic kilometers, translating to an average depth of 1.7 km.[66] 210 m goes into this 8 times, so we might expect something like 8 m of sea level rise if all the ice in Greenland melts. For Antarctica, the 26.5 million cubic kilometers corresponds to an average ice thickness of 1.9 km, which is about 70 of our 26 meter units, so we would expect about 70 meters of sea level rise in the extreme case of Antarctica losing all of its ice.

Box 9.3: A more rigorous estimate...

It is relatively easy to find references estimating sea level rise potential from complete melting of Greenland and Antarctica. One such [60] arrives at a 7.4 m rise from Greenland melting, 58 m from Antarctica, and 0.3 m from glaciers.

So why did we go through the crude estimation process? The goal was to remove the mystery.[67] Once we have an estimate of the depth and a ratio of surface areas, it is within our grasp to estimate the rise ourselves.

64: ...if uniformly distributed across the continent

65: In the spirit of an approximate estimate, we ignore the 10% difference in the density of ice vs. water and assume that one cubic meter of ice displaces one cubic meter of water.

Figure 9.17: If sea area is x times that of an ice-covered island, water level will rise by $1/x$ times the ice thickness if it all melts. The diagram shows a way to think about this, for $x = 3$: by slicing the ice into $x = 3$ layers to redistribute the volume on top of the water. In this case, a 30 m ice sheet would raise sea level by 10 m if melted.

The preceding paragraph and Table 9.6 have enough information (absolute and relative areas and volumes) to figure out.

66: Divide volume by area.

[60]: Davies (2020), *Calculating glacier ice volumes and sea level equivalents*

67: One goal of this book is to empower students to independently check more authoritative sources—much like $2+2 = 4$ can be personally verified and is not a matter of faith.

Coastal cities are already struggling to deal with the present ~0.25 m rise. This may not sound like much, and on normal days does not cause problems. But the low pressure associated with storms results in a local swell in water level,[68] so that a storm plus high tide plus heavy waves adds to the climate-change rise to destroy human structures that otherwise may have escaped harm. Even if CO_2 emissions ceased today, the warming that has already happened will continue melting ice. Temperatures will also continue to climb as increased forcing from the already-present CO_2 continues to heat the ocean as we creep toward equilibrium—a slow process. So sea-level rise may turn out to be the gift that keeps on giving for centuries more.

Sea level in the distant past—tens of millions of years ago—has been as high as 200 m over where it is now. And 20,000 years ago, during the last glacial maximum, sea level was 120 m lower than it is today [62]. So Earth is no stranger to large fluctuations in sea level. By contrast, many coastal cities do not know how to handle even a one meter rise—which is within the projections for 2100 in a number of models.

[62]: Lambeck et al. (2014), "Sea level and global ice volumes from the Last Glacial Maximum to the Holocene"

9.5 What Can Be Done?

So far, we have simply described the climate change phenomenon as an unambiguous consequence of our fossil fuel habit, but have not addressed what we might do to combat it. In this section, the author will not attempt to conceal his personal take on the matter, and will keep it short.

First, clever geo-engineering ideas ring of hubris, and seem like solving a problem of hurtling toward the ground by digging a hole ahead of the fall fast enough to keep up. Our whole problem is that we have convinced ourselves that we can outsmart nature.[69] You would stop going to a doctor who tried to treat your illness by applying superficial remedies to a resulting skin condition without addressing the underlying cause.

69: See Sec. D.6 (p. 408) in Appendix D for an extended discussion of this notion.

In this case, the underlying cause is very simple: unchecked[70] human ambitions. The combination of fossil fuels, an incessant drive for growth, destruction of forests and habitat,[71] population pressures, and an industrial approach to agriculture all play a role. This is why it has been hard to combat: it's us. It pervades all the things we do. We are our own adversaries. How do we fight ourselves? To do so requires honesty and even a collective willingness to sacrifice and prioritize planetary health over narrow short-term human interests. What is more important: that individuals attempt to fulfill all their dreams now, or that civilization endures for the long haul? Are we willing to dial back our own desires so that billions of future humans who we will never meet and countless other species on the planet can also enjoy life? We have never had to make such a difficult choice as a global whole, so it is hard to say if *Homo sapiens* can pull it off.

70: Nature just called and left this cryptic message: "Check."

71: ...eliminating carbon sinks and compromising nature's ability to adapt

The last chapters in this book, beginning with Chapter 18, deal with human factors that contribute to our prospects, ending at Chapter 20 discussing mitigation strategies at the individual level. It's not geo-engineering, but behavioral adaptation leading to reduction can have a huge impact, if broadly adopted.

9.6 Upshot: Climate Change is Serious

Despite the ease with which we can demonstrate that the measured CO_2 is from fossil fuels, and the straightforward physics governing the equilibrium temperature in the face of greenhouse gases, it is astounding that we are not fully embracing reality. But perhaps it should not be surprising. Climate change is a clear-and-present affront to some economic and political ideologies that wish we would let markets set our path, with no consequence. But operating in denial and insisting that we keep to the way things worked in the past is not our smartest play. Climate change is a stark indication that we can't just do whatever makes us the most money. Alternatives to fossil fuels are expensive and less convenient. Climate change is bad for business, and also threatens capitalism by imposing limits on our ambitions. It is little wonder that the U.S. has been among the most resistant to accepting climate change realities, being the most proud capitalist country, boasting an enormous per-capita energy demand, and having contributed the most to global CO_2 emissions throughout history (Figure 9.5).

As real as climate change is, it is less clear whether it represents an existential threat to civilization.[72] As costly as it may be to adapt,[73] the changes are gradual enough on a human timescale to possibly be manageable—though decidedly less than fun.

Resource[74] disruptions, however, can send markets into free-fall and stimulate global military actions that could be far more devastating on a shorter timescale. We are in a sort of a race to see which causes the biggest problem the soonest. With luck, a third option presents itself that does not involve such hardship. But just keep in mind that human society is a highly nonlinear construct that could become unrecognizably disrupted on a much faster scale than will be the case for the physics-governed unfolding of climate change.

72: Is it apocalyptic, or just very painful?

73: ...abandoning some coastal areas, migrating away from unsurvivable climate regions, reducing population to deal with decline of agricultural resources, adapting to new ecosystems robbed of some species

74: ...e.g., energy, water, food, mined substances

9.7 Problems

1. If you were asked to characterize a typical carbon cost of fossil fuels, in grams of CO_2 emitted per gram of input fuel, what single simple number would be a good approximation to all three types, referring to Table 9.1?

2. How many kilograms of CO_2 are produced from consuming a 40 L tank of gasoline? Compare the result to typical human mass. Gasoline is about 0.75 kg/L.

3. Typical household electricity use in the U.S. is about 30 kilowatt-hours (kWh) per day. If the electricity is produced by a natural gas plant operating at 40% efficiency, each household requires 75 kWh of energy from natural gas each day. Converting this to Joules, and using the intensity of 49 g/MJ from Table 9.1, how much CO_2 is produced daily to supply a house with electricity from a natural gas source?

4. Americans use energy at an average rate of 10,000 W. Convert this to Joules in a day, then MJ, then use a representative number[75] in the right-hand column of Table 9.1 to estimate an approximate CO_2 emission per American *per day*, in kg. Compare this to the mass of a typical person.

5. Getting more specific, Americans, on average, get about 320 MJ per person from oil each day, 265 MJ from gas, and 110 MJ from coal. Based on the CO_2 mass per energy column in Table 9.1, how much mass, in kg, of CO_2 does an American produce per day from each of these sources, and in total? Compare this daily emission to the mass of a person.

6. If carbon dioxide emission is a chief concern, switching from a coal-fired electricity plant to natural gas input is still a use of fossil fuels, but natural gas produces less CO_2 per energy unit than coal (Table 9.1). By what factor would CO_2 emissions be reduced if replacing all energy[76] we now get from coal with energy from natural gas?

7. Divide the total CO_2 increase by the annual CO_2 increase in Figure 9.3 to get a timescale in years. Compare this timescale to the period over which we have been burning fossil fuels (e.g., Figure 9.2). Do they agree? If not, what reason would you offer?

8. The right-hand panel of Figure 9.3 is an accumulation of the values on the left-hand side—representing the area under the left-hand curve, for instance. Since the gas trend on the left looks like a triangle, it is easy to approximate its area.[77] Does the result match the right-most value for gas in the right-hand plot?

9. According to the fossil fuel history shown in Figure 9.2, the world currently gets energy from coal at a rate of 5 TW. From this, figure out how many ppm_v/year we add in CO_2 from coal, following the outline in Table 9.2 and Box 9.1. Use a typical coal energy density of 6.5 kcal/g and a mass ratio of 3.67 CO_2 g/g.[78] The result should match information gleaned from Figure 9.3.

10. What is the most direct and convincing response to a "climate

75: ...considering that we use a mix

76: E.g., Even swap, MJ for MJ

77: Multiplying ppm_v/yr (height) times years (base) gives units of ppm_v, matching the plot on the right.

78: ...or equivalently, 135 g/MJ

skeptic" who names many natural sources of CO_2 and says we should not be too quick to label the CO_2 rise as *anthropogenic*?[79]

11. What letter grade would you give to the performance of the red curve in Figure 9.4 for its faithful tracking of the measured CO_2 blue curve? Describe reasons why you might deduct any points.

12. The allocation of CO_2 emissions among countries in Figure 9.5 differs pretty markedly from the distribution in Table 9.3. What does this tell us about past and present activity among countries?

13. Use a modified version of Eq. 9.2 to compute what the earth's temperature would be if it had no greenhouse gases and absorbed 100% of incident solar energy.[80] Is this warmer or cooler than the actual average surface temperature (given GHG and ~30% reflection)?[81]

14. Based on Figures 9.7 and 9.8, is the atmosphere transparent or opaque at a wavelength of 1.0 μm? What about at 6 μm? And how would you characterize the situation at 2 μm?

15. If enough ice on Earth melted and resulted in 25% reflectance instead of 29.3%, what would the equilibrium temperature become, still applying the nominal 33 K GHG contribution? How much temperature rise is this compared to the equilibrium temperature at 29.3% reflection?

16. Table 9.4 has H_2O as the leading greenhouse gas, and all of them adding to 33 °C of warming effect on Earth. Should it be our goal to reduce all of these effects to the lowest numbers possible?[82] Why or why not? How would you characterize the greenhouse effect and why it is of concern to us?

17. Treating Figure 9.9 somewhat literally, in which one out of every four infrared photons escapes without being absorbed by a greenhouse gas molecule, what would the effective upward radiation be if 400 W/m^2 left the ground, and re-radiation of the absorbed fraction was split equally between upward radiation and radiation returning to the ground?[83]

18. If the "bad" news that we are about halfway through the fossil fuels is wrong, and we are only one-quarter of the way through, and we end up using all of it, what would the ultimate CO_2 concentration be in ppm$_v$, extrapolating the increase so far? How many degrees would this turn into based on our understanding of radiative forcing and the climate sensitivity parameter?

19. Using Eq. 9.5 and $CO_{2,orig}$ = 280 ppm$_v$, together with a climate sensitivity parameter of 0.8°C per W/m^2, how much would you predict Earth's temperature to rise for CO_2 levels of 330, 380, 430, 480, 530, and 580 ppm$_v$?[84] The inputs increase by steps of 50

(linearly). Is the corresponding temperature trend linear?[85]

20. Which of the following are positive vs. negative feedback effects for climate change as a result of warming, and why?

    a) warming tundra releases methane trapped in the permafrost
    b) more water in the atmosphere creates more clouds and increases the amount of sunlight reflected from Earth
    c) an increase in forest fire activity burns lots of carbon-based material and leaves a blackened earth
    d) snow covers less of the land each winter as temperatures rise

21. Coal is, and always has been, the worst offender of the fossil fuels in terms of CO_2 emission. Based on the scenarios summarized in Table 9.5, what is the percentage reduction in final temperature rise (ΔT) based on emissions out to 2100 (in the case that we hold steady at today's fossil fuel levels) if we substitute natural gas for all uses of coal vs. if we keep things the same?

Comparing first two scenarios

22. Which of the scenarios in Section 9.3 do you deem to be most realistic, and why? Given this, how much more eventual warming is likely in store compared to the 1°C to date?

23. Verify for each panel in Figure 9.15 that the sum of the upper two arrows in the middle of each panel *and* the difference of the lower two arrows in the middle match the effective (dashed) arrow at the right (a total of 8 comparisons).

24. Construct your own panel in the manner of Figure 9.15 corresponding to the third case in Section 9.3 in which we curtail fossil fuel use by 2100, resulting in a final equilibrium ΔT of 2.6°C (thus 290.6 K ground temperature). Characterize the final equilibrium state, when net upward radiation equals solar input. After balancing the books, figure out what the GHG fractional absorption must be (the number in the "cloud" in Figure 9.15).

Tips: Upgoing radiation from the ground is computed from σT^4. Verify that you can match the first or last (equilibrium) panel of Figure 9.15 by the same technique to know if you are doing it right.

25. On a sunny day after a big snowfall, the (low winter) sun might put 500 W/m^2 onto the ground. Snow reflects most of it, but let's say that it absorbs 5% of the incoming energy. How much snow (ice) will melt in an hour from the absorbed energy if a cubic meter of snow has a mass of 100 kg?

Hint: It is easiest just to consider a single square meter of surface and figure out what fraction of a cubic meter (thus what fraction of a meter-depth) melts. Ignore any effect of air temperature.

26. If the ocean absorbs an additional 3 W/m^2 of forcing,[86] how long would it take to heat the ~4 km deep column of water directly under a particular square meter (thus 4×10^6 kg) by 1°C?[87]

27. Based on Table 9.7, if we could magically turn off infrared radiation to space, how long would it take for the sun's 240 W/m^2 average to heat the entire ocean by 1°C? It is fine to assume that the ocean covers the whole globe, for this limiting-case calculation.

Hint: multiply by area of Earth and seconds in a year to get Joules of input.

28. Let's say annual excess energy input from imbalanced radiative

forcing is 3×10^{22} J per year. How much would the temperature of the top 10 m of ocean rise in one year at this level?

29. If we somehow maintained a steady 2×10^{22} J/yr of unbalanced input indefinitely, how long would it take to:

    a) raise the entire ocean temperature by 1°C?
    b) melt all the ice?
    c) raise the entire atmosphere's temperature by 1°C?
    d) heat up the top 100 m of earth's land area?

    The sum of all these is how long it would take to get all of these tasks done, even though they would be happening in parallel.

In practice, this is not how things will go. The ocean will heat up more near the surface, allowing air to get hotter and ice to melt faster while the deep ocean stubbornly resists warming for centuries.

30. On a planet 60% covered in ocean, and 5% covered in a 3 km thick ice sheet on top of land, how much would the sea level rise if all the ice melted?

31. On a planet two-thirds covered in ocean, and 1% covered by an ice sheet melting at a rate of 1 meter per year, how fast would sea level be rising?

32. How might you express the tradeoffs between the available fossil fuel resource and climate change? In other words, if it turns out that we don't have nearly as much fossil fuel as we think, what does this mean for climate change vs. our economic/geopolitical stability and viability? On the other hand, if we have centuries of fossil fuels left, what does that mean? Which scenario is least disruptive?

33. If it were certain that the only way to provide a comfortable existence to future generations involved substantial cutbacks toward a lifestyle having far fewer energy and material comforts today, do you think humanity would voluntarily do so? Can we leave treats on the shelf, within easy reach? If so, is it only uncertainty that prevents us? If not, what do you see as the barriers?

Part III

ALTERNATIVE ENERGY

The options have not changed in 50 years.
There is nothing new under the sun.

10 Renewable Overview

We now understand that the path human civilization has traveled to this point—a path paved by fossil fuels—cannot lead very much farther before a new path must take over. One distinct possibility is a trek back toward a low-tech existence, but most people would consider such a development to be a failure. What would success look like, then? Sec. D.5 (p. 404) in the Appendices takes a look at the big picture of long-term human success, but for present purposes we will focus on the critical issue of energy: where could we get enough energy to replace the prodigious one-time gift of fossil fuels?

This chapter is very brief, only setting the stage for upcoming chapters that go into alternative energy sources in greater detail. At the end of this effort, Chapter 17 summarizes the potential of all the energy resources. For now, we describe the origins and scales of Earth's energy inputs, and finish with a reminder of how much energy we get from these sources today.

10.1 The Players

Before launching into detailed attributes of energy sources beyond fossil fuels, it is helpful to list the cast of characters.

▶ **Hydroelectric** energy (Chap. 11) traps water from a river behind a dam, forced to flow through a turbine[1] that spins a generator to make electricity.

▶ **Wind** energy (Chap. 12) spins a turbine[2] that makes electricity via a generator.

1: A turbine is basically a set of fan blades.

2: Wind turbines are sometimes called rotors or windmills.

The remnant of a fallen tree—called a nurselog—provides nutrition for a row of new trees, arranged in a colonnade. Photo credit: Tom Murphy

▶ **Solar** energy (Chap. 13) can provide direct heat or make electricity via photovoltaic (PV) panels or in a utility-scale solar thermal installation.

▶ **Biological** energy (Chap. 14) can range from food to firewood, or biofuels. These are generally a source of thermal energy, via burning, able to drive heat engines.

▶ **Nuclear Fission** (Chap. 15) relies on mining a finite supply of radioactive elements in the Earth's crust whose fission (splitting) generates heat that can make steam to run a heat engine and generator to make electricity.

▶ **Nuclear Fusion** (Chap. 15), if successful, would use abundant hydrogen resources from water to build helium nuclei—a process that would release heat to make steam for running a heat engine and generator to make electricity.

▶ **Geothermal** energy (Chap. 16) originates in Earth's hot interior, and can be used for heating or to make steam to run a heat engine and generator to make electricity.

▶ **Tidal Capture** (Chap. 16) is very similar to hydroelectricity, but based on trapped tidal basins instead of dammed rivers.

▶ **Ocean Currents** (Chap. 16) behave much like wind, and can be used similarly to produce electricity, but underwater.

▶ **Waves** (Chap. 16) bring energy to shorelines that can drive specialized generators to make electricity.

One prevalent theme connects most of these entries: lots of electricity, often by way of heat engines and/or generators. Indeed, alternatives to fossil fuels tend to excel at electricity production. But as we saw in Fig. 7.2 (p. 105), electricity makes up only 38% of energy demand in the U.S., and only 17% of the energy *delivered* to the four end-use sectors.[3] One lesson is that current energy uses that are *not* electricity-based—like transportation and industrial processing[4]—will be more difficult to replace by the alternatives above.

3: While 38.3 qBtu of the 101.3 qBtu total energy input in the U.S. is dedicated to electricity production, only 13.0 qBtu emerges from the process, which is 17% of the 76 qBtu total flowing into these end-use sectors.

4: . . . which requires a lot of heat

10.2 Alternatives vs. Renewables

Before going further, we should clarify the difference between "alternative" and "renewable" resources.

Definition 10.2.1 *Alternative Energy is a* non-fossil *source of energy. Solar, wind, and nuclear would be examples.*

Definition 10.2.2 *Renewable Energy is a source of energy that is replenished by nature, so that its use may be sustained "indefinitely," without depletion. Solar energy, for instance is not "used up" by placing a panel in the sunlight.*

The sun will continue to shine no matter how many solar panels we set out. Wind is replenished daily by the sun heating the land and driving air currents. Solar energy drives the hydrological cycle, refilling the reservoirs behind hydroelectric dams. Plants grow back to replace harvested ones—again thanks to the sun. Ocean currents and waves are also driven by the sun, via wind.

Nothing, of course, lasts forever, but the sun will continue to operate in its current mode for billions of years more, and this is long enough to count as indefinite, for our purposes. Table 10.1 classifies various sources as to whether they are alternatives or renewables, along with justifications. The items with asterisks are *technically* not renewable, but last long enough that we can treat them as such in a practical sense (see Box 10.1).

Table 10.1: Energy classification. Asterisks indicate non-replenished, but long-lasting sources.

Resource	Chapter	Alternative?	Renewable?	Reason
Petroleum	8	No	No	finite supply in ground
Natural Gas	8	No	No	finite supply in ground
Coal	8	No	No	finite supply in ground
Hydroelectric	11	Yes	Yes	Sun generates rain and refills reservoirs
Wind	12	Yes	Yes	Sun generates daily by heating Earth surface
Solar	13	Yes	Yes	Sun will last billions of years
Biomass (wood)	14	Yes	Yes	Sun grows more
Nuclear Fission	15	Yes	No	finite supply of fissile material in ground
Nuclear Fusion	15	Yes	Yes*	billions of years of deuterium; not tritium/lithium
Geothermal	16	Yes	Yes*	finite, but large; rate-limited
Tidal Capture	16	Yes	Yes*	can drive Moon away eventually
Ocean Currents	16	Yes	Yes	Sun/wind-driven
Waves	16	Yes	Yes	Sun/wind-driven

Just because a resource is renewable does not mean it is *limitless*.[5] We only have so much land, nutrients, and fresh water to grow biomass, for example. Cutting trees down faster than they grow back would result in depleting the resource—possibly permanently if the land is altered severely enough that trees do not grow back. Installing turbines throughout the ocean to capture ocean currents[6] would eventually create enough impediment to the flow that it could stop altogether.

5: Ultimately, only so much sunlight strikes the earth.

6: This would be a hugely expensive and impractical undertaking, but helps illustrate the point that renewable does not mean unlimited.

Box 10.1: About Those Asterisks. . .

The items sporting asterisks in Table 10.1 deserve additional explanation as to why they are not technically renewable, even if the depletion timescales are extremely long.

Capturing all available tidal energy would end up accelerating the moon's egress from Earth,[7] eventually causing loss of the resource.[8]

About half of the geothermal energy store represents a one-time

7: . . . now at 3.8 cm/year

8: It would take hundreds of millions of years to "accomplish" this (see Sec. D.4; p. 402).

deposit of heat left over from the collapse/formation of the earth,[9] the other half coming from radioactive decays of elements ultimately tracing to ancient astrophysical cataclysms.[10] For both the formation and radioactive contributions, the supply is not replenished after its use, although the timescale for the radioactive decays to fade away is billions of years.

Nuclear fusion needs deuterium and tritium.[11] Roughly one out of every 10,000 hydrogen atoms is deuterium, so ocean water (H_2O) will have enough deuterium to last billions of years. Tritium, however, is not found naturally and must be synthesized from lithium, in finite supply. Details will follow in Chapter 15.

9: ...a conversion of gravitational potential energy

10: ...primarily supernova explosions and neutron star mergers

11: Eventually it is hoped that only deuterium could be used.

10.3 Renewable Energy Budget

Notice that all of the unqualified[12] "Yes" entries in Table 10.1 originate from the sun. For that matter, fossil fuels represent captured ancient solar energy, stored for all these years. The sun sends energy toward the earth at a rate of 1,360 W/m². Multiplying this by the projected area[13] of the earth ($\pi R_\oplus^2 \approx 1.28 \times 10^{14}$ m²) results in 174,000 TW of solar power intercepting the earth. This number *absolutely dwarfs* the 18 TW societal energy budget of all humans on Earth. Figure 10.1 shows graphically what happens to this energy input.

12: ...i.e., no asterisk

13: See Example 10.3.1.
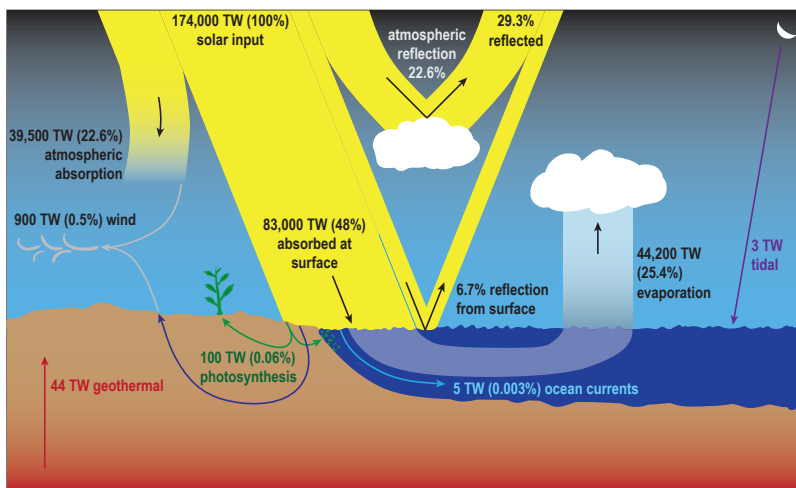
Figure 10.1: Energy inputs to the earth, ignoring the radiation piece (since that is an output channel). About 70% of incoming solar energy is absorbed by the atmosphere and land, while about 30% is immediately reflected back to space (mostly by clouds). About half of the energy absorbed at the surface goes into evaporating water, while smaller portions drive winds, photosynthesis (land and sea), and ocean currents. Additional non-solar inputs are geothermal and tidal in origin [63–65].

Example 10.3.1 Solar Input: Because we will encounter solar power flux many times in this textbook, this is a good opportunity to spell out some key numbers and concepts.

First, sunlight arriving at the top of Earth's atmosphere delivers energy at a rate of 1,360 Joules per second per square meter (1,360 W/m²), which is known as the solar constant [4].

[4]: Kopp et al. (2011), "A new, lower value of total solar irradiance: Evidence and climate significance"

Treating Earth as a sphere of radius R, it has surface area $4\pi R^2$, but the sun doesn't "see" the whole surface at once. In fact, from the vantage point of the rays of sunlight intercepting Earth, what matters is the *projection* of Earth,[14] which just looks like a disk of area πR^2. Averaging the solar input across the entire planet therefore reduces the 1,360 W/m^2 by a factor of 4 to 340 W/m^2.

Not all the sunlight arriving at the top of the atmosphere makes it to the surface, so in practice, a typical location will receive an average[15] of about 200 W/m^2. This is a number that comes up often as a typical insolation, so is worth remembering.

14: Imagine taking a picture of a sphere situated across the room. The area the sphere takes up on the photo is πR^2, not the total curved surface area of $4\pi R^2$. See also Fig. 9.6 (p. 144).

15: ... averaging over day, night, sun angles, and weather conditions.

Clouds and ice (mostly) reflect almost 30% of incoming sunlight, leaving 123,000 TW to be absorbed by land, water, and atmosphere in various forms (see Table 10.2). Virtually all of the energy hitting the surface goes to direct thermal absorption,[16] much of which then flows into evaporation of water—the starting point of the hydrological cycle. A tiny portion of the absorbed energy gives rise to wind, some of which will drive waves. An even smaller portion contributes to photosynthesis and supports essentially all life (biology) on the planet. And finally, a tiny fragment of the absorbed energy drives ocean currents. Table 10.2 tracks where the incoming solar energy goes, in several stages, also listing non-solar geothermal and tidal contributions. For comparison, the current energy scale of human activity is approximately 18 TW, while human metabolism,[17] is about 0.8 TW.

16: That is, heating (see Example 10.3.2); note that solar panels could intercept part of this energy flow.

17: Recall Ex. 5.5.2 (p. 74). 100 W per person times 8 billion people is 800 GW, or 0.8 TW.

Table 10.2: Earth's energy input budget. Symbols ⊙, ⊕, and ☾ represent Sun, Earth, and Moon, respectively. The second group breaks out the solar input into three pieces that add to the total in the row above. The third group all comes from absorbed energy—mostly at Earth's surface. The last group is not from radiant solar energy, so that percentages are in parentheses as they do not belong to the solar budget [63–65].

Category	Power (TW)	% solar	source	Comments
total solar input	174,000	100	⊙	the next 3 inputs come from here
surface absorption	83,000	47.9	⊙	heats surface; evaporates H_2O, powers life, wind, etc.
reflection to space	51,000	29.3	⊙	from clouds, ice; uncaptured energy
atmos. absorption	40,000	22.6	⊙	heats atmosphere, some to wind
evaporation	44,000	25.4	⊙ → ⊕ surf.	from surface absorption; hydrological cycle
wind	900	0.5	⊙ absorp.	from absorptions above, also makes waves
photosynthesis	100	0.06	⊙ → ⊕ surf.	fuels biology (life) on the planet
ocean currents	5	0.003	⊙ → ⊕ surf.	moves water around
geothermal	44	(0.025)	⊕	half original heat, half radioactive decays
tides	3	(0.0018)	☾, ⊙	gravitational; mostly from Moon, some from Sun

Example 10.3.2 Solar Heating: How much would a black table[18] warm up sitting in full sun for ten minutes?

A nice round-number approximation of full overhead sunlight is that it delivers 1,000 W/m^2 to the ground. If we situate a table whose top surface area is 1 m^2, has a mass of 20 kg, and a specific heat capacity

18: ... parameters defined below

of 1,000 J/kg/°C under full sun, we can proceed as follows.

The table will absorb 1,000 J per second,[19] and therefore receives 600,000 J over the course of ten minutes. Multiplying the specific heat capacity by the table mass means the table absorbs 20,000 J for every 1°C of temperature rise, and therefore would climb 30°C in 10 minutes, in this case. That's a little unrealistically high, because a real table would also have cooling influences from the air and infrared radiation. But the main point is to show how absorbed sunlight heats things up—like the Earth.

19: Because a Watt is a Joule per second and the table area is 1 m^2; the black property essentially means that it absorbs all light that hits it.

Box 10.2: Making New Fossil Fuels

We know from Chapter 8 that fossil fuels get their energy from ancient photosynthesis trapped in buried plant matter.[20] We also now have a figure for how much solar power goes into photosynthesis: 100 TW.

20: In some cases, animals ate the plants first, but the energy starts in plants.

We can compare this to the power that goes into making *new* fossil fuels right now by noting that the entire fossil fuel resource contains roughly[21] 10^{23} J (page 127), and formed over something like 100 million years, or about 3×10^{15} seconds. Dividing the two gives a power of about 3×10^7 W, or 30 MW.[22]

21: It is not important to nail down precise numbers for this exercise.

22: For reference, a single large university consumes energy at about this rate.

Three neat insights come out of this. First, we currently burn fossil fuels at a rate of about 15 TW, which is 500,000 times faster than they are being replaced! It's like short-circuiting a battery in a dramatic explosion of power. Imagine charging a phone for 2 hours and discharging it 500,000 times faster: in 0.014 seconds! Now look at the extravagant lights of Las Vegas: should we be proud of the blaze of glory or appalled?[23]

23: Also coming to mind is the Big Bay Boom in San Diego, July 4, 2012, when the entire fireworks display that was meant to last 15–20 minutes all went off in a few dazzling seconds. LMAO. Best ever!

Secondly, out of the total 100 TW photosynthetic budget on Earth, only 30 MW gets captured as fossil fuels, which is one part in three-million. Therefore, the chances that any given living matter on the planet today eventually ends up converted to fossil fuels is exceedingly slim.

Finally, if we only used fossil fuels at a rate of 30 MW,[24], then we *could* consider fossil fuels to be a renewable resource, as the sun/geology will slowly make more! So whether or not something is renewable also relies on the *rate* of use not exceeding the rate at which it is replenished.

24: This amount of power could supply only a single campus-sized consumer on Earth.

10.4 Renewable Snapshot

Table 7.1 (p. 106) already gave an account of the mix of energy use in the U.S., including many of the renewables. This section revisits those numbers, in slightly more detail.

Source	qBtu	TW thermal equiv.	% of renewables	% of total
hydroelectricity	2.77	0.093*	24	2.7
wind	2.49	0.083*	22	2.5
wood	2.36	0.079	21	2.3
biofuels	2.28	0.076	20	2.3
solar	0.92	0.031*	8	0.9
waste (incineration)	0.49	0.016	4	0.5
geothermal	0.21	0.007	2	0.2
total	11.52	0.382	100	11.4

Table 10.3: U.S. Renewable energy consumption in 2018. The last column compares to the 101.3 qBtu total U.S. consumption in 2018 [34]. Asterisks denote thermal equivalent conversions. While the second column of numbers would be more naturally expressed in GW, the choice of TW is deliberate to emphasize how small renewable usage is compared to the available resource.

In 2018, roughly 11% of energy in the U.S. came from renewable resources. Table 10.3 lists the contributions from each, the data coming from the Annual Energy Review published by the U.S. Energy Information Administration for 2018. As introduced in Box 7.2 (p. 106), the EIA adopts the practice of assigning a thermal equivalent for each source, in qBtu, even if the source had nothing to do with a thermal process. The rationale is to put everything on the same footing as fossil fuels, for more direct quantitative comparisons. In doing so, they implicitly use the average thermal-to-delivered energy efficiency for fossil fuels of 37.5%.[25] In other words, it takes 100 units of thermal fossil energy to produce 37.5 units of useful work. If, then, a solar panel delivers 37.5 units of energy over some amount of time, it will be *called* 100 units of "input" (thermal equivalent), even though it only delivered 37.5 units.[26]

Four forms dominate Table 10.3 (and Figure 10.2), each roughly equal in contribution. Lumping wood and biofuels into a generic "biomass" puts this aggregate category in the clear lead, accounting for nearly half of our renewable energy.[27] As noted, the entries in Table 10.3 are in qBtu of thermal equivalent, and these have been converted to TW, for easy comparison to Table 10.2.[28] A key take-away is how *tiny* the renewable energy numbers are compared to the natural flows in Earth's energy budget.

Figure 10.2: Same info as Table 10.3 in pie chart form.

25: 37.5% is the 2018 number from Appendix A6 of the AER.

26: Alternatively, it would have taken 100 units of fossil fuel energy to match the 37.5 units of energy delivered by the solar panel.

27: …the other half, roughly, coming from hydroelectricity and wind

28: ⚠ The items with asterisks in Table 10.3 will not match production numbers in later chapters for non-thermal resources that are not expressed in thermal-equivalent terms.

10.5 Upshot: Our Path Forward

We are now ready to plunge into learning about renewable energy resources. The topics are arranged according to ease of understanding the associated physics, which will be new to many students. So while solar is the most potent of the resources, its chapter follows those of hydroelectricity and wind, since its scheme for generating electricity[29] is likely the least intuitive of the three. Biologically-derived energy comes next—sharing its direct sunlight origin with solar. Following a foray into nuclear energy, a number of minor contributors that are unlikely to be important are relegated to a single chapter of misfits, for completeness.

29: Hydroelectricity and wind share much in common with conventional power generation schemes, in that they use turbines and generators.

After this, we will be in a position to assess the entire landscape of alternative energy options (Chapter 17). The book will then take a turn

away from physics and address how all this new information might fit into future plans at societal and personal levels.

10.6 Problems

1. Based on what you already know or suspect about the alternative energy sources listed in Section 10.1, which ones do you suspect[30] are pollution free (no emissions, no waste)?

   30: Don't worry about correctness, as this is an opportunity to think and gauge current understanding.

2. Based on what you already know or suspect about the alternative energy sources listed in Section 10.1, which ones do you suspect[31] have *no* negative environmental impacts? Explain your reasoning.

   31: Don't worry about correctness, as this is an opportunity to think and gauge current understanding.

3. Which of the truly renewable ("Yes" without asterisk) entries in Table 10.1 are limited (finite) in terms of the scale[32] we could achieve on Earth (explain the nature of the limit), and which are not limited (and why)?

   32: . . . not how long it can last; how much power is available

4. Which four entries in Table 10.1 do not ultimately trace entirely to solar energy input to Earth?

5. If, for some terrible reason, the sun ceased shining and humans managed to survive for 1,000 years longer,[33] what options would be left for obtaining energy?

   33: It's nearly impossible to imagine that survival is at all realistic in this dire situation.

6. About 30% of the 1,360 W/m^2 solar power arriving at Earth is immediately bounced back without a trace. Of the part that remains, when distributed/averaged around the whole sphere,[34] what is the average energy deposition rate per square meter into the earth system?

   34: ⚠ account for projected vs. total surface area

7. Based on the numbers in Table 10.2, Figure 10.1, and in the text, what fraction (in percent) of all biological activity on Earth do (all) humans represent, from a metabolic energy standpoint?

8. Based on the numbers in Table 10.2, Figure 10.1, and in the text, what fraction (in percent) is human societal energy production (all activities; fossil fuels, etc.) compared to all biological activity on the planet?

9. What fraction (in percent) of the solar energy *that is absorbed by Earth's surface* goes into evaporation (the hydrological cycle; refer to Table 10.2, and Figure 10.1)?

10. If we add up the five smallest pieces in Table 10.2, what fraction (in percent) does this set of energy flows represent in comparison to the total solar energy *absorbed by the earth system*?

11. The percentages of absorption and reflection in Figure 10.1 represent averages over weather conditions. On a clear day with the

sun overhead, a solar panel has access to more light than the 48% depicted reaching the ground. If we can also capture the portion that on average is reflected by the ground and clouds,[35] what might we expect for the rate of energy reaching the ground (in W/m^2) if the input at the top of the atmosphere is 1,360 W/m^2?

12. Similar to Example 10.3.2, how much warmer would you expect a 10 cm deep puddle of water[36] to get after an hour in full sun, if the water absorbs all the energy and does not lose it to the environment?

13. Comparing Table 10.2 and Table 10.3, what is the most striking mismatch,[37] in terms of large potential vs. small contribution to U.S. energy consumption?

14. In one day, a typical residential solar installation might deliver about 10 kilowatt-hours of energy. Meanwhile, a gallon of gasoline contains about 37 kWh of thermal energy. But the two ought not be directly compared, as burning the gasoline inevitably loses a lot of energy as heat. Correcting the solar output to a thermal equivalent,[38] how many gallons per day of gasoline could it displace?

15. The U.S. occupies about 2% of Earth's surface area, thus collecting only part of the 83,000 TW absorbed at the surface. Compare the amount of solar power received by the U.S. to the *total* renewable power in Table 10.3.[39] By what factor are we "underutilizing" the solar input, under the simplifying assumption that we could use all of it?

35: Recall, it's a clear day; no clouds.

36: ⓘ Water has a density of 1,000 kg per cubic meter and a specific heat capacity of 4,184 J/kg/°C. Make up whatever area you want; the answer should be the same for any pick.

37: In other words, for those sources that can be matched within the tables, which stands out as the most underutilized relative to its input numbers?

38: ⓘ Use the 37.5% factor discussed in the text.

39: …since it's almost all solar in ultimate origin

11 Hydroelectric Energy

Energy has been harnessed from flowing water for ages. Milling operations were often located on streams so that water could turn a wheel attached to grinding machinery. Today, captured water flow is a significant contributor to electricity worldwide in the form of hydroelectricity. The U.S. gets about 2.8% of its current energy (and 7% of electricity) from hydropower. Globally, hydropower accounts for about 9% of energy, or 16% of electrical production (Table 7.2; p. 107).

Hydroelectricity taps into the solar-driven evaporation cycle, relying on the gravitational energy embodied in water lifted onto the land from lower bodies of water. In other words, solar energy lifts water, giving it gravitational potential energy, which is captured and converted to electrical energy.

While hydroelectric power is a simple and low-tech form of renewable energy that has been heavily exploited for over a hundred years, it is not one that is easy to expand beyond its current level of usage. This chapter will provide a better understanding of this mainstay of the renewable portfolio and its likely role in our future.

Tidal power works in exactly the same way as hydroelectricity, but is a niche player and will be deferred to Section 16.2.

11.1 Gravitational Potential Energy

Gravitational force is incredibly weak. It may not seem so from daily experience, but consider the fact that a magnet held in your hand can lift a paperclip—overwhelming the gravitational pull of the *entire* earth! By comparison, electromagnetic forces are forty orders-of-magnitude stronger than gravitational force. We don't tend to notice because electric charges tend to balance out so that gravity is the most obvious force in our daily lives.[1]

1: Somewhat ironically, we only *feel* gravity because of a much stronger electromagnetic force that prevents us from falling through the floor. Electrons in the atoms in the floor and our feet repel each other to prevent free-fall—a weightless state in which gravity can't be felt!

Boulder Dam. Photo Credit: Tom Murphy

We know intuitively that lifting a massive object requires work, and thus energy. In fact, since work is defined as a force times distance,[2] and the force of gravity on an object follows Newton's second law, $F = ma$,[3] the force we need to exert on an object to lift it against gravity is called its weight, and is $W = mg$, where $g = 9.8\,\text{m/s}^2 \approx 10\,\text{m/s}^2$ is the acceleration due to gravity on the surface of the earth. Thus, to lift an object through height, h, requires an energy input of this force, W, times the height, h. We call this gravitational potential energy since the energy put in to lift something can be released later if the mass is allowed to fall or be lowered. An early use of gravitational potential energy was in the form of weights on chains in old clocks.

> **Definition 11.1.1** *gravitational potential energy is computed as*
>
> $$\text{G.P.E.} = mgh, \qquad (11.1)$$
>
> *where m is the mass in kg, $g \approx 10\ m/s^2$ is the acceleration due to gravity,[4] and h is the height to which the mass is raised, in meters. The result is in Joules.*

Most commonly, gravitational potential energy is converted to kinetic energy as an object falls: slowly at first but accelerating as more potential energy is converted to kinetic energy while the object gains speed (see Fig. 5.1; p. 70). Only the vertical distance matters in computing gravitational potential energy: sideways motion does not operate against the gravitational force. Sliding a crate across a flat, level floor does take work to overcome friction, but that energy is converted to heat and cannot be later returned in useful form.[5] In this case, the crate has gained no gravitational potential energy, since its height never changed.

> **Example 11.1.1** Lifting a 20 kg box of books, whose weight is therefore $W = mg \approx 200\,\text{N}$,[6] from the floor to a high shelf through a vertical distance of 2 m involves an energy expenditure of $mgh \approx 400\,\text{J}$ (Figure 11.1). We would say the box gained 400 J of potential energy.
>
> If the person doing the work is exerting energy at a rate of 200 W (200 J/s), it will take two seconds to complete the action.
>
> If the box later falls off the shelf and hits a 1.5 m tall person on the head, the box has *lost* 100 J (20 kg × 10 m/s²× 0.5 m) of potential energy (now kinetic) by the time it hits the person's head.

11.1.1 Comparison to Other Forms

To give a flavor of how weak gravitational potential energy is compared to other familiar forms of energy storage, we will consider the energy content in a standard AA alkaline battery and in a similar volume of gasoline. So we're talking about something approximately the size of
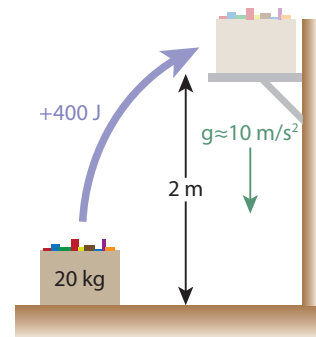
Figure 11.1: Diagram for Example 11.1.1.

a small finger. We want to know how much mass must be lifted to yield the same amount of gravitational potential energy as is contained in a battery or equivalent volume of gasoline. In the comparison, we will imagine having a hoist that can lift a large mass[7] 4 m high—about house-height.

A standard AA battery cell has a charge rating of 2.5 Ah[8] and operates at about 1.5 V. Following the development in Sec. 5.8 (p. 76), we multiply these two numbers to get 3.75 Wh, translating to 13.5 kJ. Equating this to mgh, where we know $g \approx 10\,\text{m/s}^2$ and $h = 4$ m, we find that $m \approx 340$ kg. That's really heavy—about the mass of 4–5 people.[9] Meanwhile, the AA battery is a puny 0.023 kg. Reflect for a moment on this comparison, visualizing 340 kg lifted 4 m above the ground providing the same amount of energy as a AA battery held in your hand.

Gasoline is even more extreme. At an energy density around 34 kJ per mL of volume, filling a AA-sized cup[10] with gasoline yields about 250 kJ of energy.[11] Performing the same computation, we would need to lift over 6,000 kg (6 metric tons) to a height of 4 m to get the same energy content. Typical cars have masses in the 1,000–2,000 kg range, so we're talking about something like 4 cars! One caveat is that we are not typically able to convert the thermal energy in gasoline[12] into useful work at much better than 25%, while gravitational potential energy can be converted at nearly 100%. Still, being able to lift 1,500 kg[13] to a height of 4 m using the energy in 7 mL of gasoline is rather impressive, again emphasizing that gravitational potential energy is pretty weak. It only amounts to significance when the masses (volumes) of water are rather large.

7: . . . a rock, for instance

8: The number is usually given as, e.g., 2,500 mAh (milli-amp-hours).

9: Amuse yourself by picturing 4–5 people slung haphazardly into a net and hoisted to roof height—a very odd (and grumpy?) replacement for a AA battery.

10: . . . just over 7 mL

11: Thus, gasoline is nearly 20 times as energy-dense as a AA battery *by volume*. Usually, we will discuss energy density *by mass*, in which case the ~5× denser battery provides nearly 100× less energy per gram than does gasoline.

12: . . . via combustion; see Sec. 6.4 (p. 88)

13: . . . now just one car, rather than four; it means this small volume of gasoline can propel a car up a 4 m hill

11.2 Hydroelectric Energy

The basic idea behind hydroelectricity is that water in a reservoir behind a dam (Figure 11.2) creates pressure at the base of the dam that can force water to flow through a turbine that drives a generator to make electricity—sharing elements of Fig. 6.2 (p. 90) but spinning the turbine by water flow instead. The amount of energy available works out to be the gravitational potential energy corresponding to the height of water at the lake's surface relative to the water level on the other side. It's as if dropping the water from the surface to the turbine and asking how much potential energy it gave up in the process. In reality, water is not dropping from the lake surface, but the force on the water at the turbine is determined by the height of water above it: the "pressure head," as it is called. The process is highly efficient, approaching 90% capture of the potential energy in the water delivered as electrical power from the generator.

Figure 11.2: Cross section of a dam, holding back a reservoir of water at height, h, over the downstream river.

Box 11.1: Why So Efficient?

Achieving 90% efficiency is superb! Electric motors and generators[14] can be > 90% efficient in converting between mechanical energy (rotation) and electrical energy. When coupled with low-friction turbines, dams just have very little loss—unlike thermal sources where most of the energy is unavoidably lost (for reasons covered in Sec. 6.4; p. 88).

14: Fundamentally, motors and generators are nearly identical in concept and construction.

Figure 11.3: One cubic meter of water has a mass of 1,000 kg.

Example 11.2.1 To compute the power available from a hydroelectric plant, we need to know the *height* of the reservoir and the *flow rate* of water—usually measured in cubic meters per second. The density of water is, conveniently, 1,000 kg/m^3 (Figure 11.3), so that if we consider a dam having a flow rate of 2,000 m^3/s and a reservoir height of 50 m, we can see that every second of time will pass 2×10^6 kg of water,[15] and the associated potential energy is $mgh \approx 10^9$ J. If each second delivers 1 GJ of energy, the power available is 1 GJ/s, or 1 GW. At an efficiency of 90%, we get to keep 900 MW of electrical power.

15: Flow rate times density gives mass per second: 2,000 m^3/s times 1,000 kg/m^3 = 2×10^6 kg/s

The largest hydroelectric facility in the world is the Three Gorges Dam in China, rated at an astounding 22.5 GW. The largest in the U.S. is the Grand Coulee on the Columbia River, producing a maximum of 6.8 GW. The iconic Boulder Dam (a.k.a. Hoover Dam) is just over 2 GW.

Look at the Wikipedia page on largest hydroelectric power stations [66] for a complete list.

Note that flow rates vary seasonally with rainfall, so that dams cannot always operate at full capacity. In fact, the U.S. has about 80 GW of *capacity* installed, but operates at an annual average of about 33 GW. This implies a typical "capacity factor" around 40%.

Definition 11.2.1 *A capacity factor is the ratio of actual performance over time to the peak possible performance—or average output divided by maximum output, expressed as a percentage.*

Example 11.2.2 Boulder (Hoover) Dam on the Colorado River is listed in [66] as having a capacity of 2,080 MW and an annual production of 4.2 TWh. What is its capacity factor?

We just need to turn the 4.2 TWh in a year into an average delivered power. Following the definition of a watt-hour, we note that all we really have to do is divide 4.2×10^{12} Wh[16] by the number of hours in a year: 24 times 365, or 8760.

16: 1 TWh is 10^{12} Wh.

4.2×10^{12} Wh/8760 h $\approx$ 480 MW average power. Dividing this by 2,080 MW (max capacity) gives a 23% capacity factor.

As we saw in Fig. 7.5 (p. 108) and Table 10.3 (p. 170), hydroelectricity in the U.S. accounts for 2.7% of the nation's total energy consumption, corresponding to about 33 GW of production. Globally, hydroelectric production averaged 477 GW in 2017. By comparison, Table 10.2 (p. 168)

indicates that 44,000 TW of solar input goes into evaporation and the hydrological cycle. Why, then, are we only able to use 0.477 TW (0.001%) of this bounty? Is this a great, untapped renewable resource?

11.2.1 Theoretical Potential

To understand the giant mismatch between solar input and hydroelectric development, we first need to study evaporation.

> **Definition 11.2.2** *The* **heat of vaporization** *of water is about 2,250 J per gram, meaning that every gram of water that goes from liquid to gas (vapor) requires an energy input of ~2,250 J.*

Box 11.2: Vaporization is Serious Energy

To put this in perspective, it takes 100 calories (418 J) to bring one gram of water from freezing to boiling temperature. Then it takes another 2,250 J to evaporate the water, which is a far larger quantity. This is why water in a pot does not all flash into steam once the water reaches 100°C, as it would if the evaporation energy was very small. Instead, a boiling pot will retain water for a good while as energy continues to be applied before all boiling away.
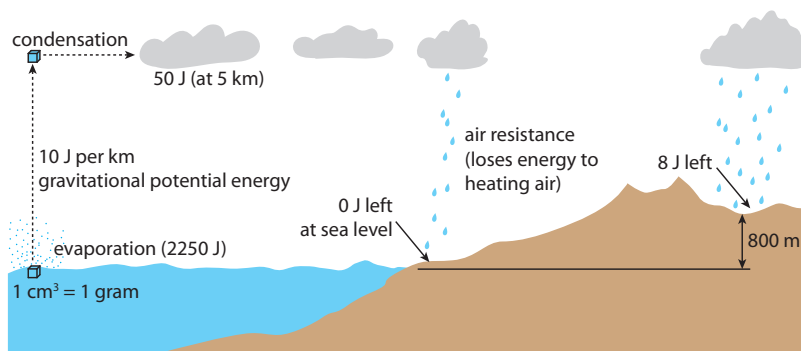
Figure 11.4: The hydrological cycle. Sunlight evaporates water from the surface, at a cost of 2,250 J per gram. Each kilometer of height the gram of water gains in forming clouds costs an additional 10 J. When rain falls on terrain, most of the gravitational potential energy is spent, but on average retains 8 J—based on an average land elevation of 800 m. The 2,250 J of evaporation energy is released as heat when the water condenses into clouds.

So let's follow the energetics of a gram of water[17] on its journey to a hydroelectric dam—most of which is represented in Figure 11.4. First, the sun injects 2,250 J to evaporate that gram. Then let's say it gets lofted to 5 km.[18] The gravitational potential energy, mgh, comes to $0.001 \times 10 \times 5000 = 50$ J. That's only 2% of the amount that went into evaporation.[19]

When the water condenses in the cloud, it *releases* 2,250 J of thermal energy into the cloud/air, then falls back to the ground as rain, offering 50 J of still-available energy. If it falls on the ocean, where it presumably started, it gives up all 50 J of gravitational potential energy into useless forms.[20] But if it falls on land—higher than sea level—it retains some gravitational potential, based on how high that land is above sea level.

17: ...one cubic centimeter

18: ...typical cloud height

19: The sun must, in total, supply 2,300 J to evaporate and lift the gram of water, and only 50 J of the 2,300 J is kept as potential energy.

20: ...heat through air resistance and collision with the ocean surface

On average, terrain is about 800 m above sea level, so each gram that falls on land has an average of 8 J left as available energy. But only 29% of the earth's surface is land, so that the gram of water we're tracking preserves about 2 J of energy, on average.[21]

We're down to only 0.1% of the input solar energy—2 J out of 2,300 J input—so that the theoretical hydroelectric potential might be about 44 TW: reduced from the 44,000 TW input. But only a small fraction of rain flows into rivers suitable for damming. And once dammed, a typical dam height is in the neighborhood of 50 m, knocking us down even further. Much of the journey from terrain to reservoir involves losing elevation in streams too small to practically dam, or just seeping through the ground. In the end, perhaps it is not surprising that we end up in the sub-TW regime globally.

Detailed assessments [67] of hydroelectric potential globally estimate a *technically* feasible potential[22] around 2 TW, but only half of this is deemed to be economically viable. Recall that 477 GW, or about 0.5 TW, is delivered globally, which is therefore about half of what we believe to be the practical limit of ~1 TW. Thus we might not expect more than a factor-of-two expansion of current hydroelectricity as possible/practical. The low-hanging fruit has been plucked already, capturing about half of the total practical resource.

Compared to the 18 TW global scale of energy use, hydroelectricity is not poised to assume a large share at this level, unless the overall scale of energy use is reduced substantially. Let's say this more visibly: **hydroelectric power cannot possibly come close to satisfying present global power demand.**

11.3 Hydropower in the U.S.

Hydroelectric power is not available to the same degree everywhere. Geography and rainfall are key factors. This brief section serves to present a snapshot of the distribution and qualities of hydroelectric power generation in the United States. We start with Figure 11.5, showing the average hydroelectric power generated in each state, the top four states being listed in Table 11.1. These four states account for 56% of hydroelectricity in the U.S., and the next states on the ranked list drop to 1 GW or lower. Most of the California generation is in the northern part of the state, effectively as part of the Pacific Northwest region.

To get a sense for how concentrated different sources are, we will make a habit of examining power density for renewable resource implementations. Figure 11.6 indicates the state-by-state density of hydroelectric power generation,[23] just dividing generation by state area. No state exceeds 0.05 W/m^2, which can be contrasted to insolation values (see Ex. 10.3.1; p. 167) of ~200 W/m^2. Globally, total land area is about

21: ... reduced from 8 J since most rain falls back onto ocean

The 90% efficiency of a hydroelectric dam is now contextualized a bit better. That last step is pretty efficient, but the overall process is extremely inefficient. Still, it takes relatively little effort to exploit, and provides real power. Efficiency is not everything.

[67]: (1997), *Study on the Importance of Harnessing the Hydropower Resources of the World*

22: ... if cost is no barrier

Table 11.1: Top hydroelectric states.

State	Production (GW)
Washington	8.9
Oregon	3.8
California	3.0
New York	2.9
U.S. Total	33

23: ... based on actual generation, not installed capacity
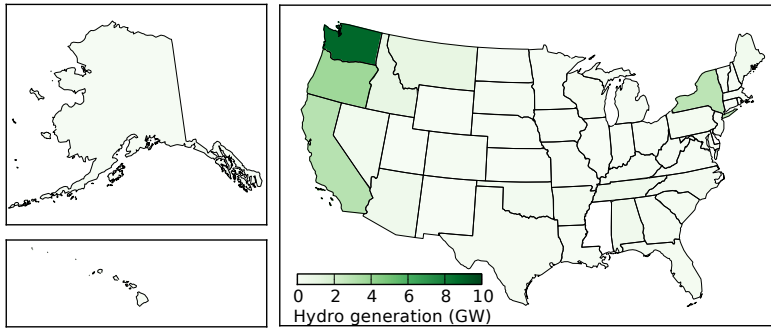
Figure 11.5: Average U.S. hydroelectric power delivered for each state, showing strongly along the west-coast, plus New York. And yes, Alaska really is that big.

1.25×10^{14} m^2, so that a total hydroelectric potential of 2.5 TW[24] would yield 0.02 W/m^2. Therefore, the state of Washington stands out as unusual, having *already developed* a generation capacity 2.5 times larger than the upper-end global average expectation. In other words, most of the world cannot emulate what nature has provided in Washington.[25] Not all places have the same available resources.

24: This is higher than estimated potential developed resources, but mathematically convenient here.

25: Washington's hydroelectric dominance owes largely to the presence of the mighty Columbia River, rather than human factors.
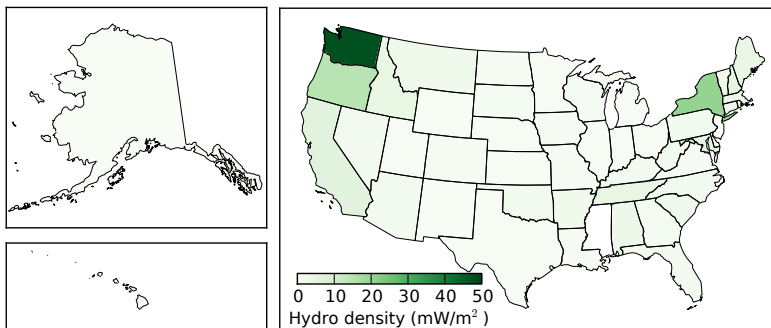
Figure 11.6: U.S. hydroelectric power per area delivered for each state, giving a sense of how concentrated the resource is. The units are milliwatts per square meter, peaking at 48 mW/m^2 for Washington.

Next, we look at hydroelectric generation per capita. Figure 11.7 shows the result. In this view, the states of the Pacific Northwest really pop up, and New York dims relative to its by-area showing. The contrast between Figure 11.6 and Figure 11.7 is effectively reflecting population density: large, sparsely-populated states[26] show up more prominently on the per-capita map than the per-area map.

26: Montana, Idaho, even Alaska

Finally, for completeness, we look at the capacity factors of hydroelectric installations, by state. The total installed capacity in the database used for these plots is 77.6 GW spread among 1,317 dams, while producing an annual average of 28.1 GW—corresponding to an overall capacity factor of 0.36. Figure 11.8 shows how this distributes around the country. Since the Pacific Northwest dominates in installed hydroelectric power, it largely determines the overall capacity factor. Iowa stands out as having a high capacity factor, but only has 0.153 GW of installed capacity.[27] Contrast this to Washington, having an installation capacity of 20.7 GW.[28]

27: ...delivering an average of 0.114 GW in 8 dams, dominated by the 0.125 GW Keokuk dam

28: ...delivering an average of 8.9 GW spread across 65 dams
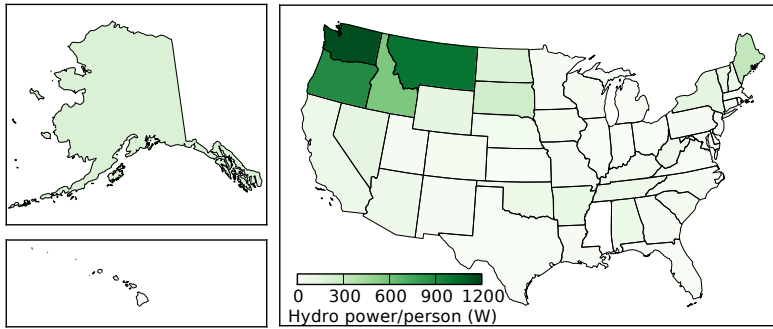
Figure 11.7: U.S. hydroelectric power per population within each state, indicating which residents receive the most hydroelectric power. The Pacific Northwest wins.
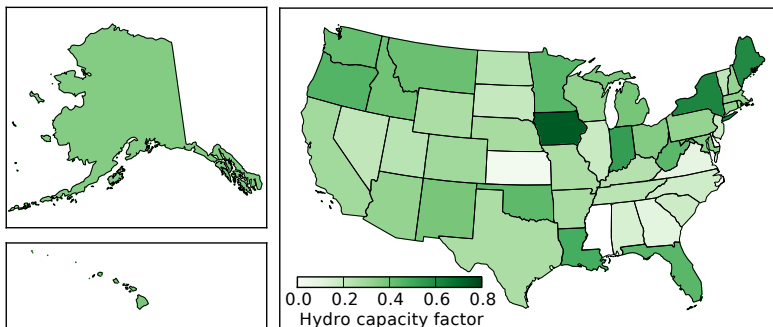
Figure 11.8: U.S. hydroelectric capacity factors by state. The national average (weighted by generation) is about 0.4, meaning that over the course of a year the dams deliver 40% of their rated capacity—subject to seasonal water flow.

11.4 Global Hydropower

This section provides a brief snapshot of hydroelectric production globally, which we saw at the end of Section 11.2.1 amounts to 477 GW. Figure 11.9 shows which countries have the most hydroelectricity, the corresponding numbers appearing in Table 11.2—including the percentage of electricity derived from hydroelectric sources within the country. Notice that Norway, Venezuela, Brazil, and Canada derive more than half their electricity demand from hydroelectricity. Keep in mind that electricity is not the whole energy story for a country, as Fig. 7.2 (p. 105) made clear.
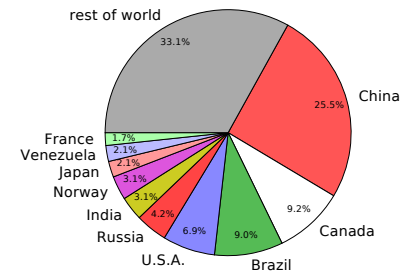
Figure 11.9: Global allocation of hydroelectric power.

Table 11.2: Top ten hydroelectric producers in the world, accounting for two-thirds of the hydroelectricity production worldwide.

Rank	Country	GW	% elec.	Rank	Country	GW	% elec.
1.	China	122	19	6.	India	15	10
2.	Canada	44	58	7.	Norway	15	96
3.	Brazil	43	63	8.	Japan	10	8
4.	U.S.	33	6.5	9.	Venezuela	10	68
5.	Russia	20	17	10	France	8	12

11.5 Upshot: Pros and Cons of Hydro

The two lists below provide some pros and cons to hydroelectric power that are relevant to our assessment of its value to our mix of renewable energy options. First, the positive attributes:

▶ Natural source, solar-driven, without waste products or pollution;[29]

▶ Technologically simple, therefore straightforward to implement and maintain;

▶ High efficiency, turning 90% of available energy into useful electricity;

▶ Good baseline of steady power over daily timescales;[30]

▶ Life-cycle CO_2 emissions only 4% that of traditional fossil fuel electricity [68];

▶ Facilities[31] can last a century or longer;

▶ While not directly related to energy, dams can provide flood control and reliable water supplies.

29: ...aside from construction and decommissioning aspects

30: ...no imposed short-term fluctuations in available power, as happens for solar or wind

31: ...the dam itself, at least; turbines and generators will need periodic replacement

And some of the downsides that may discourage further development:

▶ Silt can build up behind dams displacing the reservoir, eventually rendering them useless and dangerous;

▶ Requires the semi-permanent flooding of an ecological habitat, of varying quality and value;

▶ Seasonal variability of available power, often by a factor of ten;

▶ Defunct or poorly maintained facilities represent a dam-burst danger to downstream residents;

▶ Blocks salmon runs and impacts the health of both oceanic and forest ecosystems;

▶ As the distribution maps show, hydroelectric is not a viable option everywhere: the combination of terrain[32] and rainfall is needed.

32: ...mountains or canyons to hold the reservoir

On balance, our society has embraced hydroelectricity as a clean and reliable resource. One can view it as nature's low-hanging fruit, partly evidenced by how early it was adopted on a large scale. It is likely to remain an attractive form of energy as we face increasing pressures to migrate away from carbon-based fuels.[33] It will not, however, be able to provide an avenue for wholesale replacement of fossil fuels given its limited scale and electric-only nature. Only if our overall energy demand is reduced substantially will it form a large fraction of our portfolio.

33: ...whether due to resource limits or climate change action

11.6 Problems

1. If a 70 kg person climbs 10 flights of stairs, each flight 3 m high, how much potential energy have they gained?

2. If an 80 kg person is capable of delivering external mechanical energy at a rate of 200 W sustained over several minutes,[34] how high would they be able to climb in two minutes?

34: It is hard to keep up 200 W for too long.

3. A 10 kg box is lifted 2 m off the floor and placed on a frictionless horizontal conveyor to take it 30 m across a warehouse. At the end of the conveyor, it is lowered 1 m where it ends up on a shelf.[35]

35: The shelf is therefore 1 m off the same (original) floor.

How much *net* gravitational potential energy was given to the box from the start to the end of the process?

4. A standard AA battery cell stores about 13.5 kJ of energy. At a mass of 23 g each, how high would you have to lift the battery to get the same amount of gravitational potential energy?

(i) The result emphasizes how weak gravitational potential energy is.

5. A gallon of gasoline contains about 130 MJ of chemical energy at a mass of around 3 kg. How high would you have to lift the gallon of gasoline to get the same amount of gravitational potential energy? Compare the result to the radius of the earth.

(i) The result emphasizes how weak gravitational potential energy is.

6. Problem 5 used one gallon of gasoline to compute the equivalent height for gravitational potential energy. Does the result depend on what volume of gasoline we selected? Make an airtight argument as to why or why not. Solving symbolically[36] may be a helpful route—but not the only one.

Hint: the energy density in MJ per kg is a property of the gasoline.

36: . . . using variables/symbols

7. A typical American household uses approximately 30 kWh per day of electricity. Convert this to Joules and then imagine building a water tank 10.8 m above the house[37] to supply one day's worth of electricity.[38] How much mass of water is this, in kg? At a density of 1,000 kg/m^3, what is the volume in cubic meters, and what is the side length of a cube[39] having this volume? Take a moment to visualize (or sketch) this arrangement.

37: Pretend all the water is at this height.

38: Assume 100% conversion efficiency for mathematical convenience.

39: . . . cube root of volume

8. The biggest hydroelectric installation in the U.S. is the Grand Coulee dam on the Columbia River. The enormous flow rate reaches its maximum at 4,300 m^3/s, and the dam (reservoir) height is 168 m. At an efficiency of 90%, at what rate is this dam capable of producing hydroelectric power (in GW[40])? Don't forget the density of water and that $g \approx 10\,\text{m/s}^2$.

40: *(i)* For comparison, a large nuclear reactor typically produces about 1 GW of electrical power.

9. The Hoover Dam (also called Boulder Dam) on the Colorado River is rated at 2.08 GW when the flow is at its maximum rate of 1,280 m^3/s. How high is the reservoir if the efficiency of the installation is 90%?

10. A dam 50 meters high is constructed on a river and is delivering 180 MW at some moment in time. What is the flow rate of water, in cubic meters per second, if the facility converts gravitational potential energy into electricity at 90% efficiency?

11. A hydroelectric facility is built to deliver a peak power of 1 GW, which it manages to do for three months of the year during the spring snow-melt. But for three months in summer, it drops to 700 MW, then 500 MW for three months in fall. In winter, it drops way down to 200 MW for three months. Using the concept of the capacity factor (Definition 11.2.1), what is the annual average capacity factor for this facility?

This stepwise behavior is not exactly realistic, but need not be to apply the concept correctly.

12. While the Chief Joseph Dam on the Columbia River can generate as much as 2.62 GW (2.62×10^9 W) of power at full flow, the river does not always run at full flow. The average annual production is 10.7 TWh per year (10.7×10^{12} Wh/yr). What is the capacity factor of the dam: the amount delivered vs. the amount if running at 100% capacity the whole year?

Hint: Multiplying peak power by hours in a year will result in units similar to Wh/yr for direct comparison.

13. The Robert Moses Niagara dam in New York is rated at 2,429 MW[41] and has a high capacity factor of 0.633. How many kWh does it produce in an average day, and how many homes would this serve at the national average of 30 kWh/day?

41: ...peak power capacity

Hint: Convert average power to kW then multiply by hours.

14. The Robert Moses Niagara dam from Problem 13 is 30 m high. What is the peak flow rate, in m^3/s, if it can produce full capacity power (2.43 GW electrical output) while converting gravitational potential energy to electricity at 90% efficiency?

15. It takes 2,250 J to evaporate each gram of water, while only taking about 330 J to raise the temperature of water from room temperature to the boiling point. If it takes 10 minutes to bring a pot of water from room temperature to boiling, how much additional time will it take to boil off (evaporate) all the water if injecting energy at the same rate the whole time?

16. Starting at 44,000 TW of solar input to the hydrologic cycle, parallel the development in Section 11.2.1 by computing the power remaining at each stage[42] if, for each gram of water:

    a) water is evaporated and lifted to 5 km height;[43]
    b) 30% of the water falls on land where collection is possible;
    c) the typical land height is 800 m;
    d) only 20% of the water makes it to dammable locations;
    e) only 50 m of height (of the original 800 m average) is left for the dam.

    By this analysis, how much hydroelectric power is theoretically possible, globally?

42: Each stage will knock down the number further; report each in TW.

43: This is the largest jump, keeping only 50 J out of every 2,300 J.

17. Fig. 10.1 (p. 167) indicated that about 44,000 TW globally goes into evaporating water. We can turn this into an estimate of how much rain we expect per year, on average. The simplest way to do this is to think of a single square meter of ocean surface, receiving an average evaporation input power of 120 W.[44] Each millimeter of of water depth across our square meter has a volume of 1 L, or a mass of 1 kg. At a steady input of 120 W,[45] how many millimeters of water are drawn off in a year? That same amount will come back down *somewhere* as precipitation.

44: $44{,}000 \times 10^{12}$ W divided by 3.7×10^{14} m^2 of ocean surface is 120 W/m^2.

45: (i) The steady 120 W is already accounting for day/night: this is a time average.

12 Wind Energy

Wind energy has made significant inroads for electricity production in the U.S. and globally. Today, the U.S. gets about 2.5% of its energy (and 6.5% of electricity) from wind. Globally, wind accounts for about 2.6% of energy, or 4.8% of electrical production (Table 7.2; p. 107).

The basic technology of harnessing wind power is rather old, powering ships, milling operations, and water pumps for centuries. Today, the predominant use of wind drives generators to make electricity.

Just as hydroelectric power is related to the very basic form of gravitational potential energy, wind is connected to another simple and easy-to-understand form: kinetic energy. This chapter first develops familiarity with kinetic energy, then explores how we get energy from wind, how much we get, and future prospects.

12.1 Kinetic Energy

An object in motion carries kinetic energy equal to one-half its mass times the velocity squared.

Definition 12.1.1 *The **kinetic energy** of a mass, m, moving at velocity, v, is*

$$\text{K.E.} = \frac{1}{2}mv^2. \tag{12.1}$$

Example 12.1.1 A 70 kg person walking at a brisk 2 m/s would have a kinetic energy of 140 J.

Pushing a 50 kg ice skater from rest at a power of 75 W for 3 seconds would impart 225 J of kinetic energy, resulting in a speed of 3 m/s.

For examples like these, framed as statements and not questions, you can practice solving several types of problems by covering up one number and then solving for it using still-available information. So they provide several examples in one!

Photo credit: Tom Murphy

Often, we use energy sources to *deliver* kinetic energy, as in moving planes, trains and automobiles.

> **Example 12.1.2** A 1,500 kg car moving at freeway speeds (30 m/s) has a kinetic energy around 675 kJ.
>
> Getting up to this speed from rest in 5 seconds would require a power of 135 kW,[1] equating to 180 horsepower.[2]

1: Do the calculation yourself to follow along.

2: Recall that 1 hp is 746 W. Indeed, it takes a powerful engine to provide this level of acceleration.

But we can also go the other direction and convert kinetic energy into different forms of energy[3] for versatile use. Most commonly, we turn kinetic energy into electrical potential energy (a voltage) that can drive a circuit. At this point, the energy can be used to toast a bagel, charge a phone, or wash clothes.

3: See Table 5.2 (p. 70), for examples.

The method for converting kinetic energy into electricity is usually accomplished by transferring kinetic energy in a moving fluid[4] into rotation of a shaft by way of a turbine—essentially fan blades. The spinning shaft then turns an electric generator, which consists of the relative motion between magnets and coils of wire, and is essentially the same construction/concept as an electric motor run in reverse.

4: In this sense, "fluid" is a general term that can mean a liquid or even air.

Hydroelectric installations do the same thing—turning a shaft via blades of a turbine—even though we framed the energy source as one of gravitational potential energy. Within the dam's turbine, the water acquires kinetic energy as it flows from the reservoir to the outlet. Wind energy acts in much the same way, converting kinetic energy in the moving air into rotational motion of a fan/turbine whose shaft is connected to a generator located behind the blades.

12.2 Wind Energy

It is tempting to think of air as "empty" space, but at sea level air has a density of 1.25 kg per cubic meter ($\rho_{air} \approx 1.25\,\text{kg/m}^3$). Let this sink in visually: imagine a cubic meter sitting next to you (as in Figure 12.1). The air within has a mass of 1.25 kg (about 2.75 lb). Now draw a square meter on the ground—either literally or in your imagination. The many kilometers of air extending vertically over the top of that square meter has a mass of ~10,000 kg! For context, figure out how many cars that would be (typ. 1,500 kg ea.), or what kind of animal would be this massive.

Figure 12.1: The mass of a cubic meter of air is 1.25 kg, and the mass of atmosphere over one square meter is an astounding 10 metric tons.

What this means is that air in motion can carry a significant amount of kinetic energy, since neither its mass nor velocity are zero. If the entire earth's atmosphere moved at 5 m/s—a noticeable breeze—at a total mass[5] of 5×10^{18} kg, we'd have 6×10^{19} J of kinetic energy in air currents. If we somehow pulled all this energy out of the air—stopping its motion entirely—we might expect the atmosphere to revive its normal wind

5: 10^4 kg per square meter times the surface area of Earth ($4\pi R_\oplus^2$)

patterns over the course of 24 hours: a full day of the driving solar input around the globe. The associated power works out to 700 TW. Notice that the value for wind in Table 10.2 (p. 168) is pretty-darned close to this, at 900 TW.[6] As the margin note indicates, we should be pleased to get within a factor of two for so little work and very off-the-cuff assumptions about global average air speed (see Box 12.1 for related thoughts). Figure 12.2 shows the annual average wind velocity at a height of 80 m (typical wind turbine height) for the U.S. Note that the 5 m/s we used above falls comfortably within the 4–8 m/s range seen in the map.

See if you can confirm; own it yourself!

[6]: In the first draft of this textbook, a different data source was used for Table 10.2 that had wind at 370 TW. Even so, the 700 TW estimate corroborated the order-of-magnitude scale and was deemed a satisfactory check: within a factor of two.

United States - Annual Average Wind Speed at 80 m
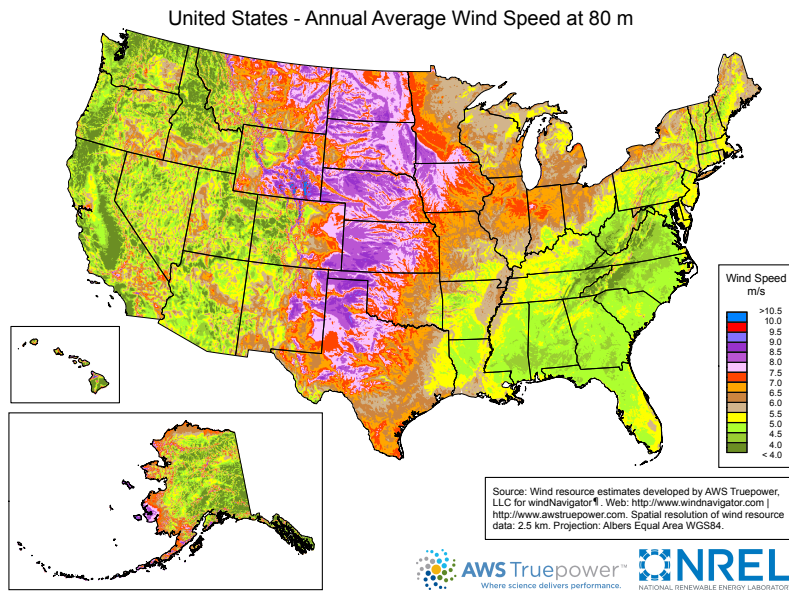
Figure 12.2: Average wind velocity at a height of 80 m across the U.S. [69]. Boundaries between colored boxes are every 0.5 m/s from 4.0 m/s to 10.0 m/s. Nothing on this map exceeds 9 m/s, and the deepest green is below 4 m/s. The plains states are the hot ticket. Note that Alaska is not to scale. From NREL.

Box 12.1: The Value of Estimation/Checking

Calculations like the one above offer a way to see if something at least checks out and seems plausible. If we had found that the whole atmosphere would have to be moving at 50 m/s to get the 900 TW figure in Table 10.2 (p. 168), we would suspect a problem, and either distrust the 900 TW number—seeking another source to confirm—or re-evaluate our own understanding. If we could get to 900 TW by only having wind speeds of 0.1 m/s,[7] we would also have cause for skepticism. When crude estimates of this type land in the general vicinity[8] of a number we see in a table, we can at least be assured that the number is not outlandish, and that our basic understanding of the phenomenon is okay. Checking understanding against presented data is excellent practice!

[7]: . . . or requiring weeks rather than a day to re-establish, once sapped

[8]: . . . say, within a factor of ten

But we can't capture the *entire* atmospheric wind, because doing so would require wind turbines throughout the volume, up to 10 km high! In fact, some estimates [70] of practical global wind installations come in as low as 1 TW—well below our 18 TW demand. Wind alone is unlikely

[70]: Castro et al. (2011), "Global Wind Power Potential: Physical and Technological Limits"

able to replace the energy currently derived from fossil fuels.

12.2.1 Wind Turbines

To understand practically-available energy, we back up and consider how much air hits a wind turbine whose rotor diameter is R. Figure 12.3 illustrates the concept. If the wind speed is v, the air travels a distance $v\Delta t$ in time interval Δt.[9]

Figure 12.3: Wind power concept. In time interval Δt at wind speed v, a volume of air encounters the rotor having the shape of a cylinder of radius R and length $v\Delta t$. Note that most wind turbines are designed to pivot about a vertical axis to face into the wind, whatever the direction.
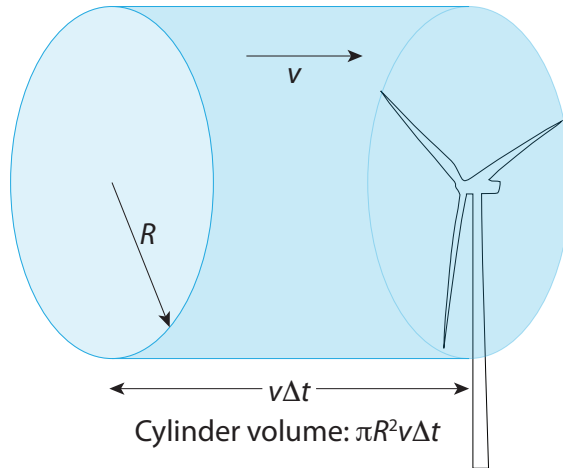
9: We can pick any value for Δt: a long interval makes a very long cylinder, while a small Δt results in a short, stubby cylinder. In the end, the value we chose for Δt will cancel out, so as not to matter.

The cross-sectional area of the wind turbine (rotor) is defined as the area swept out by the blades, so πR^2. Thus the volume of the cylinder of air interacting with the turbine over time interval Δt is the "base" (circular) area of the cylinder times its "height" (straight length, $v\Delta t$), or $V = \pi R^2 v\Delta t$. We know the density of the air,[10] so the mass of the cylinder is $m = \rho_{\text{air}} V = \rho_{\text{air}} \pi R^2 v\Delta t$. The kinetic energy *contained* in this cylinder of air is therefore K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}\rho_{\text{air}}\pi R^2 v^3 \Delta t$. Now let's get rid of that pesky Δt. Think for a moment what happens if we divide both sides by Δt: it will definitely get rid of the Δt on the right-hand-side, but what does the left-hand side mean: energy over time? Hopefully, this is familiar by now as the concept of power.

10: $\rho_{\text{air}} \approx 1.25 \, \text{kg/m}^3$

Table 12.1: Wind power scales as the cube of wind speed.

Speed	Power
0	0
1	1
2	8
3	27
4	64
5	125
⋮	⋮
10	1,000

Definition 12.2.1 *The power delivered by a wind turbine of radius R in wind speed v and operating at efficiency ε is*

$$P_{\text{wind}} = \frac{1}{2}\varepsilon\rho_{\text{air}}\pi R^2 v^3. \tag{12.2}$$

where $\rho_{\text{air}} \approx 1.25 \, \text{kg/m}^3$ at sea level. Efficiency has been inserted as ε and tends to be 40–50% for modern turbines.

Notice that the delivered power scales, sensibly, with the area of the wind turbine's blade path, but more importantly and perhaps surprisingly as the velocity *cubed* (Table 12.1). The cubed part should make you sit

up straight: that's a very strong function of velocity! It means that if the wind changes from a gentle 5 m/s to a brisk 10 m/s, the power available goes up by a factor of 8. A strong wind at 20 m/s has 64 times as much power as the 5 m/s breeze.[11] We can understand the three powers of velocity thusly: two powers come from kinetic energy, and one from the length of the cylinder. As wind speed increases, not only does the oncoming air have more kinetic energy per fixed volume, but also a larger volume encounters the turbine in a given time.

Setting $\varepsilon = 1$ in Eq. 12.2 corresponds to the *total* power present in the wind. But we can't be greedy and grab *all* of it. In fact, if we did, it would mean *stopping* the air at the wind turbine: pulling out all the kinetic energy means no velocity is left. If we did this, newly arriving air would divert around the stopped mass of air, and the turbine would no longer have access to oncoming energy. The theory has all been worked out:[12] a turbine is limited to $\varepsilon \le 16/27$ (59%) of the available energy, known as the Betz limit [72]. This is not a technological limitation, but comes from the physics of fluid flow. A second consideration enters for low-speed rotor motion, known as the Glauert limit [73], resulting in diminishing efficiency as wind speed drops.

11: Now it may be easier to understand why hurricanes can be so destructive, if their power scales as the cube of wind velocity, and velocities exceed 50 m/s.

12: A recent derivation is in [71].

[72]: Betz (1926), "Wind-Energie und ihre Ausnützung durch Windmühlen"

[73]: Glauert (1993), *The elements of airfoil and airscrew theory*
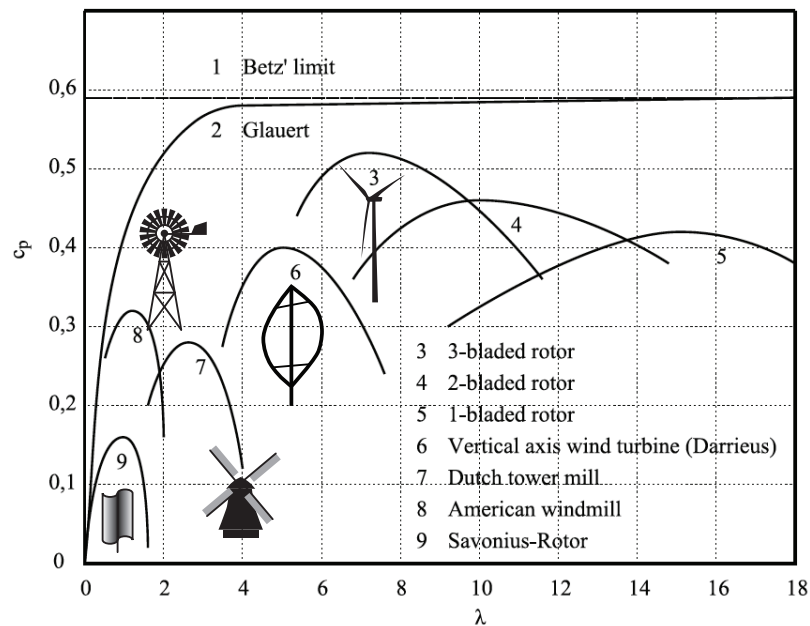
Figure 12.4: Theoretical and practical wind turbine efficiencies (ε, or c_P in the plot), for various designs. The parameter λ is the ratio of tip speed to wind speed: higher λ means a faster tip speed [74]. All designs must be below the Betz limit (horizontal line near top). At slower speeds, the Glauert limit confines performance to occupy the region to the right of the curve marked 2. Each of the 7 designs shown have arched curves, achieving maximum efficiency at a particular tip speed. Too slow, and the turbine is not transmitting much energy; too fast and drag/friction begins to dominate. Adapted from ©2010 WIT Press.

Figure 12.4 shows these theoretical limits, along with design limits from various rotor configurations. Curves reflect an optimum rotor speed for each design: speeding up produces more generator output until it gets fast enough that air drag on the blades starts to dominate. The most common modern turbine is the 3-blade design,[13] able to get roughly 50% of the energy out of the wind. Notice that the tip speed can be quite high: 6–8 times the wind speed. This can be quite alarming to birds in the area, whose cruising speed is nearer wind speed, and they

13: Not only is the three-blade design the most efficient, its lower tip speed is less dangerous than for 2 or 1-blade designs, according to Figure 12.4.

have never met something so fast before. For a modern derivation of the Betz limit and how efficiency depends on tip speed, see [71]. The largest turbines—having 150 m diameter rotors—are rated for up to 10 MW of electrical power production.

[71]: Ragheb et al. (2011), "Wind Turbines Theory - The Betz Equation and Optimal Rotor Tip Speed Ratio"

> **Example 12.2.1** How much power could you expect a small (4 m *diameter*) 3-blade wind turbine situated atop your house to deliver in a respectable 5 m/s breeze?
>
> The radius is 2 m and we'll pick a middle-of-the-road efficiency of 45%: $P = \frac{1}{2} \cdot 0.45 \cdot (1.25\,\text{kg/m}^3) \cdot \pi \cdot (2\,\text{m})^2 \cdot (5\,\text{m/s})^3$ comes to about 450 W.[14]

14: Not too impressive: hard to get much wind power on a household scale, although 10 m/s would give 3.6 kW.

Besides the limit on how much power can be pulled out of the air by a single turbine, we also find limits on how *densely* they may be populated in a given area: how much space is required between turbines so that one does not disrupt the other. Obviously, it would not serve to put one turbine directly behind another, as they would at best split the available power arriving as wind. Even side by side, it is best to leave room between windmills so that additional rows are not deprived of wind power. A rule of thumb is to separate turbines by at least 5–8 diameters side-to-side, and 7–15 diameters[15] along the (prevailing) wind direction. For the sake of illustration, Figure 12.5 shows a spacing on the denser side of the range, but otherwise we adopt the more recent recommendations and use 8 diameters side-to-side and 15 diameters deep [75]. This works out to a 0.65% "fill factor," meaning that 0.65% of the land area contains an associated rotor cross section.[16]

15: An older "rule of thumb" was 5 side-to-side and 7–8 deep, but newer work suggests as much as 8 diameters side-to-side and 15 diameters deep.

16: . . . one πR^2 rotor area for every $8D \times 15D = 120D^2 = 120 \times (2R)^2 = 480R^2$ of land area

Figure 12.5: Overhead view of wind farm turbine locations, for the case where separations are 10 rotor-diameters along the wind direction, and 5 rotor diameters in the cross-wind direction—a geometry that yields 1.6% area "fill factor." Current recommendations are for 15 and 8 rotor diameters, which is significantly more sparse than even this depiction, leading to 0.65% area fill. Note that most wind turbines can turn to face the wind direction, for times when its direction is not the prevailing one.

In order to compare to other forms of renewable energy, we can evaluate a power per unit land area (in W/m^2) by the following approach:

$$\frac{\text{power}}{\text{area}} = \frac{\frac{1}{2}\varepsilon\rho_{\text{air}}\pi R^2 v^3}{480R^2} = \frac{\pi}{960}\varepsilon\rho_{\text{air}}v^3, \qquad (12.3)$$

employing the rule-of-thumb 8×15 turbine placement scheme. Using an efficiency of 40% and $v = 5$ m/s,[17] we get 0.2 W/m^2—which is 1,000 times smaller than solar's $\sim$200 W/m^2 insolation (Ex. 10.3.1; p. 167).

17: Recall that this choice gave sensible global wind power estimates lining up with Table 10.2 (p. 168).

A final general note about wind generation is somewhat obvious: the

wind is not always blowing, and its speed varies over wide ranges. In this sense, wind is an *intermittent* power source. Just as for hydroelectric installations, wind resources are characterized by a regionally-dependent capacity factor, which is the ratio of energy delivered to what would have been delivered if the generation facility operated at full capacity at all times. Typical capacity factors for wind in the U.S.[18] are around 33%, and Figure 12.6 provides a visual sense for how this manifests in the real world: pretty erratic.

Figure 12.6: One month of wind generation from a 20 MW wind farm, illustrating the intermittent nature and why capacity factors are low [76]. The facility saturates at maximum power late in the month, self-limiting to avoid damage to the turbines. ©2010 Springer.
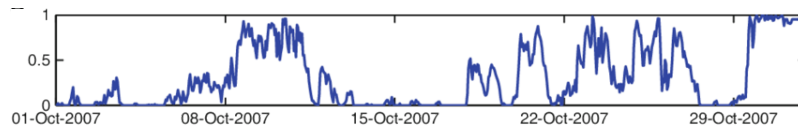
For very low wind speeds,[19] wind turbines do not have enough wind to turn at all and sit still at zero output. Furthermore, a turbine is rated at some maximum power output, which occurs at some moderately high wind speed,[20] beyond which the generator risks damage—like "redlining" a car's engine. When the wind climbs above this maximum-rated speed, the turbine is pegged at its maximum power—no longer following a v^3 relation—and deliberately twists its blades[21] to be *less* efficient as the wind speed grows so that it maintains constant (maximum) power output. When the wind speed becomes large enough to endanger the turbine, it will twist its blades parallel to the wind to allow the air to pass without turning the rotor at all, so that it no longer spins while it "rides out" the high winds.[22]

Figure 12.7: Actual data (thickly-clustered black circles) of power delivered by a turbine rated at 2 MW, as a function of wind velocity. The red curve represents the theoretical Betz limit of 59%, appearing as a cubic function of velocity—as Eq. 12.2 dictates. The better-matching blue curve corresponds to an overall efficiency $\varepsilon = c_p = 0.44$ (44%), and the green curve—which rolls over from the cubic function and saturates at higher velocities—is the manufacturer's expectation for the unit [77]. The "cut-in" velocity for this turbine is around 3.5 m/s: note the small step up from zero output in the green curve. This turbine saturates around 12 m/s: the green curve flattens out and no black circles appear above the cutoff. From ©2017 Wiley.

Figure 12.7 shows a typical power curve for a 2 MW turbine, on top of which are drawn a cubic function of velocity at the theoretical Betz limit (red curve), a cubic (blue) at 44% efficiency ($\varepsilon = 0.44$), and the green manufacturer's curve [77]. Notice that the turbine performance

[77]: Fischer et al. (2017), "Statistical learning for windpower: a modeling and stability study towards forecasting"

demonstrates the aspects covered in the previous paragraph: "cutting in" just above 3 m/s and maxing out (saturating) beyond about 12 m/s. In between, it closely follows a cubic function at an overall efficiency of 44% (blue curve).

12.3 Wind Installations

Global wind installations are rising rapidly, currently (as of 2020) above 600 GW of installed capacity.[23] Table 12.2 lists the major players, in terms of installed capacity, average generation, fraction of total energy,[24] capacity factor, and share of global wind generation. The amount of wind energy in each country depends on a combination of how much wind is available in the country, how fast electricity demand is growing, electrical infrastructure, and political interest in renewable energies.

23: A small fraction of this is realized, due to the capacity factor.

24: To compare wind to total energy, we follow the thermal equivalent convention discussed for Table 10.3 (p. 170)

Table 12.2: Global wind installations in 2018 [78–84]. The top six countries capture 85% of the global total.

Country	GW installed	GW average	cap. fac. (%)	energy fraction (%)	global share (%)
China	184	41.8	22.7	3.0	33
U.S.	97	31.4	32.4	2.7	25
Germany	59	12.7	21.4	8.3	10
India	35	6.5	18.5	2.3	5.2
Spain	23	5.4	23.5	8.3	4.3
UK	21.7	6.5	30.0	6.9	5.2
World Total	592	~ 125	21.1	2.0	100
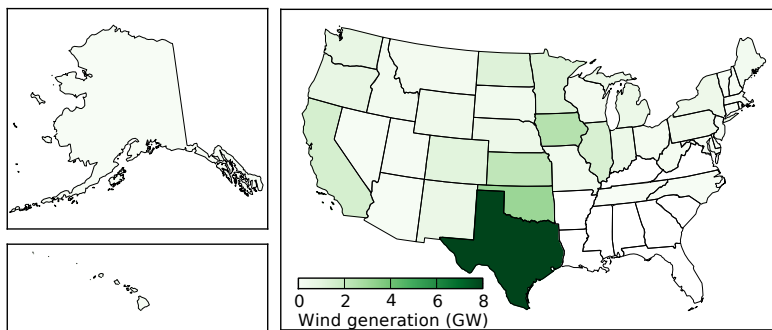
Figure 12.8: Wind power by state, in terms of average generation, in GW, in 2018. The color scale may seem unhelpful, but the unavoidable truth is that many states don't have a lot going on, and Texas is so dominant as to render other states almost insignificant. A logarithmic color scale could help, but then the important lesson on the gross disparity might go unappreciated.

In 2018, the U.S. had about 94 GW of *installed* wind capability.[25] This number has recently surpassed hydroelectric installed capacity (about 80 GW). Both are impacted by capacity factors, which for wind averages 33% in the U.S., while hydropower is just over 40%. The net effect is that the generation for the two is pretty comparable.[26] Where is the wind power in the U.S. installed? Figure 12.8 shows that Texas wins, at 8.7 GW. Oklahoma is a distant second at 3.2 GW, Iowa at 2.5 GW. California is in fifth place at 1.6 GW.

25: From tables 1.14.B and 6.2.B in [85]

26: . . . as we also saw in Table 10.3 (p. 170)
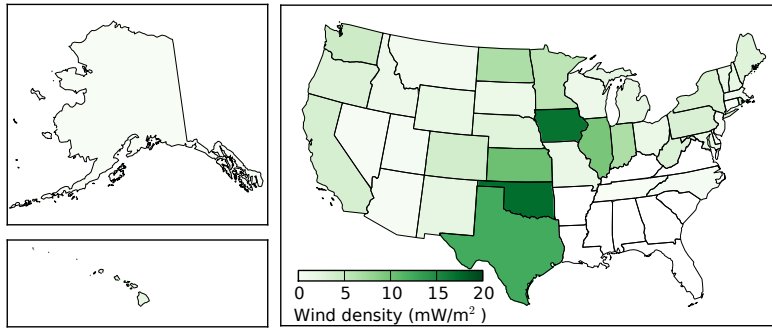
Figure 12.9: Average wind power by state, divided by state area to indicate a density of the developed resource, in milliwatts per square meter (based on 2018 data). We might expect some resemblance to Figure 12.2, based on where the resource is most favorable.

Following the flow we used in Sec. 11.3 (p. 178), we show wind generation as a function of area (Figure 12.9), to get a sense for how concentrated the installations are. Oklahoma and Iowa jump in front of Texas by this measure. Texas has more total generation than all others, but is a very large state in terms of area. Iowa, for instance produces about 30% as much wind power as Texas, but at only 20% the size. The numbers, reaching about 0.017 W/m^2 are a bit smaller than those for hydroelectricity, where two states exceeded this value.[27] We can compare these numbers to the 0.2 W/m^2 *fully-developed* potential we estimated in the paragraph following Eq. 12.3 to conclude that in principle we could expand wind by a large factor.[28]

27: . . . 0.05 W/m^2 in Washington; followed by New York at 0.02 W/m^2

28: . . . about 10 times, in the case of Iowa, but bear in mind the 0.2 W/m^2 estimate was based on 5 m/s, while Iowa scores a bit higher, according to Figure 12.2
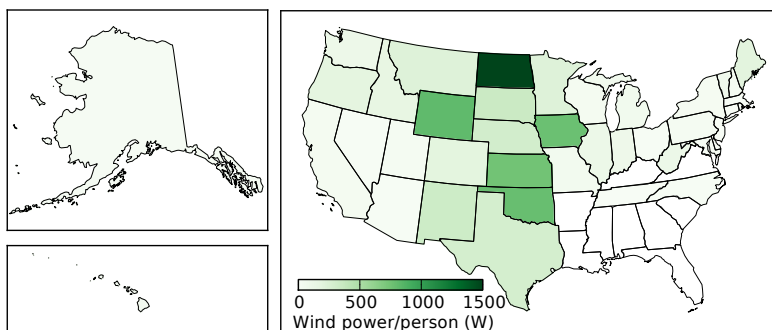
Figure 12.10: Average wind power generation by state divided by state population for an average power per person (based on 2018 data).

Next, we look at wind generation per capita in states, in Figure 12.10. Now North Dakota blows away the rest, at 1.6 kW per person, followed by four states at about half of this value. We can put this in context by noting that the average power consumption in the U.S. per capita is around 10 kW.

Finally, Figure 12.11 shows wind capacity factors, indicating where the wind is most reliable. It peaks around 41% in Kansas, but all of the plains states in general do well. The southeastern U.S. has almost no wind development,[29] as is evident in any one of these figures.

29: Lack of wind makes it a poor fit: see also Figure 12.2.
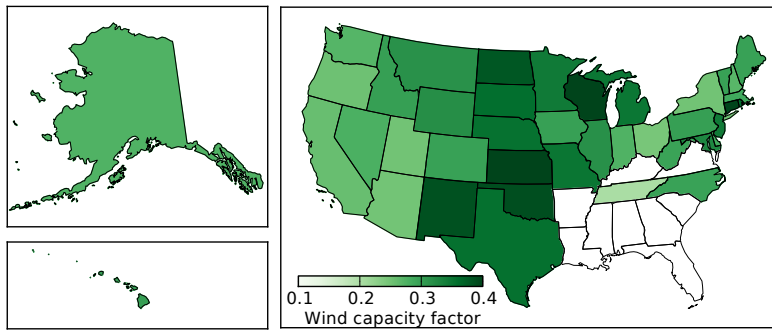
Figure 12.11: Capacity factor for wind installations by state (based on 2018 data).

12.4 Upshot: Wind is not Overblown

Wind has surged tremendously in the last decade (Fig. 7.5; p. 108), proving to be an economically viable and competitive resource. But how much could we expect to get from wind?

Putting a few of the previous results together, If the entire contiguous U.S. (area $\sim10^{13}$ m^2) were developed for wind at an estimated power density of 0.2 W/m^2—which was based on a 5 m/s average wind speed—and a capacity factor of 33%, the U.S. could theoretically produce 0.7 TW[30] from wind—roughly 20 times what is produced today. We should take this crude estimate as an extreme upper end, since it is inconceivable that we would develop wind so fully as to never be more than a few hundred meters—a few rotor diameters—away from a wind turbine, no matter where we go. Also, many areas are sub-threshold and would not support investment in wind development.

Even so, the inflated 0.7 TW estimate falls short of the current 3.3 TW energy demand in the U.S., has major intermittency problems, and is not in a form that can be well-used in all sectors, like transportation and industrial processing. While wind *alone* cannot replace fossil fuels at the current level of demand, it *can* doubtless be a significant contributor.

Globally, estimates for wind potential tend to be in the few-terawatt range, though can be as low as 1 TW for a number of practical reasons [70]. As was the case for hydroelectricity, wind is a viable player in the renewables mix, but is unable to shoulder the entire load.

Wind energy is not free of environmental concerns, disturbing landscapes and habitats. Its impact on birds[31] and bats is most worrisome, as the rotors move far faster than anything to which the wildlife is habituated. Still, compared to the environmental toll from fossil fuels, it is fairly clean—similar to the impact of hydroelectric power.

A pros and cons list will help summarize. First, the positive attributes:

▶ Wind is replenished on the planet daily by solar illumination;

30: ⚠ We're fudging things a bit here for the sake of simplicity. If turbines are built for 12 m/s, the capacity factor already has some averaging built in, so using 5 m/s *and* a capacity factor of 0.33 is unfair. On the other hand, much of the country spends substantial time below the cut-in speed for turbines, and the cruel cubic function of velocity greatly suppresses much of the land area as impractical for wind development. So the approach is a compromise that might balance out reasonably.

[70]: Castro et al. (2011), "Global Wind Power Potential: Physical and Technological Limits"

31: Domestic cats turn out to kill far more birds than wind turbines do, currently.

▶ Harnessing wind is relatively low-tech and straightforward;

▶ Wind has decent efficiency—typically 40–50%—in extracting energy from the oncoming wind;

▶ Life-cycle CO_2 emissions for wind is only 2% that of traditional fossil fuel electricity [68];

▶ Growth in the wind sector points to economic viability;

▶ Wind is able to scale up to cover a meaningful fraction of energy demand.

[68]: (2020), *Life Cycle GHG Emissions*

And the downsides:

▶ Wind is intermittent: power when nature allows, not when people demand;

▶ Wind is regionally variable: many places do not produce enough wind to support development;

▶ Wind can cause environmental disruption to habitats—especially dangerous to birds and bats;

▶ Esthetic objections to noise and degradation of scenery hamper expansion.

12.5 Problems

1. A modest slap[32] might consist of about 1 kg of mass moving at 2 m/s. How much kinetic energy is this?

32: ... how painful can a few Joules be?

2. A *hard* slap might consist of about 1 kg of mass moving at 10 m/s. How much kinetic energy is this, and how much warmer would 10 g of skin[33] get if the skin has the heat capacity properties of water, as in the definition of a calorie (Sec. 5.5; p. 73 and Sec. 6.2; p. 85 are relevant)?

33: ... corresponding to a volume of 10 mL appropriate to a slap area of 10 cm by 10 cm and to a depth of 1 mm

3. A 10 kg bowling ball falls from a height of 5 m. Using the convenient $g \approx 10\,\text{m/s}^2$, how much gravitational potential energy does it have? Just before it hits the ground, all of this potential energy has gone into kinetic energy.[34] What is the speed of the bowling ball when it reaches the ground, based on kinetic energy?

34: ... neglecting any energy flow to air resistance

4. Did the final answer for the speed of the bowling ball at the end of its drop depend on the mass?[35] Write out the math symbolically[36] and solve for velocity, v. Does the result depend on mass?

35: Try it using a different mass.

36: ... using variables/symbols

5. Thermal energy is just randomized kinetic energy on a microscopic scale. To gain some insight into this, consider one liter (1 kg) of water, and figure out how much energy it would take to heat it from absolute zero temperature[37] to 300 K assuming that the definition of the calorie (Sec. 5.5; p. 73) applies across this entire range. If this same amount of energy went into kinetic energy—hurling the water across the room—what would the corresponding velocity be?

37: ... 0 K, when the kinetic energy is effectively frozen out, or stopped

ⓘ As large as the number is, it is representative of the speeds of individual molecules within the water, and is, not coincidentally, similar to the speed of sound in water.

6. A typical house may have a floor area around 150 m² (1,600 square feet). If the floor–to-ceiling distance is typically 2.5 m, how much mass is in the air within the house? Could you lift this much mass if handed to you as bags of rocks?

7. Atmospheric pressure is about 10^5 N/m², meaning that a 100,000 N weight of air—corresponding to a mass of 10,000 kg—sits atop very square meter of the ground (at or near sea level). If the air density were constant at 1.25 kg/m³—rather than decreasing with height as it actually does—how high would the atmosphere extend to result in this weight (mass)?

ⓘ This is called the *scale height* of the atmosphere, h_0, which you may wish to compare to the tallest mountains on Earth or the height at which airplanes fly. The actual density of the atmosphere decreases exponentially as a function of height, with h_0 being the characteristic scale.

8. Comparing the pale green region in the southeastern U.S. to the purple region of the plains states in Figure 12.2, how much more power would we expect out of the same rotor placed in the plains than in the southeast (by what factor is it bigger in the plains)?

9. How much more powerful is a hurricane-strength wind of 50 m/s hitting your house than is a light breeze of 5 m/s?

10. How much *power* would a moderate-sized 50%–efficient wind turbine produce whose radius is 10 m at wind speeds of 5 m/s, 10 m/s, 15 m/s, and 20 m/s? Express the answers in kW or MW, depending on what is most natural.

11. The Betz limit says that we get to keep no more than 59% of the available wind power. If 59% of the kinetic energy in a lump of air moving at speed [38] v is removed, how fast is it going afterwards,[39] as a fraction of the original speed?

38: Make up your own velocity or solve in symbols/variables: same either way.

39: This relates to wind speed just behind a wind turbine

12. The largest wind turbines have rotor *diameters*[40] around 150 m. Using a sensible efficiency of 50%, what power does such a jumbo turbine deliver at a maximum design wind speed of 13 m/s?

40: ⚠ not radius

13. A recent news article announces the largest wind turbine yet, measuring 220 m in *diameter* and having a maximum power output of 13 MW. Using a reasonable efficiency, calculate the velocity of the wind at which maximum power is reached.

14. Compare the tip speed of a three-blade turbine operating at its optimal efficiency (as per Figure 12.4) in a moderate wind of 7 m/s to typical freeway driving speeds in the same units.

15. Traveling down the road, you carefully watch a three-bladed wind turbine, determining that it takes two seconds to make a full revolution. Assuming it's operating near the peak of its efficiency curve[41] according to Figure 12.4, how fast do you infer the wind speed to be if the blade length[42] appears to be 15 m long?

41: Hint: focus on tip speed.

42: …corresponding to radius of the rotor

Reflect on the fact that just estimating the rotor blade length and timing its revolution is enough for you to produce an estimate of power being generated!

16. Building from the result in Problem 15, how much power is this windmill delivering if its efficiency is about 50%?

17. In a way similar to Figure 12.5, replicate the statement in the text that the fraction of land covered per rotor area is 0.65% if turbines are separated by 15 rotor diameters along one direction and 8 rotor diameters along the cross direction.

18. Check that the units of Eq. 12.3[43] indeed are equivalent to Watts per square meter (W/m^2).

19. Provide a clear explanation of why the area under the blue curve in Figure 12.6 compared to the area of the whole rectangular box is an appropriate way to assess the capacity factor of the depicted wind farm?

20. What capacity factor would you estimate for the wind farm performance depicted in Figure 12.6? In other words, what is the approximate area under the curve compared to the entire box area, as explored in Problem 19? An approximate answer is fine.

21. Referring to Figure 12.7, examine performance at 5 m/s and at 10 m/s, picking a representative power for each in the middle of the cluster of black points, and assigning a power value from the left-hand axis. What is the ratio of power values you read off the plot, and how does this compare to theoretical expectations for the ratio going like the cube of velocity?

22. Figure 12.7 surprisingly has all the information required to deduce the rotor diameter! The turbine appears to produce 1,400 kW when the wind velocity is 10 m/s, and we also know it appears to operate at $\varepsilon = 0.44$. What is the rotor diameter?

23. Considering that wind turbines are rated for the maximum-tolerable wind speed around 12 m/s, and tend to operate at about 30% capacity factor, how much *average* power[44] would a 100 m diameter turbine operating at 45% efficiency be expected to produce?

24. Table 12.2 shows Germany having more than twice the wind capability as Spain, yet each gets 8.3% of its power from wind. What do you infer the difference to be between the countries?

Hint: no external research necessary: what do the numbers mean?
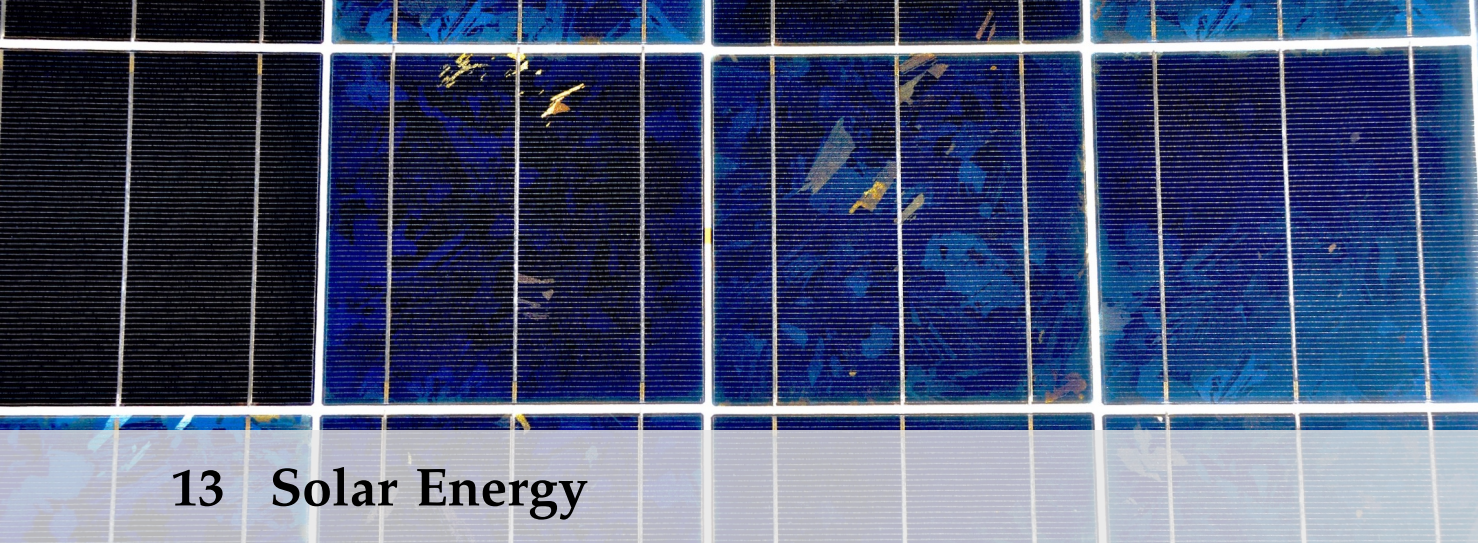
13 Solar Energy

Now we come to the main attraction. As we saw in Chapter 10, all of the strictly renewable energy options are ultimately derived from the sun. The two resources of the last two chapters—hydroelectricity and wind—represent tiny crumbs of the overall solar input to the planet.

Practically speaking, it is difficult to concoct ways to harness more than a few terawatts of power from hydroelectric or wind-based resources. Similar limitations apply to biologically-derived energy, geothermal, tidal, ocean currents, wave energy, etc. This may be worrisome considering that human society currently demands 18 TW. Meanwhile, the sun delivers 83,000 TW to the earth's surface (Fig. 10.1; p. 167, Table 10.2; p. 168). That's almost 5,000 times more than the demand. By the numbers, then, the sun seems to offer all we might ever need. In fact, the quantitative imbalance is so extreme as to make one wonder why we would ever mess around doing anything else.

Yet at present, the U.S. gets only about 0.9% of its energy from solar power, or 2.3% of its electricity. Similarly for the world, 1.2% of global energy is solar (2.1% of the electricity). It would seem to be vastly underutilized.

This chapter explains the nature of solar energy, its potential for use, practical considerations, and looks at the state of installations. While most of the focus is on photovoltaic (PV) panels that directly generate electricity from light, solar thermal power generation is also covered.

13.1 The Energy of Light

Sec. 5.10 (p. 79) introduced the basics of the energy of light. This section acts as a refresher and lays the foundation for the rest of the chapter.

Photovoltaic cells, showing grid contacts and crystal domains. Photo Credit: Tom Murphy

Light—a form of electromagnetic radiation—is composed of photons—individual particles of energy each having a characteristic wavelength—what we might call color. Photons are such tiny quanta of energy that familiar environments are awash in unfathomably large numbers of them. A typical light bulb, for instance, emits quintillions[1] of photons every second.

1: A quintillion is 10^{18}.

Definition 13.1.1 *The **energy of a single photon**, in various forms, is*

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda} \approx \frac{2 \times 10^{-19}\,\text{J}}{\lambda(\text{in }\mu\text{m})} = \frac{1.24\,\text{eV}}{\lambda(\text{in }\mu\text{m})}, \qquad (13.1)$$

where $h = 6.626 \times 10^{-34}\,\text{J} \cdot \text{s}$ (Planck's constant), and ν is the frequency of the light in Hertz (Hz, or inverse seconds).

The second form (hc/λ) is useful, as we more commonly characterize the "color" of light by its wavelength, λ. The speed of light, $c \approx 3 \times 10^8$ m/s, connects frequency to wavelength via

$$\lambda\nu = c. \qquad (13.2)$$

The third form in Definition 13.1.1 makes it easy to compute photon energy in Joules given the wavelength in microns.[2] Visible light has a wavelength around 0.4–0.7 μm (violet–to–red), so a typical photon energy, at 0.5 μm, is about 4×10^{-19} J. It's a *tiny* number!

2: One micron, or micro-meter, abbreviated μm, is 10^{-6} m.

Example 13.1.1 About how many photons strike a 0.4 m^2 patch of sidewalk per second if the overhead sun is delivering 1,000 W/m^2?

For the visible light characteristic of sunlight, we can use a convenient wavelength of 0.5 μm, amounting to 4×10^{-19} J of energy per photon. The patch of sidewalk we describe receives light energy at a rate of 400 W or 400 J/s.[3] How many 4×10^{-19} J photons does it take to amount to 400 J? Divide[4] to get 10^{21}.

3: . . . 0.4 m^2 times 1,000 W/m^2

4: . . . or try reasoning it out: 10^{19} of them would make 4 J, so we need 100× more

The final form in Definition 13.1.1 relates to the fact that photons frequently interact with electrons as we will see in Section 13.3, making it convenient to convert to another energy unit called the electron-volt, or eV (introduced in Sec. 5.9; p. 78). One electron volt is the energy it takes to move an electron through an electric potential of one Volt. The conversion is $1\,\text{eV} = 1.602 \times 10^{-19}$ J. For instance, the 0.5 μm (blue-green) photon we used in the previous example would have an energy around 2.5 eV.

Why should we care about unthinkably small quantities of light? Three reasons come to mind:

1. Eq. 13.1 elucidates that bluer[5] photons have higher energy than red photons, which is important to know;

5: . . . shorter wavelength

2. Individual photons interact with matter at the microscopic scale and are relevant to understanding solar photovoltaics and photosynthesis;
3. It's how nature really works.

13.2 The Planck Spectrum

We should first understand where photons originate, which will help us understand how solar panels work and their limitations. Until recent technological advances, photons tended to come from *thermal* sources. It's true for the white-hot sun,[6] and true for flame and incandescent light bulb filaments.[7] Likewise, hot coals, electrical heating elements, and lava are all seen to glow. Physics tells us how such hot sources radiate, as covered by the next three equations. The first (with units) is:

$$P = A\sigma T^4 \text{ (W)}. \tag{13.3}$$

We already saw this equation in the context of Earth's energy balance in Sections 1.3 and 9.2. It is called the Stefan–Boltzmann law, describing the total power (in W, or J/s) emitted from a surface whose area is A (in square meters) and temperature, T in Kelvin.[8] The constant, $\sigma \approx 5.67 \times 10^{-8}$ W/m^2/K^4 is called the Stefan–Boltzmann constant, and is easy to remember as 5-6-7-8.[9]

$$B_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \left(\frac{\text{W/m}^2}{\text{m}}\right), \tag{13.4}$$

Eq. 13.4 might look formidable, but only λ and T are variable. It describes the Planck spectrum, otherwise known as the blackbody[10] spectrum. For some temperature, T, this function specifies how much power is emitted at each wavelength, λ. Three fundamental physical constants from key areas of physics make an appearance: $c \approx 3 \times 10^8$ m/s is the familiar speed of light from relativity; $h \approx 6.626 \times 10^{-34}$ J · s is Planck's constant from quantum mechanics, and $k_B \approx 1.38 \times 10^{-33}$ J · K is the Boltzmann constant of thermodynamics.[11]

$$\lambda_{\text{max}} \approx \frac{2.898 \times 10^{-3}}{T(\text{in K})} \text{ (m)}. \tag{13.5}$$

Eq. 13.5 is called the Wien law and is a numerical solution identifying the peak of the blackbody spectrum as a function of temperature. Higher temperatures mean higher kinetic energies at a microscopic scale, so that higher-energy (shorter-wavelength) photons can be produced. This is why as objects get hotter, they move from red to white, and eventually to a blue tint.

All this may seem overwhelming, but take a breath, then just look at Figure 13.1. Everything so far in this section is captured by Figure 13.1.

6: . . . and thus stars and even the moon, which is just reflected sunlight

7: Modern lighting like fluorescent and LED sources rely on manipulating energy levels of electrons within atoms and crystals.

8: Recall that temperature in Kelvin is the temperature in Celsius plus 273 (273.15, technically).

9: The Stefan–Boltzmann constant is actually a witch's brew of more fundamental constants h (Planck's constant), c (speed of light), and k_B (the Boltzmann constant) as $\sigma = 2\pi^5 k_B^4/(15c^2 h^3)$.

10: The term blackbody effectively means a perfect emitter and absorber of thermal radiation.

11: This last one may be more familiar to students in its chemistry form of the gas constant $R = k_B N_A \approx 8.31$ J/K/mol, where $N_A \approx 6.022 \times 10^{23}$ is Avogadro's number.
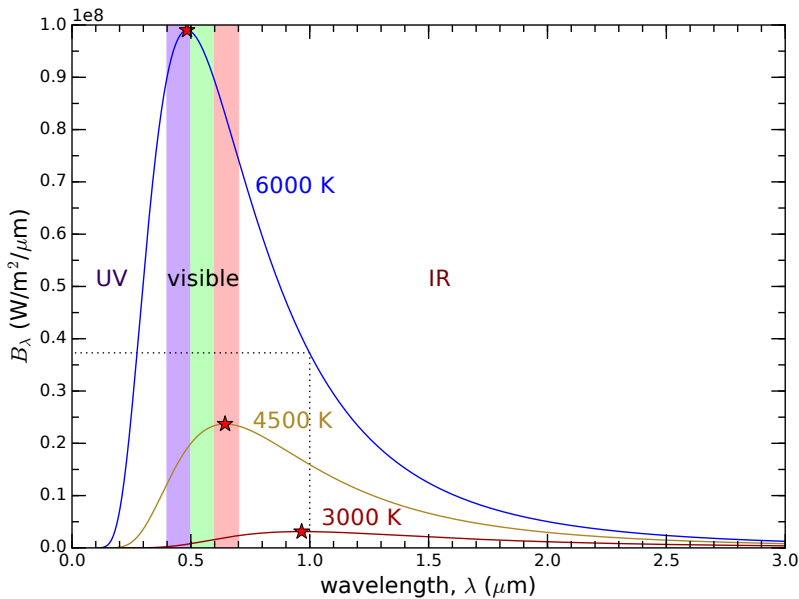
Figure 13.1: Planck spectra, or blackbody spectra for three temperatures, indicating where the ultraviolet, visible, and infrared regions lie. The shapes of the three curves (spectra) are described by Eq. 13.4, the star locations are found by Eq. 13.5, and the total power radiated, per square meter of surface is the area under each curve, as captured in Eq. 13.3. The dotted line relates to Example 13.2.2. Note the 1e8 factor on the vertical axis, meaning that the axis goes from 0 to 1.0×10^8 W/m²/μm.

The shape of each spectrum is a plot of the function in Eq. 13.4 for three different temperatures. If comparing output of Eq. 13.4 to Figure 13.1, be aware that the units have been manipulated to favor microns over meters.[12]

12: Eq. 13.4 uses units of meters for λ, but Figure 13.1 uses microns (μm, or 10^{-6} m) for convenience. Also, Eq. 13.4 delivers an answer in units of W/m² per *meter* of wavelength, but for the plot we divided by 10^6 so it would be W/m² per *micron* of wavelength. By taking care of this detail, the area under each curve in Figure 13.1 should match σT^4 as in Eq. 13.3.

Let's come at this again with numbers to help us make sense of things. Looking at the curve (spectrum) for 6,000 K, we will verify that each equation makes some sense.

Example 13.2.1 First, Eq. 13.5 says that the wavelength where emission peaks should be about $2.898 \times 10^{-3}/6000 \approx 0.483 \times 10^{-6}$ m, or 0.483 μm.

Now look at the graph to see that the peak of the blue curve is indeed just short of 0.5 μm, denoted by the red star at the top.

Example 13.2.2 Let's now verify a point on the Planck spectrum, picking 6,000 K and 1 μm to see if Eq. 13.4 lands in the same spot as indicated in Figure 13.1.

If we go through the laborious exercise of plugging in numbers to Eq. 13.4 for $T = 6000$ and $\lambda = 1 \times 10^{-6}$ (1 μm), we find[13] the overall outcome is 3.73×10^{13} W/m² per *meter* of wavelength. Once we adjust by 10^{-6} for the units on the plot (see earlier margin note), we expect 0.373×10^8 W/m² per micron.

13: Numerically, the numerator is 3.74×10^{-16}, the denominator is 10^{-30}, and the argument in the exponential is 2.4, so that the second fraction is 0.1.

Indeed, the blue curve passes through this value at a wavelength of 1 μm, as indicated by the dotted line in Figure 13.1.

Example 13.2.3 Finally, to assess Eq. 13.3, we can crudely estimate the area under the blue curve by drawing a rectangle that we think has about the same total area. We put the top of the rectangle at the top of the blue curve and ask how wide it would need to be to approximately match the area under the blue curve.

If we make it 0.5 μm wide, it seems too thin: the area is smaller than what's under the blue curve. 1.0 μm wide seems like too much area. So we pick something in the middle like 0.75 μm (Figure 13.2).

This choice makes the area about 1.0×10^8 W/m^2/μm (value at the top of the rectangle) times 0.75 μm, coming to 7.5×10^7 W/m^2. Since this is power per area, we make a minor rearrangement of Eq. 13.3 to $P/A = \sigma T^4$ and evaluate for $T = 6000$ K to find 7.35×10^7 W/m^2. Hey, not bad! So it all hangs together.

Figure 13.2: Attempts to crudely match the area under the 6000 K Planck spectrum using rectangles of three different widths. The widest (1 μm; dashed line) is too much area; the narrowest (0.5 μm; dotted line) is too small. The middle (0.75 μm; pink area) seems closest, by eye.

Now that we've batted the equations around a little bit, like a cat might do if given a new mouse toy, let's absorb some key takeaways. First, as the source gets hotter, the area under the curve[14] increases *drastically*—as the *fourth-power* of T, according to Eq. 13.3. This is seen in Figure 13.1 in that going from 3,000 K to 6,000 K[15] leads to a *tremendous* increase in area under the curve: 16 times, in fact.

Second, as an object gets hotter, it emits at shorter wavelengths, going from red-hot to yellow-hot to white-hot. The sun, at 5,800 K, peaks at 0.5 μm, in the blue-green region. We don't see it as blue-green because it emits plenty of red light as well, making a *blend*. Notice how well the 6,000 K spectrum in Figure 13.1 covers the visible colors. A cooler star at 3,000 K has a red tint to it, since the 3,000 K spectrum (Figure 13.3) shows a marked red-heavy tilt: blue is not as well represented as red is. Conversely, a hot star at 10,000 K will have a blue tint to it, since it peaks around 0.3μm and has considerably more flux at the blue end of the spectrum than at the red end.

Finally, it is worth absorbing the overall lesson that photons from a glowing source emerge as a *distribution*, spanning a wide range of wavelengths (colors). This will turn out to be important in understanding why solar panels have the efficiencies that they do.

> Follow along using Figure 13.1.

14: . . . total power emitted
15: . . . doubling temperature
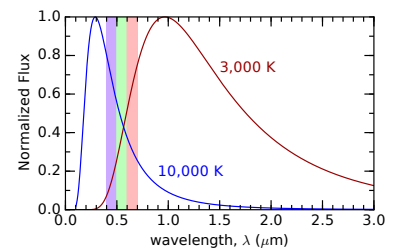
Figure 13.3: A 10,000 K star (or any black-body) has a spectrum that tilts blue in the visible spectrum, while a cooler star (object) at 3,000 K has a red slant. Spectra are normalized to the same peak for easier comparison.

13.3 Photovoltaics

We are now prepared to dig into how photovoltaic (PV) panels actually work, and what governs panel efficiency.[16] The word "photovoltaic" can be loosely read as: volts from photons, or electricity from light.

Various materials are used as the principal component in PV panels, but the vast majority are made of high-purity silicon, so we will speak in these terms alone. The basic physics will be the same for other materials as well. Getting too far into the weeds in describing the semiconductor

16: Solar thermal generation will be covered in Section 13.8.

physics is outside the scope of this course, but it is worth painting a general picture—enough to appreciate how much we can expect to get out of a PV panel.
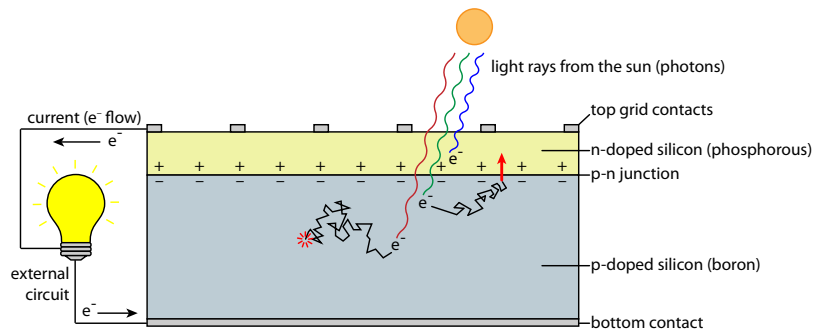
Figure 13.4 illustrates the basic scheme. The underlying idea is that adjoining two slabs of silicon into which small amounts of two kinds of impurities have been deliberately introduced[17] that either contribute extra electrons (*n*-type, for negative charge donors) or create vacancies for electrons (*p*-type, for effective positive charge donors). Putting *p*-doped and *n*-doped materials together creates a junction[18] exhibiting a *contact potential*. The result is that "donated" electrons right at the junction in the *n*-doped material decide to relocate across the junction to vacancies in the *p*-doped material, creating a wall of negative charge on the *p*-side of the junction and leaving behind "holes" (missing electrons) effectively creating positive charges[19] on the *n*-side of the junction. In the region right around the junction[20] an electric field is set up between the separated charges. *Any electron wandering into this depletion region will be swept across the junction*, across the contact potential, and will contribute to a current that is then driven around the external circuit to return to its *p*-side home.[21] Figure 13.4 shows additional salient features that will be pointed out as the story develops below.

17: ... either during or after the semiconductor crystal growth; a process called "doping"

18: So-called *p*–*n* junctions form the basis of diodes and transistors.

19: When a (negatively-charged) electron leaves an otherwise neutral medium, the medium becomes more positively charged.

20: ... called the "depletion region," as electrons have been depleted from the portion of the *n*-side adjacent to the junction

21: Once it is "home," the electron will fill a vacancy created by a sun-liberated electron to end the journey.

13.3.1 Theoretical Efficiency of Photovoltaics

We will now follow the fate of one photon as it encounters the photovoltaic material. Doing so will expose the physical process of photovoltaics and simultaneously track losses to elucidate efficiency expectations.

The basic scheme is that a photon knocks an electron away from an atom in the PV cell, and this electron has some chance of being swept across the junction upon which it contributes to a useful current.[22] The goal is to get an electron across the line. It is not unlike some sports where crossing a line is the goal, but many factors are lined up defending against successful attainment of this goal. Efficiency is related to the chance that a photon will produce a "win."

22: Current is just flow of charge, and in this case is just movement of electrons through the external circuit.

A photon leaves the hot solar surface aimed right at a PV panel on Earth. The photon can be any "color," distributed according to the Planck spectrum[23] in Figure 13.1. The most probable wavelength for a 5,800 K blackbody—according to Eq. 13.5—is ~0.5 μm, but it could reasonably be anywhere from 0.2–3 μm. The atmosphere will knock out (absorb or scatter) most of the ultraviolet light before it reaches the panel, and some of the infrared light is absorbed in the atmosphere as well. But almost 75% of the energy[24] makes it to the panel. What happens next depends on the wavelength.

First, we must understand something about the silicon material. The atoms in a typical silicon PV cell are arranged in an orderly lattice, grown as a single crystal. Expensive panels have mono-crystalline silicon, meaning that each 15 cm square cell comprising the panel is a thin slice of one giant crystal. Less expensive poly-crystalline (or multi-crystalline) panels have cells that are a patchwork[25] of randomly-oriented crystals at the millimeter to centimeter scale. But microscopically, both types are orderly crystals. Silicon has four electrons in its valence shell (outermost shell), so that a "happy" silicon atom is home to a four-outer-electron family. These electrons are said to exist in the valence band.[26] But provided a sufficient energy kick, an electron can leave home and enter the conduction band,[27] where it can freely move through the crystal and can potentially contribute to an electric current, if it finds the junction. The threshold energy level to promote an electron from the valence to the conduction band is called the band gap,[28] which for silicon is 1.1 eV $(1.8 \times 10^{-19}$ J).

Infrared photons at a wavelength of $\lambda > 1.1\,\mu$m have an energy of $E < 1.1$ eV,[29] according to Eq. 13.1. The energy falls below the band gap of silicon, and as such is not capable of promoting an electron within the silicon from the valence band to the conduction band. These longer-wavelength photons sail right through the silicon crystal as if it were transparent glass. Since these photons are not absorbed, the part of the incident energy in the infrared beyond 1.1 μm is lost. For the solar spectrum, this amounts to 23%, and is portrayed in Figure 13.5.

For the 77% of sunlight whose photons *are* energetic enough to bump an electron into the conduction band,[30] it's game-on, right? Well, not so fast—literally. Photons whose energy is *higher* than 1.1 eV have more energy than is needed to lift the electron into the conduction band. It only takes 1.1 eV to promote the electron, so a blue-green photon at 0.5 μm (~ 2.5 eV) has an excess of about 1.4 eV. The lucky electron is not just lifted out, but is given a huge boost in the process, rocketing out of the atom. It's going too fast! It knocks into atoms in the crystal and generally shakes things up a bit before settling down. We call this heat, or thermal energy:[31] its excess kinetic energy is transferred to vibrations (randomized kinetic energy of atoms) in the crystal lattice. The blue curve in Figure 13.5 reflects this loss: we get to keep all the energy at 1.1 μm (1.1 eV), thus the blue curve joins the overall black curve here.

23: The spectrum can be thought of as a probability distribution for photon wavelength, if picking out one photon.

24: This is roughly 1,000 W/m² out of the 1,360 W/m² incident at the top of the atmosphere (the solar constant, which will be derived in Section 13.4).

25: See banner image for this chapter on page 197.

26: The term "band" is used to describe energy levels. The valence band is a lower energy level.

27: ... higher energy level

28: ... difference between conduction and valence band energy levels

29: That $\lambda = 1.1\,\mu$m happens to correspond to 1.1 eV is a numerical coincidence, but perhaps convenient, in that remembering 1.1 for silicon covers it from both directions.

30: ... denoted as e^- in Figure 13.4

31: Solar panels in the sun get pretty hot.
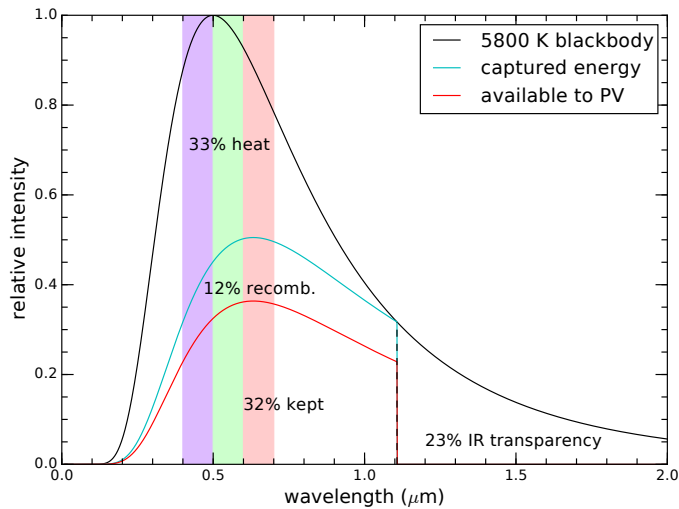
Figure 13.5: Energy budget in silicon PV cell. The areas of the four regions represent the fraction of the total incident energy going to each domain. All light at wavelengths longer than 1.1 μm (infrared; 23%) passes through the silicon without being absorbed. The photons that *are* absorbed give excess kinetic energy to electrons, losing 33% of the incident energy as heat. This effect is progressively more pronounced the shorter the wavelength. Of the remaining 44%, about a quarter disappear as electrons "recombine" with vacancies (holes) in the silicon before getting a chance to contribute to a useful current by crossing the junction, leaving 32% as the maximum theoretical efficiency.

But as the wavelength gets shorter and the energy gets higher, a greater fraction is lost to heat. Overall, 33% of the incident photon energy is lost to heat as the boosted electrons rattle the crystal before being tamed.

Now we're down to 44% of the original incident energy in the form of conduction-promoted electrons that have shaken off their excess kinetic energy. But then here's the rub: electrons are dumb. They don't know which way to go to find the junction, so aimlessly bounce around the lattice, in a motion called a random walk.[32] Some get lucky and wander into the junction, where they are swept across[33] and contribute to external current. Others fall into an electron vacancy (a hole) in a process called recombination: game over.[34] Roughly speaking, about three-quarters[35] of the electrons get lucky by wandering into the junction before being swallowed by a hole. So of the 44% available, we keep 32% (called the Shockley-Queisser limit [86]).

Another significant loss arises because some photons are absorbed in the very top layer above the junction, so that the resulting electrons do not have the opportunity to be swept across the junction to contribute to useful energy. The shorter the wavelength, the shallower the photon is likely to penetrate into the cell.[36] Meanwhile, photons around 1 μm are likely to penetrate deep—well past the junction—making it less likely that the liberated electrons will find the junction before settling into a new home (lattice site) via recombination. Figure 13.4 reflects this color-dependence, and also depicts one electron from the blue photon being generated above the junction, which will not have an opportunity to do useful work by crossing the junction.

In total, the basic physics of a PV cell is such that 20% efficiency is a reasonable expectation for practical implementations.[37] Indeed, commercial silicon-based PV panels tend to be 15–20% efficient, not far from the theoretical maximum. This may seem like a low number, but don't be disappointed! Biology has only managed to achieve 6%

32: ...sometimes called drunken walk, depicted as meandering paths in Figure 13.4

33: ...red arrow in Figure 13.4

34: ...red "poof" in Figure 13.4

35: Naïvely, 50% are lucky and wander up to the junction, and 50% make the wrong choice and go down. But even those initially going down still have a chance to wander back up to the junction before time expires and they recombine, so that effectively 75% make it.

36: Any given photon has a probability distribution of being absorbed as a function of depth. Blue photons *can* penetrate deep, but are more likely to be absorbed near the front surface. A 1 μm photon *can* be absorbed near the front surface, but it is more likely to penetrate deeper into the silicon.

37: Fancy, very expensive multi-junction PV cells may be used for special applications like in space, where size and weight are extremely important and cost is less of a limitation. These devices can reach efficiencies approaching 50% by stacking multiple junctions at different band gaps, better utilizing light across the spectrum.

efficiency in the best-case photosynthesis (algae). PV technology beats that by a factor of three! And as we'll see in Section 13.6, the only thing higher efficiency really does—besides driving up the price—is it makes the footprint (area occupied) smaller for the same power delivery. But it's already small enough to comfortably fit on most roofs, so efficiency is not a chief limitation at this point.
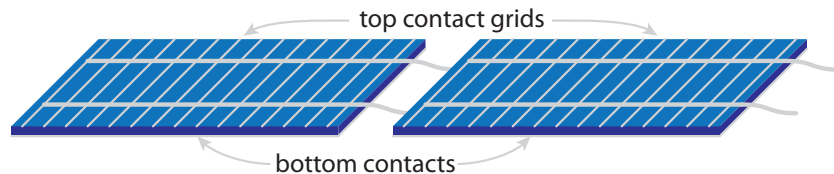
Figure 13.6: PV panels (modules) are constructed of typically 18, 36, 54, or 72 cells in series, two of which are depicted here. Cells are usually ~15 cm squares layered exactly as depicted in Figure 13.4. The bottom sides are covered by a continuous metal contact and the tops host a fine grid of metal contacts that minimize blockage of incident sunlight. Each cell presents ~0.5 V, arranged *in series* to add up to tens of volts per panel. To accomplish this, the top grid of one cell is connected to the bottom contact on the next, all down the line.

PV panels are usually constructed of many individual PV cells wired in series, as depicted in Figure 13.6. Each cell delivers maximum power when it's at a voltage around 0.45 V, and cells are usually arranged in strings of 18, adding to about 8 V. Panels commonly have 2, 3, or 4 such strings of 18—thus 36, 54, or 72 cells total—becoming 16 V, 24 V, or 32 V devices at peak power. Figure 13.7 shows typical performance curves for a PV cell (or whole panel) operating in various light levels. Recalling from Eq. 5.2 (p. 77) that electrical power is current times voltage, the power put out by a PV panel can be found as the area of the rectangle formed from the origin to the operating point somewhere on the curve. The point that maximizes area (power) is shown in Figure 13.7 as the "maximum power point." A battery being charged might hold the panel to a lower voltage, whose corresponding rectangle has a smaller area, thus operating at less than the panel's maximum power.

One serious downside of panels is that because cells are wired in series, partial shading of a *single* cell limits the current the whole panel delivers to that of the *weakest link* in the chain. In other words, 17 of 18 cells in a chain might be in full sun, but if the shadow from a roof vent, chimney, or tree shades one cell and limits its current to 10% of its full value, the whole chain[38] is knocked to 10%. Bypass diodes can isolate problem sections, but usually in chunks of 18–24 cells, so that the panel can still be seriously impaired by partial shading. Connecting panels in series also creates vulnerability to partial shadowing.

Figure 13.7: Current–voltage (*I–V*) curve for a PV cell. The cell in full sunlight will operate somewhere on the thick blue curve, and on a lower red curve under weaker illumination. The maximum power point (mppt) is about 0.45 V for silicon, while the nominal design might be for 0.35 V so that a 36-cell panel is sized to charge a 12 V battery. Rectangle area is proportional to power delivered, since $P = IV$.

38: . . . because in series, each cell shares the same current

This problem is sometimes mitigated via micro-inverters: each panel has an inverter so that higher-voltage outputs are combined in parallel.

Box 13.1: Why Not Parallel?

Given the downsides of series connection of cells, why not connect cells in *parallel*—the only other option for connecting many cells together?

Series combination adds voltages, keeping the same common current. Parallel combination shares a common (low) voltage but adds currents. The same power ($P = IV$) obtains either way. But two problems arise from a parallel combination of cells. First, the ~0.5 V voltage is too small to be useful for most devices. Second, the power lost in connecting wires scales as the square of current, so designing a system with a large current is asking for trouble.[39]

That said, PV installations often combine panels in both series *and* parallel—like 10 panels in series in parallel to another 10 in series. By this time, the voltage is plenty high to offset the losses.

39: Making things worse, the voltage drop in the lines is proportional to current, diminishing an already small voltage to even less by the time it gets to its application.

13.4 Insolation

Let's start our journey from the physics principles we covered in Section 13.2. The sun's surface is a sweltering 5,770 K, meaning that it emits $\sigma T^4 \approx 6.3 \times 10^7$ W/m^2 over its surface. The sun's radius is about 109 times that of the earth's,[40] which itself is 6,378 km at the equator. Multiplying the radiation intensity by the area gives total power output: $4\pi R_\odot^2 \sigma T^4 \approx 3.82 \times 10^{26}$ W. That's one bright bulb!

40: Why this convoluted path? Context. Building from pieces we are more likely to know/remember better engages our *understanding* and ownership of the material.

Sunlight spreads out uniformly into a sphere expanding from the sun. By the time it reaches Earth, the sphere has a radius equal to the Earth–Sun distance, which is $r_{\oplus\odot} = 1.496 \times 10^{11}$ m.[41] Spreading 3.82×10^{26} W over a sphere of area $4\pi r_{\oplus\odot}^2$ computes to 1,360 W/m^2. That's what we call the solar constant [4], and it's a number worth committing to memory.[42]

41: ... ~150 million kilometers, or 1 AU

42: See: isn't it satisfying to know that the number *comes from somewhere*? It's not just a random fact, but *connects to other pieces*. That's what the earlier margin note meant about context.

See Fig. 9.6 (p. 144) for a visual example.

Earth intercepts sunlight over the *projected* area presented to the sun: a disk of area $\pi R_\oplus^2$. Bright features like clouds and snow reflect the light back to space without being absorbed, and even darker surfaces reflect *some* of the light. In all, 29.3% of the incoming light is reflected, leaving 960 W/m^2 absorbed by the $\pi R_\oplus^2$ projected area of the planet. But now averaging the 960 W/m^2 input over the $4\pi R_\oplus^2$ *surface area* of Earth cuts the number down by a factor of four,[43] to 240 W/m^2.

43: We can understand this factor of four as two separate factor-of-2 effects determining how much solar power a particular location receives: one is simply day vs. night: half the time the sun is not up. The other half relates to the fact that the sun is not always *overhead*, so the amount of light hitting each square meter of land is reduced when the sun's rays are slanting in at an angle.

High latitude sites suffer more from low sun angles, and obviously cloudier locations will receive less sun at the surface. Taking weather into account, a decent number for the average amount of power from sunlight reaching the ground is about 200 W/m^2. This is called insolation[44]—the "sol" part of the word stemming from solar.

44: ... also called global horizontal irradiance

Table 13.1: Summary of solar power densities. Full overhead sun can be larger than the global absorbed number because the global number includes reflection from clouds, while overhead direct sun corresponds to a local cloud-free condition.

Solar Flux Context	W/m^2
Arriving at Earth	1,360
Full, overhead sun (no clouds)	~1,000
Absorbed by $\pi R_\oplus^2$	960
Absorbed by $4\pi R_\oplus^2$	240
Typical insolation, includes weather	~200
Typical delivered by 15% efficient PV	30

Table 13.1 summarizes these various power densities, the last line being typical insolation multiplied by 0.15 to represent the yield from a 15% efficient photovoltaic panel lying flat in a location receiving an insolation of 200 W/m². Figure 13.8 shows global insolation, variations arising from a combination of latitude and weather.

| 92 | 108 | 125 | 142 | 158 | 175 | 192 | 208 | 225 | 242 | 258 | 275 | 292 | 308 | W/m² |
| 2.2 | 2.6 | 3.0 | 3.4 | 3.8 | 4.2 | 4.6 | 5.0 | 5.4 | 5.8 | 6.2 | 6.6 | 7.0 | 7.4 | kWh/m²/day |
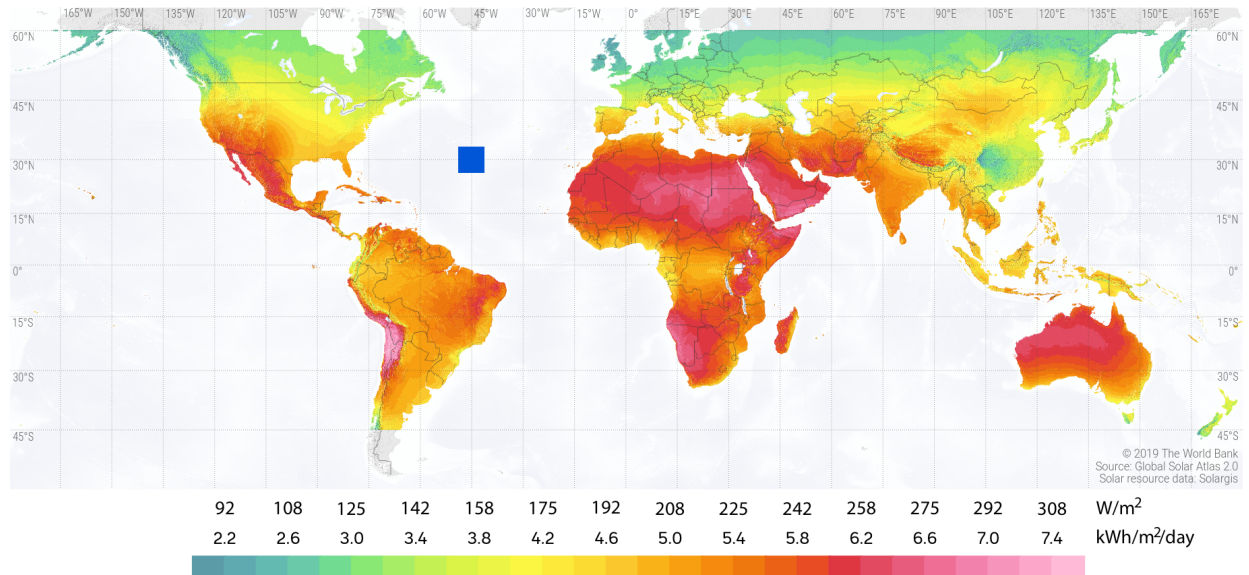
Figure 13.8: Insolation onto locally horizontal surfaces for the world (for flat plates facing directly upward), in units of W/m² and kWh/m²/day. The area of the blue square in the middle of the Atlantic ocean is enough to satisfy current global energy demand, using 15% efficient solar collection (but distributed, of course). Source: The World Bank.
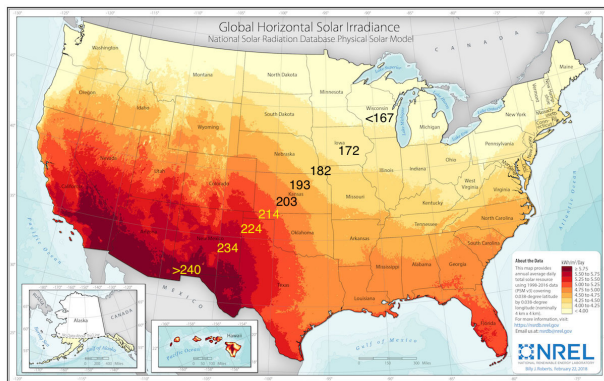
Figure 13.9: Horizontal insolation for the U.S. for a flat plate facing directly upward. Native units for the graphic are in kWh/m²/day, the break-points between colors running from 4.0 to 5.75 kWh/m²/day in steps of 0.25. These values can be converted to W/m² by multiplying by 1,000 W/kW and dividing by 24 h/day. Annotations are added once in each color band (in black or yellow) to indicate the equivalent measure in W/m² [87]. Alaska is not even close to scale. From NREL.

Figure 13.9 shows the variation of insolation across the U.S. The latitude effect is evident, but also weather/clouds make a mark, giving the southwest desert the highest solar potential. Even so, the variation from best to worst locations[45] is *not even a factor of two*.

Figures 13.8 and 13.9 are in the context of a flat surface.[46] For solar panels, it makes sense to tilt them to an angle equaling the site latitude and oriented toward the south.[47] The noon-time sun is always high in

45: ... e.g., 250 vs. 150 W/m²

46: ... as is the definition of insolation

47: ... toward the south for northern hemisphere locations; a more generally correct way to say it would be "toward the equator"

the sky near the equator, so panels there should lie flat.[48] But at high northern latitudes, the sun is lower toward the southern horizon, so the panels should tilt up to best face the sun. Tilting at an angle equal to the latitude is the best compromise, as Figure 13.10 illustrates.

48: . . . point mostly up

sun view — flat panels

sun view — titled panels

side view — titled panels
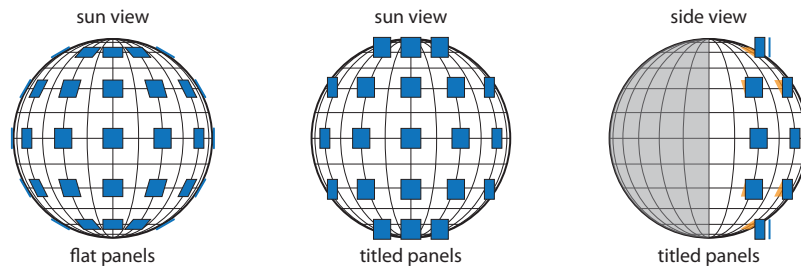
Figure 13.10: The left globe shows the sun's view of 21 panels of the same size sitting flat on the ground at their various sites. In the middle globe, the panels are all tilted up toward the equator. Notice the improvement in how much panel area is visible to the sun by doing this—especially at higher latitudes. At right is the side view, from which it is easier to appreciate why the best tilt angle is equal to the site latitude.
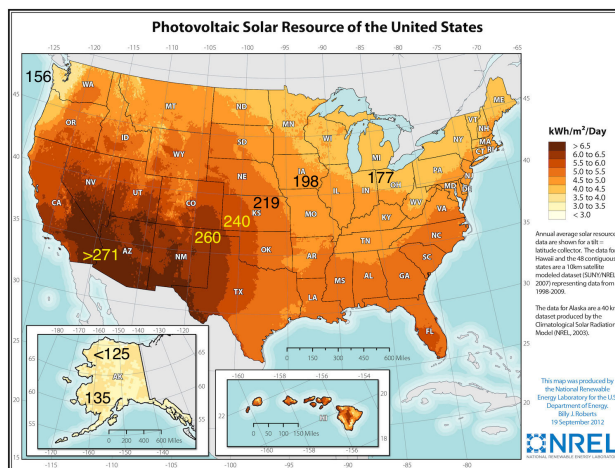
Figure 13.11: Solar potential for flat panels tilted to latitude, oriented south—relevant to PV panel installations. The graphic is presented in units of $kWh/m^2/day$, the breakpoints between colors running from 3.0 to 6.5 $kWh/m^2/day$ in steps of 0.5. Annotations are added once in each color band (in black or yellow) to indicate the equivalent measure in W/m^2 [87]. From NREL.

Tilting panels toward the equator at an angle equal to site latitude optimizes annual yield, and the results are shown in Figure 13.11. Note that the numbers in Figure 13.11 are not strictly insolations, since that's defined as what reaches *flat* ground. In this case, the area (square meters) is that of the *panel*, not of the *land*.

The fact that the numbers in Figure 13.11 are higher than in Figure 13.9 is not to say that the land offers more solar energy if the panels are tilted: just that an individual panel can get more light. But in this case, panels need to be spaced out to avoid shadowing,[49] as Figure 13.12 illustrates.

49: . . . which can be more devastating than just fractional area blocked, due to series arrangement of cells in panel modules

Some applications need to track the sun, like those that concentrate solar power, and only work when the sun is not blocked by clouds.[50] This brings us to Figure 13.13, showing the potential per square meter of collector (mirror or lens) used for the concentration (the topic of Section 13.8.2). The same pattern holds, in that the desert southwest dominates. But a look at the numbers indicates that the cloudier regions are not much better than just a flat panel facing upward (as is the case for Figure 13.9). In the southwest, where skies are often cloud-free, the boost can be

50: Photovoltaics still produce 10–50% of full capacity under cloudy skies during daylight hours, depending on how thick the clouds are: daylight still means photons.
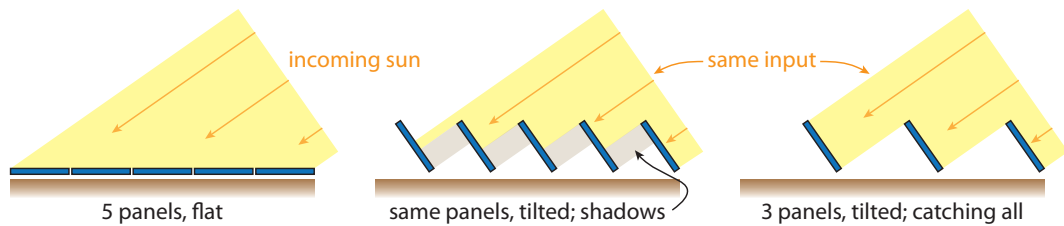
Figure 13.12: On a fixed piece of land receiving a fixed amount of sunshine at a slant angle, the amount of energy received is independent of whether the panels are flat or tilted. Just tilting the flat panels up (middle) results in self-shading. It makes the most sense to tilt and separate panels (right), one benefit being that fewer panels are needed to collect the same incident energy.

about 30% over the flat, upward-facing panel. Concentration schemes make less sense away from such regions.
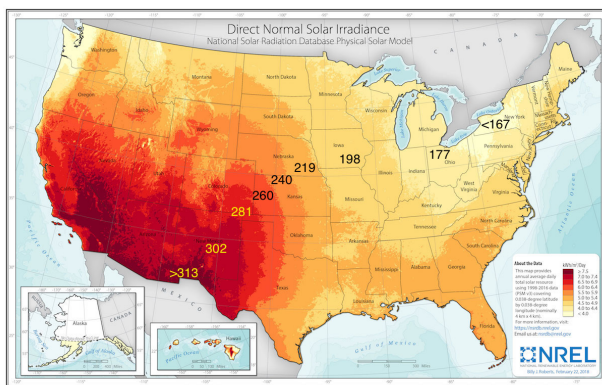
Figure 13.13: Solar potential for tracking panels, facing directly toward the sun's position and requiring a cloud-free view of the sun (concentrating collectors). The graphic is presented in units of kWh/m^2/day, the break-points between colors running from 4.0 to 7.5 kWh/m^2/day in steps of 0.5. Annotations are added once in each color band (in black or yellow) to indicate the equivalent measure in W/m^2 [87]. From NREL.

Stepping back, let's appreciate a few big-picture facets from these maps. First, numbers tend to be in the general neighborhood of 150–300 W/m^2. Burn this range in—it's a useful context. Second, the variation from the most solar-intense places in the contiguous U.S. to the weakest areas[51] is not more than a factor of two on an annual basis. This is astounding. The Mojave desert in California and the rain-forest Olympic Peninsula in Washington would seem to be practically day vs. night with respect to solar illumination. But not so much: only a factor of two.[52] Part of what this means is that if storage over *annual* timescales could be realized, solar power would become practical almost everywhere.[53]

51: ...ignoring Arctic-leaning Alaska

52: The northwest benefits from long summer days when clouds are also less likely.

53: This would require huge storage capacity: giant batteries, for instance.

Box 13.2: Hours of Full-Sun Equivalent

A useful take-away comes from the native units used in the three maps presented here: kWh/m^2/day, as opposed to our preferred W/m^2. Although they look different at a glance, kWh is a unit of energy, so kWh/day is a power, just like W. Since a kilowatt is 1,000 W and a day is 24 h, 1 kWh/day is 1,000 Wh/24 h = 41.67 W.[54] So we can multiply 6 kWh/m^2/day by 41.67 to get 250 W/m^2.

54: The hours in numerator and denominator cancel, since the kilowatt-hour is kW *times* hours.

But the main purpose of this box is to point out the following. Full overhead sunshine bathes the ground in about 1,000 W/m². [55] So if you could contrive to keep the sun directly overhead for 5 hours, you'd get 5 kWh of solar energy for each square meter on the ground. Therefore, if your site is listed as getting 5 kWh/m²/day, it's the *equivalent* amount you'd get from 5 hours of direct overhead sun. [56]

What *actually* happens is that the day is longer than 5 hours, but for much of the day the sun is at a lower angle so that the panel is not directly illuminated, and weather can also interfere. This leads to a concept of full-sun-equivalent-hours. A site getting an annual average of 5.4 kWh/m²/day might be said to get 5.4 hours of full-sun-equivalent each day. It's a pretty useful metric.

55: It's 1,360 W/m² at the top of the atmosphere, and the atmosphere blocks/scatters some of the wavelengths outside the visible part of the spectrum.

56: This equivalence relies on the convenient fact that full overhead sun is about 1,000 W/m². It would not work otherwise.

Box 13.2 leads to a crucial bit of understanding on characterizing a PV system. Panels are rated on what they would deliver when illuminated by 1,000 W/m² at a temperature of 25°C. [57] So the measure in kWh/m²/day, or full-sun-equivalent hours tells you effectively what fraction of a day the panel will operate at its rated capacity.

Example 13.4.1 A 250 W panel at a location getting 4.8 kWh/m²/day, or 4.8 full-sun-equivalent hours, is basically operating at 250 W for 4.8 hours out of every 24, or 20% of the time. So the panel delivers an *average* power of 50 W, not 250 W. [58]

The 250 W rating is referred to as "peak" Watts, sometimes denoted 250 W_p. Panels are sold this way, and now cost about $0.50/$W_p$.

57: The 1,000 W/m² is reasonable, but a photovoltaic panel in full sun will be about 30–40°C hotter than its surroundings (it gets hot!), so it would have to be very cold outside to meet the specification of 25°C panel temperature. Solar panel performance wanes when hot, and will only reach 85–90% of rated capacity in typical conditions.

58: We would need to apply a de-rating of 0.85 to 0.9 to account for typical PV temperatures in the sun, bringing the panel to about 45 W average power.

A 30-year study by the National Renewable Energy Lab [88] initiated in 1960 characterized solar potential across the U.S. and produced detailed statistics on what each location might expect to collect each month for panels in different orientations. Table 13.2 is a subset of the complete data for St. Louis, Missouri. [59] All cases in Table 13.2 correspond to a panel facing south, at various tilts (including flat, at 0° and vertical at 90°; other tilts are relative to the site latitude of $\theta \approx 39°$). From this, we see that tilting the panel at the site latitude delivers an annual average of 4.8 kWh/m²/day, matching the graphic expectation from Figure 13.11. Also shown is the monthly breakdown and how different tilts translate to performance. We will visit this table again in Section 13.6 to help us establish an appropriate size for a residential installation.

[88]: National Renewable Energy Lab (1994), *Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors*

59: ... a fairly typical solar location in the U.S.
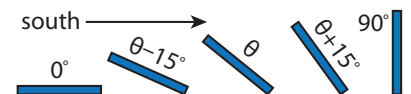
Figure 13.14: Panel tilts for Table 13.2, for $\theta = 39°$.

Angle	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
0°	2.2	2.9	3.9	5.0	5.9	6.4	6.4	5.7	4.6	3.5	2.3	1.8	4.2
$\theta - 15°$	3.2	3.8	4.6	5.4	5.9	6.3	6.3	6.0	5.3	4.5	3.2	2.7	4.8
θ	3.6	4.2	4.7	5.3	5.6	5.8	5.9	5.7	5.3	4.8	3.5	3.1	4.8
$\theta + 15°$	3.8	4.3	4.6	4.9	4.9	5.0	5.1	5.2	5.1	4.8	3.7	3.3	4.6
90°	3.5	3.7	3.4	3.1	2.6	2.4	2.6	3.0	3.5	3.8	3.2	3.0	3.2

Table 13.2: Solar exposure (kWh/m²/day) for a south-facing panel in St. Louis, MO, at various panel tilts (θ is latitude, which happens to be 39° for St. Louis). 0° means a panel lying flat, pointing straight up (like on a flat roof), and 90° means vertical, like on a (south-facing) wall (see Figure 13.14).

13.5 The Incredible Solar Potential

The potential of sunlight can be assessed by pieces we have already seen. Multiplying the solar constant of 1,360 W/m² by the projected area of Earth ($\pi R_\oplus^2$) and by 0.707 to account for the 29.3% reflection loss, we compute that Earth absorbs solar energy at a rate of 1.23×10^{17} W, or 123,000 TW (1 TW is 10^{12} W). Compared to the 18 TW societal scale, that's *huge*! Notice that the non-reflected entries back in Table 10.2 (p. 168) add[60] to this same value.

Placing solar panels on just 10% of the land (itself 29% of Earth's surface area) capturing the incoming energy[61] at 15% efficiency would produce solar power at ~500 TW: about 25 times today's 18 TW usage rate. This, in turn, means that our current energy demand could be met by covering only 0.4% of land[62] with photovoltaic panels: see Figure 13.8 for a visual representation of how much this is. Solar is the *only* currently-available resource that can come anywhere *close* to satisfying our current energy appetite.[63] And it exceeds our demand by such a huge margin! We therefore have reason to be excited about solar energy: the raw numbers are great news, indeed.

So it seems like a done deal. Solar. Let's get started! Wait, why aren't we there yet?

Naturally, solar has some downsides. First, the cost. Panel cost has dropped to something like $0.50 or less per peak-Watt. To get 10 TW of delivered (average) power at typical locations getting 20% capacity factor[64] would require about 50 TW$_\text{p}$ (Example 13.4.1 defines W$_\text{p}$), costing 25 trillion dollars. The cost of other necessary components and installation double the cost for utility-scale projects [89], bringing the cost to $50 trillion.[65] The global annual economy is not quite twice this. To outfit the world with the requisite number of panels would take about 60% of the economy for a year, or 6% over 10 years, or 1.5% continuously.[66] For comparison, the world goes through 30 billion barrels of oil each year at a cost of $50/bbl, meaning $1.5 trillion per year. Installing enough panels to fully satisfy demand would cost three-decades-worth of the *entire* global petroleum budget. So it's not going to happen fast. To put things on a personal scale, Americans use power at a rate of 10,000 W. To cover this, we would need about 50 kW$_\text{p}$ of panel per person, costing $50k per person.[67] When can we expect your payment?

Another daunting realization is that even though only 0.4% of the land is needed to match today's demand, this is comparable to the amount of area currently covered by roads and buildings. A road trip across the country conveys a sense for how vast (and boring) all that pavement can be. And pavement is a fancy form of dirt. It is true that PV panels are *also* an ultra-pure, ultra-fancy form of dirt. But it's a different level of high-tech. It becomes hard to fathom that much PV around the world.

60: ...83,000 TW plus 40,000 TW absorbed by the surface and atmosphere, respectively

61: ...out of the 123,000 TW total

62: ...1/25 of the 10% starting point

63: As we have seen, global wind may be limited to a few TW, and global hydropower would be hard-pressed to reach 2 TW.

64: 20% of 24 hours corresponds to 4.8 full-sun-equivalent hours per day.

65: The cost over the ~40 year lifetime of the panels is already competitive with conventional means, but when fuel cost is zero, *all* the cost is up-front, which presents a significant barrier.

66: ...based on 40 year effort, at which point the first panels need replacing

67: A subtlety here is that most of the 10 kW Americans use presently is in thermal form (fossil fuels), at ~35% efficiency. For non-heating applications that can use electricity, solar has an advantage. On the other hand, mitigating intermittency via storage requires a larger PV installation by as much as a factor of two. For the purposes of a crude estimate, we'll call it even and say that 10 kW per capita from PV would cover the entire demand 100% of the time.

A major impediment to solar power is its **intermittency**.[68] Figure 13.15 shows 31 days of solar capture, along with typical state-wide electricity demand. The two do not look very similar: not well matched. Demand is far more constant than the solar input, which is obviously zero at night. Even the peaks do not line up well, since demand is highest in the evening, well after solar input has faded away.

68: Recall that wind has a similar problem (Fig. 12.6; p. 190).
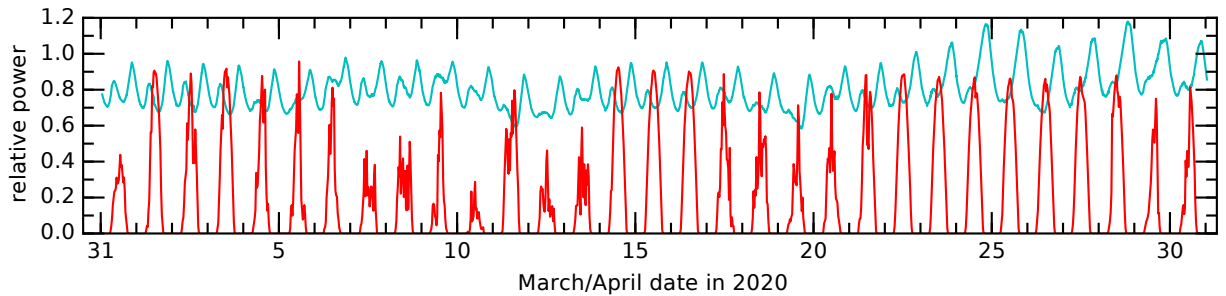
Figure 13.15: Solar input (red) and electricity demand (blue) look nothing alike. Solar data from the author's home begins 31 March 2020, while demand is for California. Tick marks denote the start of each date, at midnight. April 22–27 are essentially perfect cloudless days, while the earlier part of the month had rainy periods. Note that even a very rainy day (April 10) provides *some* solar power (15% as much as a full-sun day). Intermittent clouds cause the "hair" seen on some days. The capacity factor for the month is 19%, while the perfect six days near the end perform at 27% capacity. From this, we infer that weather caused the yield to be 70% what it would have been had every day been cloudless.

Storage is required in order to mitigate the intermittency, allowing the choppy solar input to satisfy the demand curve of Figure 13.15. We don't have good solutions for seasonal storage,[69] so a complete reliance on solar energy would necessitate over-building the system to handle months of low-sun conditions through winter (see the annual variation in Table 13.2), making it cost all that much more.

69: It's not a matter of how long the energy is stored, but a matter of sufficient *capacity* to store *enough* excess energy in summer for use during the darker winter.

Finally, all energy is not equivalent and substitutable. Solar PV cannot power passenger airplanes or power our cars down the road real-time (only via storage).[70] For all its potential, the hangups are serious enough that more than 60 years after the first demonstration of a photovoltaic cell, less than 1% of our energy derives from this ultra-abundant source.

70: It is possible to build a solar-powered aircraft or car, but not airplanes and cars as we know them (see Box 13.3). We can consider such things to be "cute" demonstrations, rather than a viable path to substitution.

Box 13.3: Why no Solar Planes?

Consider that full overhead sun delivers 1,000 W/m^2. The top surface area of a typical commercial airplane (Boeing 737) is about 450 m^2. If outfitted with the most expensive space-worthy multi-junction PV cells getting 50% efficiency, the plane would capture about 500 W/m^2 and a total of 225 kW. Sounds like a lot! The problem is that a Boeing 737 spends about 7 MW while cruising (and more during the climb). We're shy by a factor of about 25, even in optimal[71] conditions! Any solar-powered airplane[72] would be very light and very slow by air travel standards. See also Box 17.1 (p. 290) on the difficulty of battery-powered planes.

71: An airplane will not always have full overhead sun!

72: If your niece or nephew draws a solar plane in crayon, just smile, say it's very nice, put it on the refrigerator, and cry inside.

Cars have a similar problem: a top area around 10 m² equipped with the most expensive PV money can buy would get 5 kW, or less than 7 horsepower. That's about 20 times less powerful than typical cars, so again: light and slow.

13.6 Residential Solar Considerations

Despite these drawbacks, it can still make a lot of sense[73] to invest in solar photovoltaics for the home. We'll explore sizing and cost in this section.

73: The author runs most of his house from an off-grid PV system he built: a solar enthusiast when it comes down to it.

13.6.1 Configurations

A typical household uses much or most of its energy when the sun is not shining: lighting, cooking, evening entertainment, charging an electric vehicle, etc. To get around this, the system would need to have local storage,[74] or be tied to the regional electrical grid so that excess production can be exported in the daytime and electricity produced by the utility used at night or when household demand exceeds solar production. The overwhelming majority of solar installations in the U.S. are grid tied, and very few mess with batteries, which can double the cost of a system and need replacement before the system has paid for itself in electricity bill savings.

74: ...batteries: expensive, require maintenance, and periodic replacement

---

Box 13.4: Disappointing Dependence

A disappointing surprise to many who "go solar" is that their house has no power when the electrical service to the house is disrupted—even in the light of day. A grid tied system *needs* the grid to operate. Safety concerns prohibit PV systems from continuing to energize a disabled grid.

Only "off-grid" battery systems continue to work in these scenarios, but then the disappointment shifts to the price tag, maintenance, and replacement of worn-out batteries after a few thousand cycles.

While a description of the components and practical workings are beyond the scope of this book, students might be interested in an article the author wrote after first getting started tinkering with PV systems [90].

[90]: Murphy (2008), "Home photovoltaic systems for physicists"

13.6.2 Sizing and Cost

How large does an installation need to be? If the goal is to cover annual or monthly electricity use in a grid-tied system, the only two pieces of information needed are the typical electricity consumption in the

relevant period and the average solar input at that location for the period of interest.

The first can be surmised from electricity bills, usually giving a monthly total usage in kWh. We can get an approximate average scale from Fig. 7.2 (p. 105), which indicates that 42% of residential energy (11.9 qBtu per year) is from electricity. That's 5 qBtu, or 5.3×10^{18} J in one year $(3.156 \times 10^7$ s), or 167 GW. Distributed among 130 million households in the U.S.,[75] average household electricity consumption is 1,285 W. Applied over 24 hours, this makes for just over 30 kilowatt-hour (kWh) per day for an average household.[76]

The next piece is solar potential at the location of interest. We'll use the excerpted data from [88] for St. Louis, Missouri found in Table 13.2.

> **Example 13.6.1** Let's design a grid-tied PV system for an average U.S. household in an average[77] U.S. city (St. Louis). We'll orient the panels facing south and tilted to the site latitude (39°) and purchase PV panels at 18% efficiency (pretty typical).
>
> Table 13.2 indicates that for this configuration we can expect an annual average of 4.8 kWh/m²/day of input. If we're shooting for 30 kWh per day, we would need 6.25 m² of panel operating at 100% efficiency.[78]
>
> But 18% panels will require about 35 m² of panel,[79] which would be a square array about 6 meters on a side (about 20 feet) or a rectangle 5 by 7 meters, etc. The total area (400 square feet) is much smaller than a typical house footprint, so that's good news.

But panels are not marketed by the square meter. They are sold in terms of peak Watts: what the panel would deliver in 1,000 W/m² sunlight (see Example 13.4.1). How do we convert? Two ways are instructive.

> **Example 13.6.2** In one method, we multiply the 35 square-meter area from Example 13.6.1 by 1,000 W/m² and then by the PV efficiency (18% in this example) to get how much would be delivered: 6.3 kW.
>
> Alternatively, we could adopt the interpretation of 4.8 kWh/m²/day as the equivalent full-sun hours operating at peak output (Box 13.2). To get our target 30 kWh in 4.8 hours of full-sun-equivalent, we would need to produce 6.25 kW for those 4.8 hours.[80]
>
> We get the same answer either way, which is a good check.[81]

We should assume that the panels will not achieve their rated potential due to the facts that:

▶ The 25°C specification is almost never realized for a PV panel in the sun: PV panels in the sun get hot, and less efficient as a result;
▶ The panels will get a little dirty;

75: ...makes sense for a population of 330 million: translates to 2.5 people per household, on average

76: This is another case where students might suggest replacing this whole paragraph with the result. The point is to build connections, context, and tools to apply previous knowledge.

[88]: National Renewable Energy Lab (1994), *Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors*

77: ...solar-wise

78: ...Divide 30 kWh/day by 4.8 kWh/m²/day

79: ...Divide 6.25 m² by 0.18

80: 6.25 kW times 4.8 hours is 30 kWh.

81: The math is actually just the same, but we rearranged the order and interpretation.

▶ The equipment that converts panel output to AC electricity is not 100% efficient.

So it's a good idea to bump the number up by 20% or so, and order a 7.5 kW PV system for the case under study. A typical full cost (panels, electrical converters, installation) lately runs just shy of $3 per peak Watt (Figure 13.16), which in this case brings the price tag to roughly $20k. If electricity costs $0.15 per kWh—approximately the national average—each 30 kWh day costs $4.5, accumulating to $20k after 12 years. Federal and state incentives can make the payback time shorter.

What would these numbers become if trying to meet monthly instead of annual demands? December is usually the worst month for PV in the northern hemisphere, when the sun is lowest in the south, and the days are shortest. Table 13.2 backs this up, showing $3.1\,\text{kWh/m}^2/\text{day}$ for the chosen panel orientation in December. This is about two-thirds the annual average, so we would need to increase the size of the system (and thus cost and payback time) by a factor of 1.5 to produce enough in December.[82]

If sizing for an off-grid system, we need to factor in some inefficiency for battery charge/discharge and design for poorer months, so should increase by another factor of at least 1.5. The cost of batteries can be rather large, too. A good rule of thumb is to have at least three days of storage in the event of no solar input for several days during a stormy period. For our 30 kWh per day target, we would want about 100 kWh of storage. As an easy way to get a cost estimate for storage, the Tesla powerwall 2 is 13.5 kWh[83] and costs about $7k apiece. If we follow along, the cost of the off-grid PV system for 30 kWh/day at an installation cost of $3/W will be $7500\,\text{W} \times 1.5 \times 1.5 \times \$3 \approx \$50\text{k}$ for panels/installation plus $56k for batteries.[84] Then the batteries may be in need of replacement every 10–15 years.[85]

If this seems rather alarming, don't worry—there's a trick to making it much more affordable/practical: **don't demand 30 kWh per day**! Even though we picked 30 kWh/day due to the fact that it is the average American electricity demand, it is a worthwhile challenge[86] to seek ways to use *far* less energy than the average. Trying to make solar fit our present expectations may be the wrong approach. Also, expecting to get away from fossil fuels *and* have it be cheaper may be unrealistic.

13.7 Photovoltaic Installations

The Energy Information Administration's Electric Power Monthly (EPM) [85] provides detailed statistics on power generation in the U.S. Photovoltaic data is available in the EPM's tables 1.17.B and 6.2.B. In the usual way, we first look at installed capacity, based on the actual average *delivered* power. Figure 13.17 shows the situation in the U.S. California is
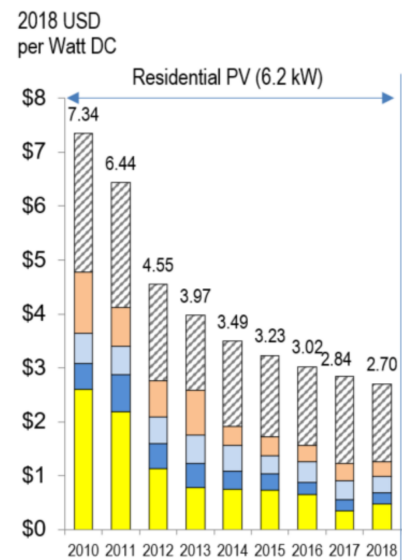
Figure 13.16: Evolving price of PV installation per peak Watt. Yellow is for the panels; the two blues for electronics; peach is labor; and hashed is for the utility hookup, inspection, taxes, profits [89, 91]. From NREL.

82: ... resulting in over-production in summer

83: ... so we would need about 8

84: It is often the case that battery cost is comparable to the rest of the system, roughly doubling the total cost.

85: Good batteries generally last a few thousand full charge cycles.

86: See Chapter 20 for examples.

[85]: U.S. Energy Inform. Admin. (2020), *Electric power monthly*
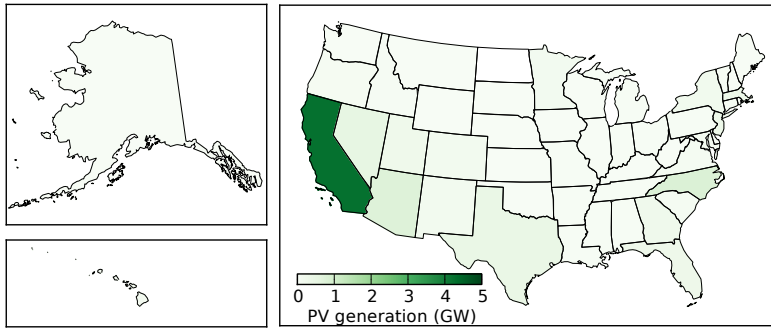
Figure 13.17: Photovoltaic power production by state, in GW, in 2018.

rocking it! The average solar power in California was 4.3 GW in 2018, far ahead of the next biggest: North Carolina at 0.82 GW. For California, this is 13% of its electricity. But electricity production is 38% of all energy in the U.S., so we might say that California gets about 5% of all its energy from solar. This is far ahead of other states.[87] The U.S. as a whole gets about 0.9% of its energy from solar.

87: North Carolina got about 5% of its electricity from solar in 2018, or less than 2% of all its energy.
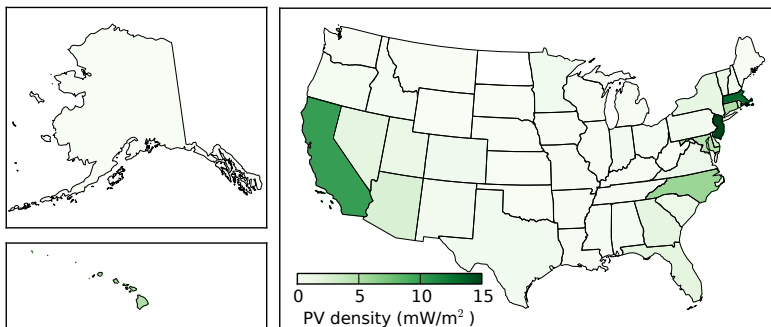
Figure 13.18: Photovoltaic power production areal density by state, in milliwatts per square meter.

Next, we divide by area to get power density from photovoltaic installations. A site having an insolation of 200 W/m^2 and 15% efficient panels has access to 30 W/m^2 of production capability (Table 13.1). Figure 13.18 shows how much we're actually getting. New Jersey has its moment in the sun, here. A few sites (NJ, MA) are pushing[88] 15 mW/m^2, which is a factor of 2,000 lower than the full potential. What this says is that only 1/2,000 of the land (0.05%) is covered by solar panels. This sort-of makes sense, right?

88: Compare to 50 mW/m^2 for hydroelectricity in Washington state (Fig. 11.6; p. 179) and 17 mW/m^2 for wind in Iowa (Fig. 12.9; p. 192).

On a per-population basis (Figure 13.19), Nevada shines brightest, at 180 W per person.[89] The southwestern U.S. is doing well overall, as is North Carolina on this measure.

89: . . . still small compared to the American metric of 10,000 W/person

Finally, we look at capacity factor: how much was generated compared to installed capacity (Figure 13.20). We expect something like 20%, corresponding to 4.8 full-sun-equivalent hours per day. The best states top out at about 0.27, equating to about 6.5 full-sun-equivalent hours per day. States at higher latitude and/or having more clouds will do
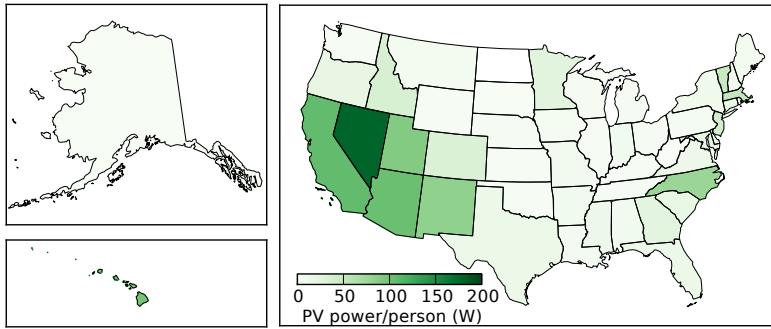
Figure 13.19: Per capita photovoltaic power production by state, in Watts per person.
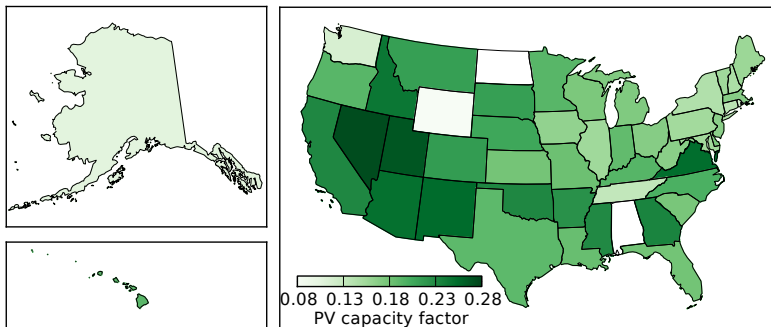
Figure 13.20: Photovoltaic capacity factors by state. While we see lots of darker green, it's because everybody has similarly *low* numbers, due to unavoidable nighttime and low sun angles. Somebody tell Wyoming, North Dakota, and Alabama to get with the program!

more poorly on this measure. Alaska clocks in just over 0.1, mapping to about 2.5 hours per day, on average.

Country	installed (GW$_p$)	average (GW)	% of all energy	global share (%)
China	175	~18	1.2	27
U.S.	62	10.6	0.9	16
Japan	56	6.5	3.5	10
Germany	46	5.0	3.3	7.5
India	27	4.1	1.5	6
World	510	67	1.5	100

Table 13.3: Top five global producers of PV power in 2018, accounting for two-thirds of the world's total production [92, 93]. The installed PV corresponds to peak watts (W$_p$), or production in full overhead sun.

Globally, two-thirds of the photovoltaic capacity is represented by five countries, shown in Table 13.3. Note that delivered power is significantly lower than installed capacity because of the low capacity factor for solar.

13.7.1 Pros and Cons of Photovoltaics

Before advancing to solar thermal generation, let's summarize the major advantages and disadvantages of solar photovoltaics. First, the good stuff:

▶ PV taps into a super-abundant resource—the only renewable that has such a margin;

▶ PV technology has no moving parts or steam; panels are robust and last a long time;

▶ PV is one of the few resources that can fit on a rooftop and provide self-contained electricity generation;

▶ PV efficiency is rather good: close to theoretical expectations and much better than biology has managed at getting energy from sunlight;

▶ PV technology works well, and despite expense has been deployed on rooftops across the world;

▶ Life-cycle CO_2 emissions are 15 times smaller than that of traditional fossil fuel electricity [68];

[68]: (2020), *Life Cycle GHG Emissions*

▶ PV is often a good solution when utility electricity is far away.

And now the less attractive aspects:

▶ PV is intermittent, and not well-matched to energy demand; it would be hard to "balance" the electrical grid if too much of the input came from such an intermittent source, and storage is difficult;

▶ PV is still expensive[90] relative to prevailing energy resources—especially important in terms of up-front cost;

90: Cost *has* been a major barrier, but may cease to be so as prices fall further.

▶ Electricity alone is not well-suited to many of our current energy demands, like transportation and industrial heat/processing;

▶ Stand-alone operation requires batteries, at least doubling the cost and adding maintenance/replacement demands;

▶ Even partial shading can be disproportionally disruptive;

▶ PV manufacturing involves environmentally unfriendly chemicals;

▶ PV deployment can harm habitats if installed in undeveloped areas.

13.8 Solar Thermal

Photovoltaics (Section 13.3) convert sunlight directly into electricity, but this is not the only way to harness energy from the sun. Solar energy can also be used for *heat*. We'll first have a brief look at home heating, then turn to electricity generation from solar heat.

13.8.1 Passive Solar Heat

Full sun delivers something like 1,000 W/m^2 at the earth's surface. Now imagine a window in a house intercepting 1.5 m^2 of sunlight, in effect admitting 1,500 W into the home—like a space heater, and it's free! Depending on window construction, some of the infrared energy may be blocked, so maybe not all 1,000 W/m^2 will make it inside, but a sizable portion will. Clever design has south-facing windows for receiving low-angle winter sun, but an overhang to keep out the high summer sun

(Figure 13.21). A dark and massive absorber[91] inside the house capturing the heat can continue to provide warmth through the evening hours. The Passive House designs mentioned in the context of Box 6.1 (p. 87) attempt to maximize solar capture so that little active heating is required.
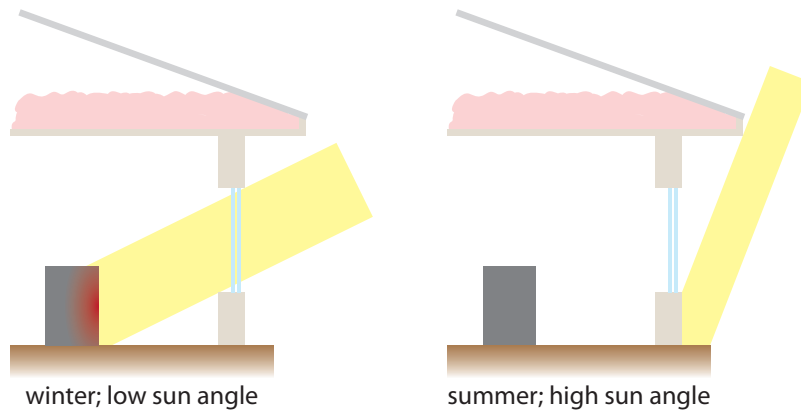
91: . . . dark rock or brick works well

winter; low sun angle   summer; high sun angle

Figure 13.21: A well-designed house has thick walls, thick insulation, and double-paned windows. Even better, it can have south-facing windows that admit sunlight in the winter but not in the summer (the overhang shields the window). A large, dark thermal mass—stone or brick works well—can absorb energy and continue to release heat into the evening.

13.8.2 Solar Thermal Electricity

While 1,000 W/m^2 is nice, the power is too diffuse to get anything very hot and create a large enough ΔT to allow the operation of an efficient heat engine (Sec. 6.4; p. 88). More complex arrangements can *concentrate* solar power—think of a magnifying glass—to heat up a liquid in pipes. Figure 13.22 shows an example of a parabolic reflector that can track the sun to concentrate light onto the energy-absorbing central pipe. This shape can be extruded along a long cylinder—a "trough"—following the pipe.
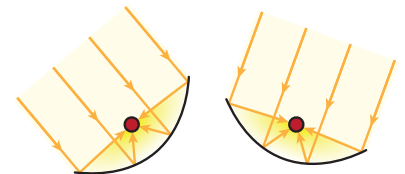
Figure 13.22: Solar trough cross sections showing the focusing of sunlight onto a central pipe. The troughs can be oriented to follow the sun.
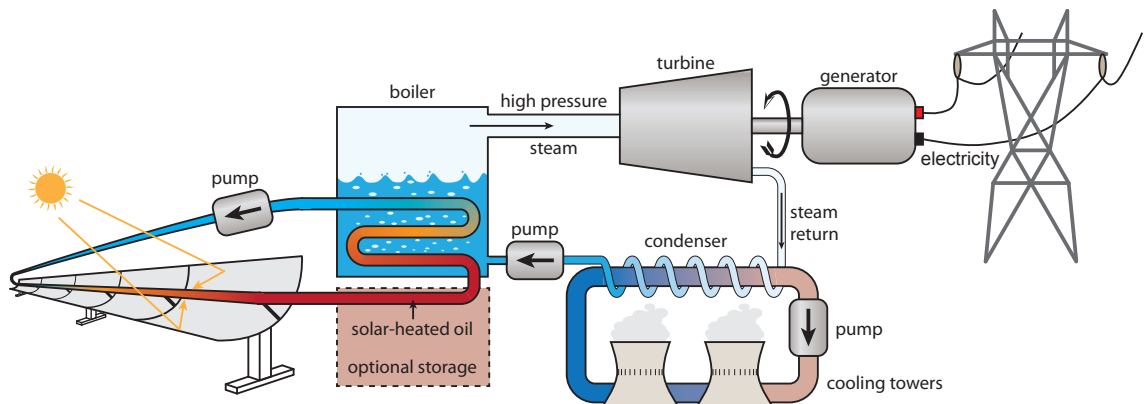
Figure 13.23: A common solar thermal power scheme uses parabolic "trough" reflectors to focus sunlight onto a central pipe, which carries oil that can be heated to very high temperatures for making steam to run a traditional electrical power plant very much like that of Fig. 6.2 (p. 90). Optional thermal storage can save heat for later use.

Figure 13.23 shows a schematic representation of a typical solar thermal (ST) collector, and a picture of one appears in Figure 13.24. A curved reflector tilts to track the sun, concentrating light onto a long pipe in

front of the reflector carrying a fluid (usually oil) that can be heated to a high temperature by the absorbed sunlight. The hot oil pipes can then be run through water to boil it and make steam, thereafter driving a traditional steam power plant. Such ST arrangements are sometimes called concentrated solar power (CSP). Another common variant—called a "power tower"—is shown in Figure 13.25, in which an array of steerable flat mirrors on the ground direct sunlight to the top of a central tower to make steam.

As for efficiency, solar thermal is at face value similar to PV: 15–20% is fairly typical. Broken down, roughly 50–75% of the available energy successfully transfers to the fluid, and then the heat engine delivers about 25–30% efficiency. But these numbers only apply if we count just the area of the reflective *collector*. Because they have to track the sun, and self-shadowing is to be avoided, only a small amount of the *land area* is occupied by the reflectors. Characterization of real facilities indicates that only 3% of the solar energy hitting the patch of land corresponding to the power plant is exported in the form of electrical energy.

But efficiency is not everything. 3% of a gigantic resource like solar energy input can still be tremendously large. It translates to over 6 W/m^2 for a standard insolation of 200 W/m^2, which is about thirty-times better than wind, per land area. While a field of PV panels outperform an ST installation by a factor of 5–6, the technologically simpler solar thermal designs can be more cost effective than PV. Reflectors and oil pipes are low-tech cheap devices, compared to photovoltaic material. The production cost for solar thermal is estimated to be about $0.06/kWh, which is lower than the typical retail cost of electricity, but still a factor of two higher than fossil fuel electricity production costs.

One disadvantage of solar thermal is that concentration only works when the sun itself is visible in the sky: no obscuring clouds. One way to think of it is: if you can't see your shadow, solar concentration will not work. Meanwhile, PV panels will still produce a meaningful amount of daytime electricity from the bright sky and clouds even if the sun itself is not "out."

Balancing this disadvantage is the fact that solar thermal has some built-in storage capacity, in that the heated oil can be "banked" for some hours[92] and continue to produce electricity even during the passage of a cloud or for a few hours into the evening. In this sense, it can better match the peak of electrical demand (early evening: Figure 13.15) than can PV, which goes to zero once the sun sets.

As seen in Figure 13.13, the desert southwest is the best place in the U.S. for solar thermal electricity generation. It makes sense that deserts would be good spots, since effective concentration requires no interference from clouds. Incidentally, transmitting electricity over intermediate distances (across regions) is fairly efficient: typically better than 90% for distances shorter than ~1,000 km.

Figure 13.24: Parabolic trough-based ST plant, in which part of the power generation facility is seen in the background. Reflectors must be spaced out to prevent self-shadowing. From U.S. DoE.

Figure 13.25: One of three "power towers" of the Ivanpah facility in California. By Craig Dietrich.

92: . . . thus the "optional storage" block in Figure 13.23

In terms of implementation, solar thermal is a small player. In 2018, only four states produced solar thermal power, 68% from California and 22% from Arizona. Table 13.4 provides some context, comparing ST to PV in each of the four states that have any solar thermal. For the entire U.S., less than 0.1% of electricity derives from solar thermal, and PV is about 25 times bigger on the whole. Globally, ST averages about 1.1 GW[93] (2016), about half in Spain and a third in the U.S.[94]

State	ST MW avg.	ST % elec.	PV MW avg.	PV % elec.	ST/PV %
California	281	1.25	4,285	19.0	6.6
Arizona	89	0.08	765	5.1	11.6
Nevada	35	0.09	552	12.1	6.3
Florida	6	0.002	326	1.2	1.8
U.S. total	410	0.086	10,565	2.2	3.9

93: …0.006% of global demand

94: …therefore not much left in the rest of the world

Table 13.4: Solar Thermal (ST) generation in the U.S. in 2018, compared to photovoltaic (PV); MW is megawatts.

13.8.3 Pros and Cons of Solar Thermal

Summarizing the pros and cons for solar thermal (ST), starting with the good aspects:

▶ ST taps into a super-abundant resource—the only renewable that has such a margin;
▶ ST technology is low-tech and inexpensive, using well-developed power plant technologies;
▶ ST has built-in short-term storage capacity for covering evening power demands;
▶ Life-cycle CO_2 emissions are 20 times smaller than that of traditional fossil fuel electricity [68].

[68]: (2020), *Life Cycle GHG Emissions*

And the less great stuff:

▶ ST requires direct sunlight; intolerant of clouds;
▶ ST is only possible at utility-scale, requiring a power plant;
▶ ST has a lower land-area efficiency than PV panels;
▶ Some disruption will be imposed on the local environment/habitat.

13.9 Upshot for Solar

Hands down, solar is *the only renewable resource* capable of matching our current societal energy demand. Not only can it reach 18 TW, it can exceed the mark by orders of magnitude. Finding space for panels is not a limitation. The efficiency of PV panels is perfectly respectable based on physics expectations, and beats the best that biology has done by a factor of 3–4. The efficiency is high enough that roof space tends to be more than sufficient to satisfy the demands of individual houses.

Pros and cons are listed separately for PV and ST in Section 13.7.1 and Section 13.8.3, respectively.

Holding solar back is its intermittency[95] and high up-front cost. Intermittency can be solved by battery storage, but this can double the cost and require maintenance and periodic battery replacement. Additionally—as for many of our renewable options—all of our society's demands[96] are not well met by electricity generation.

Sizing up a PV installation is fairly straightforward. Having first determined how many kWh per day are to be produced, on average, divide this by the kWh/m^2/day value for the site,[97] which is essentially the number of hours of full-sun[98] equivalent, and tends to be in the 4–6 hour ballpark. This says how many kilowatts the array should produce in full sun (peak Watts). For instance, if only 10 kWh/day are needed,[99] and the region in question gets 5 kWh/m^2/day, the system needs to operate at a peak power of 2 kW$_p$, costing about \$6k to purchase and install (grid tied). Inflating by 20% offsets unaccounted losses[100] to better match real conditions.

Solar thermal energy is another way to run a traditional steam-based power plant, using relatively low-tech mirrors and pipes to concentrate solar energy into a heat-carrying fluid that can later make steam. Effective efficiencies are relatively low,[101] but on the bright side, the low-tech nature makes it fairly cheap, and the technique can accommodate some degree of thermal storage for use some hours into the evening. Anything[102] starting from solar input has the potential to be a major player, given the ~100,000 TW scale of solar energy incident upon the earth.

13.10 Problems

1. If we had two monochromatic (single-wavelength) light sources—a green one at $\lambda = 0.5\,\mu$m and a near-infrared one at $\lambda = 1.0\,\mu$m— each emitting photons at an energy rate of 1 W,[103] how does the number of photons emerging per second from each source compare? Is it the same number for each because both are 1 W sources, or is it a different number—and by what factor, if so?

2. Overhead sunlight arrives on the surface of the earth at an intensity of about 1,000 W/m^2. How many photons per second strike a solar panel whose area is 1.6 square meters, if the typical wavelength is $\lambda = 0.5\,\mu$m?

3. Using the setup in Problem 2, how many photons enter your pupil every second if you look directly at the sun? When doing so, your pupil restricts to a diameter of about 2 mm.

4. The dimmest stars we can see with our eyes are thirteen orders-of-magnitude[104] dimmer than the intensity of the sun. Building off of Problem 3, how many photons enter your eye per second at this edge of detectability?

5. Warm humans at ~300 K and glowing-hot light bulb filaments at ~2,400 K both radiate according to Eq. 13.3. How much more power per unit area (W/m^2) does an incandescent filament emit compared to human skin, roughly?

6. The outcome of Problem 5 indicates that a hot light bulb filament emits thousands of times more power per unit area than human skin. Yet both a human and a light bulb may emit a similar amount of light[105]—both around 100 W. Explain how both things can be true?

105: Humans emit in the infrared, so we don't see it with our eyes.

7. At what wavelength does the Wien Law say the Planck spectrum will peak for a temperature of 4,500 K? Express your answer in microns and compare to Figure 13.1 for confirmation.

8. Human bodies also glow by the same physics as the sun or a light bulb filament, only it is too far out in the infrared for the human eye to see. For familiar objects (and human skin) all in the neighborhood of 300 K, what is the *approximate* wavelength of peak blackbody radiation, in microns?

ⓘ Note that the spectrum is spread over so many microns and we're being approximate about the temperature, so relax your answer to an easy, round number.

9. We might describe the efficiency of a light bulb as the fraction of its total light output that falls within the *visible* range. If using a *thermal* source[106] why would you expect it to be impossible to reach 100% efficiency at any source temperature, based on what Figure 13.1 shows?

106: ... like the sun or an incandescent light bulb; one that obeys Eq. 13.4

10. Using a technique similar to that in the text, approximate the height and width of a rectangle that has comparable area to the spectrum for $T = 4500$ K in Figure 13.1. Compute the area of your rectangle and compare to the expectation from σT^4.

11. What are two reasons that blue photons are disadvantaged in terms of having their energy contribute to useful current in silicon photovoltaics?

12. Which photons are most responsible for heating up a silicon photovoltaic panel in full sun: blue photons or infrared photons (beyond 1.1 μm)?

13. If a blue photon having 3.3 electron-volt of energy liberates an electron in silicon, whose band gap is 1.1 eV,, what fraction of the photon's energy is "kept" by the electron once it settles down from the excess?[107]

107: The rest going into excess kinetic energy which just heats up the PV cell.

14. If a 2.5 electron-volt photon liberates an electron from silicon with a 1.1 eV band gap, how much kinetic energy does the emerging electron have? Express in both eV and Joules, and then determine the velocity of the electron if the electron mass is 9×10^{-31} kg.

15. Briefly summarize the sequence of events[108] that results in a

108: ... consistent with the relevant physics, of course

successful contribution to PV current, starting with a green photon leaving the sun and ending with an electron crossing the junction.

16. Many people have an instinctive reaction to discount the < 20% PV panel efficiency as disappointingly low—perhaps thinking they should hold out for higher. Present a multi-point argument about why the efficiency is actually pretty good, and why in practice it is plenty good enough to be practical.

17. If aiming for a particular power output[109] from a PV array, describe explicitly/quantitatively how PV panel efficiency interacts with the physical size (area) of the array. For instance, what happens if the efficiency doubles or is cut in half, while keeping the same target output?

    109: Make up your own number if it helps.

18. Make the connection between Figure 13.4 and Figure 13.6 by drawing a zoom-in of the bottom left corner of one of the cells in Figure 13.6.

19. Figure 13.7 shows operational curves of a PV cell for different levels of illumination. If the illumination is low and the panel continues to operate at maximum power,[110] which changes the most compared to full-sun operation: the voltage or the current? Why might lower light (fewer photons) directly connect to a lower current based on the physics of PV operation?

    110: . . . largest rectangle that fits in curve

20. Replicate the calculation on page 206 (showing work) that starts with the surface of the sun being 5,770 K and finds that we receive 1,360 W/m^2 at Earth.

21. According to Figure 13.8, which continent appears to have the most solar potential? How would you rate China? Do the largest populations and/or largest energy consumers in the world tend to be well-aligned to the best solar resources?

22. Examine Figure 13.9 to determine the insolation at the "four corners" location where Arizona, New Mexico, Utah, and Colorado touch. Express this in both kWh/m^2/day and in W/m^2, showing how to convert from one to the other.

    Just using this location as an unambiguous spot on the map.

23. What is a typical value for hours of full-sun-equivalent[111] of solar exposure in the U.S. based on the map and native units in Figure 13.9? Explain how you arrive at this.

    111: . . . full sun meaning 1,000 W/m^2

24. A 30 year study by the National Renewable Energy Lab[112] indicates that in San Diego, a typical year delivers an annual average of 5.0 kWh/m^2/day of insolation for a flat panel facing straight up. Convert this to W/m^2.

    112: . . . called the Redbook study: [88]

25. The same study mentioned in Problem 24 finds that worst year in San Diego delivered an annual average of 4.7 kWh/m^2/day

and the best gave 5.2 kWh/m²/day for a flat horizontal panel. What, then, is the range of annual average insolation values in units of W/m² for San Diego, and what percentage variation is this, roughly (in round numbers)?

26. The study from Problem 24 finds that a flat panel facing south and tilted at various angles[113] relative to the horizontal produce the following annual average yields in units of kWh/m²/day:

Angle	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
0°	3.1	3.9	4.9	6.1	6.3	6.4	6.9	6.5	5.4	4.4	3.4	2.9	5.0
$\theta - 15°$	4.1	4.8	5.6	6.4	6.3	6.3	6.8	6.7	6.0	5.3	4.5	3.9	5.6
θ	4.7	5.3	5.8	6.3	5.9	5.8	6.4	6.5	6.1	5.7	5.1	4.6	5.7
$\theta + 15°$	5.1	5.5	5.7	5.9	5.2	5.1	5.6	5.9	5.8	5.8	5.4	5.0	5.5
90°	4.5	4.3	3.9	3.2	2.4	2.1	2.2	2.9	3.6	4.4	4.6	4.5	3.5

What tilt delivers the best yield for the year, and how much better is this (in percent) than a flat plate facing straight up? What tilt appears to result in minimal seasonal variation?

27. What if you adjusted the tilt of panels throughout the season to maximize yield? Reproduce the table above, but only writing in the highest number for each month.[114]. What average does this track produce for the year, and how much improvement (in percent) does this represent compared to the best fixed-tilt performance?

28. If typical insolation is 200 W/m², how much land area would be needed for a 15% efficient flat PV array supplying an average of 10 kW of power—which is the U.S. individual share? If arranged in a square, how large is the side-length of this array? Compare its size or area to something familiar.

ⓘ This exercise provides insight into PV at a personal-scale that would cover an American's total *share* of energy demand across all sectors.

29. If typical insolation is 200 W/m², how much land area would be needed for a 15% efficient flat PV array supplying an average of 10,000 W for every person in the U.S. (population 330 million). If arranged in a square, how large is the side-length of this array? Draw it on top of a state of your choice, to scale.

ⓘ This exercise provides insight into the total PV area needed to cover America's *total* energy demand across all sectors.

30. Based on what is presented in the text,[115] why is solar power still such a minor player if it is so hugely abundant and the technology has been around for a long time? What are some of the challenges?

31. You look at a PV panel and mentally estimate it to measure about 0.8 m by 1.5 m. Knowing that PV panels tend to be 15–20% efficient, you guess that it is 18% efficient. How much power would you expect it to deliver in full sun (1,000 W/m² incident)?

32. According to the table in Problem 26, San Diego can expect an annual average solar yield of 5.7 kWh/m²/day when the panel is tilted to the site latitude and facing south.[116] If a household seeks to produce a modest 8 kWh per day using 16% efficient panels,

how large will the array need to be? Express as an area in square meters, and in side length for a square of the same area.

33. Using the parameters from Problem 32, interpreting the solar yield as daily hours of peak-sun-equivalent (at solar exposure of $1 \, \text{kW/m}^2$) what should the array size be in terms of peak Watts in order to deliver 8 kWh per day? How much would the system cost to install at \$3.00 per W_p?

34. One way to look at solar payback time time is to note that an installed system will cost something like \$3,000 for each kW_p (peak capacity), and that you'll produce x kWh from that 1 kW_p array if your region gets x hours of full-sun-equivalent on average. Since each kWh of electricity costs something like \$0.15, it becomes straightforward to compute the value per day as $0.15x$, and determine how long to match the \$3k investment. The result is independent of the actual array size, depending only on the cost per W_p, the solar yield at your location, and the cost of electricity. What would the payback time be, in years, if the cost is $3/W_p$,[117] the yield is 6 hours per day of full-sun-equivalent, and electricity in your region costs \$0.15/kWh?

117: ... consistent with the first sentence

35. From Table 13.3, compute the capacity factors for the countries listed, and for the whole world, based on average vs. installed power. What is a characteristic range of numbers, and why[118] is it so low? Which country does the best, and which does the worst? What clues does Figure 13.8 offer as an explanation?

118: Hint: a large part is insolation vs. over-head sun.

36. How much power would a large window measuring 2 m wide and 1.5 m tall admit if the sun were shining straight[119] into the window from a cloudless sky? How many kilowatt-hours does this translate to over a four hour period, and how much is it worth monetarily compared to electricity at \$0.15/kWh?

119: ... e.g., what is the most one might expect for direct sun?

37. Solar photovoltaics are practical for individual homes, but solar thermal is only to be found in large utility-scale installations. What is the practical reason why we should not expect solar thermal installations on peoples' rooftops for electricity generation?

38. Solar thermal has a fairly low efficiency in terms of land area of about 3%, compared to 15–20% for PV. Many would shake their heads and say that's too low to be of any use. What is the counter-argument that it may be fine?

14 Biological Energy

The renewable energy options discussed thus far have been rather different from the chemically-stored thermal energy provided by fossil fuels. These sources—hydroelectricity, wind, and solar—are good at making electricity, but are intermittent to various degrees and are not directly suitable for transportation, except via bulky batteries.

Biologically-based energy is more similar to fossil fuels in that it is a form of chemical energy burned to create thermal energy. We will focus on two major forms: solid biomass and liquid biofuels. The latter is well-suited to transportation: one of the few renewable energies that can make this claim. In some cases, the same plant can produce either food or bio-energy—depending on whether it is eaten by another biological form or by a machine.

Ultimately, biologically-based energy is a form of solar energy, creating chemical storage by means of photosynthesis. Fossil fuels are also an ancient form of biofuel, deriving from photosynthetic energy captured millions of years ago. So sunlight is the actual energy source, and photosynthesis is the mechanism by which the energy is stored in chemical form.

14.1 Photosynthesis

This textbook will not focus on the complex mechanisms responsible for photosynthesis, but rather will describe the net result and efficiency. Photosynthesis involves the absorption of individual solar photons that ultimately facilitate the movement of electrons in order to change bonding structures, forming sugars, cellulose, and other materials used to construct a plant. The fundamental chemical reaction is depicted in
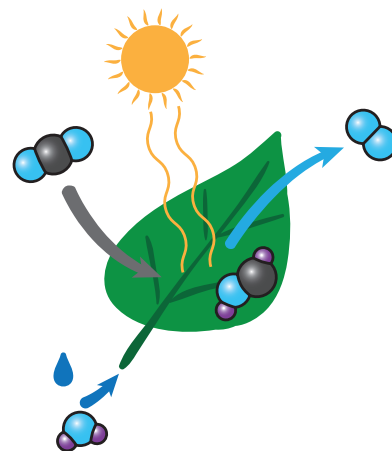
Figure 14.1: Cartoon version of photosynthesis, providing a graphical representation of Eq. 14.1. Water, CO_2, and sunlight are inputs. The leaf "exhales" oxygen and keeps sugar (only part of the final sugar molecule is pictured here).

Photo Credit: Rudy and Peter Skitterians.

[Figure 14.1](#) and represented as a formula in [Eq. 14.1](#), in which the product is a basic unit of a larger sugar molecule, like glucose ($C_6H_{12}O_6$).

$$CO_2 + H_2O + light \rightarrow CH_2O + O_2 \qquad (14.1)$$
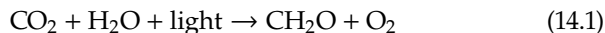
In sentence form: energy from light transforms carbon dioxide and water into a building block of sugar and releases oxygen back to the air.

Box 14.1: Where Do Plants Get Their Mass?

A valuable question to ponder: where do plants get their mass? Is it from the soil? Is it from water? Is it from the air? Take a moment to think about it. We can rule out soil on the observation that massive trees are not sitting in holes of excavated earth. Yes, the roots displace some of the soil, but a fallen tree reveals little root volume compared to the trunk and branches. And while living plant material contains significant water, completely dry plant matter[1] has plenty of mass without water.

1: . . . dry wood, for instance

Plant matter contains substantial carbon content,[2] and we now connect the knowledge that plants' leaves "breathe in" CO_2 and release O_2, as in [Eq. 14.1](#). Every time this happens, the plant steals a carbon atom from the air, spitting the oxygen right back out. The carbon is stuck into a sugar or other structural molecule and stays in the plant. Thus plants obtain their dry mass out of "thin air."

2: . . . which we combust when burning wood

In terms of efficiency, plants tend to convert sunlight into stored chemical energy at a rate of 0.01–6%. The range is rather large due to limitations in water and nutrients. A well-watered and fertilized corn field might reach 1.5% efficiency. Algae tend to top the charts at 5–6%. Dry climates might have ample sunlight, but too little water for efficient use of the available light. [Box 14.2](#) provides an example of how one might estimate what fraction of incident solar energy is turned into chemical storage by a potato plant.

250 W/m² average

0.5 m² area

4×0.5 kg

Figure 14.2: The potato plant in [Box 14.2](#).

Box 14.2: Example Photosynthetic Efficiency

Let us consider a potato plant ([Figure 14.2](#)) as an example by which to estimate photosynthetic efficiency. The potato plant might have a leafy footprint of 0.5 m²—a square roughly 0.7 m on a side or a circle about 0.8 m in diameter—and produce four 0.5 kg potatoes, or 2 kg of starchy material. Carbohydrates have an energy density of 4 kcal/g, so the potato plant has stored 8,000 kcal, or about 32 MJ.[3]

3: Recall 4,184 J per kcal.

If the typical five-month growing-season ($\sim 1.25 \times 10^7$ s) has insolation[4] of 250 W/m², the plant collects 125 W[5] times 1.25×10^7 s, or 1.6×10^9 J, while making 32 MJ of spuds. The photosynthetic

4: . . . summer, averaging day/night and weather

5: 250 W/m² times the plant area

---

efficiency is calculated as the ratio of output to input: about 2% in this case.

14.2 Biomass

Biological mass, or biomass, has long been utilized to supplement our energy needs, via controlled use of fire starting hundreds of thousands of years ago. Burning wood or other plant matter, and in some places dried animal feces[6] counts as utilization of biomass. Wood provides about 4 kcal of energy per gram when burned, or about 16 MJ/kg—much like proteins or carbohydrates[7] in our diet. Burning of biomass is most typically used for heating and cooking within individual homes.

6: ...such as cow dung

7: Fats and hydrocarbons like fossil fuels are 2–3 times more energy dense.

> **Example 14.2.1** A 10 kg bundle of dry firewood is used to heat a home that requires 4,000 W to stay warm. How long will the wood last?[8]
>
> Every gram of wood has 4 kcal or about 16 kJ of energy. We get 4,000 W by burning one gram every 4 seconds: 16 kJ/4 s is 4 kW. Each kilogram will therefore take about 4,000 s (a little over an hour) and the whole bundle will be gone after 11 hours.

8: ...assuming a wood stove or other efficient device to prevent most heat from just escaping through the chimney

In the U.S. in 2018, 2.36 qBtu of the 101.25 qBtu total came from burning wood, and an additional 0.5 qBtu came from incinerating waste products [34]. Thus about 2.8% (0.1 TW) of U.S. energy comes from biomass. Out of the 11.41 qBtu of all renewables in 2018, biomass accounted for 25% of the U.S. renewable budget.[9]

[34]: U.S. Energy Inform. Administration (2011), *Annual Energy Review*

9: ...while 23% was hydro; 22% was wind; 20% were biofuels, 8% was solar, and 2% was geothermal; see Table 10.3 (p. 170)

Globally, biomass use is estimated to be more important, at 6%,[10] constituting more than a third of global renewable resources (Fig. 7.8; p. 109). The high use of biomass in the wider world is a reflection on the difference between developed countries like the U.S. and developing countries that are more likely to rely on more primitive forms of energy like firewood and animal dung. Since most biomass around the world is burned for individual use, emissions controls are essentially non-existent, resulting in high levels of pollution—smoke and harmful chemicals that would be scrubbed out of a power plant's exhaust.[11]

10: ...down considerably from ~25% in 1950

11: Note that CO_2 is common to both, and is *not* scrubbed out of power plant exhaust, comprising the bulk of the emissions.

> **Box 14.3: Life is Thin and Precious**
>
> The total mass of living organisms on Earth is estimated to be about 2 trillion tons [94]. Having a density similar to that of water, 2×10^{15} kg, if spread uniformly around the planet, would stack to 4 mm high! Put another way, a random line projecting upward from the surface of the earth would go through about 4 mm of living matter, on average. That's a pretty thin shell of life!
>
> If we tried to substitute our 18 TW global power demand by burning

[94]: Elhacham et al. (2020), "Global human-made mass exceeds all living biomass"

biological matter,[12] we would run through *all* the currently-living mass—land and sea—in a short 15 years!

Can you imagine burning through all of Earth's forests and animals in 15 years? That's the rate at which we use energy today—illustrating the disparity between the biological resources on the planet and our staggering energy appetite. We can't expect to maintain our pace based on biomass and biofuels, and still have a vibrant natural planet.

12: About a quarter of the biomass is "dry" combustible material, at about 4 kcal/g.

14.3 Biofuels

Biofuels deserve their own category because the origins and end uses are different enough to warrant distinction. While the biomass sources from Section 14.2 tend to be in solid form, biofuels—as treated here—are *liquid*. Liquid fuels are instantly a big deal because they have the energy density and versatility to be used in transportation applications. An airplane can't very well fly on firewood, hydroelectricity, solar, wind, ocean currents, geothermal, or nuclear energy.[13] matter. Biofuels therefore occupy a special place in the pantheon of renewable resources as the most obvious viable replacement for petroleum—the dominant fossil fuel responsible for 92%[14] of transportation in the U.S.

13: See, for instance, Box 13.3 (p. 212) and Box 17.1 (p. 290).

14: Another 5% is from biofuels, usually blended into gasoline.

In the U.S. in 2018, 2.28 qBtu (2.3%; 0.08 TW) came from biofuels [34], which is very similar to the amount from biomass (wood, waste). Out of the 11.41 qBtu of all renewables, biofuels account for 20% of the U.S. renewable budget (Table 10.3; p. 170).

[34]: U.S. Energy Inform. Administration (2011), *Annual Energy Review*

Most prominently, ethanol is the chief biofuel, accounting for about 80% of the total. It is an alcohol that can be produced by fermenting the photosynthetically-produced sugars in the plant and then distilling the result.[15] Structurally, ethanol is very similar to ethane[16] except that the terminating hydrogen on one end of the chain is replaced by a hydroxyl group (OH; shown in Figure 14.3).

15: . . . also how "moonshine" alcohol is made

16: . . . C_2H_6: the second in the alkane sequence of methane, ethane, propane, butane, . . . , octane, etc.

Though it is not necessary to fully understand the chemistry,[17] combustion of ethanol—for comparison to the fossil fuel reactions in Eq. 8.1 (p. 121)—goes according to

17: Appendix B provides some background on chemical reactions and associated energy.

$$C_2H_5OH + 2O_2 \rightarrow 2CO_2 + 3H_2O + 29.7\text{kJ/g}. \qquad (14.2)$$
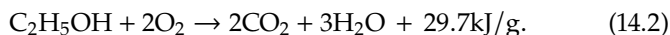
In other words, ethanol combines with oxygen via combustion (burning) producing carbon dioxide and water, also releasing energy. It is almost like the photosynthesis reaction (Eq. 14.1) in reverse.
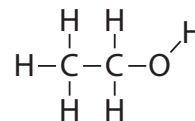
Figure 14.3: Ethanol is similar to ethane, but replacing the hydrogen at the end with hydroxyl (OH).

The energy density works out to 7.1 kcal/g, which is considerably lower than octane (representing gasoline) at 11.5 kcal/g (Table 8.2; p. 121). In terms of CO_2 production, the reaction generates 88 g of CO_2 for each 46 g of ethanol, coming to 1.9 g/g—which is lower than the 3.09 factor

for octane. In terms of CO_2 energy intensity, ethanol produces 64 g of CO_2 for every 1 MJ of energy: exactly the same as petroleum (Table 8.2). Generally speaking, biofuels—and other forms of biomass—are often considered to be carbon-neutral,[18] as the carbon released upon burning was taken in from the atmosphere in the process of photosynthesis, making it a cycle.

18: In practice, fossil fuels are used in the cultivating of biofuels, so it's not a perfectly clean, closed cycle in present form.

Most of the ethanol in the U.S. is blended into gasoline into E10, E15, or E85 products meaning 10%, 15%, or 85% ethanol. Not all vehicles are equipped to handle the more corrosive ethanol, and those that are ("flex-fuel" vehicles) might expect lower energy performance due to the fact that ethanol has lower energy density than gasoline.

Both the lower energy density and lower carbon mass per input fuel mass can be attributed to the oxygen atom hosted by the ethanol molecule.[19] Ethanol can derive from a number of plants. In the U.S., corn is the most common feedstock. Brazil uses sugar cane, which requires tropical climates.

19: See Sec. B.3 (p. 379) for more insight into chemical reactions and energy content.

14.3.1 EROEI

Before going further, we introduce a crucial metric for evaluating the merit of any energy source: the EROEI.

> **Definition 14.3.1** *EROEI, or Energy Returned on Energy Invested, is a measure of how profitable an energy source is in terms of energy, expressed as a ratio. For instance, a 9:1 EROEI means 9 units were extracted or produced for an investment of 1 unit, leaving a net gain of 8 units of energy. 1:1 is break-even, deriving no net energy benefit*

EROEI (sometimes just EROI) is one level above *efficiency*. Efficiency asks how much of a source is converted into a form useful to us. EROEI asks about what went into *obtaining* that source in the first place. It's still an efficiency, of sorts. Maybe a pre-efficiency.

By and large, energy does not come for free. Oil has to be actively drilled; hydroelectricity requires construction of a dam; solar panels are fabricated in an industrial process requiring energy input. So the question is: how much energy do we *get out* compared to the amount we had to *put in*? If we extract less energy than we invest, we *lose* net energy and probably should not bother.[20] If we only get a little more out, we still may question the investment.

20: In some cases, it may still make sense to pursue EROEI < 1 resources if the resulting form is otherwise hard to obtain, like food energy. As another example, we might use coal to process biofuels, in effect converting a less useful solid to a more useful liquid, even if losing energy in the process. But such desperate measures will not be favored if alternatives exist.

> **Example 14.3.1** Let's say an oil drilling operation uses petroleum products (like gasoline) as its only energy input for drilling and extracting oil. In one year, the operation pumps 12,000 barrels of oil, and in the effort uses 800 barrels of oil as energy input. What is the EROEI?
>
> In this case, we just have to arrange output to input as 12,000:800, and reduce to 15:1.

Early oil wells were shallow and under pressure, producing "gushers" that exceeded 100:1 in EROEI. To understand what this means, imagine

using oil as the energy source for the original exploration, building the equipment, running the drill, and collecting/storing the product. An EROEI of 100:1 means that for every barrel of oil that goes *into* the process, 100 barrels come *out*. That's a good deal. A high EROEI means *nearly* "free" energy: low effort for high reward.

As we progress to more challenging oil resources,[21] the EROEI drops— now around 10–15:1 for conventional oil and as low as 3:1 for tar sands [95]. Table 14.1 provides one set of EROEI estimates for various sources. Note that estimates vary due to difficulties in proper accounting of all energy inputs, so don't take these numbers literally—just as approximate guides.

21: ... deep water, fracking, tar sands

[95]: Hall et al. (2014), "EROI of different fuels and the implications for society"

Source	EROEI Est.	Source	EROEI Est.
Hydroelectric	40+	Solar PV	6
Wind	20	Soy Biodiesel	5.5
Coal	18	Nuclear Fission	5
Oil	16	Tar Sands	3–5
Sugar Cane Ethanol	9	Heavy Oil (Can., Ven.)	4
Natural Gas	7	Corn Ethanol	1.4

Table 14.1: EROEI estimates for various sources [96]. For example, Wind has an estimated EROEI of 20:1. See Table 7.1 (p. 106) for a refresher on how much energy we get from various sources. Canada and Venezuela tend to have heavy oil deposits.

If life were a video game, we would look at Table 14.1, decide that hydroelectric and wind are "the best," cursor over to them and "plus" those two up until we're getting all our energy from these low-energy-investment sources. But of course the world is constrained, placing real limits to what is possible. We saw in Chapter 11 and Chapter 12 that hydroelectricity and wind cannot be expected to provide more than a few terawatts, leaving a large shortfall. Meanwhile, solar has the largest raw potential. In other words, it is useful to appreciate the EROEI of various resources, but EROEI is not the sole determining factor of what is practical. A low EROEI can be tolerable if abundance makes up for it.

For resources whose energy investment is mostly up-front, before production begins, the resulting EROEI depends critically on *how long* the resource will provide energy. After all, the energy return gets larger the longer the facility can operate, while the investment part may be essentially done and unchanging. It can be difficult to predict how long a resource will last, which is part of why EROEI estimates are just that: approximate guidelines.

Example 14.3.2 Let's say a particular wind turbine achieves a 20:1 EROEI after operating for a 40 year lifetime. How many years-worth of its energy output went into constructing and installing it?

Each year the turbine produces some amount of energy, which we can call E (in Joules, for instance). In this case, it will produce $40E$ Joules in its lifetime. Since EROEI is 20:1, it must have taken $40E/20 = 2E$ Joules of input energy to create.[22] At a rate of E per year, it will produce $2E$

22: ... because the delivered energy is 20 times the input energy

in 2 years, therefore taking two years to produce as much as went into its manufacture—paying for itself, energetically.

In a self-supporting sense[23] the *net* energy is $x - 1$ for an EROEI of x:1. In other words, an EROEI of 1.25:1 only "really" produces 0.25 units of exportable energy for every one unit invested, if that one invested unit *comes from* the 1.25 units extracted in a closed system. In this case, for every one unit netted,[24] 4 units went in and 5 came out–only 1 of the 5 free and clear.

23: ...if the energy extracted is then used as the input to extract more

24: ...multiplying the 0.25 net by 4

Example 14.3.3 A self-contained operation to produce ethanol manages to only use its own ethanol to run the entire operation of growing, harvesting, and processing the crops to produce ethanol. In one year, the operation produced a total of 250,000 L of ethanol at an EROEI of 1.25:1. How much ethanol were they able to export/sell from the operation?

The 1.25 number is associated with *total* production, which is 250,000 L in this case. Multiplying both sides of the 1.25:1 ratio by 200,000 results in an output:input ratio of 250,000:200,000 meaning that the operation required 200,000 L of input. Thus the operation was able to deliver 50,000 L to market.

Low EROEI cuts into the effective available resource, demanding investment of precious energy. As conventional resources are exhausted, forcing us to lower-EROEI deposits, even if we keep up with energy demand in absolute terms,[25] the net energy available will decline as a greater fraction of the harvest must go back into extraction.

25: ...e.g., same number of barrels of oil produced each year

Example 14.3.4 What would have happened to an early agricultural society if the EROEI of growing food[26] slipped below 1:1, if all of the energy used to harvest the food came from workers and animals fed by the same food?

26: ...a form of energy

At 1:1, every unit of energy extracted requires one unit of investment. Then 100% of the energy is spent acquiring energy, leaving no energy for other functions of the society (shelter, defense, etc.). Such a marginal existence could not be maintained, so some minimum exists below which the society becomes non-viable.

Note that many of the entries in Table 14.1 have low numbers, translating to a tough life in which a substantial fraction of the total energy resource is dedicated to continued energy procurement. Biological forms of energy are not superstars in this regard.

Box 14.4: Eating Our Fossil Fuels

Relatedly, and in a familiar context, the food industry in the U.S. today expends about 10 kcal of mostly fossil-fuel energy for every 1 kcal of

food energy consumed [97]. In a sense, we are *eating*[27] our fossil fuels! It also points to an EROEI of 0.1:1, which is well below break-even. Obviously in times prior to fossil fuels, when we used human and animal labor in our agricultural pursuits, an EROEI less than 1:1 would spell starvation: more energy going in than was recouped from the land. Today, fossil fuels give us a temporary exception, so that we can afford to lose useful energy in the bargain, turning 10 units of fossil fuel energy into one unit that we eat. We might view this as a negative aspect of the Green Revolution.

14.3.2 EROEI of Biofuels

Various estimates exist for the EROEI for different biofuels. Unfortunately for the U.S., the corn ethanol industry is estimated to have an EROEI of anywhere from 0.8:1 to 1.6:1. The former would mean it's a net loss of energy, and that we would have more energy available if we did not spend any of it trying to get ethanol from corn. Biodiesel (a non-ethanol biofuel produced from vegetable oils or animal fat) is estimated to have an EROEI of 1.3:1 [98]. Sugar cane may be anywhere from 0.8:1 to 10:1 [99] (see Table 14.2).

To explore an example of how this all plays out, let's say that corn ethanol provides an EROEI of 1.2:1—in the middle of the estimated range. This means that in order to get 1.2 units of energy out, one unit has to go in. Or for every 6 units out, 5 go in. If we use *that same* resource as the energy input—in other words, we use corn ethanol as the energy input to grow, harvest, distill, and distribute corn ethanol—then we get to "keep" one unit for external use out of every 6 units produced. For the U.S. to replace its 37 qBtu/yr oil habit with corn ethanol, it would take six times this much, or 220 qBtu (2.3×10^{20} J) of corn ethanol production each year. If the growing season is 5 months, the solar input is 250 W/m^2 on average, and the corn field is 1.5% efficient at turning sunlight into chemical energy, then each square meter of corn-land produces 4.9×10^7 J of energy[28] and we would therefore need about 5×10^{12} m^2 of land for corn. This is an area 2,200 km on a side (Figure 14.4)! The U.S. does not possess this much arable land (estimated at about 30% of this). About 4×10^{11} m^2 of land in the U.S. is currently used for corn production, which is 8% of what would be needed. And of course we must still feed ourselves. In 2018, 31% of U.S. corn production went into ethanol. We would somehow need to ramp corn ethanol production up by a factor of 40 to derive our current liquid fuels from corn in a self-sufficient way. Don't expect to see this fantasy materialize.

Box 14.5: Why Do Corn Ethanol?

If corn ethanol has such low EROEI, why is it pursued in the U.S.?

[97]: Pfeiffer (2006), *Eating Fossil Fuels*

27: . . . or at least subsidizing the energy

Table 14.2: Summary: EROEI of biofuels.

Source	EROEI
sugar cane ethanol	0.8–10
soy bean biodiesel	5.5
biodiesel	1.3
corn ethanol	0.8–1.6
algae-derived	0.13–0.71

[98]: Pimentel et al. (2005), "Ethanol production using corn, switchgrass, and wood; biodiesel production using soybean and sunflower"

[99]: Murphy et al. (2011), "Order from Chaos: A Preliminary Protocol for Determining the EROI of Fuels"
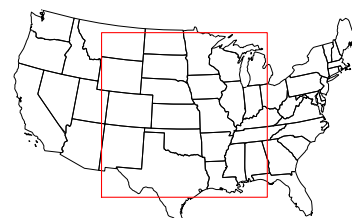
Figure 14.4: Area of corn growth needed to displace U.S. petroleum demand if at EROEI of 1.2:1. This is far larger than agriculturally productive land in the U.S.

28: 150 days times 86,400 seconds per day times 250 W/m^2 times 0.015 gives Joules per square meter produced.

> Why do we have mandates to introduce ethanol into fuel blends?
>
> Don't assume that the world is always scientifically rational and run by "adults" whose interests extend beyond personal gain. Many political factors enter: votes from midwestern swing-states,[29] job dependencies, influences from a powerful industry, the appearance of "green" (carbon-neutral) energy[30] all play a role.

29: ...where corn is grown

30: ...compromised by low EROEI if using fossil-fuel inputs to run production

A fundamental reason why the EROEI for biofuels tends to be low is that processing the material into ethanol requires a fair amount of energy input in the form of heat. Burning biomass, by contrast, does not have this requirement. Also, burned biomass is often gathered from untended (natural) environments that required little deliberate energy input on the parts of humans. Therefore, low EROEI is more a problem for biofuels than biomass.

14.4 Upshot for Biomass and Biofuels

Wood has always provided a source of heat for people, and will continue to do so. Its use occupied a much higher fraction of energy resources hundreds of years ago before being supplanted by fossil fuels. Still, several percent of U.S. energy comes from wood (and over 5% globally). Wood represents a renewable resource that can often be locally obtained, and will likely continue steady use,[31] potentially assuming a greater fraction again if overall energy expenditure declines.

31: ...subject to availability in the face of deforestation

Biofuels are special due to their liquid nature, as a potential replacement for oil to drive transportation. Because photosynthesis is not terribly efficient, and the EROEI of biofuels tends to be on the low side, the amount of land needed to replace petroleum is anywhere from daunting to prohibitive. This is even before addressing the crunch an extensive expansion would place on water resources or food supply, or the degradation of arable land that may result from depleting nutrients in the soil. Algae may represent another approach, but so far the process appears to be well below break-even in terms of EROEI (from 0.13–0.71:1 [100]). It is difficult to see a meaningful path forward for wholesale replacement of liquid fuels using biological resources.

[100]: Saad et al. (2019), "Algal Biofuels: Current Status and Key Challenges"

A final perspective is that the total biological scale on the planet is estimated to be 100 TW (Table 10.2; p. 168), which is not outrageously more[32] than the current 18 TW scale of the human endeavor. Can we really imagine commandeering 20% of all life on Earth to serve our energy needs? It would actually need to be substantially more than this, given EROEI limits. It may be that Earth does not possess enough biology to offer a substitute for our current fossil fuel appetite—even if we *tried* to use it *all*.

32: ...compared to solar or wind budgets, for instance, which are 5,000 times and 50 times our demand–not just 5 times as is the case for all biology

We conclude by listing some pros and cons for biologically-derived energy, beginning with the advantageous aspects:

▶ Biofuels offer a possible liquid fuel substitute to support transportation needs;

▶ Biological energy relies on dependable solar input, replenished as harvested stocks grow back;

▶ Biofuels represent a form of storage of solar energy, mitigating intermittency;

▶ Methods for growing and harvesting crops are well established;

▶ Burning biomass is low-tech and likely to remain part of our energy portfolio.

And the less savory aspects:

▶ It is difficult to scale biological energy to meaningful multi-terawatt levels;

▶ Heavy reliance on biological energy co-opts earth's biology and displaces natural habitat;

▶ Cultivating biofuels competes with food production for water and land resources;

▶ Low EROEI for biofuels reduces net energy available;

▶ Smoke and other pollutants from burning biomass can be problematic.

14.5 Problems

1. A large tree might have a trunk 0.5 m in diameter and be 40 m tall. Even though it branches out many times, pretend all the wood fits into a cylinder maintaining this 0.5 m diameter for the full height of the tree. Wood floats,[33] so let's say it has a density around 800 kg/m^3. How many kilograms of CO_2 did this tree pull out of the atmosphere to get its carbon, if we treat the tree's mass as 50% carbon?

2. Using the geometry and density of the tree in Problem 1, if the resulting wood has an energy density similar to carbohydrates (4 kcal/g), and the tree spent 50 years accumulating this bulk while receiving an average of 250 W/m^2 of solar input over 5 months each year in a leafy area averaging 200 m^2 to receive sunlight, what is the net photosynthetic efficiency of the tree?

3. The U.S. gets 2.4 qBtu per year of energy from burning biomass (mostly firewood). At an energy density of 4 kcal per gram, and a population of 330 million, how many 5 kg logs per year does this translate to per person?

33: ⓘ Water has a density of 1,000 kg/m^3.

Hint: carbon dioxide is 44/12 times the mass of plain carbon.

Now if you could just think of a way to put the answer on a *log* scale, ha ha.

4. How many logs of firewood per day (whose parameters are specified in Problem 3) would you need to burn to provide 5,000 W of heating to a house?

5. Replicate the conclusion of Box 14.3 by assuming one-quarter of the 2 trillion tons (5×10^{14} kg) of mass is combustible at 4 kcal/g. How long—in years—would this amount of energy last if burning at 18 TW?

6. Given the energy densities of ethanol vs. octane (gasoline), how many liters of ethanol does it take to replace one liter of gasoline, if the densities are 789 g/L for ethanol and 703 g/L for octane?

7. If 50 kJ of energy are spent to extract 1 MJ of energy content in the form of coal, what is the EROEI?

8. Re-express an EROEI of 1.5:1 in terms of how many total units of energy must be produced in order to extract one unit of *net* energy in a self-supporting operation.

9. Imagine that the extraction of a low-EROEI biofuel is performed using energy derived from that biofuel alone—in other words, a self-contained operation not using any other (external) form of energy. We can think of the situation thusly: for each hectare of land producing fuel for external use, some additional land must be dedicated to raising the energy used to perform the extraction operation—like an overhead. If the EROEI is 1.5:1,[34] how much *total* land area would need to be devoted to the endeavor for every hectare (or any area unit you wish) that contributes to net production? See Problem 8 for a related scenario.

   34: You can always multiply both sides by the same factor to make both sides integers, if this is easier to understand.

10. It takes a certain energy investment to fabricate a solar panel. Referring to Table 14.1, figure out how many years of the panel's output energy it takes to "pay off" the investment if the EROEI estimate assumed operation for 30 years.

11. Our modern food industry has an EROEI of 0.1:1. In pre-industrial settings, when energy investment for food production was in the form of muscle power (animal and human), why would a 0.1:1 EROEI for food have been untenable? Next, describe the conditions for an exact break-even food EROEI of 1:1. What would this mean in terms of where effort/energy goes? What would this leave for building shelters, cathedrals, or esthetic pursuits?

12. Parallel the development in the text for the land required for corn ethanol if it is self-sufficient[35] in the case for EROEI of 1.5:1—near the optimistic end of the range. How much larger is this than the area now devoted to corn, and how does it compare to total arable land in the U.S.?

   35: ...i.e., relying on its own energy to re-invest in its extraction

13. Using the setup for Problem 12, how large would the required corn

area be in terms of the side length of a square in units of kilometers. Draw this to approximate scale on a crude representation of the contiguous U.S. And this is at the optimistic end.

14. What fraction of the earth's 100 TW biological budget (all life on the planet) do you think is justifiable to use in service of human energy needs? Explain your reasoning. What does this become in TW, and how does it compare to our 18 TW current appetite?

If the EROEI is less than 2:1 (as it is for many biofuels), we would cut your estimate in TW by more than half to account for the diverted energy used for extraction.

15 Nuclear Energy

Most of the energy forms discussed thus far derive from sunlight—either contemporary input or fossilized storage. The *fuel* for nuclear energy is truly ancient, predating the solar system. The heavy elements participating in fission were produced by astrophysical cataclysms (most likely merging neutron stars), while the hydrogen building blocks for fusion originated in the Big Bang itself. In brief, fission involves the splitting of heavy nuclei into smaller pieces, while fusion builds larger nuclei from smaller ones.

While only fission has been successfully implemented as a source of societal energy, both types essentially boil down to the same thing: a source of heat to make steam and drive a heat engine. How and why nuclear material generates heat will be a primary focus of this chapter. Many practical concerns surround nuclear power, such as safety, weapons, waste, and proliferation of dangerous material. Self-pride for the impressive accomplishment of mastering nature well enough to implement nuclear power may not adequately justify continued reliance upon it—even if it is not a direct emitter of CO_2.

Understanding nuclear energy requires a longer journey than was needed for hydroelectricity, wind, and solar photovoltaics. We first learn about the nucleus and its many configurations, how nuclei transform from one to another through radioactive decay, the role $E = mc^2$ plays, and finally dig into the workings of fission and fusion.

15.1 The Nucleus

First, what is a nucleus? Every (neutral) atom consists of a positively-charged nucleus surrounded by a cloud of negative electrons (Figure 15.1).

Cooling towers of the decommissioned Satsop nuclear power plant in Washington. Photo credit: Tom Murphy

The nucleus is about 100,000 times smaller than the electron cloud,[1] but contains 99.97% of the atom's mass in a super-dense nugget composed of protons (positive charge) and neutrons (no charge). While electromagnetic forces vehemently resist the close congregation of positively-charged protons, the strong nuclear force overpowers this objection and sticks the protons and neutrons together in a stable existence.

1: ... which defines the size of the atom
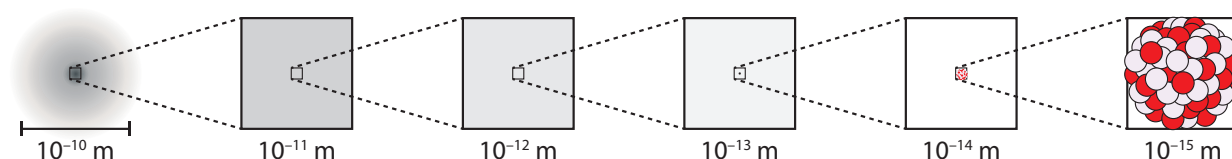
Figure 15.1: Zooming in on an atom in steps of 10×. At left, we see the entire extent of the atom's electron cloud. For a while, no nucleus is visible, being 100,000 times smaller than the atom itself. The nucleus within an atom is like a small dust grain in a bedroom.

By convention, the number of protons is labeled as Z and the number of neutrons as N. The total number of nucleons[2] is called the mass number: $A = Z + N$. It's just counting.

2: ... a name describing *either* protons or neutrons: any nuclear constituent

Picking carbon as an example, all carbon atoms have $Z = 6$: six protons.[3] Most carbon atoms (98.93%) have $N = 6$, making $A = 12$. But some isotopes carry a different number of neutrons. In natural carbon samples, 1.07% have $N = 7$, making $A = 13$. We label such an isotope as ^{13}C ($A = 13$), or sometimes $^{13}_{6}$C ($A = 13$; $Z = 6$), or even in some cases the fully-described $^{13}_{6}$C$_7$ ($A = 13$; $Z = 6$; $N = 7$). The latter two forms are somewhat redundant—though sometimes appreciated/helpful—because *all* carbon atoms have $Z = 6$, and $N = A − Z$ always. Therefore, ^{13}C says it all, provided you can easily find or remember the Z number for carbon.[4] The general pattern for an isotope of element X is $^{A}_{Z}$X$_N$. Other common designations are, for example, C12, C13, U238, or C-12, C-13, U-238 as alternatives to ^{12}C, ^{13}C, and ^{238}U, respectively.

3: One might say this is what *defines* the carbon atom.

4: ... just the sequential number labeling boxes in the periodic table: Fig. B.1 (p. 375)

Example 15.1.1 Write down all the various ways of designating the isotope of plutonium (Pu; 94 protons) that has mass number $A = 239$.

First, the math. $A = 239$ and $Z = 94$ so $N = A − Z = 145$. Starting at the simple end and working up, we can label this Pu239, Pu-239, ^{239}Pu, $^{239}_{94}$Pu, and finally $^{239}_{94}$Pu$_{145}$.

The physicist's version of the periodic table is called the Chart of the Nuclides, and contains a wealth of information. The basic layout idea is introduced in Figure 15.2, for the extreme low-mass end of nuclides.

Definition 15.1.1 *A **nuclide** is any unique combination of nucleons, so that every nucleus is one of the possible nuclides. For example, the ^{12}C nucleus is one nuclide, while ^{13}C is a distinct, different nuclide.*

Figure 15.2: Lower left start of the Chart of the Nuclides, shown pictorially in terms of the number of protons (red) and number of neutrons (lavender) in each nuclide. Gray boxes are stable nuclides, and H3 (tritium) is semi-stable for a decade or so.

Figure 15.3 provides a full view of the chart layout: neutron number, N, runs horizontally and proton number, Z, runs vertically. Stable nuclei are indicated by black boxes at some particular integer value of N and

Z. Notice how they bend away from the *N* = *Z* line, preferring to be neutron-rich. This can be traced to the fact that protons repel each other due to their electric charge, so the nucleus can be more tightly bound if fewer protons than neutrons are present—balanced against another penalty for being too far away from *N* = *Z*.
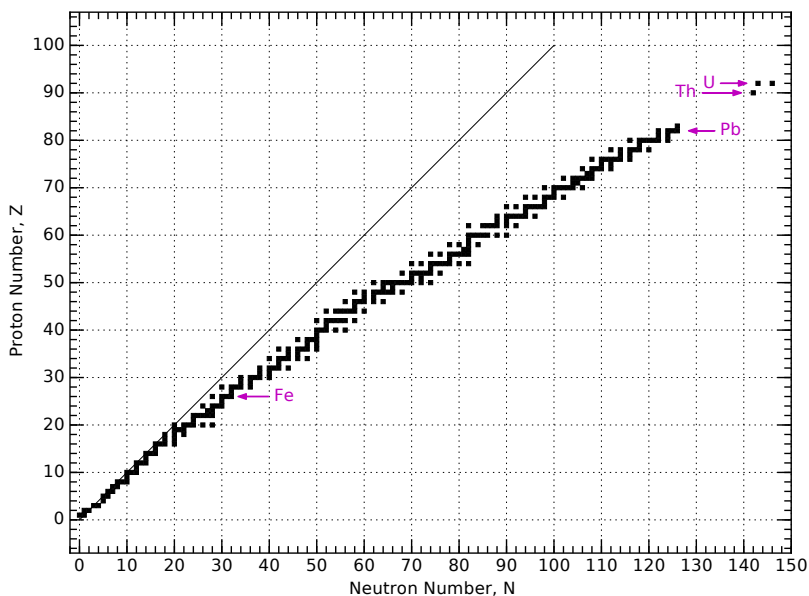
Figure 15.3: Layout of the Chart of the Nuclides, showing positions of naturally-occurring nuclei (stable or long-lived enough to be present on Earth). Stable nuclei tend to have more neutrons than protons—especially for heavier nuclei. This is why the track of stable nuclei bends away from the *N* = *Z* diagonal line. Arrows point to important elements of iron, lead, thorium, and uranium at *Z* values of 26, 82, 90, and 92, respectively.

Figure 15.4 shows the lower-left corner of the chart in much greater detail.[5] For each element (horizontal row), properties of all known isotopes are listed—even those that are radioactive and do not persist for even a small fraction of a second before decaying. Stable isotopes are denoted by gray boxes. The mass of each, in atomic mass units (a.m.u.)—defined so that the neutral ^{12}C atom is exactly 12.0000 a.m.u.—is given, and the natural abundance as found on Earth, in percent. The Chart of the Nuclides lets us peak inside the periodic table in great detail, as Example 15.1.2 suggests.

5: Even this level of detail is short of what can be found in the actual Chart of the Nuclides, which also provides quantitative values for neutron absorption, nuclear spins, excited states, additional decay paths and associated energies.

> **Example 15.1.2** From the Boron row (*Z* = 5) in Figure 15.4, we can see that 19.9% of boron is found in the form of ^{10}B, while the other 80.1% is ^{11}B.
>
> The weighted composite mass is therefore $0.199 \times 10.0129370 + 0.801 \times 11.0093055$, yielding 10.81103 a.m.u., which is the number presented as the molar mass on the periodic table.[6]

6: ... and in the summary information in the blue box at the left of each row

Because the Chart of the Nuclides has neutron number, *N*, increasing from left to right, and proton number, *Z*, increasing vertically, nuclei having the same mass number, *A* = *Z* + *N*, are arranged on diagonals. Notice that in the region shown in Figure 15.4, we never find more than one stable element at each mass number (constant *A*).

> Follow *A* = 12, for instance, from O12 through Be12, crossing through C12 as the only stable element of this mass.

Legend boxes:

H1 / 99.9885 / proton / 1.007825032 — isotope / % abundance / stable; name (for some) / mass, a.m.u.

Li8 / 0.840 s / β⁻ / E 16.0045 — isotope / half life (radioactive) / primary decay path / mass or decay energy (MeV)

Top Half Coloring: Half Life Guide
- < 1 day
- 1 to 10 days
- 10 to 100 days
- 100 days to 10 yr
- 10 yr to 500 Myr
- > 500 Myr or stable

Chart of the Nuclides (low-mass end):

Z = 8 — O (Oxygen, 15.9994):
O12	O13	O14	O15	O16	O17
~1e-21 s	0.0086 s	70.6 s	2.037 m	99.757	0.038
2p	β⁺	β⁺	β⁺		
12.03440	E 17.77	E 5.1430	E 2.754	15.99491462	16.9991315

Z = 7 — N (Nitrogen, 14.0067):
N11	N12	N13	N14	N15	N16
1e-21 s	0.0110 s	9.97 m	99.632	0.368	7.13 s
p	β⁺	β⁺			β⁻
11.0268	E 17.338	E 2.2204	14.00307401	15.00010890	E 10.420

Z = 6 — C (Carbon, 12.0107):
C8	C9	C10	C11	C12	C13	C14	C15
2.0e-21 s	0.127 s	19.29 s	20.36 m	98.93	1.07	5715 yr	2.450 s
4p+α	β⁺	β⁺	β⁺			β⁻	β⁻
8.03768	E 16.498	E 3.6478	E 1.983	12.00000000	13.00335484	E 0.156475	E 9.772

Z = 5 — B (Boron, 10.811):
B7	B8	B9	B10	B11	B12	B13	B14
3e-22 s	0.770 s	8e-19 s	19.9	80.1	0.02020 s	0.0174 s	0.013 s
3p+α	β⁺	p+2α			β⁻	β⁻	β⁻
7.0299	E 17.979	9.013329	10.0129370	11.0093055	E 13.369	E 13.437	E 20.64

Z = 4 — Be (Beryllium, 9.012182):
Be6	Be7	Be8	Be9	Be10	Be11	Be12	Be13
5.0e-21 s	53.3 days	~7e-17 s	100	1.5 Myr	13.8 s	0.0215 s	v. short
2p+α	e⁻ capt. to Li7	2α		β⁻	β⁻	β⁻	n?
6.01973	E 0.86182	8.0530509	9.0121821	E 0.556	E 11.51	E 11.71	E 17.1

Z = 3 — Li (Lithium, 6.941):
Li4	Li5	Li6	Li7	Li8	Li9	Li10	Li11
8e-23 s	~3e-22 s	7.59	92.41	0.840 s	0.1783 s	2e-21 s	0.0086 s
p	p+α			β⁻	β⁻	n	β⁻
4.0272	5.0125	6.015122	7.016004	E 16.0045	E 13.606	10.03548	E 20.62

Z = 2 — He (Helium, 4.002602):
He3	He4	He5	He6	He7	He8	He9	He10
0.000137	99.999863	7e-22 s	0.807 s	3e-21 s	0.119 s	v. short	2e-21 s
	alpha (α)	n+α	β⁻	n	β⁻	n	2n
3.016029310	4.002603250	5.0121	E 3.508	7.02803	E 10.65	9.0438	

Z = 1 — H (Hydrogen, 1.00794):
H1	H2	H3	H4	H5	H6
99.9885	0.0115	12.32 yr	8e-23 s	v. short	3e-22 s
proton (p)	deuteron (D)	β⁻ triton (T)	n	n	3n or 4n
1.007825032	2.014101778	E 0.018591	4.0278	5.035	6.0449

Z = 0 — n (neutron, 1.0086649):
n1
10.25 m
neutron (n)
β⁻
E 0.782353
1.008664923

Z increases vertically (blue numbering at left); N → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (green numbering at bottom).

radioactive but found naturally:
C14 / 5715 yr / β⁻ / E 0.156475

1 a.m.u. = 1.66054e-27 kg
1 a.m.u. = 931.49432 MeV/c²
1 MeV = 1.6022e-13 J

Bottom Half Coloring: Neutron Absorption (Barns)
- or < 10
- 10 to 100
- 100 to 500
- 500 to 1000
- >1000

Figure 15.4: Chart of the Nuclides for the low-mass end. Neutron number, N, increases toward the right (green numbering at bottom) and proton number, Z, increases vertically (blue numbering at left). Scientific notation is expressed as, e.g., 8e–23, meaning 8×10^{-23}. A wealth of information is included: spend some time studying the surrounding guides to learn what data each box contains.

15.2 Radioactive Decay

When one nuclide, or isotope changes into another, it does so by the process of radioactive decay. Stable nuclides have no incentive to undergo such decays, but unstable nuclides will seek a more stable configuration through the decay process.

The black squares in Figure 15.3, or gray squares in Figure 15.4 are *stable*,[7] leaving all others as *unstable*,[8] meaning that they will undergo radioactive decay to a different nucleus after some time interval that is characterized by the nuclide's half life.

Definition 15.2.1 *The half life of a nuclide is the time at which the probability of decay reaches 50%. A large sample of such nuclides will be reduced to half the original number after one half-life. Each subsequent half-life interval removes another half of what remains.*

7: ... or long-lived enough to be found in nature

8: Nuclides are unstable if a lower energy (more stable) configuration is within easy reach, better balancing desire for $N = Z$ against the cost of proton repulsion.

Figure 15.4 lists a half-life[9] for each unstable nuclide. For example, the half-life for a neutron (n1 in Figure 15.4) is 10.25 minutes, meaning that a lone neutron has a 50% chance of surviving this long. The process is statistical, so an individual neutron might only last 3 seconds, or might still be around in 15 or even 60 minutes. The predictive power is sharpened the larger the sample is: half will remain after 10.25 minutes.

9: . . . in units of seconds, minutes, hours, days, or years

Example 15.2.1 If starting with 16 million separate neutrons, we would expect 8 million to still be present after 10.25 minutes, 4 million after 20.5 minutes, 2 million after 30.75 minutes, and down to 1 million neutrons in 41 minutes.

Correspondingly, a single isolated neutron has a 50% chance of still being around in 10.25 minutes, a 25% chance of lasting 20.5 minutes, and a 6.25% chance of surviving 41 minutes. Every half-life interval cuts the probability of survival in half again.

Table 15.1 summarizes these results, adding jumps to 10 and 24 half lives for illustration, ending at one neutron.

Table 15.1: Decay of 16 million (M) neutrons, having a half life of 10.25 minutes, mirroring Example 15.2.1. Time is in minutes. The number remaining at each step is given, as well as the probability of any particular neutron surviving this long. After about four hours, only one would be expected to remain (and not for much longer).

Time (min)	Half Lives	Remain	Prob.
0	0	16 M	100%
10.25	1	8 M	50%
20.5	2	4 M	25%
30.75	3	2 M	12.5%
41.0	4	1 M	6.25%
⋮	⋮	⋮	⋮
102.5	10	15,625	0.1%
⋮	⋮	⋮	⋮
246	24	~1	1/16M

Luckily, radioactive decays don't go just any which way, but stick to a very small menu of possible routes. When a decay happens, the nucleus always spits *something* out, which could be an electron, a positron, a helium nucleus (called an alpha particle), a photon, or more rarely might spit out one or more individual protons or neutrons. Because these particles can emerge at high speed (high energy), they are like little bullets firing at random times and directions into their surroundings. These bullets are potentially damaging to materials and biological tissues—especially DNA, able to cause mutations and/or initiate cancerous growth. The primary decay mechanisms pertaining to the vast majority of decays are listed below and accompanied by Figure 15.5.
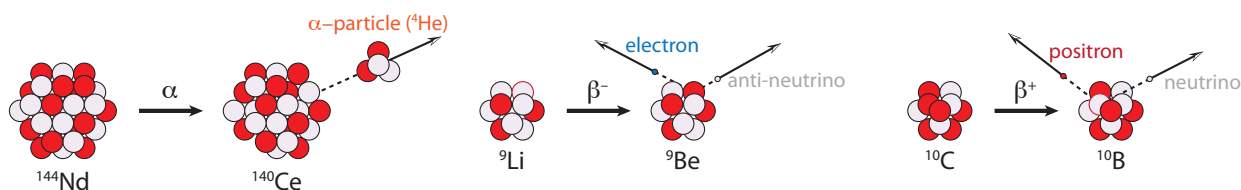
Figure 15.5: Radioactive decay mechanisms for α, β^-, and β^+. Protons are colored red, and neutrons light purple. The total nucleon counts are correct for the two beta decays, but only schematic for the larger ^{144}Nd nucleus used to illustrate alpha decay, which is predominantly seen only in heavier nuclei (aside from ^{5}Li and ^{8}Be). The positron is an anti-electron: a positively-charged antimatter counterpart to the electron. Neutrinos are sometimes called "ghost" particles for their near-complete non-interactivity with ordinary matter.

1. **Alpha decay** (α), in which a foursome of two protons and two neutrons—essentially a ^{4}He nucleus—leaps out.[10] When this happens, the nucleus reduces its N by two, reduces its Z by two, and therefore A by 4. On the chart of the nuclides, it moves two squares left and two squares down (see Figure 15.7). For example, ^{8}Be decays this way, essentially splitting into two ^{4}He nuclei;

10: Helium is found mixed in with natural gas, and derives from alpha particle decay of elements in the earth's interior.

Follow along on Figure 15.4.

2. **Beta-minus** (β^-) decay is a manifestation of the weak nuclear force, in which a neutron within the nucleus converts to a proton, and in the process spits out an electron (β^- particle, really just e^-) to conserve total electric charge, and a neutrino—which we will ignore.[11] The mass number, A is unchanged, but N goes down one and Z goes up one (gaining a proton and losing a neutron). Thus on the chart of nuclides the motion is one left, one up. It's like a chess move (Figure 15.7);

3. **Beta-plus** (β^+) decay, like β^-, is a manifestation of the weak nuclear force, in which a proton within the nucleus converts to a neutron, emitting a positron (β^+, or e^+, or anti-electron; a form of antimatter) again maintaining charge conservation, and an ignored neutrino. Similar to β^- decay, A is unchanged, but Z is reduced by one and N gains one. On the chart, the move is diagonal: down one and right one (Figure 15.7).

4. **Gamma decay** (γ) happens when a nucleus is in an excited energy state, having been rattled by some other decay or bombardment, and it emits a high-energy photon, called a gamma ray, as it settles into a lower energy state (Figure 15.6). For γ decays, Z, N, and A do not change, so the nucleus does not morph into another flavor, and thus does not move on the Chart of the Nuclides.

Figure 15.7 demonstrates the motion of each of these decays on the Chart of the Nuclides, and Table 15.2 summarizes the nucleon arithmetic.

11: Perhaps it is fair to ignore neutrinos since they ignore us. Neutrinos interact so infrequently with matter that a neutrino could fly through light years of rocky (Earth-like) material before being expected to hit something (interact). This extreme non-interactivity earns it the title of "ghost" particle.
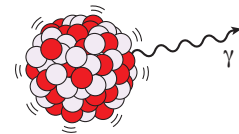
Figure 15.6: Gamma decay of an excited nucleus.
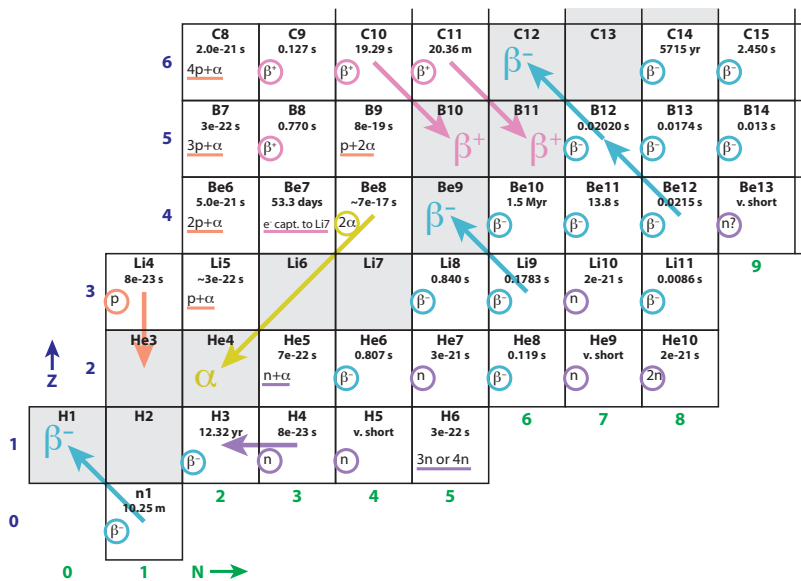
Figure 15.7: Radioactive decays shown as moves on the "chess board" of the Chart of the Nuclides. The different decay types are color-coded to match Figure 15.8, and are only shown in a few representative squares. Decays frequently occur in a series, one after the other (a decay chain), as hinted by the double-sequence starting at ^{12}Be and ending on ^{12}C. Note that the square of every unstable nuclide indicates a decay type, even if arrows are not present.

Table 15.2: Summary of decay math on nucleon counts.

Decay	$Z \rightarrow$	$N \rightarrow$	$A \rightarrow$
α	$Z - 2$	$N - 2$	$A - 4$
β^-	$Z + 1$	$N - 1$	unchanged
β^+	$Z - 1$	$N + 1$	unchanged
γ	unchanged	unchanged	unchanged

Example 15.2.2 What will the fate of ^{8}He be, according to Figure 15.4?

We can play this chess game! According to the chart, the primary decay mechanism of He8 is β^- with a half-life of about a tenth of a second. It will become ^{8}Li, which hangs around for about a second before undergoing another β^- decay to ^{8}Be. This one lasts almost no time at all ($\sim 10^{-16}$ s) before α decay into two alpha particles (two ^{4}He). Such a sequence is called a decay chain.

As is evident in Figure 15.8, unstable isotopes *above* the stable track in Figure 15.3 tend to undergo β^+ decays to drive toward stable nuclei, while those *below* the track tend to experience β^- decays to drive up toward the stable track. The α decays are more common for heavy nuclei (around uranium), which drive toward the end of the train of stable elements in Figure 15.3, ending up around lead (Pb). We can understand the abundance of lead as a byproduct of heavy-element decay chains.

legend:
- stable
- alpha (α)
- beta-minus (β^-)
- beta-plus (β^+) or e^- capture
- spontaneous fission
- neutron emission (n)
- proton emission (p)
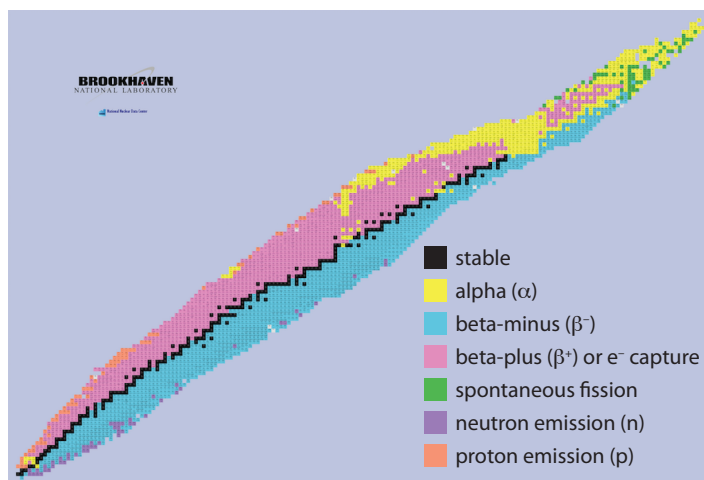
Figure 15.8: Another view of the Chart of the Nuclides, color coded to indicate prevailing decay modes as a function of position on the chart. Note that β^+ sometimes captures an electron rather than emitting a positron, but amounting to the same thing, essentially. From U.S. DoE.

Box 15.1: The Weak Nuclear Force

An aside worth making is that having discussed beta decays, governed by the weak nuclear force, we have now covered all four known forces of nature: gravity, electromagnetism, the weak nuclear force, and the strong nuclear force. That's it: a small menu, really. The latter three are unified into a Standard Model of Physics, but gravity—described by General Relativity—has defied all attempts at "grand unification," or a "theory of everything" trying to unite all four forces under a single theoretical framework. One implication is that known physics offers no other "magic" solutions to our energy needs. No new forces have come to light in more than half-a-century, despite dramatic advances in tools to probe the fundamental nature of physics.

15.3 Mass Energy

Energy—whatever the form—*has mass* and actually changes the weight of something, although almost imperceptibly. A hot burrito has more mass than the exact same burrito—atom for atom—when it's cold.[12] Most of us are familiar, at least casually, with the famous relation $E = mc^2$. More helpfully, we might express it as

$$\Delta E = \Delta mc^2, \tag{15.1}$$

where the Δ symbols indicate a *change* in energy or mass, and $c \approx 3 \times 10^8$ m/s is the speed of light. Using kilograms for mass results in Joules for energy. Because c^2 is such a large number (nearly 10^{17}), the mass change associated with daily/familiar energy quantities is negligibly small. Box 15.2 explains why $E = mc^2$ is valid for all energy exchanges—not just nuclear ones—but generally results in mass changes too small to measure in non-nuclear contexts. Earlier, we discussed conservation of energy. More correctly, we observe conservation of mass-energy. That is to say, a system *can* actually gain or lose net energy if the mass changes correspondingly. In the case of nuclear energy release, the "new" energy comes at the expense of *reduced mass*.

12: The burrito is also ever-so-slightly more massive if it has kinetic energy, gravitational potential energy, or any form of energy. A battery is more massive when charged, even if no atoms or electrons are added. Incidentally, charging a battery does not mean literally adding electrical charges (adding particles), but amounts to rearranging electrons among the atoms within the battery.

Box 15.2: $E = mc^2$ **Everywhere**

Physics is not selective about when we might apply $E = mc^2$. It always applies, to every situation. It's just that outside of nuclear reactions it does not result in significant mass differences.

For example, after we eat a 1,000 kcal burrito to fuel our metabolism, we expend the energy[13] and lose mass according to $\Delta m = \Delta E/c^2$. Since $\Delta E \sim 4$ MJ (1,000 kcal), we find the associated mass change is 4.6×10^{-11} kg, which is ten orders-of-magnitude smaller than the mass of the burrito itself.[14] So we'd never notice, even though it's really there.

When we wind up a mechanized toy, coiling a spring, we put energy into the spring and the toy *actually gets more massive*! But for every Joule we put in, the mass only increases by about 10^{-17} kg. Forgive us for not noticing. Only in nuclear contexts are the energies large enough to produce a measurable difference in mass.

13: ...ultimately given off as thermal energy to our environment

14: This amount of mass corresponds to that of a tiny length of hair that is shorter than it is wide.

Example 15.3.1 Since mass and energy are intimately related, it is common to express masses in *energy* terms. How would we express 12.0 a.m.u. in MeV (a unit of energy; see Sec. 5.9; p. 78)?

1 a.m.u. is equivalent to 1.66×10^{-27} kg (last row of Table 15.4), so 12 a.m.u. is 1.99×10^{-26} kg. To get to energy, apply $E = mc^2$, computing to 1.8×10^{-9} J of energy. Since 1 MeV is 1.6×10^{-13} J, we end up with

11,200 MeV corresponding to 12 a.m.u. (1 a.m.u. is 931.5 MeV).

In practice, and perhaps surprisingly, atoms (nuclei) weigh *less* than the sum of their parts due to binding energy. In order to rip a nucleus completely apart and move all the nucleons far from each other, energy must be *put in* (left part of Figure 15.9). And any change in energy is accompanied by a change in mass, via $\Delta E = \Delta m c^2$. All the energy that must be injected to completely dismantle the nucleus *weighs something*! So the mass of the individual pieces after dismantling the nucleus is effectively the mass of the original nucleus *plus* the mass-equivalent of all the energy that was put in to tear it apart (middle panel of Figure 15.9). Therefore, binding energy effectively *reduces* the mass of a nucleus, which we will now explore quantitatively.
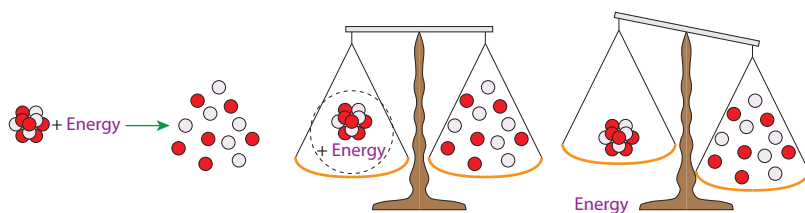

Figure 15.9: One must add energy to overcome nuclear binding energy in order to bust up a nucleus into its constituent nucleons (left). Thus, the collective mass of a nucleus *plus* the mass associated with the energy it takes to break it apart (via $E = mc^2$) must be equal to the sum of the masses of the constituent parts (middle). Therefore, if we compare the mass of the nucleus *alone* (removing the energy's mass from the scale) it must be less than the mass of the loose collection of nucleons (right).

A careful look at Figure 15.4 reveals that lighter stable nuclei (gray-squares) at the lower left of the chart have a mass a little larger than the corresponding mass number, but by the upper right—around oxygen—the mass has edged just lower than A. Table 15.3 shows this trend, confirmable in Figure 15.4 for the first four nuclides in the table. The difference between mass and A is most negative around iron, then turns around and becomes positive again for heavy elements like uranium.

What is going on here? If the mass of a nucleus were just the sum of its parts, we would expect the total mass to just track linearly as we add more pieces. In fact, if we try to build a neutral carbon atom out of 6 protons, 6 neutrons, and 6 electrons, the sum, according to Table 15.4, should be 12.099 a.m.u., not 12.000. The discrepancy is due to nuclear binding energy, as was introduced in Figure 15.9.

Table 15.3: Example mass progression.

Nuclide	A	mass (a.m.u.)
^{2}H	2	2.014
^{4}He	4	4.003
^{12}C	12	12.000
^{16}O	16	15.995
^{56}Fe	56	55.935
^{235}U	235	235.044

Particle	a.m.u.	10^{-27} kg	MeV/c^2
proton	1.0072765	1.6726219	938.2720882
neutron	1.0086649	1.6749275	939.5654205
electron	0.00054858	0.000911	0.510999
(a.m.u.)	1.0000000	1.660539	931.494102

Table 15.4: Constituent masses of atomic building blocks, expressing the same basic thing in three common units systems.

Nuclear binding energy is *incredibly* strong[15] and is able to overpower the natural electric repulsion between positively charged protons and stick them together in an unwilling bunch. The strong nuclear force only acts over a tiny range within about 10^{-15} m:[16] it is very powerful

15: ...relating to what we call the strong nuclear force

16: The whole atom is around 10^{-10} m in scale

on short length scales, but ceases to operate much beyond the confines of the nucleus. Think about binding energy this way: if we tried to pry a proton or a neutron (nucleon, generically) away from a nucleus, we would encounter a very powerful force opposing the action. But let's say we persist, and *do work* in extracting the nucleon by the usual recipe of force times distance. It is *so much* work, in fact, that $\Delta E = \Delta mc^2$ becomes relevant, measurably altering the mass.

Table 15.5 walks through some example calculations, one of which is traced in Example 15.3.2. Because the ^{1}H nuclide is just a lone proton, it has no binding energy.

Grab a calculator and follow Example 15.3.2 yourself!

Nucleus	$\Sigma m_{p,n,e}$	actual m	Δm	Δmc^2 (MeV)	MeV per nucleon
^{1}H	1.007825	1.007825	0	0	0
^{2}H	2.016490	2.014102	0.002388	2.22	1.11
^{4}He	4.032980	4.002603	0.030377	28.29	7.07
^{12}C	12.09894	12.000000	0.098940	92.16	7.68
^{56}Fe	56.46340	55.934942	0.528447	492.25	8.79
^{235}U	236.9590	235.043920	1.915065	1783.85	7.59

Table 15.5: Example nuclear binding energy calculations. The second column is the simple sum of masses of protons, neutrons, and electrons, per Table 15.4. Next is measured mass, then the difference. The difference is re-cast in MeV, representing the *total* binding energy of the nucleus, inexorably rising with the size of the nucleus. The final column divides by the mass number to get binding energy per nucleon, which peaks around iron. See Example 15.3.2 to understand how these numbers are computed.

Example 15.3.2 Following the entry in Table 15.5 for ^{56}Fe, we first multiply the individual proton, neutron, and electron masses from Table 15.4 by the 26 protons, 30 neutrons, and 26 electrons comprising ^{56}Fe to get a sum-of-parts value of 56.46340 a.m.u..[17]

The *actual* mass, as it appears for ^{56}Fe in the Chart of the Nuclides is 55.934942 a.m.u., which is smaller by 0.528447 a.m.u.[18]

Since 1 a.m.u. is 1.660539×10^{-27} kg, we can convert this mass difference into kilograms, then multiply by c^2, where $c = 2.99792458 \times 10^8$ m/s to get the associated energy in units of Joules. Traditionally, nuclear physics adopts a more convenient scale of electron-volts, and in particular, the MeV.[19] To get our mass-energy difference from Joules to MeV, we divide by 1.6022×10^{-13} J/MeV, and this is the 492 MeV number appearing in the Δmc^2 column of Table 15.5.

Finally, we divide by the number of nucleons in the nucleus—$A = 56$ in this case—to determine how much binding energy is present *per nucleon*—the significance of which will soon become clearer.

17: Find this in Table 15.5.

18: These numbers also appear in Table 15.5.

19: 1 MeV is 10^6 eV, and 1 eV is 1.6022×10^{-19} J (Sec. 5.9; p. 78).

Therefore, the difference between the sum-of-parts mass and actual nucleus mass in Table 15.5 provides a measure of how much binding energy holds the nucleus together.[20]

Notice that the first entry in Table 15.5 for the single-proton hydrogen atom has *no* binding energy in the nucleus: the lonely proton has no other nucleon to which it might bind. But deuterium (^{2}H) has a proton and a neutron, held together by 2.2 MeV of binding energy. The binding

20: . . . thus how much energy would need to be supplied to completely *unbind* the entire nucleus, as in Figure 15.9

energy per nucleon in the last column of Table 15.5 starts out small, but soon settles to the 7–9 range for most of the entries. It is extremely insightful to plot the binding energy per nucleon as a function of the nucleon mass number, A, which we do in Figure 15.10.
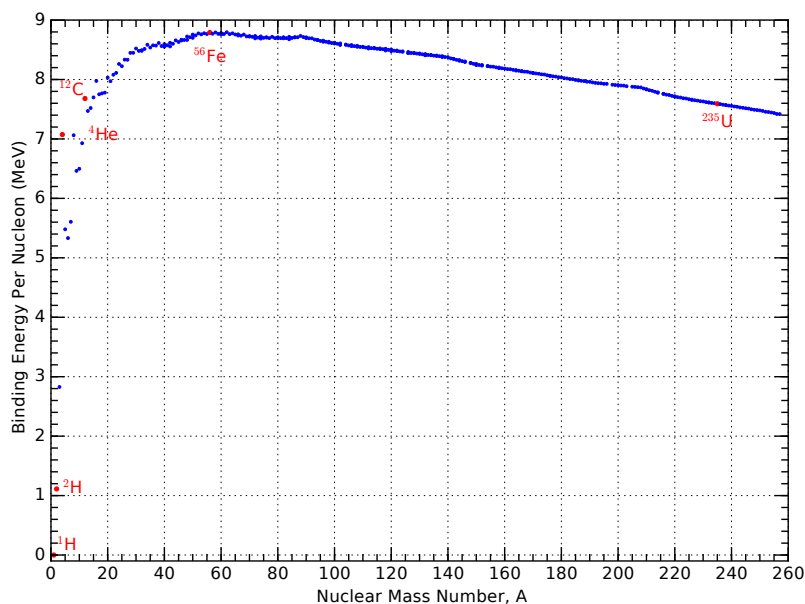

Figure 15.10: Binding energy per nucleon as a function of total mass number, A. The nuclei featured in Table 15.5 are indicated as red points. Note in particular that ^{56}Fe sits at the peak of the curve. Fusion operates from left to right, building larger nuclei, and fission goes from right to left, tearing apart nuclei. Only actions that *climb* this curve are energetically favorable, meaning that fusion is profitable on the the left-hand side, and fission makes sense on the right: each driving toward the peak binding energy per nucleon.

The value of Figure 15.10 is hard to over-emphasize. Key take-aways are:

1. Most nuclei are at around 8 MeV per nucleon, meaning that it would take an average of about 8 MeV of energy to rip out each member (proton or neutron) from a nucleus;
2. The peak is at ^{56}Fe,[21] meaning that this is the most tightly bound nucleus;[22]
3. The slope on the left side is *much* steeper than the slope on the right side, after the peak, which speaks to why fusion (building from small to big) is more potent than fission (tearing apart very massive nuclei);
4. Fusion in stars does not build elements beyond the peak around iron, since to go beyond the peak is not energetically favorable.

21: Actually, ^{62}Ni wins by a hair at 8.795 MeV/nuc, but is somewhat overlooked because it is only 0.006% as abundant as ^{56}Fe, whose binding energy per nucleon is essentially tied for the top at 8.790 MeV/nuc.

22: A peak exists because nucleons initially find advantage in binding together, but ultimately the increasing number of mutually-repelling protons makes the environment less appealing for larger nuclides.

It can be helpful to think of Figure 15.10 upside-down, as in Figure 15.11, turning the iron "peak" into a trough. A ball will roll toward and settle near the bottom of the trough, which is what both fusion and fission do, but from opposite directions.

15.4 Fission

Having covered some fundamentals, we are ready to tackle aspects of nuclear energy. Really it is very simple. Enough nuclear material in a
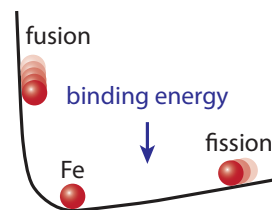

Figure 15.11: Turning the binding energy curve upside-down makes it easier to conceptualize fusion and fission driving toward the most tightly bound point (iron), like a ball might roll.

small space will get hot, for reasons detailed below. The heat is used to boil water into high-pressure steam, which then turns a turbine and generator (Figure 15.12). Note that a nuclear fission plant has much in common with a coal-fired power plant, as evidenced by the similarity of Figure 15.12 to Fig. 6.2 (p. 90). Only the source of heat is much different in origin.
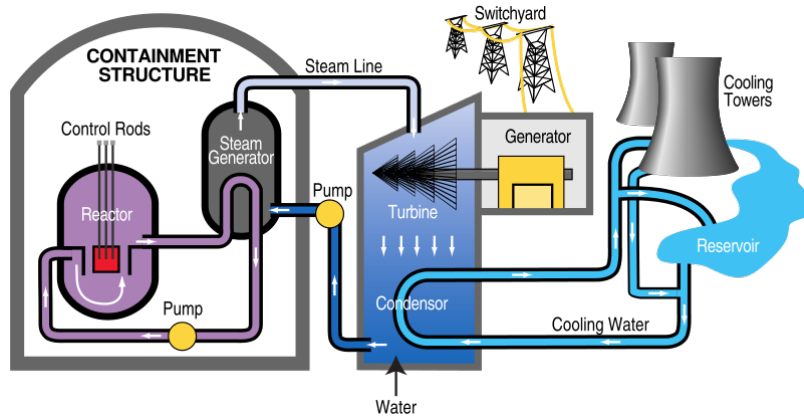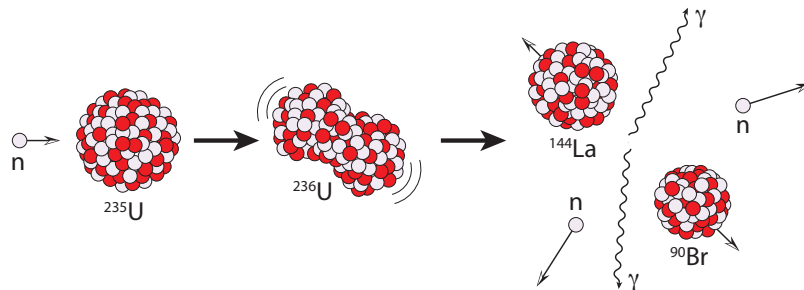

Figure 15.12: Typical nuclear power plant design, bearing much resemblance to the generic scheme from Figure 6.2. Details on the reactor core will follow in Section 15.4.4. Source: TVA.

15.4.1 The Basic Idea

Out of all the nuclides, three are amenable for use in a fission reactor. Two are isotopes of uranium: ^{233}U and ^{235}U; and one is plutonium: ^{239}Pu. Of these, only ^{235}U is found in nature, so we will concentrate on this one, returning later to the other two when we talk about breeder reactors in Section 15.4.4.2.

What makes ^{235}U (and the other two) special is that a slow[23] neutron—one just bumping around at a speed governed by the local temperature, and thus called a thermal neutron—can walk up to and stick[24] to the nucleus and cause it to split into two large chunks—depicted in Figure 15.13. Other nuclei would not break up, just accepting the new neutron and possibly converting a neutron to a proton via β^- decay.

23: This is in contrast to a fast neutron that tends to bounce rather than stick to the nucleus.

24: No forces prevent a neutron from approaching a nucleus. Happening to hit the tiny nucleus is the only barrier.

Figure 15.13: Fission schematic for ^{235}U, showing one of many possible outcomes—in this case ^{90}Br and ^{144}La plus two neutrons (an example case treated in detail in the text). The intermediate state, ^{236}U, created when ^{235}U absorbs a neutron, is highly unstable and will spontaneously break into (always) two different-size large fragments ("daughter" nuclei) and perhaps some extra neutrons. Gamma rays and kinetic energy (high-velocity fragments) are also released. Note that at each stage, the total number of nucleons is always 236.

When the nucleus breaks up, the pieces fly out at high speed, carrying kinetic energy that will be deposited in the local material as they bump

their way to a halt. Gamma rays[25] are also released. By catching all of this energetic output, the surrounding material gets very hot and can be used to make steam.

15.4.2 Chain Reaction

As we have seen, in order to get fission to happen, we need ^{235}U and some wandering neutrons. Once fission commences, the breakup of the nucleus usually "drips" a few spare neutrons, like crumbs left after cutting a piece of bread. The left-over neutrons provide a replenished source of neutrons ready to initiate more fission events. Now the door is open for a chain reaction, in which the neutrons produced by the fission events are the very things needed to stimulate additional fission events.

When the nucleus splits, any extra neutrons come out "hot" (high speed), which tend to bounce off uranium nuclei without sticking. They need to be slowed down, which is accomplished by a moderator: basically light atoms[26] that can receive the neutron impact as a sort of damping medium. Then the main trick is to prevent a runaway that could occur if *too many* neutrons become available; in which case it's a party that can get out of control. So nuclear plants employ control rods containing materials particularly effective at absorbing (trapping) neutrons. The colors of the lower halves of some squares in the Chart of the Nuclides (Figure 15.4) indicate neutron capture cross section. Boron (^{10}B) is a favorite choice to soak up neutrons and tame (or even halt) the reaction. The goal is to maintain a chain reaction that produces a net balance of **exactly one** unabsorbed slow neutron per fission event, available to attach itself to a waiting ^{235}U nucleus.

15.4.3 Fission Accounting

The nucleus (uranium in the present discussion) always breaks up into two largish pieces, possibly accompanied by a few liberated spare neutrons. Because of the way the track of stable elements curves on the Chart of the Nuclides, the resultant pieces are likely to be neutron rich, to the right of the stable nuclei. To understand this, refer to Figure 15.14 and the associated caption.

The math always has to add up: nucleons are not created or destroyed during a fission event. They just rearrange themselves, so the total number of neutrons stays the same, as does the total number of protons. *After* the split, β^- decays will carry out flavor changes, but we'll deal with that part later.
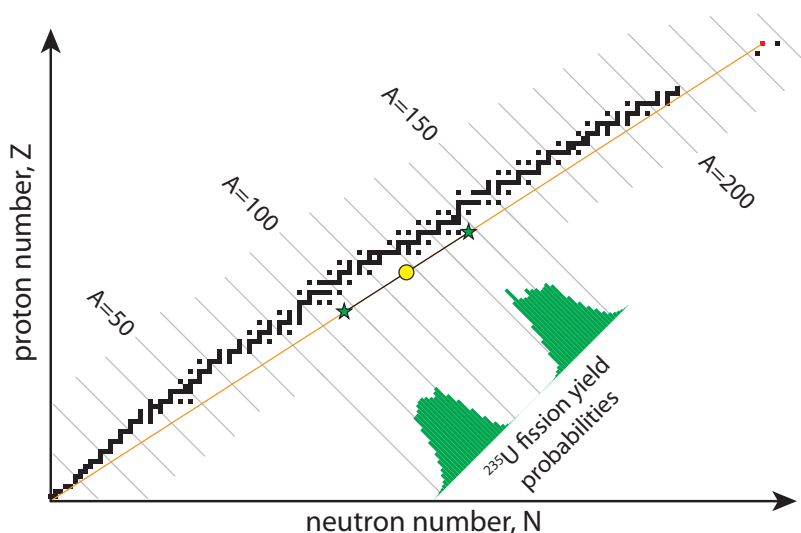
Figure 15.14: Fission of ^{235}U (small red square, upper right) tends to produce two neutron-rich fragments. If it split exactly in two, the result would lie at the midpoint of the orange line connecting ^{235}U to the origin, at the yellow circle. In practice, an equal split is highly unlikely, as one fragment tends to be around $A \sim 95$ and the other around $A \sim 140$, as depicted by the probability histogram in green. The two green stars separated along the orange line represent a more likely outcome for the two fragments. As long as the green stars are located so that the yellow circle is exactly between them, the accounting of proton and neutron number is satisfied. Because the orange line lies to the right of the stable nuclei, the fission products tend to be neutron-rich and undergo a series of radioactive β^- decays before reaching stability, which could take a very long time in some cases.

Example 15.4.1 If one of the two fragments from the fission of a ^{235}U nucleus ($Z = 92$) *after adding* a thermal neutron winds up being ^{90}Br ($Z = 35$), what is the other nucleus going to be?

The other fragment will preserve total proton count, so $Z = 92 - 35 = 57$, and as such is destined to be the element lanthanum. Which isotope of lanthanum is produced depends on how many neutrons escape the split. Table 15.6 summarizes the particle counts of the various players.

If no spare neutrons are left over, the lanthanum must have $N = 144 - 55 = 89$ neutrons,[27] in which case its mass number will be $A = 146$, so ^{146}La. If two neutrons are set free, then the lanthanum will only keep 87 neutrons and be ^{144}La, as depicted in Figure 15.13.

Typically, about 2–3 neutrons are left out of the final fragments, and can go on to promote additional fission events in the chain reaction.

27: ^{235}U had $A - Z = 235 - 92 = 143$ neutrons, plus the thermal neutron addition.

	^{235}U	^{90}Br	^{146}La	^{145}La	^{144}La	^{143}La
A	235	90	146	145	144	143
Z	92	35	57	57	57	57
N	143	55	89	88	87	86
n	1		0	1	2	3

Table 15.6: Possible outcomes for Example 15.4.1 if we set one of the daughter particles to be bromine-90, forcing the other daughter to be lanthanum. Different isotopes of lanthanum will result for differing numbers of spare neutrons left after the break-up (last row).

Being a probabilistic (random) process, each fission can result in a large set of possible "daughter" nuclei—only one set of which was explored in Example 15.4.1. As long as the masses all add up, and the two-hump probability distribution in Figure 15.14 is respected, anything goes. In other words, we have no control over exactly what pieces come out. Figure 15.15 provides a graphic illustration of four different possible pairs of daughter fragments. The counting requirement is satisfied by having the products located diametrically opposite from the ^{235}U midpoint (yellow circle). The positions of the stars will distribute along A-values

according to the probability distribution (multi-colored histogram). Note the completely distinct peaks, conveying that virtually *every* fission event results in just two fragments: one bigger and one smaller. At least that aspect of fission is predictable, even if we can't say precisely which nuclei will be left after an individual fission event.
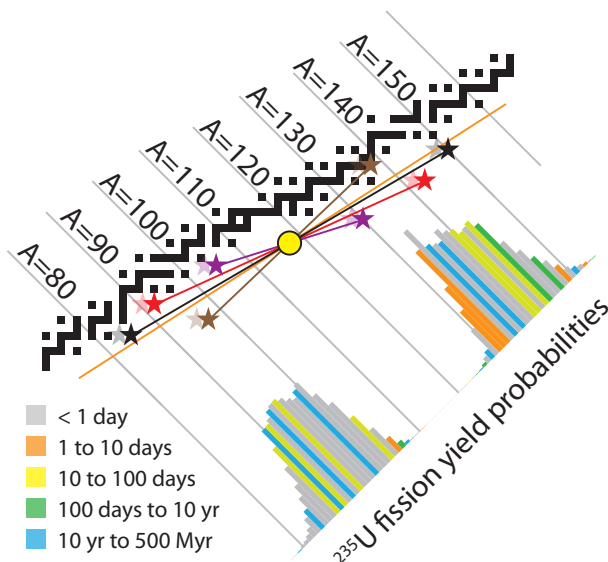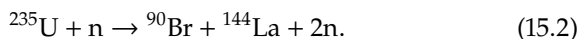

Figure 15.15: Various fission product outcomes are possible, indicated here by four sets of colored star pairs and connecting lines. The average position of each pair is the yellow circle (the stars are diametrically opposite the circle), which guarantees that the total number of neutrons and protons is unchanged from the parent nucleus to the daughter nuclei. To the extent that additional neutrons are left behind like crumbs, the stars will displace to the left of their indicated positions a bit, as hinted by the lighter-shaded "ghost" stars, whose offsets from the nominal star positions will also vary depending on how many neutrons are left out of the two final fragments. The coloring of the histogram indicates radioactive lifetime for the decay chain of a neutron-rich fragment at each mass number, matching the half-life color scheme used in Figure 15.4.

Let us now examine the energetics, using the result from Example 15.4.1, in which ^{235}U breaks into ^{90}Br and ^{144}La, plus two spare neutrons.[28] To be explicit, the reaction we will trace is

28: . . . also matching the scenario in Figure 15.13 and the penultimate column of Table 15.6

$$^{235}\text{U} + \text{n} \rightarrow {}^{90}\text{Br} + {}^{144}\text{La} + 2\text{n}. \tag{15.2}$$

Table 15.7: Mass details of Eq. 15.2, tracking before and after masses in both a.m.u. and MeV units. The input mass of around 236 a.m.u. is reduced by about 0.185 a.m.u., or 0.08%.

Constituent/Stage	mass (a.m.u.)	mass (MeV/c^2)
^{235}U	235.04392	218,942.0
n	1.00866	939.6
input mass	236.05259	219,881.6
^{90}Br	89.93069	83,769.9
^{144}La	143.91955	134,060.2
2n	2.01733	1,879.1
output mass	235.86757	219,709.3
mass change	0.18502	172.3

The masses of each piece, according to the Chart of the Nuclides, appear in Table 15.7. Again, we find that the mass sums don't equal: the final parts are lighter than the inputs. The fission managed to lose 0.185 a.m.u. of mass, corresponding to 172 MeV of energy (via $E = mc^2$; see Example 15.3.1). That's a 0.08% change in the mass, and converts to an energy density of roughly 17 *million* kcal/g, making the process over a million

times more energy–dense than our customary ~ 10 kcal/g chemical energy density. See Box 15.3 for an example of how to compute this.

Box 15.3: Nuclear Energy Density

The example corresponding to Table 15.7 is said to correspond to 17 million kcal/g, but how can we get here? The mass change of 0.185 a.m.u. corresponds to a mass in kilograms of 3.07×10^{-28} kg, according to the conversion that 1 a.m.u. is 1.6605×10^{-27} kg (Table 15.4). Multiply this by c^2 to get energy in Joules, yielding 2.76×10^{-11} J.[29] In terms of kcal, we divide by 4,184 J/kcal to find that this fission event yields 6.6×10^{-15} kcal.

We now just need to divide by how many grams of "fuel" we supplied, which is 236.05 a.m.u. (Table 15.7), equating to 3.92×10^{-25} kg, or 3.92×10^{-22} g. Now we divide 6.6×10^{-15} kcal by 3.92×10^{-22} g to get 16.8×10^6 kcal/g. Blows a burrito out of the water.

29: This result, by the way, is the same as 172.3 MeV in Table 15.7 using the conversion that 1 MeV is 1.6022×10^{-13} J.

Example 15.4.2 Considering that the average American uses energy at a rate of 10,000 W, how much ^{235}U per year is needed to satisfy this demand for one individual?

Since we have just computed the energy density of ^{235}U to be 17×10^6 kcal/g (Box 15.3), let's first put the total energy in units of Joules, multiplying 10^4 W by 3.155×10^7 seconds in a year and then dividing by 4,184 J/kcal to get kilocalories. The result is 75 million kcal, so that an American's annual energy needs could be met by 4.5 g[30] of ^{235}U. That translates to about a quarter of a cubic centimeter, or a small pebble, at the density of uranium. Pretty amazing!

30: 75 million kcal divided by 17 million kcal/g is 4.5 g.

We can take a graphical shortcut to all of Section 15.4.3, which hopefully will tie things together in an instructive way.

Example 15.4.3 Refer back to Figure 15.10 (and/or Table 15.5) to see that ^{235}U has a binding energy of about 7.6 MeV per nucleon. Where we end up, around $A \approx 95$ and $A \approx 140$, the binding energies per nucleon are around 8.7 and 8.4 MeV/nuc at these locations, respectively.

Multiplying the binding energy per nucleon by the number of nucleons provides a measure of *total* binding energy: in this case 1,790 MeV for ^{235}U, about 825 MeV for the daughter nucleus around $A \approx 95$, and 1,175 MeV for $A \approx 140$.[31] Adding the latter two, we find that the fission products have a total binding energy around 2,000 MeV, which is greater[32] than the ^{235}U binding energy by about 210 MeV—somewhat close to the 172 MeV computed for the particular example in Table 15.7.

31: 7.6×235; 8.7×95; and 8.4×140

32: Binding energy *reduces* mass, so larger binding energy means lighter overall mass.

The graphical method got us pretty close with little work, and hopefully led to a deeper understanding of what is going on. The rest of this

paragraph explains the discrepancy, but should be considered non-essential reading. The fission process typically results in a few spare neutrons. Each left-over (unbound) neutron deprives us of *at least* 8 MeV in unrealized binding potential,[33] *and* the subsequent β^- decays from the neutron-rich daughter nuclei to stable nuclei also release energy not accounted in Table 15.7. Both of these contribute to the shortfall in comparing 172 MeV to 210 MeV, but even without this, we got a decent estimate just using the graph in Figure 15.10.

15.4.4 Practical Implementations

As we saw above, nuclear fission involves getting fissile nuclei—generally ^{235}U—to split apart by the addition of a neutron. The following criteria must be met:

1. presence of nuclear fuel (^{235}U);
2. presence of neutrons, provided as left-overs from earlier fission events;
3. a moderator to slow down neutrons that emerge from the fission events at high speed;
4. a high enough concentration of nuclear fuel that the slowed-down spare neutrons are likely to find fissile nuclei;
5. neutron absorbers in the form of control rods that can be lowered into the reactor and act as the main "throttle" to set reaction speed (thus power output), and also prevent a runaway chain reaction;
6. a containment vessel to mitigate radioactive particles (gamma rays, high-speed electrons and positrons) from escaping to the environment.

Figure 15.16 shows a typical configuration.
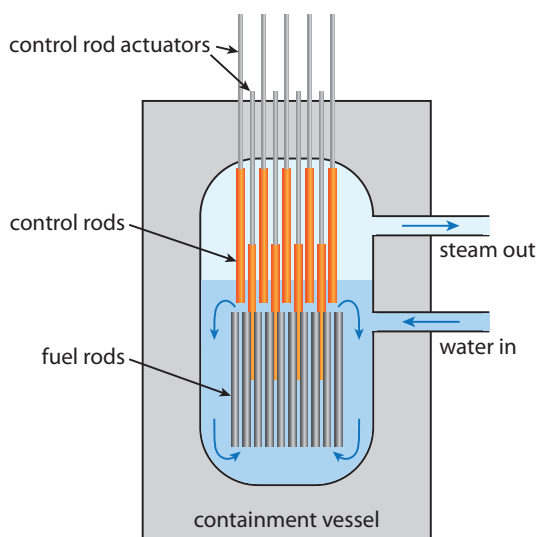
Figure 15.16: Typical boiling water reactor design. A thick-walled containment vessel holds water surrounding ^{235}U fuel rods. The water acts as the moderator to slow neutrons and also circulates around the rods to carry heat away, boiling to form steam that can run a standard power plant. Control rods set the pace of the reaction based on how far they are inserted into the spaces between fuel rods. Extra control rods are poised above the reactor core ready to drop quickly into the core in case of emergency—suddenly bringing the chain reaction to a halt.

In the design of Figure 15.16, called a boiling water reactor, the water acts as both the neutron moderator and the thermal conveyance medium. Nuclear fuel (uranium) is arranged in fuel rods, providing ample surface area and allowing water to circulate between the rods to slow down neutrons and carry the heat away. Neutron-absorbing control rods—usually containing boron—set the reaction speed by lowering from the top.[34] An emergency set of control rods can be dropped into the core in a big hurry to shut down the reactor instantly if something goes wrong. When the emergency rods are in place, neutrons have little chance of finding a ^{235}U nucleus before being gobbled up by boron.

34: . . . always this direction, so that gravity does the pulling rather then relying on some other drive force

As of 2019, the world has about 455 operating nuclear reactors, amounting to an installed capacity of about 400 GW.[35] The average *produced* power—not all are running all the time—was just short of 300 GW. The thermal equivalent would be approximately three times this, or 1 TW out of the 18 TW we use in the world. So nuclear is a relevant player. See Table 15.8 for a breakdown of the top several countries, Fig. 7.7 (p. 109) for nuclear energy's trend in the world, and Fig. 7.4 (p. 107) for the U.S. trend.

35: From this, we glean that reactors average roughly 1 GW each.

Country	# Plants	GW inst.	GW avg.	% elec.	global share (%)
U.S.	95	97	92	20	31
France	56	61	44	71	15
China	49	47	38	5	13
Russia	38	28	22	20	8
Japan	33	32	8	8	3
S. Korea	24	23	16	26	5
India	22	6	5	3	2
World Total	455	393	295	11	100

Table 15.8: Global nuclear power in 2019 [101], listing number of operational plants, installed capacity, average generation for 2019 (Japan currently has stopped a number of its reactors), percentage of *electricity* (not total energy), and fraction of global production (these 7 countries accounting for over 75%). Notice the close match between number of plants and GW installed for most countries, indicating that most nuclear plants deliver about 1 GW.

Nuclear plants only last about 50–60 years, after which the material comprising the core becomes brittle from exposure to damaging radioactivity and must be decommissioned. The median age of reactors in the U.S. is 40 years, and all but three are over 30 years old. Additional challenges will be addressed in the sections that follow.

When nuclear energy was first being rolled out in the 1950s, the catch phrase was that it would be "too cheap to meter," a sentiment presumably fueled by the stupendous energy density of uranium, requiring very small quantities compared to fossil fuels. The reality has not worked out that way. Today, a 1 GW nuclear power plant may cost $9 billion to build [102]. That's $9 per Watt of output power, which we can compare to the cost of a solar panel, at about $0.50 per W (Fig. 13.16; p. 215), or utility-scale installation at $1 per Watt [89]. While it seems that solar[36] wins by a huge margin, the low capacity factor of solar reduces average power output to 10–20% of the peak rating, depending on location. Meanwhile, nuclear reactors tend to run steadily 90% of the time—the off-time used for maintenance and fuel loading. So nuclear fission costs about $10 per delivered Watt, while solar panels are $2.5–5 per delivered

[102]: Union of Concerned Scientists (2015), *The Cost of Nuclear Power*

36: Recall, for context, that solar is not among the *cheaper* energy resources. Like solar, nuclear power is dominated by up-front costs, rather than fuel cost.

Watt and installed utility-scale systems are \$5–10 per Watt. In short, nuclear power is not an economic slam dunk.

15.4.4.1 Uranium

So far, we have ignored a crucial fact. Only 0.72% of natural uranium on Earth is the fissile ^{235}U flavor. The vast majority, 99.2745%, is the benign ^{238}U.[37] The ratio is about 140:1, so for every ^{235}U atom pulled out of the ground, 140 times this number of uranium atoms must be extracted. The origin of the disparity is a story of astrophysics and eons, covered in Box 15.4.

37: A trace amount, 0.0055%, is in ^{234}U.

Box 15.4: Origin of Uranium

The Big Bang that formed the universe produced only the lightest nuclei. By-and-large, the result was 75% hydrogen (^{1}H) and 25% helium (^{4}He). Deuterium (^{2}H) and ^{3}He were produced at the 0.003% and 0.001% levels, respectively, and then the tiniest trace of lithium. No carbon or oxygen emerged, which must be "cooked up" via fusion in stars.

Fusion in stars does not "climb over" the peak of the binding-energy curve in Figure 15.10, so stops in the vicinity[38] of iron. From where, then, did all of the heavier elements on the periodic table derive? Exploding stars called supernovae and merging neutron stars appear to be the origin of elements beyond zinc.

38: Iron has $Z = 26$; stars tend not to produce elements beyond zinc ($Z = 30$) by fusion.

The relative abundance of ^{235}U and ^{238}U on Earth can be explained by their different half-lives of 0.704 Gyr and 4.47 Gyr, respectively. Even if starting at comparable amounts, most of the ^{235}U will have decayed away by now. Solving backwards[39] to when they would have been present in equal amounts yields about 6 Gyr, which is older than the age of the solar system (4.5 Gyr) and younger than the universe (13.8 Gyr). This is a reasonable result for how old the astrophysical origin might be—allowing a billion years or so for the material to coalesce in our forming solar system.

39: This follows almost the exact same logic and process as carbon-14 radioactive dating, but using much longer half life nuclei to date Earth's building blocks!

Uranium is not particularly abundant. Table 15.9 provides a sense of how prevalent various elements are in the earth's crust. Uranium is more abundant than silver, but the *useful* ^{235}U isotope is four times rarer than silver, and only about 5 times as abundant as gold. Proven reserves of uranium [103] amount to 7.6 million (metric) tons available, and we have used 2.8 million metric tons to date. The implication is that we could continue about 3 times longer than we have gone so far on proven reserves. But nuclear energy has played a much smaller role than fossil fuels, so maybe this isn't so much.

[103]: (2020), *List of Countries by Uranium Reserves*

Evaluating the uranium reserves in energy terms is the most revealing

Element	Abund.	Element	Abund.	Element	Abund.
silicon	282,000	carbon	200	thorium	9.6
aluminum	82,300	copper	60	uranium	2.7
iron	56,300	lithium	20	silver	0.075
calcium	41,500	lead	14	^{235}U	0.02
titanium	5,650	boron	10	gold	0.004

Table 15.9: Example material abundances in the earth's crust, in parts per million by mass.

approach. First, we take 0.72% of the 7.6 million tons available to represent the portion of uranium in the form of ^{235}U. Enrichment (next section) will not separate *all* of the ^{235}U, and the reactor can't burn all of it away before the fuel rod is essentially useless. So optimistically, we burn half of the mined ^{235}U in the reactor. Multiplying the resulting 27,300 tons of *usable* ^{235}U by the 17 million kcal/g we derived earlier yields a total of 2×10^{21} J. Table 15.10 puts this in context against fossil fuel proven reserves from page 127. We see from this that proven uranium reserves give us only 20% as much energy as our proven oil reserves, and about 5% of our total remaining fossil fuel supply. If we tried to get all 18 TW from this uranium supply, it would last less than 4 years! This does not sound like a salvation.

Table 15.10: Proven reserves, in energy terms.

Fuel	10^{21} J
Coal	20
Oil	10
Gas	8
^{235}U	2

Proven uranium reserves would last 90 years at the *current* rate of use, so really it is in a category fairly similar to that of fossil fuels in terms of finite supply. To be fair, proven reserves are always a conservative lower limit on estimated total resource availability. And since fuel cost is not the limiting factor for nuclear plants, higher uranium prices can make more available, from more difficult deposits. Still, even a factor of two more does not transform the story into one of an ample, worry-free resource.

15.4.4.2 Breeder Reactors

In its native form, ^{235}U is too dilute in natural uranium—overwhelmingly dominated by ^{238}U—to even work in a nuclear reactor. It must be enriched to 3–5% concentration to become viable.[40] Enrichment is difficult to achieve. Chemically, ^{235}U and ^{238}U behave identically. The masses are so close—just 1% different—that mechanical processes have a difficult time differentiating. Centrifuges are commonly used to allow heavier ^{238}U to sink faster[41] than ^{235}U. But it's inefficient and usually requires many iterations to work up higher concentrations. The process is also lossy, in that not all of the ^{235}U finds its way to the enriched pile.[42]

But what if we could use the *bulk* uranium, ^{238}U, in reactors and not only save ourselves the hassle of enrichment, but also gain access to 140 times more material, in effect? Doing so would turn the proven reserves of uranium into about 7 times more energy supply than all of our remaining fossil fuels. Well, it turns out that despite its not being

40: Uranium bombs need at least 20% ^{235}U concentration, but typically aim for 85% to be considered *weapons grade*.

41: . . . in gaseous form

42: Depleted uranium is defined as containing 0.3% or less in the form of ^{235}U, which is not a huge reduction from the 0.72% starting point.

one of the three fissile nuclei, we can *convert*[43] ^{238}U into the fissile ^{239}Pu the following way.

43: . . . called transmutation

1. A ^{238}U may absorb a wandering neutron to become ^{239}U.
2. ^{239}U, whose half life is 23.5 minutes, undergoes β^- to become ^{239}Np in short order.
3. ^{239}Np also undergoes β^- with a half life of 2.4 days to become fissile ^{239}Pu.
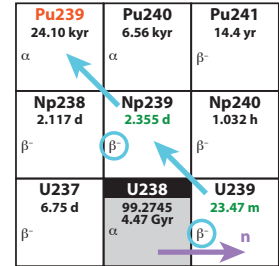
Figure 15.17 highlights this process in a simplified region of the Chart of the Nuclides, while Figure 15.18 shows complete details for the entire region around the fissile materials—the ones with red isotope names—which can be used to track the sequence outlined above.

Figure 15.17: Breeder route to ^{239}Pu.
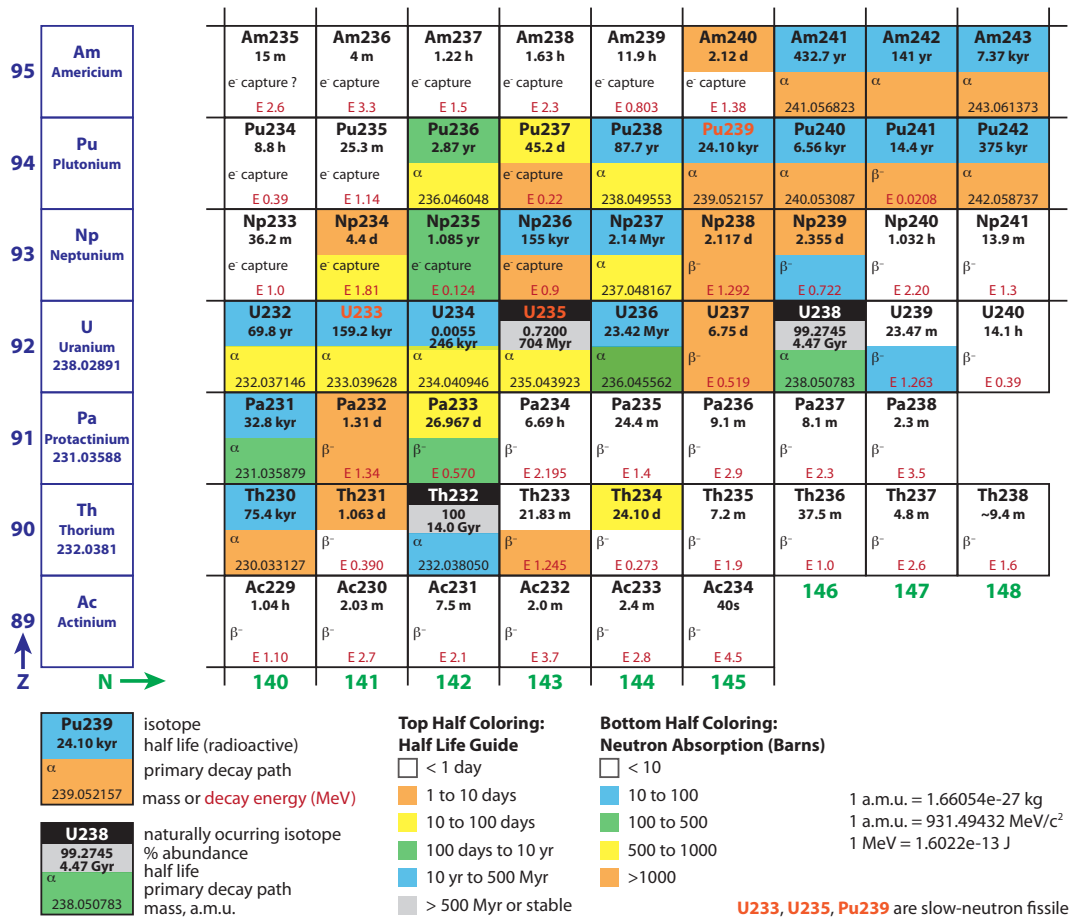
Figure 15.18: Chart of the Nuclides in the fission region. See also Figure 15.4 for the lower-left corner.

The result is that sterile ^{238}U can be turned into fissile ^{239}Pu that can be used in fission reactors. This process of transmuting an inert nucleus into a fissile one is called breeding, and is how we get any plutonium at all.[44] A nuclear reactor is a great place to introduce ^{238}U to neutrons: both are already in attendance. In fact, breeding happens as a matter of course in a nuclear reactor: it is estimated that one-third of the fission

44: . . . e.g., for weapons

energy in ordinary nuclear reactors comes from plutonium breeding and subsequent fissioning—without any extra effort. Special reactor designs enhance plutonium production, allowing the fuel rod to be "harvested" for plutonium. Usually, the plutonium is destined for use in weapons, but in principle reactors could be designed to efficiently produce and use plutonium from the ^{238}U feedstock. Downsides will be addressed in Section 15.4.6 on weapons and proliferation.

Box 15.5: Thorium Breeding

Another form of breeding merits mention. Notice that thorium[45] is more abundant than uranium in Table 15.9. But like ^{238}U, it is not fissile. However, applying the breeding trick, the absorption of a neutron by ^{232}Th ends up as ^{233}U—the last of our three fissile nuclei—in about a month's time. This provides an avenue for an even *greater* energy store than exists in ^{238}U via breeding to ^{239}Pu, by virtue of greater abundance. Unlike the plutonium route, thorium breeders are less susceptible to weapons and proliferation concerns.[46] That said, thorium reactors are more complex than uranium reactors, so that technical hurdles have thus far prevented any commercial scale application of the technique, leaving us unclear whether thorium represents a viable nuclear path.

45: ...of which 100% is the desired ^{232}Th isotope

46: ...although, radioactive waste is still problematic

15.4.5 Nuclear Waste

As we saw in our description of the fission process, the fragments distribute over a range of masses in a randomized way (Figure 15.15). The results are generally neutron-rich, and will migrate toward stable elements via β^- decays over the ensuing seconds, hours, days, months, and years. Some will go fast, and some will take ages to settle, depending on half-lives. Radioactive waste is dangerous to be around because the high-energy particles (like sub-atomic "bullets") spewing out in all directions can alter DNA, leading to cancer and birth defects, for instance.

The lighter of the two fission fragments has a 59% chance of landing on a stable nucleus within a day or so. For the heavier fragment, it's a 45% chance. The rest get hung up on some longer half-life nuclide, and could remain radioactive for a matter of weeks or in some cases millions of years. The colors in the fission probability histograms in Figure 15.15 provide a visual guide for the mass numbers that reach stability promptly (gray) vs. those that get hung up for a long time (blue is more than 10 years). For example, the histogram element at $A = 90$ is blue because ^{90}Sr—discussed below—stands in the way of a fast path to stability.

Figure 15.19: Decay activity of fragments from 1 kg of fissioned ^{235}U over time, on a log–log plot. The vertical axis is the power of radioactive emission, in W, for a variety of relevant isotopes—each having their own characteristic half life. The black line at the top is the total activity (sum of all contributions), and some of the key individuals are separated out. The dashed line for actinides is an approximate representative indicator of the role played by heavy nuclides formed in the reactor by uranium absorption of neutrons. Minor tick marks are at multipliers of 2, 4, 6, and 8 for each axis. As a matter of possible interest, the exponential decays of each element on this log–log plot have the functional form of exponential curves drawn upside-down.
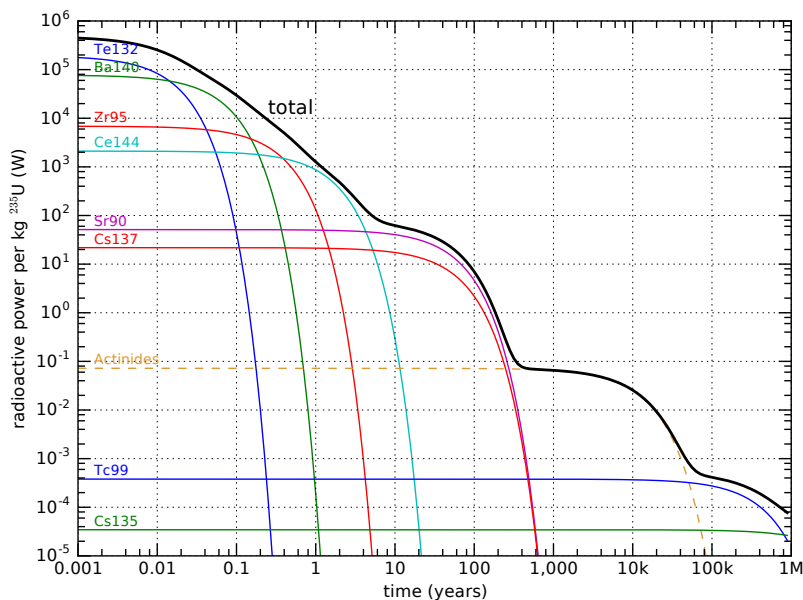
Figure 15.19 shows how the fission decays play out over time. For the first month or so out of the reactor, the spent fuel is really "hot" radioactively, but falls quickly as ^{95}Zr and then ^{144}Ce dominate around one year out. At about 5 years, the pair of ^{90}Sr and ^{137}Cs begin to dominate the output for the next few-hundred years. Some of the products survive for millions of years, albeit at low levels of radioactive power. In addition to the daughter fragments, uranium in the presence of neutrons transmutes into neptunium, plutonium, americium, and curium via neutron absorption and subsequent β^- decays, represented approximately and collectively in Figure 15.19 by a dashed curve labeled Actinides.[47]

47: Breeder reactors can "burn" the actinides, reducing some of the long-term waste threat, but will unavoidably still be left with all the radioactive fission products.

The bottom line is that fission leaves a trash heap of radioactive waste that remains at problematic levels for many thousands of years. When nuclear reactors were first built, they were provisioned with holding tanks—deep pools of water—in which to place the waste fuel until a more permanent arrangement could be sorted out (Figure 15.20). We are still waiting for an adequate permanent solution for waste storage, and the "temporary" pools are just accumulating spent fuel. Transporting the spent fuel is hazardous—in part because it could fall into the wrong hands and be used to make "dirty" bombs—and no one wants a nuclear waste facility in their backyard, making the problem politically thorny. On the technical side, it is difficult to identify sites that are geologically stable enough and have little chance of groundwater contamination. Underground salt domes offer an interesting possibility, but political challenges remain daunting.
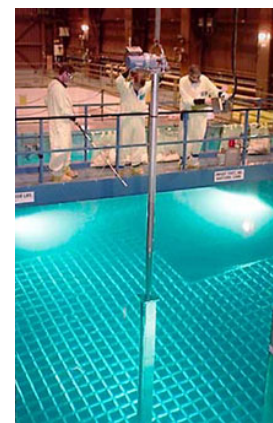

Figure 15.20: A spent fuel rod being lowered into a storage grid in a pool of water at a nuclear power plant. Source: U.S. DoE.

15.4.6 Nuclear Weapons and Proliferation

Nuclear bombs are the most destructive weapons we have managed to create. The first bombs from the 1940s were based on either highly enriched ^{235}U or on ^{239}Pu. For uranium bombs, the idea is shockingly simple. Two separate lumps of the bomb material are held apart until detonation is desired, at which point they are slammed together.[48] It's not the collision that creates the explosion, but a runaway process based on having a high concentration of fissile material and no neutron absorbers present to control the resulting chain reaction. The concept is critical mass. The combined lump exceeds the critical mass, and explodes.[49]

As simple as nuclear weapons are to build, the bottleneck becomes obtaining fissile material. Plutonium does not exist in nature, since its 24,100 yr half-life means nothing is left over from the astrophysical processes that gave us uranium and thorium (Box 15.4). We only still have the latter two thanks to their long half lives. So fissile material has to start with uranium. But as we have seen, natural uranium is only 0.72% fissile (^{235}U). In order to be explosive, the uranium must be enriched to at least 20% ^{235}U, and generally much higher (85%). Reactor fuel, at 3–5% ^{235}U will experience meltdown if the critical mass is exceeded, but will not explode. Enrichment is technically difficult, and attempts to acquire and enrich uranium are monitored closely. Often we hear of countries pursuing uranium enrichment, claiming that they are only interested in domestic energy production—a peaceful purpose. And it is true that the first step in nuclear power generation is also enrichment. So it is very difficult to ascertain true intentions. Once a country has the ability to enrich uranium enough for a nuclear plant, they can in principle keep the process running longer to arrive at weapons-grade ^{235}U.

While we worry about ^{235}U falling into the wrong hands, perhaps more disturbing is ^{239}Pu. Having a much shorter half-life than ^{235}U (24 kyr vs. 704 Myr), it is more dangerous to handle.[50] But plutonium is otherwise easy to deal with, since it requires no enrichment and can be chemically separated to achieve purity. It is the material of choice for nuclear weapons.

Serious pursuit of breeder reactors effectively means manufacturing lots of plutonium, leading to proliferation of nuclear materials: it becomes harder to track and keep away from mal-intentioned groups. The world becomes more dangerous under a breeder program. Thorium breeding (Box 15.5) is less risky in this regard because the ^{233}U prize is mixed with a ridiculously dangerous ^{232}U isotope that puts plutonium to shame, so working with it is pretty deadly, which may deter would-be pursuit of this material by rogue groups.

A related concern involves proliferation of the abundant radioactive waste from fission plants, which could be mixed into conventional explosives[51] to radioactively contaminate a city or local region—poisoning

48: For plutonium, this process is fouled by the presence of ^{240}Pu, forcing a different approach in which a sphere below critical mass is imploded to create high density.

49: Never stack lumps of fissile material together on a shelf, or a nasty surprise may be in store.

50: . . . much higher rate of radioactive decay

51: . . . called a "dirty bomb"

water, food, and air. In short, nuclear fission carries many perils on a number of fronts.

15.4.7 Nuclear Safety

A properly operating nuclear facility actually emits less radioactivity than does a traditional coal-fired power plant! As is true for many materials mined from the ground, coal contains some small amount of radioactive elements found in the earth's crust: principally thorium, uranium, and potassium. Lacking any shielding or protection, the exhaust from a coal plant distributes these products into the atmosphere. Nuclear plants, by contrast, have no exhaust,[52] and carefully control the exposure to radioactivity.

52: Note that cooling towers often have a plume of water vapor above them, but this is the result of evaporative cooling, and not exhaust in the usual sense.

However, things can go wrong. The U.S. had a scare in 1979 when a six-month-old nuclear plant at Three Mile Island in Pennsylvania (Figure 15.21) suffered a loss-of-cooling incident that resulted in severe damage to (meltdown of) the core. But the containment vessel held and no significant radioactivity was released to the environment. Workers at the plant received a dose equivalent to an extra 100 days of natural[53] exposure. So we dodged a bullet.

53: We are unavoidably exposed to radiation in our daily lives from air, water, food, Earth, and the cosmos.

Chernobyl was not so lucky in April 1986 when an ill-conceived test went sideways and resulted in an actual explosion of the core. This scenario was previously thought to be impossible, but it was a steam explosion, not a nuclear blast—so more like a "dirty bomb" that scattered radioactive material across the region. Thirty-one people died in the immediate aftermath, and about 200 people got acute radiation sickness. It is estimated that in the long term, 25,000 to 50,000 additional cancer cases will result, but this number is controversial and it is hard to tease Chernobyl-caused cancer/deaths apart from the much larger number of background cancer cases. The town of Chernobyl is still abandoned and only recently has begun to allow strictly limited incursions.

Figure 15.21: Three Mile Island nuclear plant in Pennsylvania. The two reactor cores are in the foreground of the larger cooling towers behind. Source: U.S. DoE.

The most recent major accident was the Fukushima Daiichi plant in Japan following the Sendai earthquake in March 2011, resulting in the evacuation of 200,000 people and agricultural loss. The earthquake caused the three operating reactors to shut down (safely), while diesel-fueled generators ran to power pumps maintaining cooling flow over the hot fuel rods. The core of a reactor is still very hot after fission stops and continues to generate heat as daughter nuclei decay, so cooling flow must be maintained or the core can melt. The ensuing tsunami[54] ruined the plan to keep the cores cool, as the generator rooms flooded, causing the cooling flow to fail. The cores of all three reactors melted down and hydrogen gas explosions created a major release of radioactivity. Perhaps in contrast to the Chernobyl plant, Fukushima was designed by General Electric and operated by a well-educated high-tech society. No one is exempt from risk when it comes to nuclear reactors.

54: ...within 10 minutes of the earthquake

15.4.8 Pros and Cons of Fission

Collecting the advantages and disadvantages of fission, we start with the positive aspects:

► Nuclear fuel has extraordinary energy density, about a million times better than chemical energy density;
► Nuclear fission is proven technology providing a substantial fraction of electrical energy at present;
► Life-cycle CO_2 emissions for nuclear fission is only 2% that of traditional fossil fuel electricity [68];
► Breeder reactors could provide thousands of years of fuel, by way of uranium and thorium (undeveloped as yet).

[68]: (2020), *Life Cycle GHG Emissions*

And for the downsides:

► Radioactive waste is dangerous for thousands of years, and no clear solution to its disposal or long-term storage has emerged.
► Conventional uranium fission has limited fuel supply,[55] measuring in decades;
► Breeder reactors exacerbate the waste issue and promote proliferation of nuclear materials;
► Development of nuclear energy technology prepares an easy step to immensely destructive nuclear weapons;
► Accidents happen even to the best-managed reactors, the consequences often being severe for a region.

55: . . . in the absence of breeder reactor implementation

Nuclear fission is a complex topic that has compelling advantages and worrisome faults. Not surprisingly, attitudes are highly mixed. One survey [104] indicates that adults in the U.S. oppose building more nuclear plant by a slim 51% to 45%, while scientists overall favor advancing nuclear plants by a 2:1 margin,[56] and physicists surveyed favored nuclear by 4:1. Scientists are much more likely to view climate change as a serious threat than the U.S. population as a whole, and therefore are likely to be attracted to energy resources that do not emit CO_2. Of the physicists surveyed, it would be a mistake to assume that even the majority know the topic as thoroughly as it is covered in this chapter—given the degree of specialization within the field. Among those who understand the topic thoroughly[57] it is almost certain you'd find a healthy split: those for whom the perils outweigh advantages, and those who are concerned enough about climate change to accept the "lesser of two evils," and/or who are enthusiastic about the technology as a glowing example of our mastery over nature's hidden secrets.

[104]: Pew Research (2015), "Elaborating on the Views of AAAS Scientists, Issue by Issue"

56: . . . a clear, but not overwhelming, result

57: We might also acknowledge an intrinsic psychological appeal for complex topics that have been mastered: a sort of pride in the privileged comprehension that might transfer to warm feelings for the subject.

15.5 Fusion

Given that fission has problems of finite uranium supply, radioactive waste, proliferation and weapons, and safety issues, its future is uncertain.
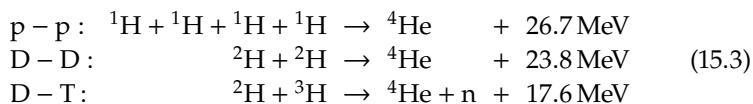
Fusion, on the other hand, is not plagued by most of these issues. It's main problem is that it is incredibly difficult and has been in the research stage for 70 years. Other than that, it has many (virtual) virtues. To be clear, the world does not have and never has had an operational fusion power plant. It *may* belong to the future, but is not guaranteed to ever become practical.

First, the basics. We have alluded to the fact that fusion builds from the small to the big. Putting four ^{1}H nuclei together, at 1.007825 a.m.u. each and forming ^{4}He at 4.0026033 a.m.u. leaves a difference of 0.0287 a.m.u.—0.7% of the total mass—which amounts to 153 million kcal/g.[58] This is *almost ten times* as large as the amount for fission (17 million kcal/g; Box 15.3), making it ten-million times more potent than chemical reactions. Recall that fusion's better performance can be related to the steepness of the left-hand-side of the binding-energy-per-nucleon curve of Figure 15.10.

What makes fusion so difficult is that getting protons to stick together is incredibly hard. Their electric repulsion is so strong that they need to be approaching each other at a significant fraction of the speed of light (about 7%) in order to get within reach of the strong nuclear force that takes over at distances smaller than about 10^{-15} m. The corresponding temperature is a billion degrees.[59] Even the center of the sun is "only" 16 million degrees. The sun has the advantage of being enormous, though. So even at a comparatively chilly 16 million degrees, some rare protons by chance will be going extra fast and have enough oomph to overcome the repulsion and stick together. It's like winning the lottery against very long odds, but the sun is large enough to buy ample tickets so the process still happens often enough.[60] We don't have such a luxury in a terrestrial laboratory setting, so we need higher temperatures than what exists in the center of the sun!

Using ^{2}H nuclei (deuterons, labeled D) instead of ^{1}H (protons) in what is called a D–D fusion reactor, allows operation at 100 million degrees instead of 1 billion. And colliding one deuteron with a triton[61] (^{3}H nucleus, labeled T; 12.3 year half-life), *only* requires 45 million degrees for a D–T fusion reactor. For this reason, only D–T fusion is currently pursued.

For all three types, the relevant reactions[62] are:

$$\begin{array}{lllll} p - p: & {}^1H + {}^1H + {}^1H + {}^1H & \rightarrow & {}^4He & + \ 26.7\,\text{MeV} \\ D - D: & {}^2H + {}^2H & \rightarrow & {}^4He & + \ 23.8\,\text{MeV} \\ D - T: & {}^2H + {}^3H & \rightarrow & {}^4He + n & + \ 17.6\,\text{MeV} \end{array} \qquad (15.3)$$

But the 45 million degrees required for D–T fusion is still frightfully hard to achieve. No containers will withstand temperatures beyond a few thousand degrees. Containment—or confinement—is the big challenge then. The multi-million degree plasma[63] cannot be permitted to touch the
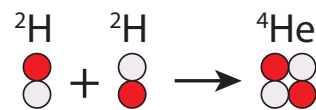

Figure 15.22: Fusion concept: helium from deuterium.

58: The calculation is that 0.0287 a.m.u. corresponds to $\Delta m = 4.8 \times 10^{-29}$ kg, or $E = \Delta mc^2 = 4.2 \times 10^{-12}$ J (26.7 MeV). We convert the Joules to kcal by dividing by 4,184, and then divide by the input mass in grams (4.03 a.m.u. times 1.6605×10^{-24} g/a.m.u.) to get 153 kcal/g. Starting with two deuterium nuclei reduces energy yield a bit to to 137 kcal/g, and for deuterium-tritium reactions it's down to 81 million kcal/g.

59: For temperatures this high, it does not matter whether we specify Kelvin or Celsius, as the 273 degree difference is nothing compared to a billion degrees. The scales are therefore essentially identical here.

60: This is no accident: if the center were too cool, the sun would contract in the absence of radiation pressure until the center heated up from the compression and nuclear fusion ignited—just enough to hold off further contraction. It finds its own equilibrium right at the edge of fusion. In the case of the sun, all it takes is one out of every 10^{26} collisions to stick in order to keep the lights on.

61: If only the UCSD mascot were named after *this* triton. . .

62: . . . allowing beta decays to change protons to neutrons in the process

63: Plasma is a hot ionized gas where electrons are stripped off the nuclei. The sun qualifies as a plasma.

walls of the chamber, despite its constituents zipping around at speeds around 1,000 km/s! This feat can be sort-of managed via magnetic fields bending the paths of the fast-moving charged particles into circles. But turbulence in the plasma plagues attempts to confine the D–T mixture at temperatures high enough to produce fusion yield.

Box 15.6: Successful Fusion

Note that besides stars as an example of successful fusion, we *have* managed to create artificial fusion in a net-energy-positive manner in the form of the **hydrogen bomb**. This is indeed a fusion device, but we could not call it *controlled* fusion. It actually takes a fission bomb (plutonium) right next to the D–T mixture in a hydrogen bomb to heat up the D–T enough to undergo fusion. It's neat (and awful) that it works and is demonstrated, but it's no way to run a power plant.

If a 45 million degree plasma could be confined in a stable fashion, the heat generated by the reactions[64] could be used to make steam and run a traditional power plant—replacing the flame symbol in Fig. 6.2 (p. 90) with something much fancier. The scheme, therefore, requires first heating a plasma to unbelievable temperatures in order for the plasma to self-generate enough *additional* heat through fusion that the game shifts to one of keeping the plasma *cool* enough to produce a steady rate of fusion without blowing itself out. In this scenario, the heat extracted from the cooling flow makes steam. It's the most elaborate[65] possible source of heat to boil water. It may be a bit like working hard to develop a light saber whose only use will be as a letter opener.

64: . . . in the form of radioactive release back to the plasma

65: Should we be proud if we succeed, or embarrassed at the lengths we had to go to?

15.5.1 Fuel Abundance

Deuterium—an isotope of hydrogen—is found in 0.0115% of hydrogen,[66] which means that the occasional H_2O molecule is actually HDO.[67] Therefore sea water is chock-full of deuterium. The global 18 TW appetite would need 3×10^{32} deuterium atoms per year for D–D or 2×10^{32} each of deuterium and tritium atoms per year for D–T. Running with this latter number for the comparatively easier D–T reaction, we would need to process 9×10^{35} water molecules each year to find the requisite deuterium. This corresponds to 26 million tons of water, which is a cubic volume about 300 m on a side. Yes, that's large, but the ocean is larger. Also, it corresponds to a volume of 0.16 billion barrels per year, which is about 200 times smaller than our annual oil consumption. Thus, the volume required should be not at all challenging.[68] The ocean volume is 60 billion times larger than our 300-m-sided cube, implying that we have enough deuterium for 60 billion years. The sun will not live that long, so let's say that we have sufficient deuterium on Earth.

66: See the Chart of the Nuclides abundance information in Figure 15.4.

67: . . . one ^{1}H, one ^{2}H and one oxygen

68: Ocean water is *far* easier to access than underground oil deposits, after all.

Tritium, however, is essentially nowhere to be found, as it has a half-life of 12.3 years. We can generate tritium by adding a neutron to lithium and stimulating an α decay. So the question moves to how much lithium we have. Proven reserves are at about 15 million tons, currently produced at about 30,000 tons per year.[69] We would need 2,300 tons[70] of lithium per year to meet our 2×10^{32} tritium atom target (for 18 TW). In the absence of competition[71] for lithium resources, the associated R/P ratio timescale is 6,500 years. Yes, that is a comfortably long time, but not eons. The thought is that this would buy time to solve the D–D challenge.

15.5.2 Fusion Realities

It is clear why people get excited by fusion. It seems like an unlimited supply that can last thousands if not billions of years at today's rate of energy demand. For some perspective, think about what else we know that lasts billions of years. We already have a giant fusion reactor parked 150 million kilometers away that requires no mining, servicing, or any attention whatsoever. In this sense, the sun is essentially as inexhaustible as fusion promises to be, but already working and free of charge. Photovoltaic panels plus batteries work *today* and have already shown a possible path to eternal energy. The author built his own off-grid solar setup on a budget that's tiny compared to the fusion enterprise.

As for the fusion enterprise, an effort called ITER (Figure 15.23) in southern France is an international effort currently constructing a plasma confinement machine that aims to commence experimental D–T fusion by the year 2035 via occasional 8-minute pulses of 0.5 GW thermal power. This machine is a stepping stone that is not designed to produce electricity. Estimates for construction cost range from $22 billion to $65 billion. By comparison, a nuclear fission plant costs $6–9 billion to build. Admittedly, the first experimental facility is going to cost more, but it is hard to imagine fusion ever being a real steal, financially. Even if the fuel is free, *so what*? Solar is the same.

An effort in the U.S. called the nuclear ignition facility (NIF) is pursuing a different approach to fusion research: attempting to implode a tiny sphere of D–T mixture by blasting it with 192 converging laser beams, crushing it to enormous pressure exceeding that in a star's interior, leading to an explosive release of heat. The building, mostly taken up by gigantic lasers, is the size of three football fields and has so far cost something to the tune of $10 billion. Again, this experimental facility is not provisioned to harness any net energy gain[72] to create electricity.

Let's say that by the year 2050, we will have mastered the art and can build a 1 GW electrical-output[73] fusion plant for $15 billion. That's $15 per Watt of output, which we can compare to a present-day solar utility-scale installation cost of $1 per peak Watt [89]. Applying typical capacity factors[74] puts fusion at twice what solar costs *already*, today.

69: Most lithium is used in batteries; the R/P ratio in this case is 500 years.

70: . . . only 8% of current annual production

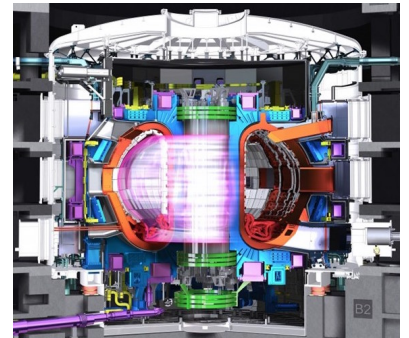71: Otherwise, we're still looking at the 500 year R/P ratio.

Figure 15.23: ITER tokamak cut-away where the plasma would be created. The white outer chamber is the size of a six-story building. From the ITER Organization.

72: . . . the prospects for which are dubious

73: . . . thus ~3 GW thermal, given typical heat engine efficiency

74: . . . 10–20% for PV and perhaps 90% for fusion?

Fusion is therefore a complicated and not particularly cheap way to generate electricity. Meanwhile, we are not running terribly short on renewable ways to produce electricity: solar; wind; hydroelectric; geothermal; tidal. Liquid fuels for transportation represent a greater and more pressing challenge, and fusion does not directly address this aspect any better than other options for electrical production. Fusion is by far the most complex power generation scheme we have ever attempted, evidenced by the 70 year effort to bring it to fruition that is still underway. How many physics PhDs will it take to keep a fusion plant running? Sometimes, we get stuck pursuing a flawed vision of the future, and have trouble reevaluating our options. Imagine being a middle-aged physicist or engineer in the 1950s. In your lifetime, you would have seen the advent of the car, airplane, radio, television, nuclear fission, among a blur of other technology advances. The next frontier was obviously fusion, so let's crack that one! At this point, 70 years later, maybe we should ask: why?

And let's point out that fusion is not without its waste challenges. It is still a radioactive environment, albeit not one that produces dangerous direct products (^{4}He is okay!). It *does* involve a radioactive fuel source (tritium), and it *does* embed the containment vessel with high energy particles and neutrons that over time compromise the integrity of the vessel so that it must be discarded as a radioactively-charged hunk of metal.[75] By comparison, solar, wind, and other renewable sources based on the sun have no such problems. All of the nastiness is created in the sun, and stays in the sun.

75: Transmutation of the nuclei in the material will create radioactivity.

15.5.3 Pros and Cons of Fusion

Collecting the advantages and disadvantages of fusion, we start with the positive attributes:

▶ Fusion would enjoy an inexhaustible supply of deuterium, easily accessed, outlasting the sun;
▶ The fusion reactor would serve as a heat source for tried-and-true steam-driven power plant technology.

And now the not-so-good aspects:

▶ Stable plasmas are exceedingly hard to generate at the requisite temperatures;
▶ 70 years of effort have not yet borne fruit as an energy supply;
▶ Tritium is not available, and must be fabricated from a limited supply of lithium;
▶ Fusion still contends with radioactive fuel (tritium) and a containment vessel that is radioactively contaminated.

The smaller number of positive points is not in itself an indicator of imbalance, since the first point is huge. One elephant can balance dozens of kids on a playground see-saw.

15.6 Upshot on Nuclear

Nuclear fission is a real thing: it can and does produce a significant fraction the world's power. A number of substantive challenges stand in the way of scaling up significantly.[76] For conventional nuclear fission as it has been practiced thus far, the proven reserves of uranium only last 90 years at today's rate of use, and less than 4 years if we tried to get all 18 TW from fission. Radioactive waste is an unsolved problem that persists for hundreds to thousands of years. Breeder programs can extend the resource by large factors (into the 500 or 1,000 year range under an 18 TW nuclear-breeder effort). But proliferation and bomb dangers become more pronounced—not to mention an even more pressing waste issue and greater accident rates given the profusion of operating reactors. It can be difficult to get excited about a nuclear future. It is very cool that we figured out how to do it. But just because we *can* do something does not mean it is a good idea to scale it up.

Fusion is a harder prospect to pin down. At present, it is not on the table, having never been demonstrated in a viable reactor capable of producing commercial-scale electricity. But even if we did manage it, how could it compete economically, as complex as it is? Even if the fuel itself is free,[77] it may turn out to be the most expensive form of electricity we could muster. Fusion is not without radioactivity concerns, and placed side-by-side, solar can look a lot better—intermittency being the crippling drawback, necessitating storage.

Nuclear options cause us to grapple with the question: who are we? What is our identity? What are our aims, and where do we see ourselves going? Are we plotting a course for a Star Trek future, in which case it seems we have little choice but to adopt the highest-tech solutions. Or are we aiming for a more modest future more aligned with natural ecosystems on Earth? So even if we *can* do something, does it mean we're obligated to? Sometimes the costs may be too high.

76: See [105] for a short article summarizing the various challenges.

Pros and cons are listed separately for PV and ST in Section 15.4.8 and Section 15.5.3, respectively.

77: . . . as it also is for solar power, which does not mean solar power is cheap

15.7 Problems

1. If an atom were scaled up to be comparable to the extent of a mid-sized campus, how large would the nucleus be, and what sort of familiar object would be similar?

2. In parallel to Example 15.1.1, what are all the ways to label the radioactive isotope carbon-14?

3. How many neutrons does the isotope ^{56}Fe contain?

4. Use the information in the boxes for ^{12}C and ^{13}C in Figure 15.4 to determine the weighted composite mass of a natural blend of carbon—showing work—and compare this to the number in the left-most box for carbon in the same figure.

5. In Figure 15.4, what are the only mass numbers, *A*, for which no stable nuclei exist?

6. What are the only three long-lived radioactive isotopes in the portion of the Chart of the Nuclides appearing in Figure 15.4, and which one lives the longest (how long)?

7. Cosmic rays impinging on our atmosphere generate radioactive ^{14}C from ^{14}N nuclei.[78] These ^{14}C atoms soon team up with oxygen to form CO_2, so that plants absorbing CO_2 from the air will have about *one in a trillion* of their carbon atoms in this form. Animals eating these plants[79] will also have this fraction of carbon in their bodies, until they die and stop cycling carbon into their bodies. At this point, the fraction of carbon atoms in the form of ^{14}C in the body declines, with a half life of 5,715 years. If you dig up a human skull, and discover that only one-eighth of the usual one-trillionth of carbon atoms are ^{14}C, how old do you deem the skull to be?

   78: Nitrogen is the principal constituent in Earth's atmosphere.

   79: . . . and/or eating the animals that eat these plants

   The wording is long because without context, it's just math. The real learning is in the *application* of math to the world.

8. If a friend creates a nucleus whose half-life is 4 hours and gives it to you at noon, what is the probability that it will *not* have decayed by noon the following day?

9. In close analog to the half-lives of ^{235}U and ^{238}U, let's say two elements have half lives of 4.5 billion years and 750 million years.[80] If we start out having the same number of each (1:1 ratio), what will the ratio be after 4.5 billion years? Express as *x*:1, where *x* is the larger of the two.

   80: . . . a factor of 6 different

10. Control rods in nuclear reactors tend to contain ^{10}B, which has a high neutron absorption cross section.[81] What happens to this nucleus when it absorbs a neutron, and is the result stable? If not, track the decay chain until it lands on a stable nucleus.

    81: . . . as indicated by the orange lower-half of the corresponding box in Figure 15.4

11. If someone managed to create a ^{14}B nucleus, what would its fate be? Track the decay chain on Figure 15.4—indicating the type of decay at each step—until it reaches stability, and indicate how long each step is likely to take.

12. A particular nuclide is found to have lost 3 neutrons and 1 proton after a decay chain. What combination of α and β decays could account for this result?

    82: Region descriptions can include references to the mass range (e.g., low mass or high mass), above or below the stable elements (proton-rich or neutron-rich).

13. How would you qualitatively describe the overall sense from Figure 15.8 in terms of where[82] on the chart one is likely to see α

decay, β^- decay, β^+ decay, and spontaneous fission?

14. In a year, an average American uses about 3×10^{11} J of energy. How much mass does this translate to via $E = mc^2$? Rock has a density approximately 3 times that of water, translating to about 3 mg per cubic millimeter. So roughly how big would a chunk of rock material be to provide a year's worth of energy if converted to pure energy? Is it more like dust, a grain of sand, a pebble, a rock, a boulder, a hill, a mountain?

15. The world uses energy at a rate of 18 TW, amounting to almost 6×10^{20} J per year. What is the mass-equivalent[83] of this amount of annual energy? What context can you provide for this amount of mass?

83: ⓘ This is how much mass would have to "disappear" each year to satisfy current human demand.

16. How much mass does a nuclear plant convert into energy if running uninterrupted for a year at 2.5 GW (thermal)?

17. A large boulder whose mass is 1,000 kg having a specific heat capacity of 1,000 J/kg/°C is heated from 0°C to a glowing 1,800°C. How much more massive is it, assuming no atoms have been added or subtracted?

18. Replicate the computations in Table 15.5 for ^{4}He, paralleling the ^{56}Fe case in Example 15.3.2. Along the way, report the Δm in kg and the corresponding ΔE in Joules, which are not in the table.

19. To illustrate the principle, let's say we start with a nucleus whose mass is 200.000 a.m.u. and inject 1,600 MeV of energy to completely dismantle the nucleus into its constituent parts. How much mass would the final collection of parts have?

   a) the exact same: 200.000 a.m.u.
   b) less than 200.000 a.m.u.
   c) more than 200.000 a.m.u.

20. Using the setup from Problem 19, compute the mass of the final configuration in a.m.u., after adding energy to disassemble the nucleus.

Hint: convert MeV to Joules, then kg, then a.m.u.

21. Referring to Figure 15.10, what is the *total* binding energy (in MeV) of a nucleus whose mass number is $A = 180$?

Hint: Fig. 15.10 is binding energy *per nucleon*.

22. Explain in some detail what happens if control rods are too effective at absorbing neutrons so that each fission event produces too few unabsorbed neutrons.

23. Which of the following is true about the fragments from a ^{235}U fission event?

   a) any number of fragments (2 through 235) can be produced
   b) a small number of fragments will emerge (2 to 5)
   c) two nearly identical fragments will emerge

    d) two fragments of distinctly different size will emerge

    e) the fission is an alpha decay: a small piece having $A = 4$ is emitted

24. A particular fission of ^{235}U + n (total $A = 236$) breaks up. One fragment has $Z = 54$ and $N = 86$, making it ^{140}Xe. If no extra neutrons are produced in this event, what must the other fragment be, so all numbers add up? Refer to a periodic table (e.g., Fig. B.1; p. 375) to learn which element has the corresponding Z value, and express the result in the notation ^{A}X.

25. Follow the same scenario as in Problem 24, except this time *two* neutrons are left out of the final fragments. What is the smaller fragment this time, if the larger one is still ^{140}Xe?

26. Provide three examples of probable fragment size pairs (mass numbers, A) from the fission of ^{235}U + n, making up your own random outcome while respecting the distribution of Figure 15.15 in determining A values. For the sake of this exercise, assume no extra neutrons escape the fragments.

> Hint: no need to identify elements; just settle on pairs of A values that add up correctly.

27. Paralleling the graphical approach in Example 15.4.3 using Figure 15.10, what total energy would you expect to be released in a fusion process going from two deuterium (^{2}H) nuclei to ^{4}He, in MeV?

> Hint: Don't forget to count both ^{2}H.

28. Both nuclear and coal electric power plants are heat engines. What is the fundamental difference between these two, comparing Fig. 6.2 (p. 90) to Figure 15.12?

29. If a nuclear plant is built for $10 billion and operates for 50 years under an operating cost of $100 million per year, what is the cost to produce electricity, in $/kWh assuming that the plant delivers power at a steady rate of 1 GW for the whole time?

> Hint: express the plant power in kW and multiply by hours in 50 years to get kWh produced.

30. Since each nuclear plant delivers ~1 GW of electrical power, at ~40% thermodynamic efficiency this means a *thermal* generation rate of 2.5 GW. How many nuclear plants would we need to supply all 18 TW of our current energy demand? Since a typical lifetime is 50 years before decommissioning, how many days, on average would it be between new plants coming online (while old ones are retired) in a steady state?

> Hint: how many days will one plant live, then how many plants per day?

31. Extending Problem 16 toward what *actually* happens, we know from Table 15.7 that the change in mass (which was close to 1 kg in Prob. 16) is only 0.08% of the ^{235}U mass.[84] Furthermore, a fresh fuel rod is only 5% ^{235}U—the rest being ^{238}U. So how much total uranium[85] must be loaded into the reactor each year, if all the ^{235}U is used up?[86]

> 84: 0.185 out of 235 a.m.u.
>
> 85: Treat the two isotopes as having the same mass: the rod has 20 times more uranium than just the ^{235}U part.
>
> 86: It's not, actually, so this answer is a lower limit on the actual mass that has to be loaded in. So much for the ~1 kg answer from Problem 16.

32. Problem 15 indicated that we need the mass-equivalent of fewer than 10 tons[87] of material to support the world's annual energy

> 87: One ton is 1,000 kg.

needs. But given realities that only 0.08% of mass is converted to energy in nuclear reactions, that only 0.72% of natural uranium is fissile ^{235}U, and that only half of the ^{235}U is retrievable[88] and "burned" in reactors, how many tons of uranium must be mined per year to support 18 TW via conventional fission, assuming for the sake of this problem that 5 tons of mass need to convert to energy via $E = mc^2$?

33. Based on the abundance of ^{235}U in the earth's crust (Table 15.9), how many kilograms of typical crust would need to be excavated and processed per year to provide the ~ 0.005 kg of ^{235}U you need for your personal energy (as in Example 15.4.2)?

ⓘ Of course mining does not work this way, instead seeking concentrations.

34. In crude terms, proven uranium reserves could go another 90 years at the present rate of use. But the world gets only about a tenth of its electricity from nuclear. What does this imply about the timescale for the uranium supply if the world got *all* of its electricity from conventional (non-breeding) nuclear fission?

35. Replicate the calculation and show the work that if we have 2×10^{21} J of proven uranium reserves under conventional fission, we would exhaust our supply in less than 4 years if using this source to support the entire 18 TW global energy appetite.

36. Use Figure 15.18 to reconstruct the breeder route from ^{232}Th to ^{233}U by describing the associated nuclei and decays (and half-lives) involved.

Hint: start by adding a neutron to ^{232}Th

37. For spent nuclear fuel a few decades old, what isotopes are responsible for most of the radioactivity, according to Figure 15.19?

38. Let's say that spent fuel rods are pulled out of the holding pool at the nuclear facility ten years after they came out of the core. Based on the total radioactive power from waste products (black line on Figure 15.19), *approximately* how long will you have to wait until the radioactivity level is down by another factor of 1,000 from where it is at the time of extraction?

39. Operating approximately 450 nuclear plants over about 60 years at a total thermal level of 1 TW, we have had two major radioactive releases into the environment. If we went completely down the nuclear road and get all 18 TW[89] this way, what rate of accidents might we expect, if the rate just scales with usage levels?

40. On balance, considering the benefits and downsides of conventional nuclear fission, where do you come down in terms of support for either terminating, continuing, or expanding our use of this technology? Should we pursue breeder reactors at a large scale? Please justify your conclusion based on the things you consider to be most important.

41. The sun is a fusion power plant producing 3.8×10^{26} W of power. How many kilograms of mass does it lose in a year trough pure energy conversion? How does this compare to the mass of a spherical asteroid 50 km in diameter whose density is 2,000 kg/m³?

42. Based on the fractional mass loss associated with turning four hydrogen atoms into a helium atom, what fraction of the sun's mass would it lose over its lifetime by converting all its hydrogen into helium, under the simplifying assumption that it starts its life as 100% hydrogen?

43. The three fusion forms in Eq. 15.3 each have different energy outputs. Looking at Figure 15.10,[90] how would you *qualitatively* describe why the three reactions differ in this way?

90: Tritium is not labeled, but visible just below 3 MeV on the left side.

44. Based on the calculation that 18 TW would require an annual cube of seawater 300 m on a side to provide enough deuterium, what is your personal share as one of 8 billion people on earth, in liters? Could you lift this yourself? One cubic meter is 1,000 L.

45. What are your thoughts about fusion? Are you excited, skeptical, confused, all of the above? Please offer your thoughtful assessment of the role you imagine fusion playing in our future—your best guess.

16 Small Players

This short chapter serves to round out the menu of renewable energy options. While all of the entries are viable, at various levels of demonstration and implementation, none of them can scale up to be an important contributor to global power at the relevant level of many terawatts. In this sense, we might call the remaining resources "cute." Because of this, items are kept brief—not warranting unjustified attention. Likewise for graphical adornment: the chapter is a little bare, in part because the subjects do not deserve much in the way of undue promotion.

Skipping this chapter will not sacrifice much in the way of knowledge vital to our future. It simply fills in the gaps and addresses some of the "but what about *insert-scheme*?" questions that may arise. The next chapter, Chapter 17, highlights the pros and cons of the full range of alternative energy resources, so this installment largely exists to provide a basis for the conclusions of that more important capstone.

16.1 Geothermal Energy

The interior of Earth is still hot from the initial collapse of mass in the formation of the planet. We can think of this as the thermal conversion of gravitational potential energy as pieces fell into the gravitational field of the earth, converting first to kinetic energy and then into thermal energy (heat) after collision. But that's only half of the story. The other half is radioactive decay of long-lived unstable elements within the earth [106]. Of the radioactive decay part, 40% is from uranium (^{238}U), 40% is from thorium (^{232}Th), and 20% is from potassium (^{40}K). As was indicated in Table 10.2 (p. 168), a total of 44 TW of geothermal power flows through the earth's crust. Divided by total surface area, this amounts to less than

[106]: Johnston (2011), *Radioactive decay accounts for half of Earth's heat*

Lava seeping along east of Puʻu ʻŌʻo crater in Hawaii as an evident display of geothermal energy. Photo Credit: Tom Murphy

$0.1\ \text{W/m}^2$, making it less than one-two-thousandth of the solar input average.[1]

16.1.1 Sustainable Harvest

Let's evaluate what it would take to tap into the steady 44 TW geothermal heat flow in order to provide some or all of today's 18 TW energy demand. In this way, we could be assured of steady flow for billion-year timescales, as the half lives of the radioactive elements measure in the billions of years and the residual heat of formation is still slowly leaking out after 4.5 billion years. On the face of it, 44 TW compared to 18 TW would suggest that geothermal has margin to spare.

In order to utilize geothermal energy for most of the things we do, we really need to turn the heat into a versatile form of energy like electricity. The only exception would be for heating water and air spaces. Otherwise, it is not hot enough to perform most industrial process demands, like melting metals. We know how to turn a temperature difference into electricity, via a heat engine (Sec. 6.4; p. 88). We saw that the theoretical efficiency of a heat engine is

$$\varepsilon_{\text{max}} = \frac{T_{\text{h}} - T_{\text{c}}}{T_{\text{h}}} = \frac{\Delta T}{T_{\text{h}}}, \tag{16.1}$$

⚠ Temperatures must be in Kelvin.

where the hot and cold thermal reservoirs are at temperatures T_{h} and T_{c}, and ΔT is the difference.

A typical temperature gradient in the earth's crust is 25°C per kilometer, meaning that the temperature rises by another 25°C for each kilometer of depth. A heat engine constructed to operate between the surface (288 K average) and 1 km down (313 K) could expect a *maximum* thermodynamic efficiency of 8%. Now imagine an ambitious deployment across 100% of land (29% of the globe): 8% of 29% of the original 44 TW flow is down to a paltry 1 TW.[2] And that's if achieving the *theoretical maximum* efficiency limit[3] and somehow accessing the *entire* flow of heat 1 km down through a network of pipes across *every* bit of land area on Earth. Can you imagine the scale of this effort? And for all that, it would fall well short of 1 TW, considering practical limitations. Don't hold your breath for this long-term-sustainable form of geothermal energy to provide an answer to our 18 TW appetite.[4]

2: $44\ \text{TW} \times 0.08 \times 0.29 \approx 1\ \text{TW}$.

3: Typically, we fall short by a factor of two.

4: In fact, the continental share of the steady geothermal flow—29% of 44 TW—is only 13 TW and not enough to satisfy demand even at the impossible-to-realize efficiency of 100%.

> **Box 16.1: Deeper and Oceans, Too**
>
> While the ambitious global-scale 1 km deep geothermal network was disappointing in its output, the mathematical solution is to "just"[5] use the *entire* globe and dig deeper, allowing a larger thermal gradient and therefore higher efficiency, according to Eq. 16.1. Allowing for

5: ⚠ Beware of the word "just."

engineering challenges that may limit us to half the theoretical efficiency, we would have to capture 36 TW of theoretical flow to end up at 18 TW. Now we need a theoretical efficiency of 82%,[6] translating to T_h of 1,600 K, which would be about 50 km down: deeper than the earth's crust is thick.

6: 36 out of 44

For context, the deepest mine is less than 4 km deep, and the deepest drill hole is about 12 km.[7] So outfitting 100% of Earth's surface—including under the oceans—with a dense thermal collection grid 50 km down sounds like pure fantasy.

7: Drilling stopped because technical challenges prevented going deeper. The project goal was 15 km.

16.1.2 Geothermal Depletion

The previous section was framed in the context of accessing the 44 TW steady geothermal flow, sustainable for billions of years—finding that we cannot expect to satisfy demand by that route. But when did we ever exhibit collective concern for long-term sustainable solutions? The human way is more about exploiting a resource fully, not worrying about consequences even decades down the line. In that sense, geothermal energy has more to offer—at least on paper.

A one-time extraction of thermal energy under out feet—not worrying about replenishment—amounts to *mining* thermal energy, in much the same way that we mine copper, or fossil fuels. Using a rock density of 2,500 kg/m³ and a specific heat capacity of 1,000 J/kg/°C (Sec. 6.2; p. 85), each cubic meter of rock has an extra 60 MJ of thermal energy for each kilometer deeper we go—based on a gradient of 25 °C/km, as before. Is that a lot? It's about the same as the energy in 2 L of gasoline. The energy density works out to 0.006 kcal/g, to put in familiar units (see Table 16.1).

Table 16.1: Energy densities of familiar energy substances. For hydroelectricity, a 50 m dam is assumed, and for geothermal, the depth is 1 km.

Substance	kcal/g
Gasoline	11
Fat (food)	9
Carbohydrates	4
TNT explosive	1
Li-ion battery	0.15
Alkaline battery	0.11
Lead-acid battery	0.03
Geothermal (1 km)	0.006
Hydroelectric (50 m)	0.0001

So it's no screaming-good deal, but it's still energy, and the earth's crust has a heck of a lot more rock than it does oil. To appreciate the scale, the land area of the lower-48 states is approximately 10^{13} m². A 1-meter-thick slice of earth under the U.S. at a depth of 1 km therefore contains 60 MJ/m³ times 10^{13} m³, or 6×10^{20} J of energy. It's a big number, but recall that 1 qBtu is about 10^{18} J, so we're talking about ~600 qBtu. The U.S. uses about 100 qBtu per year of energy, but at an average efficiency of 35% in heat engines, so that we seek about 35 qBtu of *useful* energy. As we saw, the geothermal resource, at lower temperature, is less potent in terms of efficiency. If achieving half of the theoretical 8% efficiency for the 1 km ΔT of 25°C, a one-meter-thick slice would provide about 24 qBtu of useful work.[8] Reaching the 35 qBtu goal would require a slice about 1.5 m thick, at 24 qBtu per meter.

8: ... 4% efficiency times 600 qBtu thermal resource for one meter

To summarize, we would need to *completely* remove all the heat from all the rock 1 km below our feet in a 1.5 m-thick layer every year. Once we cool the underground rock, it will take a long time for the surrounding

heat to diffuse in, so we'd have to keep moving down, year after year. This would mean completely replacing the collection network (whatever comprises it—this is all fantasy) every few years. It's ludicrous to imagine we would endeavor to go to such extremes, plowing through the deep earth to remove every scrap of thermal energy in a massive never-ending effort.[9] Such a colossal scheme makes oil drilling seem like child's play.

9: It would not be surprising if the EROEI is abysmally low or even a net energy drain.

Granted, as one drills deeper, the thermal energy goes up *and* the efficiency increases as ΔT climbs. The result is *quadratic*, in that the energy yield 3 km down is 9 times that at 1 km down.[10] At the same time, drilling gets more challenging and at some point exceeds the current state of the art. By the time the temperature reaches 150°C—which is thought of as a minimum viable temperature for traditional geothermal ventures—drilling technology runs into limitations.

10: ...because the hotter rock contains 3× more thermal energy *and* that energy can be converted to electricity 3× more efficiently

---

Box 16.2: There Will Be Hype

As long as an idea is not outright impossible, the world is big enough and competitive enough that enterprising individuals will be able to generate interest and investment in ideas that seem viable and can be touted to have great promise. Whether that idea is truly capable of benefiting humanity as a "good" idea is not fully evaluated. Instead, if it can make money in the short term,[11] then it may get a green light.

11: ...or at least attract investment

So be wary of claims by people or companies whose financial interests lie in the perception of success and promise. Even media coverage that *should* be objective is often quantitatively sloppy,[12] and has a much easier time finding enthusiasts willing to devote time and quotes than non-enthusiast experts who are too busy pursuing their own projects to waste time poking holes in shoddy ideas.

12: ...lacking a staff physicist

16.1.3 Geothermal Reality

Enough, then, about "pipe dreams" of massive geothermal exploitation on a planetary scale. Geothermal energy is not *all* fantasy, as some places are able to capture significant energy from this resource.

In a few locations, hot magma is brought near the surface, offering rare cracks of access to high temperatures. An electrical power plant, as depicted in Fig. 6.2 (p. 90), does not care particularly where the thermal energy derives, as long as it's hot enough to make steam. The ideal site has:

▶ magma near the surface—volcanic regions, for instance;
▶ fractured rock above the magma in which water can flow;
▶ water temperatures in excess of 180°C (under pressure);
▶ a caprock above the fractured rock, able to trap pressurized steam.

The most common scheme—labeled "hydrothermal"—is to drill two holes into the ground near each other, injecting water into one and collecting pressurized steam from the other. Fractures in the rock permit water and/or steam to flow between the two holes. Alternatively, but far less common, a fluid[13] can be run through a closed loop that passes through the hot medium. By either direct use of the steam in the hydrothermal case, or generating steam from the hot fluid in the closed-loop case, the resulting steam can be used to run a turbine and generator in the usual way.

13: ...not necessarily water now

A newer form, called "binary" geothermal uses two fluids: water in the ground as in other schemes, but a second fluid having a much lower boiling point to make a steam analog at lower temperatures. This opens additional power generation possibilities at temperatures below 100°C, but of course will suffer the inevitable efficiency hit when T_h is lower, according to Eq. 16.1.

Globally, roughly 10 GW of electricity is produced from geothermal energy [107], and an estimated additional 28 GW of direct heating is obtained from this source [108]. Together, these account for 0.4% of the 18 TW global energy budget, after a thermal equivalent adjustment.

[107]: Intern'l Renewable Energy Agency (2018), *Renewable Energy Statistics 2018*

[108]: (2020), *Geothermal Heating*

Country	GW installed	GW produced	% elec.
U.S.	3.5	1.9	0.4
Philippines	1.9	1.3	27
Indonesia	1.5	1.2	4
New Zealand	1.0	0.85	15
Mexico	1.0	0.7	3
Italy	0.9	0.7	1.5
Iceland	0.7	0.6	30
World Total	12.6	9.4	0.4

Table 16.2: Global geothermal electricity production in 2016 [85, 107, 109]. Note that the percentage is the fraction of *electricity*, not total energy, contributed by geothermal. The capacity factor tends to be relatively high for this non-intermittent resource (see Problem 6).

Table 16.2 lists the top 7 producers of geothermal electricity, capturing 72% of the global total. Note that many are on the Pacific Rim, sometimes called the "ring of fire" for its volcanic activity. Iceland gets 30% of its electricity[14] from geothermal sources. But in absolute terms, it is a small amount of energy. Considering that a single nuclear plant puts out about 1 GW, the countries in Table 16.2 have the equivalent of 1–2 nuclear plants in the form of geothermal (compare to Table 15.8; p. 256).

14: And electricity is only about one third of Iceland's energy demand.

The U.S. gets an average of 1.9 GW of electrical production[15] from geothermal sources [85]. 72% of this is produced in California—almost all at a site called The Geysers in the northern part of the state—accounting for ~6% of the state's electricity. Another 22% of U.S. geothermal electricity is produced in in Nevada. The rest is in Utah, Hawaii, Oregon, Idaho, and New Mexico, in that order (7 states total).
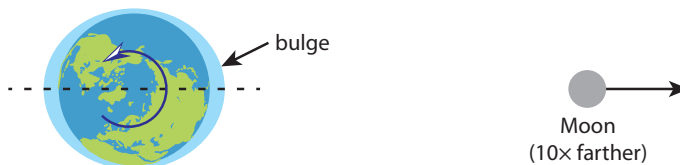
15: ...~0.4% of total electricity

[85]: U.S. Energy Inform. Admin. (2020), *Electric power monthly*

Geothermal is just a small player. The fact that a country like Iceland can produce a large fraction of its electricity this way mostly tells us that Iceland is on a geological hot-spot and is not very populated. We should

not expect geothermal to assume a large role in the energy landscape of tomorrow.

16.2 Tidal Capture

Table 10.2 (p. 168) indicates that the earth currently receives 3 TW of power in the form of tidal energy. Gravity from the moon (and the sun[16]) tug on the earth, pulling slightly harder on the nearest side, and less enthusiastically on the far side. This deforms the otherwise spherical Earth[17] into a *prolate ellipsoid*, which is a fancy way to say oval—somewhat like an egg. The bulges that form on either side (near and far; see Figure 16.1) as a consequence are more pronounced in the ocean than the land, but both indeed deform.

The earth rotates "under" these bulges, since the bulges point along the Earth–Moon axis and take a month to make a complete revolution in space, while the earth rotates once per day. At a fixed position on Earth, then, the experience is two high tides and two low tides each day.[18] As the tide comes in and flows out, friction between land and water results in energy dissipation, to the tune of 3 TW.[19]

The idea for capturing this natural flow of energy is to allow the tide to enter a bay or inlet—raising the water level by perhaps several meters—then closing the exit, trapping the water behind a wall. At this point, the situation is very much like a hydroelectric dam (Sec. 11.2; p. 175), but at a much lower water height than is typical for hydroelectric dams. All the same, exiting water can be forced through turbines turning generators to create electricity. Draining through turbines over a 6-hour period allows the process to begin again at low tide, opening the gates to refill the inlet as the tide comes back in.

The amount of energy depends on the area of the captured body of water and the height of water trapped behind the wall. We use the familiar gravitational potential energy: $E = mgh$ to calculate the energy involved (Sec. 11.1; p. 173). If the height of water trapped behind the wall at high tide is called h, the height of water behind the wall smoothly transitions from $h \rightarrow 0$ as the water drains out, so that the *average* height of water behind the wall is $h/2$, and the gravitational potential energy available really looks like $mgh/2$. The mass is density ($\rho =$ 1,000 kg/m^3) times volume, and volume is the area of the captured water surface, A, in square meters, times the initial water height, h. To get power, we

16: The tidal influence from the sun is sub-dominant, at 45% the strength of lunar tides, so for simplicity we will describe just lunar tides from here on.

17: Actually, an even larger (21 km) effect is the spinning motion of the earth flattening it into an *oblate ellipsoid*.

Figure 16.1: The moon (pictured closer here to fit on page) raises bulges on the near and far side of the earth, while the earth rotates underneath, causing two high tides and two low tides each day.

18: The sun also contributes, sometimes adding to the peaks and troughs (near new and full moon, when Earth, Moon, and Sun are arranged along a line), and sometimes moderating the tides by filing in the gaps a bit (at quarter moon phases).

19: An aggressive global-scale tidal capture enterprise could actually increase the total tidal energy budget, which would have the side effect of pushing the moon further away, slowly, as is described in Sec. D.4 (p. 402).

divide the energy available by the time over which we let out the water: nominally 6 hours.[20] Collecting it all, we have:

$$P = \varepsilon \frac{\Delta E}{\Delta t} = \frac{\varepsilon m g \frac{h}{2}}{\Delta t} = \frac{\varepsilon \rho A g h^2}{2 \Delta t}, \tag{16.2}$$

where $g \approx 10\,\text{m/s}^2$ is the acceleration due to gravity and ε is the efficiency of converting gravitational energy into electrical energy.

> **Example 16.2.1** The Rance tidal capture station in France has a capture area of $A = 22.5\,\text{km}^2$, and a height capacity of 8 m. If operating off of a 7 m high tide and draining for 6 hours, what is the expected power delivered at an efficiency of 90% (as is typical for hydroelectric power)?
>
> Expressing A in square meters yields $22.5 \times 10^6\,\text{m}^2$. The time is 21,600 seconds, leading to a calculated value of 230 MW.

Only two large tidal facilities operate in the world today: Rance in France (Example 16.2.1) has a rated capacity of 240 MW, and produces an average of 57 MW. Thus the capacity factor is about 24% due to the fact that it can only generate tidal power half the time[21] and not all high tides are at the full design height. The h^2 in Eq. 16.2 indicates particular sensitivity to height—due to the double-whammy that lower height means less trapped mass *and* lower pressure head.

The other facility is Sihwa Lake in South Korea, a 254 MW facility that came online in 2011. Much like Rance, and for the same reasons, its capacity factor is 25%, averaging 63 MW. Its basin is 30 km² and has similar operating height as the French installation. The Sihwa Lake facility cost $560 million to build, making it $9 per average Watt of delivered power. This puts it roughly in line with the cost of nuclear power (page 256), and a little higher than utility-scale PV, currently.

Two other large tidal stations in the 300–400 MW capacity range are in the works for the UK and Scotland. But it's not something that works well everywhere: best suited for areas that have large tidal swings and large inlets having narrow mouths that are easy to dam. It's a niche player now and always will be. After all, the 3 TW global budget for tidal energy suggests that it is not an energy jackpot.

16.3 Ocean Currents

Steady currents throughout the volume of the ocean[22] are estimated to total 5 TW (see Table 10.2; p. 168). This is not much more than total tidal dissipation on the planet, at 3 TW. Already, we don't hold out much hope for an energy bonanza. Wind in the atmosphere, by contrast, has

almost 200 times as much power. And we saw that getting even a few terawatts out of wind is a challenge, globally.

The physics is very similar to that of wind (Chapter 12). Kinetic energy from the motion of the water is transferred to rotational motion in a turbine to turn a generator to make electricity. Echoing Eq. 12.2 (p. 187), we have

$$P_{\text{current}} = \frac{1}{2}\varepsilon\rho_{\text{water}}\pi R^2 v^3. \tag{16.3}$$

The efficiency, ε is limited by the same physics as for wind, so 50% is a reasonable maximum. Recall that $\rho_{\text{water}} = 1,000\,\text{kg/m}^3$.

So we still have the quadratic dependence on rotor radius and the crucial cubic power of velocity. Ocean currents tend to be far slower than wind velocities, but the density of water is about 800 times the density of air, so that for the same rotor area, water moving at 1 m/s has similar power as air moving at 9 m/s.

One problem is that the giant land-based wind turbines whose rotors move at high speeds relative to the wind would not be practical in water, so individual units would more likely be on the ten-meter scale and not a 100-meter scale. Individual rotors would therefore likely generate a few hundred kilowatts apiece, requiring thousands of rotors to make up a typical 1 GW power plant output.

> **Example 16.3.1** A 10 m diameter rotor sits in a brisk 2 m/s current. At an efficiency of 50% (similar to wind), how much power would the turbine deliver?
>
> All the pieces[23] fall right into Eq. 16.3 to yield about 160 kW.

23: Just be careful about diameter vs. radius.

Among the advantages, marine currents are very steady and dependable, so that capacity factors would be high. This is in contrast to solar, wind, and even hydroelectric and tidal resources.

However, marine environments are not kind to underwater structures,[24] making it hard to imagine the headache it would be to maintain the moving parts in good condition—all in a difficult operating environment for human workers. Additionally, marine life might not be so happy to get whacked by a rotor blade.

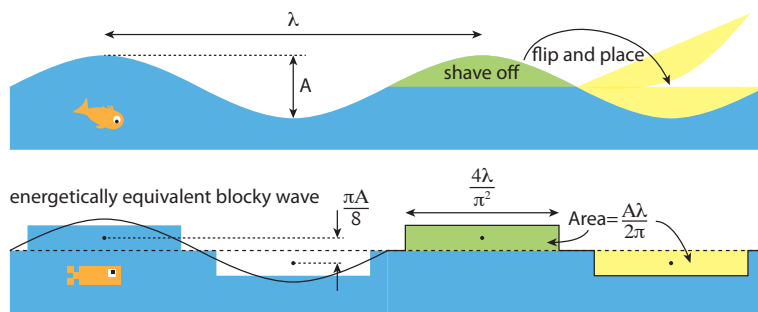24: Look at pier pilings and sunken ships to get an idea.

In the end, we should not expect this sector to contribute substantively to global energy, given the small scale of total energy available and the practical difficulties associated with harnessing any sizeable piece of it.

16.4 Wave Energy

Waves present an interesting case to study, being composed of a continuous interplay between potential and kinetic energies—water particles

executing circular paths as they gain and lose gravitational potential energy. Meanwhile, the energy in a packet of waves travels efficiently onward, until reaching a barrier like a shoreline, where they break in a display of kinetic energy. It is hard to stare at a coastline being pounded by surf without admiring the power of nature. What if we harnessed that power?

As a means of estimation, let's imagine sinusoidal[25] waves 1 m in amplitude (peak-to-trough) arriving every 6 seconds, traveling at about 3 m/s. Crests must then be 18 m apart, since the distance between crests is their rate (3 m/s) times time (6 seconds). We call this the wavelength, symbolized by λ (Greek lambda). See Figure 16.2 for the layout.

Figure 16.2: Sinusoidal waves have amplitude A, and wavelength λ. In terms of harvesting the gravitational potential energy, we can think of it as lopping off the crest and flipping it over onto the trough to level the water surface. In doing so, we move some mass, m, down a height h to get mgh of energy. The block-equivalent is shown below, where the area and average height of the sinusoidal trough/crest has been faithfully captured by rectangles of height $\sim0.39A$ and length $\sim0.41\lambda$. From these, it is possible to figure out the potential energy associated with the wave.

In order to figure out the energy involved, we need the mass of water raised and a height to which it is raised. Notice that in Figure 16.2, the potential energy in the wave can be extracted by making the water flat again, which is equivalent to taking all the water from the crest and putting it into the trough. We just need to know how much water we're moving, and through what height. Figure 16.2 has done the fancy math already and redrawn the wave as rectangular chunks that have equivalent area as the sinusoidal crest and trough and also the same average (midpoint) height relative to the average surface height (dashed line). From this, we learn that the wave crest has area $A\lambda/2\pi$ and the height of the displacement is $\pi A/8$, where A is the wave amplitude from the top of the crest to the bottom of the trough.

To assess potential energy as mgh, we need three pieces. We already know $g \approx 10\,\text{m/s}^2$, and we now know that $h = \pi A/8$. The mass is a density, ρ, times a volume. We already have the area of the crest cross section as $A\lambda/2\pi$. To get a volume, we need a length *along* the wave, which we are free to make up as a variable we'll call ℓ. The mass, m, of our block of water is then $\rho A\lambda\ell/2\pi$. Putting this together, the gravitational potential energy (GPE) associated with putting the water back to a flat state along a length, ℓ, of one wave is

$$E_{\text{GPE}} = mgh = \frac{\rho A\lambda\ell}{2\pi}g\frac{\pi A}{8} = \frac{\rho\lambda\ell g A^2}{16}. \tag{16.4}$$

⚠ Author's note: we'll be going through a bit of a derivation here—not because wave power is particularly important, but because physicists sometimes can't help it. Feel free to skip past all the equations, as this is meant as a sort of entertainment. What can I say: the introduction had a warning about skipping the whole chapter, yet here you are.

It's not an equation to remember, just a way to keep track of the physics as we build toward a final/useful result. Next, we want to understand the power delivered as the waves come and come. We get a new one every $\Delta t = 6$ s. And we also reasoned before, in slightly different form, that $\Delta t = \lambda/v$, where v is the velocity of the wave: 3 m/s in our example. The power looks like:

$$P_{\text{GPE}} = \frac{E_{\text{GPE}}}{\Delta t} = \frac{\rho \lambda \ell g A^2}{16 \Delta t} = \frac{\rho \ell g A^2 v}{16}. \tag{16.5}$$

This is how much power the potential energy part of waves contributes as the waves pile onto the shore. But waves also have kinetic energy. It turns out that kinetic energy and potential energy are balanced in a wave—which is perpetually sloshing back and forth between potential and kinetic forms, much as happens in the motion of a pendulum. So the total power is just double P_{GPE}, or

$$P_{\text{tot}} = \frac{\rho \ell g A^2 v}{8}. \tag{16.6}$$

It is a little awkward to have to specify the length of the wave, but we needed it to make sense of the mass involved. At this point, let's switch to expressing the *power per unit length* of the wave, or P/ℓ.

$$\frac{P_{\text{tot}}}{\ell} = \frac{\rho g A^2 v}{8}. \tag{16.7}$$

Notice that this expression does not actually depend on the wavelength, in the end. The only measures of the wave that enter are the amplitude and velocity.[26]

26: The velocity of near-shore waves is set only by the depth, d, of the water ($v = \sqrt{gd}$).

For our example of 1 m amplitude and 3 m/s velocity, we compute a power per unit length of 3,750 W/m. Okay, this is a new unit, and it looks vaguely encouraging. Blow dryers, toaster ovens, space heaters, or similar power-hungry appliances consume about 1,800 W of power when running full blast, so 3,750 W/m is roughly equivalent to having *two* such appliances plugged in and running for every meter of length along the wave, or coastline. It seems like a bonanza: Our collective hair will be dry in no time! Take a moment to picture a beach cluttered with a power-hog appliance plugged in every 0.5 m all down the beach, all running at full power. That's what the waves can support, and it seems pretty impressive.

⚠ We will use this 3,750 W/m figure from here on for our rough analysis, but it should be borne in mind that larger wave amplitude has a quadratic effect on power, and waves are not all 1 m peak-to-trough!

But what we care about, in the end, is how much *total* power the waves can deliver: how many terawatts? So we need to multiply the wave P/ℓ value by a length along the wave, or a shoreline length.

Example 16.4.1 How much wave power arrives on the U.S. Pacific coast if the whole coastline is experiencing 1 m amplitude waves at a

wave speed of 3 m/s?

The foregoing text already worked out that P/ℓ for these wave parameters is 3,750 W/m. Now we just need to multiply by a coastline length. The Pacific coast of the U.S. is approximately 2,000 km long. Multiplying 3,750 W/m by 2,000,000 m yields 7.5 GW.

Getting 7.5 GW on the Pacific coast, and maybe a similar amount on the Atlantic coast for a total of 15 GW is nothing to sneeze at. But consider that the U.S. electricity demand is about 450 GW, and various alternatives already top the upper limit of wave potential, as shown in Table 16.3.[27] Also, to get 15 GW from waves would require extracting *all* the wave energy from the U.S. coasts. Sorry surfers. Sorry marine life who depend on the waves for stirring nutrients and other functions a physicist can only guess. The point is that when a *fully developed* wave energy resource only provides a few percent of demand, while a promising thing like solar is already roughly matching it and has ample room to grow, we can be pretty confident that wave power will not become an important player.

But let's make a quick estimate of global potential to compare to the 18 TW demand. The question becomes one of how much coastline[28] receives wave energy.

We can play a crude trick by recognizing that two big clumps of connected land occupy the eastern and western hemispheres, each having an east coast and a west coast, running roughly pole to pole. So the exposed coastlines roughly ring the globe twice. Our familiar unit of length, the meter, was actually chosen to yield approximately 10,000 km from equator to pole, so it takes 40,000 km to circle the earth once, and we therefore estimate 80,000 km of wave-receiving coastline. Multiplying 3.75 kW/m by 80,000 km yields 0.3 TW, or less than 2% of global demand, if *fully* developed.

3,750 W/m seemed so promising at first, especially compared to insolation numbers we saw in Chapter 13 that tend to be around 200 W/m^2. But a huge difference here is that wave density is a *linear* measure (Watts per *meter* of coastline) versus an *areal* measure in the case of insolation (Watts per *square* meter of land). A country has *far* more square meters than linear meters. So wave power, even if fully developed around all the world's coastlines, could not amount to very much, and is therefore placed in the "boutique" category, along with the other occupants of this chapter.

We have not even mentioned the technologies that can convert wave power to electricity—because what's the point? Suffice it to say that the same relative motion of magnets past coils of wire that create electrical energy in a generator can work in applications that do not spin all the way around, as is usually the case. A back-and-forth motion like one cylinder inside another, or a joint bending back and forth can be configured to generate electrical power as well.

Table 16.3: Alternative electricity scales in the U.S.

Resource	GW average
Solar	11
Wind	31
Hydro	33
Nuclear	92
All Elec.	450

27: Note that wind and solar are growing year by year, so their ultimate numbers will be significantly higher, still.

28: Keep in mind that many coastlines are protected from waves by their orientation or lack of exposure to long stretches of open water.

Work this out for yourself, maybe by thinking of something small like a house first, to understand why.

16.5 Hydrogen

Combustion of hydrogen gas produces energy, and indeed is sometimes used as rocket fuel and in some demonstration vehicles. It can also be combined with oxygen in a fuel cell (not combustion) to generate electricity. Twenty years ago, references to a *hydrogen economy* replacing fossil fuels were common. So why has it been left out of this book, shoved unceremoniously into the last part of a chapter on miscellaneous small players? One key reason: hydrogen **is not a source** of energy. Pockets of hydrogen gas are not found underground.[29]

29: It is too reactive to survive on its own, instead bound in H_2O and CH_4, for instance.

Hydrogen can be produced from water given enough energy input, typically via *electrolysis*, which splits H_2O into hydrogen and oxygen. Electrolysis efficiency is typically 65–80% in terms of capturing input energy in the form of stored hydrogen. So hydrogen should be thought of as energy **storage**—like a chemical battery—rather than as an energy *source*. Combustion of hydrogen to drive a car, for instance, would suffer the usual heat engine inefficiency of ~25%. Combined with electrolysis inefficiency, total efficiency is down to 15–20%. A standard battery does far better in round-trip[30] storage efficiency: 60–90%. Fuel cells are far more efficient than combustion—around 65%. Combined with the electrolysis step, the overall efficiency is still well short of battery storage, at around 50% overall.

30: . . . meaning charge and discharge energy cycle

Hydrogen in gaseous form is bulky and hard to package—especially in mobile applications like cars. The alternative to gaseous storage is as a cryogenic liquid, adding other complications. Hydrogen is more dangerous than gasoline as an explosion hazard. Fuel cells are not particularly robust and have difficulty in cold weather. On balance, hydrogen is unlikely to come roaring onto the scene as a fossil fuel replacement. An article [110] summarizes the pros and cons well.

[110]: Volkswagen (2019), *Hydrogen or battery? A clear case, until further notice*

16.6 Upshot on Small Players

The topics discussed in this chapter are presented more out of duty to completeness than as set of hopeful candidates for the energy of tomorrow. The first four are physically viable *sources* of real energy. Geothermal and tidal (to a lesser extent) contribute to today's energy mix, and new development is underway to expand both.

Geothermal energy is only practical in a small number of places that provide proximate access to high-temperature magma. Plans to extract geothermal energy from "normal" locations have not yet materialized, and the challenges are substantial. We should not expect geothermal energy to contribute at the terawatt scale, in practice.

Tidal energy, like geothermal, is very location-dependent—requiring large inlets with large tidal amplitude that can be closed off easily. But

some locations *are* well-suited to supplement local power generation from this resource. The possibilities for terawatt-scale extraction are essentially non-existent, so tidal will remain a small player.

Ocean currents and wave power are sometimes explored in small demonstration projects, but never yet at scales that add significantly to the total energy mix. Terawatt-scale implementation is not to be expected.

Hydrogen is not a source of energy, but a possible means of energy storage. Various challenges make it less than exciting as a fuel of the future.

As much as anything, this chapter may go to show that physics can be used to explore all sorts of ideas, operating within a finite set of rules and options. Humans have had a very long time to explore the world and identify possible sources of energy.[31] The options on the table are now well known, and any new ideas can be assessed quantitatively— usually only to show that the resource is small compared to present-day demand.

31: . . . which has been understood to be a pressing concern for at least half-a-century

16.7 Problems

1. A typical college campus is probably about one square kilometer in area. How much power out of Earth's 44 TW geothermal budget passes through the campus area, assuming uniform distribution across the 5.1×10^8 square kilometers of earth's surface? How does this compare to a typical college electrical demand of about 20 MW?

2. If it were possible to achieve 60% of the *theoretical* maximum heat engine efficiency for a geothermal plant, how deep would it have to drill to access high enough temperatures to match the 35% efficiency of fossil fuel power plants if the thermal gradient is 25 °C/km? Have we drilled this far before?

   Hint: using T_c = 288 K, solve for T_h at the target theoretical maximum efficiency, then figure out ΔT.

3. At 4 km down, we expect the ΔT to be about 100°C, and each cubic meter of rock would contain about 250 MJ of thermal energy. If 50% of the maximum theoretical efficiency were achieved from an ambient environment at 288 K, how much rock thickness would have to be depleted in a year to satisfy a 1 km² campus whose output electricity demand is 20 MW?

   Hint: breathe. Realized efficiency; amount of *thermal* power needed; thermal energy in a year; how many cubic meters; how thick for the campus area?

4. Your friend just visited a geothermal power plant, and is excited by the facts that it is environmentally clean, not intermittent like solar or wind, can last ages, totals more than twice our 18 TW budget, and it really works—seen with their very own eyes. What are the key points you might offer to temper their unvarnished enthusiasm?

5. What are your thoughts on whether we should "mine" geothermal heat in a way that could theoretically last hundreds or thousands of years, but not tens of thousands of years before depleting the resource? Such activity would not be strictly sustainable in the long haul, but could seem abundant for many generations. Should we care about this? Where do you land?

6. Work out the capacity factors of geothermal installations for various countries and the world as a whole from the data in Table 16.2.

7. The average global production, in GW, for hydroelectricity, wind, solar, and nuclear are 477, 125, 67, and 393, respectively (Tables 11.2, 12.2, 13.3, and 15.8). Add geothermal production from this chapter to the list and express each of the five as percentages[32] and draw an approximate pie chart to help put these alternative electricity resources in context.

   32: ...so that the five add to 100%

8. The Sihwa tidal power plant has a reservoir area of 30 km^2 and is rated to generate 254 MW of electrical output. If generation efficiency is 90%, what initial water height does the power rating correspond to, assuming a 6 hour discharge time?

9. On the basis of fluid power scaling as the cube of velocity, show the supporting math for the claim in the text that a water current at 1 m/s delivers the equivalent power (per rotor area) as a wind speed of about 9 m/s.

10. If placed in a steady current of 1 m/s, and at a generation efficiency of 40%, how large would an ocean current rotor be (diameter) to satisfy the 10,000 W demand of the average American? Put this scale in the context of some familiar object or space.

11. The running example for waves in the text delivers 3,750 W/m of energy along a coastline. It sounds like a lot. But if we put the 40 million residents of California along its 1350 km coastline, how many meters does each person have, and how much power from waves?

12. Let's imagine waves hitting the entire 2,000 km Pacific coastline of the U.S. that are different from those evaluated in the text. This time, the waves have 50 m wavelength, arriving every 10 seconds, and 2 m crest-to-trough amplitude. How much power does the coast receive under these more active conditions?

17 Comparison of Alternatives

This book has explored the energy surge of the last ∼ 150 years, largely casting it as a story of fossil fuels. Combining the finite nature of fossil fuels with their environmental cost makes it clear that we cannot expect fossil fuels to carry us into the indefinite future—and the sooner we transition away, the better. We have explored aspects of the chief alternatives in the preceding chapters. This chapter gathers key results.

One striking realization is that no fundamentally new energy technologies have emerged in the last half-century. Hydroelectricity, nuclear fission, wind, and solar photovoltaics, had all been invented. Fusion has been in the research phase the entire time. It would be unwise to expect a miraculous new entry into the field of energy—a knight in shining armor.[1] It is not as though scientists and engineers have not prioritized energy research: energy has been recognized as a lynchpin of modern life all this time. The peak of U.S. oil production in 1970 followed by two global oil crises that decade served as a wake-up call. Alarm bells have been ringing over climate change almost the entire time. Interest has been keen, and many ideas[2] have been entertained. Scratching our heads to generate more ideas is not so promising at this stage.

Perhaps, then, the table is set. We know the actors. We know the pros and cons, having many decades of experience with each of the technologies. Where does this leave us? This chapter[3] aims to pull it all together to take stock. The chapters to follow will begin to address how we might move forward in light of our predicament.

17.1 The Alternative Energy Matrix

In exploring potential replacements for fossil energy, it soon becomes apparent that fossil fuels are unparalleled in many respects. Even though

1: Extensive understanding of fundamental physics offers no hopeful revolutionary energy source—especially on relevant timescales.

2: . . . most untenable

3: The content of this chapter is an edited reprisal of the author's contribution to *State of the World 2013: Is Sustainability Possible?* by The Worldwatch Institute [111]. Copyright © 2013 Worldwatch Institute. Reproduced by permission of Island Press, Washington, DC. Originally, the content appeared in a *Do the Math* blog post called The Alternative Energy Matrix.

Imagery from the groundbreaking movie *The Matrix*, by J. Rue.

viewed as a *source* of energy from the ground, fossil fuels are perhaps more aptly described as nearly perfect energy *storage* media, at energy densities that are orders of magnitude higher than anything achieved thus far in the best available battery technology. The storage is nearly perfect because it is reasonably safe, not especially corrosive, easy to transport,[4] lightweight yet dense enough to work in airplanes (see Box 17.1), and indefinitely storable—indeed, for millions of years—without loss of energy. No alternative storage technique can boast all the same benefits, be it batteries, flywheels, hydrogen, or ethanol.

4: . . . via pipelines, often

Box 17.1: Electric Airplanes?

Box 13.3 (p. 212) demonstrated that an airplane receiving solar energy in ideal conditions could gather only 4% of the power typical of a cruising airliner, and would not be able to get off the ground on direct solar input. But what about battery storage?

The best lithium-ion batteries store 0.17 kcal/g, which is 65 times less energy-dense than gasoline, at ~11 kcal/g. A Boeing 737—the workhorse of the airline industry—has an empty weight around 35 tons, and accommodates about 15 tons of fuel and 15 tons of passengers/cargo. A comparable energy storage in the form of batteries—even allowing a factor of three difference in thermal efficiency versus electric[5] efficiency—would take 300 tons of battery: *far* in excess of the entire plane's maximum takeoff weight. Or the airplane could just accept a reduced range by a factor of 20: down to 200 km. Recharge time could easily exceed flight time. It's not the same. See Sec. D.3 (p. 397) for additional analysis.

5: This may be too generous, since performance will depend on the propulsion mechanism: propellers?

Electric flight is not *impossible*. Model airplanes, drones, and other cute demonstrations may get airborne. But this is not at all the same thing as practical air travel!

This chapter aims to build a relatively complete list of ways in which we might tap into Earth's various energy flows and stores (as in Table 10.2; p. 168), most of which have been addressed in some detail in Chapters 11–16.

In order to make comparisons, it is helpful to create a matrix of energy source properties so that the relative strengths and weaknesses of each are obvious at a glance. (See Figures 17.1 and 17.2.) The matrix is presented as a color-coded figure based on 10 different criteria.[6] Blue, yellow, and red can be loosely interpreted as good, neutral, and deficient, respectively. Yellow boxes are often accompanied by brief reasons for their neutrality—the reasons for blue or red extremes often being obvious. While some criteria are quantitative, many are subjective. The following 10 properties are assessed for this comparison.

6: The criteria are defined by the author: this method of comparison is not common practice—even if something like it perhaps should be.

▶ **Abundance**. Not all ideas, however clever or practical, can scale to meet the needs of modern society. Hydroelectric power cannot expand beyond about 10% of current global demand, while the solar potential reaching Earth's surface is easily calculated to exceed this benchmark by a factor of about 5,000. Abundant

sources are coded blue, while niche ideas like hydroelectricity that cannot conceivably fulfill a quarter of global demand are colored red. Intermediate players that can satisfy a substantial fraction of demand are coded yellow.

▶ **Difficulty**. This field tries to capture the degree to which a resource brings with it large technical challenges. How many PhDs does it take to run the plant? How intensive is it to maintain an operational state? This one might translate into economic terms: difficulty serves as a crude proxy for expensive.

▶ **Intermittency**. Colored blue if the source is rock-steady or available whenever it is needed. If the availability is beyond our control, then it gets a yellow at least. The possibility of substantial underproduction for a few days earns red.

> Storage can offset this concern, at increased cost. Seasonal storage is particularly impractical, which hampers solar resources.

▶ **Demonstrated**. To be blue, a resource has to be commercially available today and providing significant energy to society. Proof of concept on paper, or prototypes that exhibit some of the technology, do not count as demonstrated.

▶ **Electricity**. Can the technology produce electricity? For most sources, the answer is yes. Sometimes it would make little sense to try.[7] For other sources, it is impractical.

> 7: For instance, biofuels *could* be used to make electricity, but renewable liquid fuels are too rare to squander in this way.

▶ **Heat**. Can the resource produce direct heat? Colored yellow if only via electric means.

▶ **Transport**. Does the technology relieve the looming decline in oil production? Anything that makes electricity can power an electric car, earning a yellow score. Liquid fuels are blue. Bear in mind that large-scale migration to electric cars is not guaranteed to happen, as the cars may remain too expensive or impractical to be widely adopted, among other challenges related to grid infrastructure for mass-scale charging.

> See additional discussion on page 292 and in Sec. D.3 (p. 397).

▶ **Acceptance**. Is public opinion[8] favorable to this method? Is resistance likely, whether justified or not? This dimension encompasses environmental concerns, threats to health and safety, and unsightliness in natural settings.

> 8: ...judging by U.S. attitudes

▶ **Backyard**. Is this something that can be used domestically, in someone's backyard, rooftop, or small property, managed by the individual? Distributed power adds to system resilience.

▶ **Efficiency**. Over 50% earns blue. Below about 10% gets red. It is not the most important of criteria, as the abundance score implicitly incorporates efficiency expectations, but we will always view low efficiency negatively.

Environmental impact has no column in this matrix, although the "acceptance" measure captures some of this. Climate change is an obvious negative for fossil fuels, but not so much as to have resulted in curtailed global demand thus far (see Fig. 8.2; p. 118). None of the alternatives presented here contribute directly to carbon dioxide emissions, earning an added advantage for all entries.

Each energy source can be assigned a crude numerical score, adding one

point for each blue box, no points for yellow boxes, and deducting a point for each red box. Certainly this is an imperfect scoring scheme,[9] giving each criterion equal weight, but it provides *some* means of comparing and ranking sources.

	abundance	difficulty	intermittency	demonstrated	electricity	heat	transport	acceptance	backyard?	efficiency	Score
Natural Gas	for now						buses, trucks / via electric			for heat / elec/transport	8
Petroleum	for now				mis-spent					for heat / elec/transport	7.5
Coal	for now						via electric (and trains?)			for heat / elec/transport	7.5

Figure 17.1: Fossil fuel matrix of attributes. Blue is good (+1 point); yellow is neutral (0 points).

The conventional fossil fuels each score 7–8 out of 10 possible points by this scheme, displayed on the right side of Figure 17.1. Some attribute ratings are divided into heating versus electricity production for a few of the scoring categories.

The overall impression conveyed by this graphic is that fossil fuels perform rather well in almost all criteria. Because fossil fuels collectively supply about 80% of global energy usage, they are each classified as having intermediate abundance. But even this is not a permanent condition—providing significant motivation for exploring alternatives in the first place. Getting energy out of fossil fuels is trivially easy. Being free of intermittency problems, fully demonstrated, and versatile enough to provide heat, electricity, and transportation fuel, fossil fuels have been embraced by society and are frequently used directly in homes. Efficiency for anything but direct heat is middling, typically registering 15–25% for automotive engines and 30–40% for power plants.

The commonly discussed alternative energy approaches display a wider range of ratings. Immediately, some overall trends are clear in Figure 17.2. Very few options are both abundant *and* easy. Solar photovoltaic (PV) and solar thermal are the only exceptions. A similar exclusion principle often holds for abundant and demonstrated/available—again satisfied only by solar PV and solar thermal. This uncommon combination plays a large role in the popularity and promise of solar power.

Intermittency mainly plagues solar and wind resources, although many natural sources (hydroelectric, tidal, wave, biofuels) present mild inconvenience due to intermittency.

Electricity is easy to produce, resulting in many options. Since the easiest and cheapest will likely be picked first, the less convenient forms of electricity production are less likely to be exploited.[10]

Transportation needs are hard to satisfy. Together with the fact that oil production will peak before natural gas or coal, transportation may appear as the foremost problem to address. Electric cars are an obvious—albeit expensive—solution. Besides being unclear how we might all afford[11] electric cars, the technology has a number of drawbacks relative

	abundance	difficulty	intermittency	demonstrated	electricity	heat	transport	acceptance	backyard?	efficiency	Score
Solar PV						via electric	via electric				5
Solar Thermal			some storage				via electric				5
Solar Heating			some storage								4
Hydroelectric			seasonal flow			via electric	via electric	not universal	micro-hydro		4
Biofuel/Algae		gunk/disease		some R&D	mis-spent				small scale?		4
Geothermal/Electricity	hotspots						via electric				4
Wind						via electric	via electric	noise, birds, eyesore			3
Artificial Photosynth.		catalysts		active devel.	mis-spent				?		3
Tidal			daily/monthly variations			via electric	via electric				3
Conventional Fission		high-tech					via electric	waste/fear			2
Uranium Breeder		high-tech		military			via electric	proliferation			2
Thorium Breeder		high-tech					via electric	waste/fear			2
Geothermal/Depletion		deep drill		rarely?				deep wells	impractical		2
Geothermal/Heating		deep drill		rarely?				deep wells	impractical		1
Biofuel/Crops	food / cellulosic	annual harvest	seasonal	ethanol, etc. R&D effort	mis-spent			food/land competition	small beans		1
OceanThermal		access/ maintenance				via electric	via electric				1
Ocean Current		access/ maintenance				via electric	via electric				1
Ocean Waves			storms/lulls	many one-off designs		via electric	via electric	eyesore			1
D-T Fusion	lithium	future-tech					via electric	trit/neutron contamination			1
D-D Fusion		farther future					via electric				1

Figure 17.2: Alternative energy matrix of attributes. Blue is good (+1 point); yellow is neutral (0 points), and red is bad (−1 point). Only green-labeled entries contribute more than a few gigawatts to our energy today, out of the 18,000 GW total. This comparison is meant to help differentiate attractive directions for future energy implementation.

to fossil fuels and does not lend itself to air travel or heavy shipping by land or sea (see Sec. D.3; p. 397). A car filling its gasoline tank is transferring energy at an astounding rate of 15 MW, equivalent to 3,000 homes running air conditioning. Limited range and slow charge times[12] do not permit electric transport to simply replace transportation as we currently know it.

Few of the options face serious barriers to acceptance, especially when energy scarcity is at stake. Some energy sources are available for individual implementation, allowing distributed power generation as opposed to centralized resources. For example, a passive solar home having PV panels, wind power, and some method to produce liquid fuels on-site would satisfy most domestic energy needs in a self-sufficient manner.

Cost is not directly represented in the matrix, although the difficulty rating may serve as an imperfect proxy. In general, the alternative methods have difficulty competing against cheap fossil fuels. It is not yet clear whether the requisite prosperity needed to afford a more expensive energy future at today's scale will be forthcoming, as prosperity historically has been closely tied to the availability of natural resources,

12: A 220 V 40-Amp circuit, which is on the high end of practicality for a residence, will charge a single car at a rate of about 20 miles of range per hour.

and these are precisely what our populated planet strains the most.

17.2 Tally for Individual Alternative Sources

A single chapter cannot adequately detail the myriad complex considerations that went into the matrix in Figure 17.2. The previous chapters address a number of the considerations, but many of the quantitative and qualitative aspects for each were developed at the Do the Math website. The key qualities of each resource in relation to the matrix criteria are discussed in this section, focusing especially on less obvious characteristics.

Solar PV (Sec. 13.3; p. 201): Covering just 0.4% of Earth's land area with PV panels that are 15% efficient satisfies global annual energy demand, qualifying solar PV as abundant. PV panels are being produced globally in excess of 100 gigawatts (GW) peak capacity per year,[13] demonstrating a low degree of difficulty. Most people do not object to solar PV on rooftops or over parking areas, or even in open spaces.[14] Solar panels are well suited to individual operation and maintenance. Intermittency is the Achilles' heel of solar PV, requiring storage solutions if adopted on a large scale. To illustrate the difficulty of storage, a lead-acid battery large enough to provide the United States with adequate backup power would require more lead than is estimated to be accessible in the world and would cost approximately $60 trillion at today's price of lead [112]. Lithium or nickel-based batteries fare no better on cost or abundance. Pumped storage is limited by a small number of suitable locales.

13: . . . translating to about 10–15 GW of average power production added per year

14: . . . especially sun-saturated deserts

[112]: Pickard (2012), "A Nation-Sized Battery?"

Solar Thermal (Sec. 13.8.2; p. 219): Achieving comparable efficiency to PV but using more land area, the process of generating electricity from concentrated solar thermal energy has no problem qualifying as abundant—although somewhat more regionally constrained. It is relatively low-tech: shiny curved mirrors, tracking on (often) one axis, heat up oil or a similar fluid to drive a standard heat engine. Intermittency can be mitigated by storing thermal energy, perhaps even for a few days. A number of plants are already in operation, producing cost-competitive electricity. Public acceptance is no worse than for PV, but the technology generally must be implemented in large, centralized facilities.

Solar Heating (Sec. 13.8.1; p. 218): On a smaller scale, heat collected directly from the sun can provide domestic hot water and home heating. In the latter case, this can be as simple as a south-facing window. Capturing and using solar heat effectively is not particularly difficult, coming down to plumbing, insulation, and ventilation control. Technically, solar heating potential might be abundant, but since it is usually restricted to building footprints (roof, windows), it gets a yellow rating. Solar heating does not lend itself to electricity generation or transport, but it has no difficulty being accepted and almost by definition is a backyard-ready technology.

Hydroelectric (Chapter 11): Despite impressive efficiency, hydroelectric potential is already well developed in the world and is destined to remain a sub-dominant player on the scale of today's energy use. It has seasonal intermittency,[15] does not directly provide heat or transport, and can only rarely be implemented on a personal scale. Acceptance is fairly high, although silting and associated dangers—together with habitat destruction and the forced displacement of people—do cause some opposition to expansion and have resulted in removal of some hydroelectric facilities.

15: A typical hydroelectric plant delivers only 40% of its design capacity.

Biofuels from Algae (Sec. 14.3.2; p. 234): Because algae capture solar energy—even at less than 5% efficiency—the potential energy scale is enormous.[16] Challenges include keeping the plumbing clean, possible infection,[17] contamination by other species, and so on. At present, no algal sample that secretes the desired fuels has been identified or engineered. No one knows whether genetic engineering will succeed at creating a suitable organism. Otherwise, the ability to provide transportation fuel is the big draw. Heat may also be efficiently produced, but electricity production would represent a misallocation of precious liquid fuel.

16: However, low EROEI may make the enterprise non-viable.

17: . . . for example, a genetic arms race with evolving biological phages

Geothermal Electricity (Sec. 16.1; p. 275): This option makes sense primarily at rare geological hotspots. It will not scale to be a significant part of our entire energy mix. Aside from this, it is relatively easy, steady, and well demonstrated in many locations. It can provide electricity, and obviously direct heat—although often far from locations demanding heat.

Wind (Chapter 12): Wind is neither super-abundant nor scarce, being one of those options that can meet a considerable fraction of present needs under large-scale development [70]. Implementation is relatively straightforward, reasonably efficient, and demonstrated the world over in large wind farms. The biggest downside is intermittency. It is not unusual to have little or no regional input for several days in a row. Objections to wind tend to be more serious than for many other alternatives. Wind turbines are noisy and tend to be located in prominent places (ridgetops, coastlines) where their high degree of visibility alters scenery. Wind remains viable for small-scale personal use.

[70]: Castro et al. (2011), "Global Wind Power Potential: Physical and Technological Limits"

Artificial Photosynthesis: Combining the abundance of direct solar input with the self-storing flexibility of liquid fuel, artificial photosynthesis is a compelling future possibility [113]. Being able to store the resulting liquid fuel for many months means that intermittency is eliminated to the extent that annual production meets demand. A panel in sunlight dripping liquid fuel could satisfy both heating and transportation needs. Electricity can also be produced, but given an abundance of ways to make electricity, the liquid fuels would be misallocated if used in this way. Unfortunately, an adequate form of artificial photosynthesis has yet to be demonstrated in the laboratory, although the U.S. Department of Energy initiated a large program in 2010 toward this goal.

[113]: Andreiadis et al. (2011), "Artificial Photosynthesis: From Molecular Catalysts for Light-driven Water Splitting to Photoelectrochemical Cells"

Tidal Power (Sec. 16.2; p. 280): Restricted to select coastal locations, tidal power will never be a large contributor to global energy. The resource is intermittent on daily and monthly scales but in a wholly predictable manner. Extracting tidal energy is not terribly hard—the efficient technology is similar to that found in hydroelectric installations—and has been demonstrated in a number of locations around the world.

Conventional Fission (Sec. 15.4.4; p. 255): Using conventional uranium reactors and conventional mining practices, nuclear fission does not have the legs for a marathon. On the other hand, it is certainly well demonstrated and has no intermittency problems—except that it cannot accommodate intermittency on the demand side (variable load). Compared to other options, nuclear power qualifies as a high-tech approach—meaning that design, construction, operation, and emergency mitigation require more advanced training and sophistication than the average energy producer.

Acceptance is mixed. Germany and Japan plan to phase out their nuclear programs by 2022 and the 2030s, respectively, despite being serious about CO_2 reduction. Public unease also contributed to a halt in licensing new reactors in the United States from 1978–2012. Some opposition stems from unwarranted—yet no less real—fear, sustained in part by the technical complexity of the subject. But some opposition relates to political difficulty surrounding proliferation and the onerous radioactive waste problem that no country has yet solved to satisfaction.

Uranium Breeder (Sec. 15.4.4.2; p. 258): Extending nuclear fission to use plutonium synthesized from ^{238}U, which is 140 times more abundant than ^{235}U, gives uranium fission the legs to run for at least centuries if not a few millennia, ameliorating abundance issues. Breeding has been practiced in military reactors, and indeed some significant fraction of the power in conventional uranium reactors comes from incidental synthesis of plutonium (^{239}Pu) from ^{238}U. But no commercial power plant has been built to deliberately tap the bulk of uranium for power production. Public acceptance of breeder reactors will face even higher hurdles because plutonium is more easily separated into bomb material than is ^{235}U, and the increase in radioactive waste from an expansion of nuclear power causes trouble.

Thorium Breeder (Box 15.5; p. 260): Thorium is more abundant than uranium and only has one natural isotope,[18] qualifying it as an abundant resource. Like all nuclear options, thorium reactors fall into the high-tech camp and include new challenges[19] that conventional reactors have not faced. A few small-scale demonstrations have been carried out, but nothing in the commercial realm; bringing thorium reactors online at scale is probably a few decades away, if it happens at all. Public reaction will likely be similar to that for conventional nuclear: not a deal breaker, but some resistance on similar grounds. It is not clear whether the novelty of thorium will be greeted with suspicion or enthusiasm. Although thorium also represents a breeding technology (making fissile

18: . . . thus enrichment is not a bottleneck

19: . . . such as a liquid sodium medium for the reactor core

^{233}U from ^{232}Th), the proliferation aspect is severely diminished for thorium due to a highly radioactive ^{232}U by-product[20] and virtually no easily separable plutonium.

20: . . . making it deadly to rogue actors

Geothermal Heating allowing Depletion (Sec. 16.1.2; p. 277): A vast store of thermal energy sits in Earth's crust, permeating the rock and moving slowly outward. Ignoring sustainable aims, boreholes could be drilled a few kilometers down to extract thermal energy out of the rock faster than the geophysical replacement rate, effectively mining heat as a one-time resource. In the absence of water flow to distribute heat, dry rock will deplete its heat within 5–10 meters of the borehole in a matter of a few years, requiring another hole 10 m away from the previous, in repeated fashion. The recurrent large-scale drilling operation across the land qualifies this technique as moderately difficult.

The temperatures are marginal for running heat engines to make electricity with any respectable efficiency,[21] but at least the thermal resource will not suffer intermittency problems during the time that a given hole is still useful. Kilometer-scale drilling hurdles have prevented this technique from being demonstrated at geologically normal (inactive) sites. Public acceptance may be less than lukewarm given the scale of drilling involved, dealing with tailings and possibly groundwater contamination issues on a sizable scale. While a backyard might accommodate a borehole, it would be far more practical to use the heat for clusters of buildings rather than for just one—given the effort and lifetime associated with each hole.

21: . . . especially given that many easier options are available for producing electricity

Geothermal Heating, Steady State (Sec. 16.1.1; p. 276): Sustainable extraction of geothermal heat—replenished by radioactive decay within the Earth—offers far less total potential, coming to about 13 TW of flow if summed across all land. And to get to temperatures hot enough to be useful for heating purposes, boreholes at least 1 km deep would be required. It is tremendously challenging to cover any significant fraction of land area with thermal collectors 1 km deep. As a result, a yellow score for the abundance factor may be generous. To gather enough steady-flow heat to provide for a normal U.S. home's heating demand, the collection network would have to span a square 200 m on a side at depth, which is likely unachievable.

Note that technologies known as "geothermal" heat pumps are not accessing an energy resource; they are simply using a large thermal mass against which to regulate temperature.

Biofuels from Crops (Sec. 14.3; p. 230): While corn ethanol may not even be net energy-positive, sugar cane and vegetable oils as sources of biofuel fare better. But these sources compete with food production and arable land availability. So biofuels from crops can only graduate from "niche" to moderate scale in the context of plant waste or cellulosic conversion. The abundance and demonstration fields are thus split: food crop energy is demonstrated but severely constrained in scale. Cellulosic matter becomes a potentially larger-scale source but is undemonstrated.[22] Growing and harvesting annual crops on a relevant scale constitutes a massive, perpetual task and thus scores yellow in difficulty—also driving down EROEI.

22: . . . to the point that perhaps this should even be red

If exploiting fossil fuels is akin to spending a considerable inheritance, growing and harvesting our energy supply on an annual basis is like getting a manual labor job: a most difficult transition. The main benefit of biofuels from crops is the liquid fuel aspect. Public acceptance hinges on competition with food or even land in general. Because plants are only about 1–2% efficient at harvesting solar energy, this option requires the commandeering of massive tracts of land.

Ocean Thermal: The ocean thermal resource uses the 20–30°C temperature difference between the deep ocean[23] and its surface to drive a ridiculously low-efficiency heat engine. The heat content is not useful for warming any home (it's not hot). But all the same, it is a vast resource due to the sheer area of the solar collector (i.e., the ocean). Large plants out at sea will be difficult to access and maintain, and transmitting power to land is not easy. The resource suffers seasonal intermittency at mid-latitudes, but tropical installations would obviate this effect. No relevant/commercial scale demonstration exists. Like many of the other sources, this one produces electricity[24] only. In terms of acceptance, few likely care what we put to sea: out of sight, out of mind. Ocean thermal is not a backyard solution!

23: …a few hundred meters down

24: …far from demand locations

Ocean Currents (Sec. 16.3; p. 281): Large-scale oceanic currents are slower than wind by about a factor of ten, giving a kilogram of current 1,000 times less power than a kilogram of wind. Water density makes up the difference to make ocean current comparable to wind in terms of power per rotor area. Not all the ocean has currents as high as 1 m/s, so the total abundance is put in the same category as wind, although this is likely far too generous and probably should be red. Placing machinery underwater (corrosive) and far from demand classifies this option as difficult. On the plus side, the current should be stable, eliminating intermittency worries, unlike wind. None of our electricity mix comes from ocean currents at present, so it cannot be said to have been meaningfully demonstrated. For the remaining categories: it is electricity only; we might expect little resistance to underwater installations; and no backyard opportunity.

Ocean Waves (Sec. 16.4; p. 282): While they seem strong and ever-present, waves are a linear-collection phenomenon, and not an areal phenomenon. As a consequence, not that much power arrives at shores all around the world (a few TW at best). It is not particularly difficult to turn wave motion into useful electricity at high efficiency, and the proximity to land makes access, maintenance, and transmission far less worrisome than for the previous two ocean-based cases. Intermittency—largely seasonal—is moderate as storms and lulls come and go. Prototype concepts abound, and a few are being tested at commercial scale. So this is further along than the previous two oceanic sources, but not so much as to earn a blue box. Push-back would be moderate from people whose ocean views are spoiled, or who benefit from natural wave energy hitting the coast. Few people have access to waves in their backyard.

D–T Fusion (Sec. 15.5; p. 264): The easier of the two fusion options, involving deuterium and tritium, represents a longstanding goal under active development since 1950. The well-funded international effort, ITER, plans to accomplish a 480 second pulse of 500 MW power[25] by 2035. This defines the pinnacle of hard. Fusion brings with it numerous advantages: enormous power density; only moderate radioactive waste products; and abundant deuterium.[26] Fusion would have no intermittency issues, can directly produce heat and derivative electricity, but like the others does not directly address transportation. The non-existent tritium can be knocked out of lithium with neutrons, and even though we are not awash in lithium, we have enough to last many thousands of years in the absence of demand for batteries. We might expect some public opposition to D–T fusion due to the necessary neutron flux and associated radioactivity. Fusion is the highest-tech energy we can envision at present, requiring a team of well-educated scientists/technicians to run—meaning don't plan on building one in your backyard, unless you can afford to have some staff PhDs on hand.

D–D Fusion (Sec. 15.5; p. 264): Replacing tritium with deuterium means abundance of materials is no concern whatsoever for many billions of years. As a trade, it's substantially harder than D–T fusion.[27] D–D fusion requires higher temperatures, making confinement that much more difficult. It is for this reason that we make a single exception to the scoring scheme and give D–D fusion a −2 score for difficulty.

17.3 Student Rankings

In Winter 2020, students at UC San Diego did Problem 11 from this chapter, which asks them to come up with their own weighting for the ten attributes in the comparison matrix, rather than the simple but poorly-justified practice of giving each category the same weight. This section reports the result from that exercise.

On average, the weights did not deviate dramatically from uniform, the adjustments being within a factor of 1.5 for all cases: ranging from 0.66 to 1.47, when each student's weighting was re-normalized after the fact to add to ten points. The attributes getting a clear boost were abundance, efficiency, and transportation capability. Those getting lower weight were the backyard criterion, acceptance, and ability to produce heat.[28] What this says is that students, on the whole, value solutions that can get the job done efficiently and preserve transportation, not caring as much about individual resilience or acceptance.[29]

Table 17.1 summarizes the outcome of this experiment, in which we see broad agreement with the nominal uniform-weighting scheme, mostly serving to downgrade the scores of some entries, and only promoting one (thorium breeding). Note that the exercise preserved the blue/yellow/red color scheme of the figures and only changed relative

25: Compare to a real power plant: years-long "pulse" at 3,000 MW thermal power.

26: ... though no natural tritium

27: ... or we would not even consider D–T

28: One wonders if students in colder climates would disregard heating capability.

29: In other words, centralized power less bound by public opinion is okay.

Table 17.1: Result of student weighting of the comparison matrix attributes. Notable gaps between scores appear between fossil fuels and solar, between solar and the middle group, and between the middle group and the final group (right). When differences between student scores and the nominal scores in Figures 17.1 and 17.2 exceed rounding error, the result is shown as a Δ, helping highlight the main differences between student scoring and the nominal uniform-weight result.

Resource	Score	Δ	Resource	Score	Δ	Resource	Score	Δ
Gas	7.3	−0.7	Hydroelectric	3.6		Conv. Fission	1.6	
Oil	6.8	−0.7	Algae/Biofuel	3.6		Geotherm. Depl.	1.4	
Coal	6.7	−0.8	Geotherm. Elec.	3.3	−0.7	D–T Fusion	1.4	
			Solar Heat	3.2	−0.8	Wave	1.3	
			Tidal Capture	3.0		Ocean Thermal	1.3	
Solar Thermal	5.4		Wind	2.9		Ocean Currents	1.0	
Solar PV	5.1		Uranium Breeder	2.8		D–D Fusion	1.0	
			Artif. Photosyn.	2.8		Geotherm. Heat	0.7	
			Thorium Breeder	2.5	+0.5	Biofuel/Crops	0.4	−0.6

importance of the attributes—thus much of the behavior is "baked in" based on the author's judgment of color assignment. Still, shifting emphasis on the attributes will shuffle the order somewhat, and that itself is interesting. Any ambitious reader is welcome to mess with the color scheme as well to see what happens.

17.4 Upshot: The Fossil Fuel Gap

The subjective nature of this exercise certainly allows numerous possibilities for modifying the box rankings in one direction or the other. The matrices embody some biases, but no attempt by anyone would be free from bias. The result, in this case, is dramatic. Even allowing some manipulation, the **substantial gap** between the fossil fuels and their renewable alternatives would require excessive "cooking" to close.

A key take-away from this chapter is that we can devise methods to compare disparate sources of energy in a systematic way. The outcome does not provide an authoritative answer, but what it *can* do is:

► help guide our thinking;
► expose gaps that we might not otherwise appreciate;
► bring attention to the complexity of energy choices: it's more than how many terawatts are available.

One lesson is that a transition away from fossil fuels *does not appear at this time to involve superior substitutes*, as has been characteristic of our energy history. The alternatives might be *good enough*, even if not *as good as* what we are accustomed to using. Fossil fuels represent a generous one-time gift from the earth. From our current vantage point, it is not clear that energy—vital to our economic activity—will be as cheap, convenient, and abundant as it has been during our meteoric ascent to the present. If not for finite supply and the CO_2 problem, fossil fuels would continue to satisfy our energy needs—shifting focus to various other global-scale

problems resulting from human pressures[30] on the planet. Because of the downsides of fossil fuels and the inferiority of the substitutes on a number of fronts, it is unclear how we might patch together an energy portfolio out of the alternatives that allows a continuation of our current lifestyle. Even if the physics allows it, many other practical and economic barriers may limit options or successful implementations.

Adding to the hardship is the fact that many of the alternative energy technologies—solar, wind, nuclear power, hydroelectric, and so on—require substantial up-front energy investment to build and deploy. If society waits until energy scarcity forces large-scale deployment of such alternatives, it risks falling into an "energy trap" in which aggressive use of energy needed to develop a new energy infrastructure leaves less available to society in general—a political non-starter. If there is to be a transition to a sustainable energy regime, it is best to begin it now.[31]

30: ...which could conceivably get only *worse* if humans "solved" energy, effectively unconstrained

See Sec. 18.3 (p. 310) for more discussion of the energy trap.

31: We can't count on better options showing up, and may need to get busy migrating to available choices. Fossil fuels not only drive climate change, but create an addiction that is best tapered slowly rather than risk chaos and war if we remain completely reliant on a dwindling crucial resource.

17.5 Problems

1. What is your overall assessment of our energy future given the presentation of alternatives to fossil fuels presented in this chapter? What key take-aways do you form?

2. What, to you, is the biggest surprise in the rank order of alternatives in Figure 17.2?

3. One consideration not present in the alternative energy matrix is EROEI. Identify at least one example that would likely receive a boost (move up in the rankings) if EROEI is considered, and at least one example that would likely move down as a result of considering it. Refer back to Chapter 14, both tables and text.

4. (Suggest double-credit problem) Count up the number of blue, yellow, and red squares for all ten attributes (columns) in Figure 17.2. Score each attribute the same way as energy resources were scored: +1 for blue; nothing for yellow; −1 for red.

   a) What category has the highest score? What does this tell you about alternative energy in terms of what is easy?
   b) What category has the lowest score? What does this tell you about what attribute is hardest to satisfy in energy production?
   c) Which two categories have the largest number of blue squares and which two have the lowest number? What do you learn in terms of what is easy and not so easy?
   d) Which category has the largest number of red squares and which *two* have the lowest number? What do you learn in terms of what is hardest and what is least difficult to satisfy?

5. Why is it so hard to satisfy transportation needs? What is it about fossil fuels that has made transportation so much easier than electrified transport would be?

6. Verify—showing work—the claim on page 292 that a car refueling at a gasoline station is transferring energy at a rate in the neighborhood of 15 MW if the transfer rate is about 6 gallons (23 L) per minute[32] and gasoline contains about 34 MJ per liter.

32: Time it yourself some time.

7. What is at about the backyard[33] attribute that earns so many red scores? What does this tell you about access to energy and the complexity of its capture/use?

33: ...just shorthand for energy on a personal scale at home

8. What two resources are crippled by intermittency? How might that challenge be mitigated so that it is less problematic? What are some difficulties in this solution?

9. If biofuels turn out to be hampered by low EROEI values, the only transportation-friendly option left is artificial photosynthesis, which was not covered in this book because it's not a "real thing" at this time—as captured by the two red squares for difficulty and demonstrated. Two questions:

   a) What other resources are in a similar "not real yet" state, based on the same red squares?
   b) What is most encouraging about the prospect of obtaining liquid fuels from sunlight? What problems does it solve for solar power and for transportation?

10. Figure 17.2 left out firewood as an energy resource, even though it has been with us for a long time and will no doubt continue to provide home heating. What would the ten colors be for this resource of intermediate abundance in the context of home heating? What would its score be, and which sources is it then tied with? Justify each of your color choices.

⚠ Different students may get different results, but this will provide some appreciation for considerations that enter the creation of such a scoring scheme.

11. (Suggest triple-credit problem) For the sake of simplicity and transparency, the scoring system used a simple uniform weighting for each of the 10 attributes, but it is unlikely all ten are equally important. Make up your own weighting scheme.[34] For each energy resource, blue cells *add* the corresponding category's weight, and red *subtracts*, and yellow does nothing. Re-score the matrix using your own weights—including the fossil fuels. Now re-order the matrix from highest to lowest score and comment on any major upheaval from the nominal ordering. What are the big surprises? Did the fossil fuel gap persist?

34: A category might be worth 2 points, 0.3 points, 5 points, or whatever you wish; need not add to 10

First, make sure you can reproduce the scoring in the present table based on all weights being 1.0.

Part IV

Going Forward

We have layered an artificial world atop the natural one.
Which do you think will stand the test of time?
The sooner we dovetail back to the natural,
the greater our chances for success become.

18 Human Factors

Energy has occupied center-stage in this book because it is the physical currency of activity. Fossil fuel availability permitted the Industrial Revolution and all that came with it. Human civilization has never before faced the prospect of such a crucial global resource either disappearing or being abandoned due to other ills (e.g., climate change).

Despite the mixed suitabilities of alternative energy sources presented in Chapter 17, *some* physically viable avenues *are* open to us. In other words, physics itself does not preclude development of an energy landscape consisting primarily of solar, wind, and hydroelectric power, plus a few small contributions from a variety of other sources. Life may be different; less air travel or transportation in general, as one possibility. The enormous solar potential combined with decent storage technologies could *conceivably* maintain our 18 TW appetite without fossil fuels. Or we could scale back and not expect to live in such a profligate manner, viewing our current ways and expectations in the broader human experience as anomalous, temporary, and not important to preserve.

In any case, the barriers are not completely due to physical constraints—though we must heed limits where they exist. A practical concern is the expense of a non-fossil energy infrastructure: both in energy and financial terms. It is possible that the prosperity required to afford such a dream is physically unavailable.[1] Another limitation involves people and how they work collectively, which is the focus of this chapter.

1: If it were easy to conjure wealth, energy, and material goods, why would any person or country be poor?

18.1 Personality

Discussion of personality may seem out of place in a physics-based textbook. But how we react to a looming crisis—both individually

Easter Island stands as a testament to humans' ability to do really dumb things, like cutting every tree on the island. Who cut down the last one? Photo by Sophie Hurel.

or collectively—becomes a central question. Physics and nature are indifferent to the choices we make, and whether or not these choices serve our long-term interests. Success or failure therefore depends more on our attitudes and reactions to physical reality than it does on physics itself. While this section addresses how personality preferences might influence human reaction to crisis, it is by no means the only determining factor. Education, socio-economic status, financial interests, and many other attributes can play a role. Yet underneath it all, we are humans, and possess various traits that influence our reactions.

A common fallacy is that a collection of people, possessing access to the same set of facts,[2] will respond similarly. Having only occupied one person's head, each of us cannot easily comprehend the reactions another might have to a situation. A variety of metrics have been devised to characterize different personalities and preferences, which play a role in shaping reactions. The Big–5 and Myers–Briggs are among the most well known. The former provides a scale for each of five traits:

1. Openness: creative and curious vs. careful and regimented
2. Conscientiousness: degree of organization and efficiency
3. Extraversion: socially energetic vs. preference for quiet or alone
4. Agreeableness: engaging and sympathetic vs. aloof or cranky
5. Neuroticism: easily rattled vs. confident and stable

The Myers–Briggs scheme[3] has four categories that are usually represented as binary (either/or) labels, but in truth also exist on a continuum as do the Big–5 traits. Each category in Myers–Briggs acquires a one-letter label, in the end generating 16 possibilities.[4] The categories are:

1. I/E: Introvert vs. Extravert: are parties draining or energizing?
2. N/S: iNtuiting vs. Sensing: preference for abstract vs. concrete direct-sensory information
3. T/F: Thinking vs. Feeling: preference for intellectual/logical or emotional reasoning
4. J/P: Judging[5] vs. Perceiving: decisiveness vs. keeping options open

If you have never taken a Myers-Briggs type test, the following three citations include links to online tests [114–116]. These sites, and many others, offer descriptions of the types. For instance, here is a link[6] to a description of the INTJ type from [114]. Other types are linked from that page.

Many scientists find themselves in the INTX camps.[7] Technically speaking, the last category (J/P) reflects whether a person relies more heavily on the N/S axis (if P) or the T/F axis (if J). In the case of INTP, abstraction/theory is valued over logic, and the reverse for INTJ. Figure 18.1 provides an indication of the frequency of each type.[8]

A survey of visitors to the author's Do the Math website, which presents a quantitative look at energy and resource use much in the same spirit as

2: . . . itself a perplexing challenge in an ecosystem of polarized information flow

3: Myers–Briggs often triggers a reaction among knowledgeable individuals that it is an invalid, or debunked personality measure. If you are one of those, please reserve judgment until seeing how it is framed, and accompanying data.

4: . . . ignoring the inconclusive "X" label that can take the place of any letter when someone is not strongly to one side or the other in a category

5: . . . not meaning judgmental

6: . . . can also access from citation link

7: Experimentalists are more likely to be J and theorists are more likely to be P.

8: Keep in mind that real people can blend attributes and do not always fall cleanly into a given "box," so the 16 tidy categories is somewhat artificial.
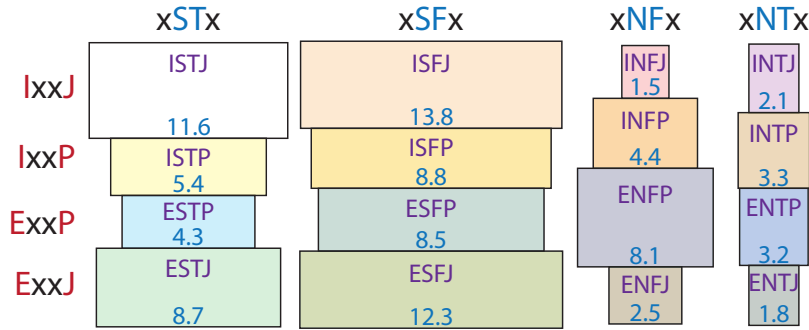
this text, found that a staggering 44% of about 1,000 survey respondents shared the *same* INTJ type as the author, despite INTJs constituting only about 2% of the population. The over-representation of INTJ is thus about a factor of 20, and beyond question a statistically significant result (see Figure 18.2). The adjacent INTP cohort accounted for an additional 21% of visitors, despite being about 3% of the population. Thus about two-thirds of the site visitors represented 5% of the population for more than a factor-of-ten anomaly. While the Myers–Briggs scheme is not free of criticism (see Box 18.1), the result from *Do the Math* is pretty convincing that the Myers–Briggs scheme is measuring *something* relevant. Had the site asked for astrological sign, we can be pretty sure that representation would have been essentially uniform: no statistical anomalies. Myers–Briggs is evidently telling us *something*.
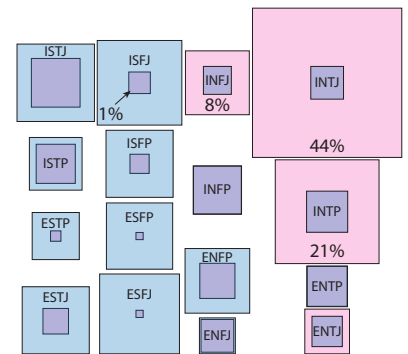
Figure 18.2: Stunning Myers–Briggs personality disparity between Do the Math readership and the general population. Areas of blue boxes represent frequency of types in the general population, as a variant of Figure 18.1. Pink areas reflect the readership of Do the Math, and overlaps appear purple. Visible pink means over-representation on the site, and visible blue indicates under-representation. 73% of the readership occupies the top right corner of the personality types, even though these three boxes only capture 7% of the population. A result this extreme, involving over 1,000 respondents, is absurdly far from random. Percentages are shown in the three most overwhelmingly over-represented boxes, plus a 1% box for reference.

What if objection to the Myers–Briggs framework correlated strongly with Myers–Briggs type?

> **Box 18.1: Criticism of Myers–Briggs**
>
> Much of the criticism of the Myers–Briggs personality type indicator owes to the binary nature of each result, unlike the Big–5 percentage for each category. The argument is that people are more complex than this, and do not fall into 16 distinct and tidy categories. But any Myers–Briggs test will indicate where you fall on the spectrum from the hard extremes—based on the number of answers in the survey toward each direction. Some people fall in the middle of a category and get an X designation. 16 rigid types is illusory and unnecessary.
>
> It is easy enough to accept this valid criticism yet still value the fact that people *do* spread out in their preferences. Also, some of the dimensions map onto the more widely-embraced Big 5 scheme.

The most important aspect of personality that bears on our energy/resource challenges, and a key reason for covering the topic at all in this book, is that the Myers–Briggs "S" types constitute 73% of the population, yet are most resistant to the argument that unprecedented disruptive changes are on the horizon. S-types place more value on direct sensory input than on abstraction. Things directly experienced—seen with their own eyes, heard by their own ears, personally touched, smelled or tasted—carry much greater weight than "theory." Presented with

warnings about climate change, a hard-over S-type [118] might hold up a snowball (as Senator Inhofe famously did in front of Congress in 2015) to demonstrate—in their mind—how preposterous this global warming talk is. When warned about potential hardships following fossil fuel availability, an S-type may look out the metaphorical window and sense that everything seems to be fine.

[118]: Weiler et al. (2012), "Personality type differences bet. Ph.D. climate researchers and the general public: implications for effective communication"

Box 18.2: Scientists Have a Type

Scientists tend to be NT types, combining a preference for abstraction and models with a favoring of logical thought. Note that not all scientists are alike; it is good to have diversity, so it is possible to find any personality type within the science community. But the NT types will be perhaps more attracted to scientific pursuits as a good match to their personality preferences. The NT combination is the most rare, comprising 10% of the population (see Figure 18.3). Meanwhile, SF—the diametric opposite of NT—is over four times as prevalent.

When it comes to forming rational plans for future unseen challenges, NT types are well suited to the task. Ironically, if N–type predictions of future threats were faithfully heeded and mitigated, the threats would be averted and their failure to materialize would make the N–types look like terrible predictors to the S–type crowd! Failure by success.[9]

Figure 18.3: Distribution of N/S and T/F traits among the population.

18.1.1 Consequences and Coping

The S-type asymmetry[10] in human populations may confer an overall adaptive advantage. In stable times, recent history and apparent conditions provide reliable guidance to the likely future. It is understandable, therefore, that humankind would have difficulty adapting early to an upcoming reversal of fortunes, or even acknowledging its possibility. The prevailing narrative of growth and progress are so firmly rooted in society that the mere suggestion of a more primitive future[11] is discordant enough to be rejected by cultural antibodies—alien enough to resist comprehension, as if spoken in a foreign language.

The growth narrative's firm grip is easy to understand: Earth has always been large enough to accommodate human cravings, for countless generations. Yet 8 billion people competing for finite resources[12] at an unprecedented rate, climate change, deforestation, fisheries collapse, species loss, and a host of other crises signal that the prevailing narrative may simply be *wrong*[13] (Figure 18.4).

Timely, effective mitigation is possible only by seriously entertaining a radically new paradigm *before* widespread disruption becomes unmistakably evident. Is it possible to implant a wholly different narrative

9: This irony can also plague successful governance: people take for granted clean air, water, and food ensured by invisible, effective government. This quiet success creates complacent room for anti-government sentiments, potentially culminating in its failure.

10: . . . nearly 3 to 1

11: . . . fewer resources and possessions, no space colonies, closer to nature

12: See Fig. 8.1 (p. 116) for a visual reminder.

13: Recall that Chapters 1 and 2 made a compelling case.

WRONG
WRONG
WRONG WRONG
WRONG WRONG ... YOU'RE-WRONG ... YOU'RE-WRONG ... YOU'RE-WRONG
WRONG ... WRONG

Figure 18.4: The growth narrative is. . .*wrong*. Sung to the tune of a clock chime striking three; inspired by Dr. Cox in S1:E9 of *Scrubs*.

about the long-term relationship of humans to this planet, or is it cooked in to human nature[14] that we fail this challenge?

One aspect of this dilemma is a pattern of unwillingness to accept personal responsibility for our predicament. Our own habits and expectations place demands on planetary resources that lead to global-scale challenges larger than we have ever faced. People have a tendency to blame others for their plights. In this complex world, it is never difficult to identify some other contributor to our problems: capitalists, socialists, liberals, conservatives, environmentalists, illegal immigrants, other religions, other powerful countries. What fraction of the blame might we assign to ourselves, and is it honest/accurate? In the end, we as humans must accept responsibility for the conditions we—and our expectations—create.

The human penchant for blaming and even demonizing "others" might lead to resource wars in the face of hardships imposed by limited resource availability. Such a path is lamentable on many levels, not least of which is that precious resources and energy would be channeled toward destructive acts rather than using them to build a better future. Are humans capable of mounting a transformative effort of global cooperation on a scale even greater than that of, say, World War II if we are not fighting an enemy other than ourselves and our own resource demands? Can we identify a precedent in which human societies have done so in the past at a large/relevant scale?

The first step in avoiding these pitfalls is awareness of the roles that human personality and psychology play in these problems, as this section has attempted to point out. One thing that became evident to the author on the basis of the Do the Math survey was that like seems to attract like: the communication style of the blog was a magnet for those of the same or adjacent types. But the message had startlingly little grip on the S crowd—especially the population-dominant xSFx types (second column in Figure 18.2). Perhaps a concerted effort to recruit all personality types to communicate important messages will better reach a broader audience, in terms that are more resonant with recipients.

One other coping mechanism is simple: time. The world a child is born into is by definition "normal" to the child. Future generations who do not grow up spoiled by abundant fossil fuel energy will not fight for a bygone lifestyle, and will simply adapt to the world as they find it—where the S–types shine. One way or another, nature will settle on a solution, and humans will be part of that solution, whatever its form. Ideally, we can guide ourselves into a mutually agreeable coexistence.

18.2 Policy vs. Individual Action

From where does power[15] stem? Monarchs of old claimed divine provenance as a source of power. Authoritarians might use a combination of personality and strong military backing. Some countries put on a show of democracy, while exercising power to thwart rivals and control the legal apparatus—resulting in sham elections.

In a functioning democracy, power is meant to come *from the people*. Votes send ineffective or crooked politicians packing, replacing them with folks promising to be faithful servants to the people and to the country's constitution. If democracy works as planned, then how far can a politician be from the sentiment of the people[16] and still get elected (or re-elected)? Since presumably politicians cannot afford to stray far from the views of their constituents, are democratic politicians leaders or followers?

What would happen in a functioning democracy if the government told people what to eat, how many kids to have, or what temperature to set in homes? If people are not *already* willing to make the changes on their own,[17] how can a democratic government impose "responsible" choices when those choices involve unwanted sacrifice in some form? If democratically-elected politicians are constrained to only offer "better," more personally attractive choices, a democratic world may no longer have the flexibility necessary to address our fundamental challenges.

So if democratic governments turn out to be ineffective at promoting substantive change involving reduction, what can be done if indeed that's the necessary course of action? Citizens are free to take charge of their own choices, for the greater good. Yet, individual action as a response to our predicament often provokes criticism and debate. The argument is that only policy has the teeth to bring about effective change. If person A uses less of a resource, it just leaves person B free to use more of it.

But consider that *voting* is also an individual action. One person's vote seldom makes any measurable difference: it is a tiny drop in the bucket. Yet, if everyone[18] concluded that the individual action of voting was wasted and meaningless, the result would be disastrous: gross underrepresentation and distortion of the will of the people.

We understand in the context of voting that individual action leads to collective representation, and tend to practice this right even though policy does not demand that we do so. We see it as part of a civic responsibility—as the right way to participate in our society. Our system can tolerate a certain fraction of individuals who won't donate time to their civic responsibility, but things fall apart if we have too many such people.[19] Next time someone challenges your individual efforts to improve the world in some way, labeling them as meaningless, ask them to be consistent with their espoused principles and please refrain from

15: Wait, Watt? For once, we are not talking about Joules per second.

16: If 73% of the people are S-type personalities, then our policies will lean toward satisfying that contingent.

17: . . . and in fact, already *changing* habits on their own accord

It feels icky to point out flaws in democracy, as this form of government has admirably lifted countless individuals up and promoted enlightened progress. It really is the best in many respects—except for this fundamental flaw in a world of constraints and possible reductions.

18: . . . or even a large majority

19: The technical term for these people is: jerks.

voting in the future, lest they be labeled hypocrites. Often such objections are rooted in defense: they *want* stuff, and your giving something up is a threat to their perceived moral "right" to have it. The sense of sanctimony and righteousness of the individual making sacrifices—even if not intentional—can be very offputting.

One other aspect of individual action is that it could influence others to follow, thus amplifying the individual's effect. This approach is perhaps most effective if others see benefits for themselves, and are not made to feel bad for not already being "woke" to the right side of the practice.

18.3 The Energy Trap

If we unwisely mount a response only *after* we find ourselves in fossil fuel decline—as crisis responders, not proactive mitigators—we could find ourselves in an energy trap: a crash program to build a new energy infrastructure requires up-front energy, for decades. If energy is already in short supply, additional precious energy must be diverted to the project, making peoples' lives seem even harder/worse.[20] A democracy will have a hard time navigating this decades-long sacrifice.

20: Amid consternation over energy shortfalls and likely high prices, pulling more energy away from people will not be popular.

21: . . . with interest

Let's flesh this concept out a bit more. In the financial world, money can be borrowed on the promise of paying it back.[21] In this way, something is created from nothing, essentially. Modern monetary systems are based on *fiat currency*, rather than being tied to physical gold or silver. This means money can be "willed" into existence by the financial system. Energy does not work that way. To build a hydroelectric dam, solar panels, wind turbines, or a nuclear plant, all the energy must be available up front. Nature offers no financing!

Recall from Sec. 14.3.1 (p. 231) that the EROEI, or energy returned on energy invested, describes the ratio of output energy over the lifetime of the resource to the input energy needed to secure it in the first place. For many cases, like a hydroelectric dam, nuclear plant, wind turbine, or solar panel, most of the energy *input* happens before any energy is delivered. In other cases, like biofuels, the investment may be more drawn out and seem more like an efficiency. In the context of the energy trap, we will focus on the input as an up-front investment.

invest one unit to create resource

get 6 units of output over 30 years…

…as if first 5 years spent repaying "loan"

Figure 18.5: Energy "payback" time for 30 year resource with EROEI of 6:1.

22: Any EROEI estimate must assume some resource lifetime in order to compute the amount of energy delivered.

> **Example 18.3.1** If a solar panel has an EROEI of 6:1, that "1" unit has to be paid *up front*, even though ultimately the panel will more than pay for itself, energetically. How many years of the panel's energy needs to be available up front if the panel lasts 30 years in the sun?[22]
>
> The 6 in the EROEI figure relates to the total output of the resource. So we equate 30 years of operation to the number 6, meaning that 1 "unit" is 5 years of output (Figure 18.5). Since the input is 1 unit (in

6:1 construction; see Sec. 14.3.1; p. 231), we conclude that it takes 5 years of the panel's output energy to fabricate the panel. So its first five years are spent paying off the "loan," in a sense.

As another example, development of a resource that will last 40 years and whose EROEI is 10:1 will require 4 years of its energy output ahead of time[23] to bring it to fruition.

> **Example 18.3.2** In order to replace the current 15 TW[24] now derived from fossil fuels with a renewable resource whose lifetime is 40 years and EROEI is 10:1,[25] what options might you suggest for diverting the 15 TW into construction and how long would it take under those options?
>
> It takes four years of the ultimate resource output to create the resource in this scenario. In one extreme, *all* 15 TW from fossil fuels could be diverted into the effort over a four year period[26] to develop 15 TW of the new resource. Or half of the 15 TW fossil resource could be dedicated to the effort over 8 years, or a quarter over 16 years, or 10% over 40 years.[27] Choosing this last path for a 40-year resource means "starting over" at this juncture, essentially forever re-investing 10% of available energy into perpetuating a resource with EROEI of 10:1.

Imagine now that we find ourselves having reduced access to oil,[28] driving prices up and making peoples' lives harder. Now the government announces a 16 year plan to divert 25% of energy into making a new infrastructure in an effort to reduce dependence on fossil fuels. That is a huge additional sacrifice on an already short-supply commodity. Voters are likely to respond by tossing out the responsible[29] politicians, installing others who promise to kill the program and restore relief on a short timescale. Election cycles are short compared to the amount of time needed to dedicate to this sort of major initiative, making meaningful infrastructure development a difficult prospect in a democracy. And this is before addressing the likely contentious fights about *what* the new infrastructure should be, out of the table of imperfect[30] options.

Now it is perhaps more apparent why this is called an energy trap: short-term political and economic interests forestall a proactive major investment in new energy, and by the time energy shortages make the crisis apparent, the necessary energy is even harder to attain. Short-term focus is what makes it a trap.[31]

One wonders how democracies will fare in the face of declining resources. The combination of capitalism and democracy have been ideal during the growth phase of our world: efficiently optimizing allocation of resources according to popular demand. But how do either work in a decline scenario, when the future is not "bigger" than today, and may involve sacrifice? We simply do not yet know. This is a giant unauthorized experiment that is not operating from a script. Chapter 19 will return to this notion.

23: ... from a resource that is *already* producing energy

24: ~80% of the 18 TW total

25: ... or a combination of resources having similar EROEI and lifetime

26: ... leaving nothing for societal needs

27: Even a 10% diversion will "hurt" and be unwelcomed.

28: ... as the most plausible example; the first to peak

29: ... in the true sense of the word

30: See Chapter 17, for instance.

31: Is this a human limitation?

18.4 Fermi Paradox Explained?

Having discussed some of the top-level challenges facing our technological society, it is too tempting to speculate on how universal our trajectory might be. We do so in the context of the Fermi Paradox [119], which asks: **if** the universe produces a reasonably high probability of intelligent life developing, why have we seen no credible evidence[32] of advanced civilizations? Yes, this is a bit of a tangent,[33] but it could be interpreted as a relevant data point in the likelihood that we maintain an advanced civilization for a long duration.

The setup to the question is a "big if." The usual approach to estimating the number of intelligent species in our galaxy[34] is via the *Drake Equation* [120]. The equation is simple, just multiplying the number of stars by a chain of probabilities: that the star has planets at all; that one of the star's planets is "habitable;" that the environment is benign and that life has had enough time to develop; and finally that the life develops into an intelligent form, capable of communicating. Also factored in is the likelihood that any such species would endeavor to reach out, and how long their civilization lasts in that state so that we might overlap[35] in time. For the Milky Way galaxy, 100 billion stars—almost all of which we now know are likely to have planets—gives a huge start so that even if the chance of intelligent life arising is one in a million, we're left with quite a margin.

But maybe our trajectory is pretty typical. By the time an intelligent species arises, the many millions or billions of years of life leading up to that event may ubiquitously lead to deposits of fossil fuels.[36] The first species smart enough to utilize the planet's fossil fuels does so with reckless abandon. Because evolution does not skip steps, we should not expect to find a species *wise* enough to *refrain* from rapid fossil fuel use emerging before a species who is *just* smart enough to use them, but not smart enough *not* to. So the "intelligent" species short-circuits the battery[37] in a blaze of glory that may even involve baby-step excursions into space before either climate change or other resource/planetary limitations removes the fossil fuel source that made it all possible. Lacking wisdom and foresight, solid plans are not in place to handle the withdrawal, which does not go well and leaves the species in a crippled lower-tech state. Rebuilding from the ashes is then much less likely to explode without that one-time elixir that made it all possible the first time.[38] A simple, and possibly quite satisfying life may await, but it may not involve traveling or communicating across space for others to learn of our existence.

This notion is, of course, highly speculative and of little practical value. Except it may allow us to think "bigger" than ourselves and ponder whether it is surprising that we might fail to achieve a Star Trek future. It is at least *somewhat* relevant to note that the universe we peer into does *not* have an evident intergalactic intelligent presence. Perhaps the

32: ...in the form of communications or alien visits

33: Normally, this kind of tangent might go in a box, but it is big enough to be awkward in that format, thus a whole section.

34: ...or the whole universe, if thinking more broadly

35: The notion of "overlap" is complicated here by the 100,000 light-year scale of the galaxy, so that by the time a signal arrives at Earth, the civilization may be long gone already.

36: ...buried remnants of life

37: Fossil fuels can be thought of as a solar-charged battery that we are discharging almost a million times faster than the time it took to charge it: see Box 10.2 (p. 169).

38: Also, easily accessed surface metal deposits have long disappeared, bringing into question whether a Bronze Age would be possible to replicate.

combination of evolution and fossil fuels makes our path—including a possible decline to follow—a very natural and expected outcome, leaving the universe a quiet place.

For extended thought on this subject, see Sec. D.6 (p. 408).

18.5 Upshot on Humanity

We face unprecedented pressures on resources and on our environment, as human population *and* standard of living both surge on a finite planet. Nature will not allow this trend to continue indefinitely. While much of this book has pointed out limitations of one form of energy or the other, we should be clear that physics does not preclude a satisfying route to the future that operates within planetary limits. It would appear that the real barrier is human limitations in accepting physical constraints. Failure (as a whole) to:

1. process abstract information;
2. anticipate situations that have never yet arisen;
3. make individual sacrifices[39] for the greater good, even if not mandated;
4. recognize that awaiting a clear-and-present crisis may leave us unable to mount a timely response;
5. acknowledge that our loneliness in the universe might constitute evidence that intelligent species don't routinely "make it."

39: For how many generations in a row are humans capable of leaving tempting global resources "on the shelf," while being perfectly capable of exploiting them?

Of course, *individuals* in a society may not share all of these shortcomings. But if these failings are *collectively* prevalent, the more cognizant individuals have too little sway.

Can we collectively overcome these limitations? Can we use the gift of intelligence[40] to bypass built-in limitations? Only by being well aware of the barriers do we have any meaningful chance of managing them. This chapter aimed to at least raise awareness so that readers can begin to think about the role that human nature plays in the challenges ahead.

40: Are we smart enough to recognize and mitigate our shortcomings?

18.6 Problems

1. If each of the four axes in the Myers–Briggs type happened to be equally[41] likely—that is, 50% chance of being I and 50% chance of E, etc., and uncorrelated, then how probable would you expect each of the 16 types to be, in percent?

41: ⚠ they are not, in reality

Hint: the total should add to 100%.

2. Approximate probabilities for the four different axes in the Myers–Briggs type appear as follows:[42]

42: You can recreate this table by adding numbers in Figure 18.1.

Axis	% Share	% Share
I/E	I: 51	E: 49
S/N	S: 73	N: 27
F/T	F: 60	T: 40
J/P	J: 54	P: 46

If we imagine that the four axes do not correlate with each other,[43] we could approximate the fraction of the population that is ESFP as $0.49 \cdot 0.73 \cdot 0.60 \cdot 0.46 = 0.103$, or approximately 10%.[44] What would you expect the most abundant personality type to be, and what percentage of the population would be this type? What is the rarest, and what percentage would you expect in this type? How do the results compare to the probabilities in Figure 18.1?

43: ⚠ in practice, they do

44: Actual frequency is 8.5%, which is close but not exact.

Hint: Largest factors yield the largest product.

3. Reproduce the numbers in Figure 18.3 from the percentages in Figure 18.1. The visual width of each of the four columns in Figure 18.1 also speaks to this.

4. In reference to box sizes in Figure 18.2, answer the following questions:

   a) Which three types are vastly over-represented[45] in the survey?
   b) Which three types turn out to be almost perfectly represented?
   c) Which column is most poorly represented (what common letters in four associated types)?
   d) Which two types are most vastly under-represented, and by roughly what factor,[46] based on area of the squares?
   e) What relationship do you notice between the types in comparing the two most over-represented types and the two most under-represented types?
   f) How would you explain to a fellow student who disparages the Myers–Briggs scheme as little better than astrology what this figure/result means in terms of far-from-random outcomes and predictiveness?

45: ...i.e., far more respondents than is consistent with the general population frequency of Figure 18.1

46: Form a rough guess on ratio of areas.

5. Imagining for a moment that the distribution of human personality types is adaptive[47] in an evolutionary sense, we might try to understand the asymmetry between "S–types" and "N–types." When trying to plot a course for the future, what advantages does each type bring, and what disadvantages? Another way to frame the question is: how can the contributions of each help things go right, and how might each one contribute to wrong decisions?

47: ...emerging to best serve collective survival interests

6. How might you propose getting people of all personality types on board with a collective campaign to fight a credible future threat that may involve sacrifice and the recognition that our own habits are part of the problem?

7. Let's say you have decided to reduce your footprint[48] in some way. You run into someone who challenges your choice, pointing

48: E.g., allow a larger indoor temperature range as it gets colder/warmer outside; eating less meat; conserving water via shower habits

out that your effort is so small in the grand scheme of things that it cannot make a credible difference, and others will just use more—offsetting your sacrifice. Do you believe that is correct? If not, what argument would you offer in support of your decision?

8. Let's say that the U.S. were willing to divert a one-time investment of 10 qBtu out of its 100 qBtu annual energy budget toward building a new energy infrastructure having a 10:1 EROEI and a 40 year lifetime. How many qBtu will the new resource produce in its lifetime, and now much per year? How many years before the amount of energy put in is returned by the output?

Hint: you never need to convert qBtu: it's just some energy unit.

9. If some country or the entire world committed to a one-decade program to replace fossil fuels with solar photovoltaics at an EROEI of 6:1 (Table 14.1; p. 232) based on a 36 year panel lifetime expectation, what fraction of that region's[49] energy would have to be poured into this effort?

49: . . . country or world, depending on the chosen confines of the problem

10. Imagine that we hit energy decline as a result of less energy available each year in traditional fossil forms, experiencing 5% less energy each year[50] A new renewable energy infrastructure effort will require up-front energy, reducing the available energy even further.[51] Imagine yourself as a politician wanting to get elected *after* such a program has been started, and you think you can get elected by pledging to kill the program. What is your pitch to the voters to get elected?

50: . . . a steep decline

51: . . . maybe by another 10% for a 40-year plan at EROEI of 10:1 and a 40-year lifetime resource

11. The Milky Way has about 100 billion stars (10^{11}). If 50% of stars have planetary systems, 10% of those have a rocky planet in the habitable zone, 10% of those are in benign[52] environments, 0.1% manage to produce life of some form, and 0.1% of those result in intelligent life, how many instances of intelligent life might we expect to emerge in our galaxy?

52: . . . stable star without flares and cosmic rays

Note that we have no solid understanding of the probability that life emerges or that having done so intelligent life will form, so these numbers could be optimistic or pessimistic.

12. What would it take, in your view, to overcome the collective human failings summarized in Section 18.5? How do we crack this predicament?

19 A Plan Might Be Welcome

Having discussed some of the human factors related to accepting and mitigating challenges, we now turn to the question of what humanity's goal might be if we could collectively start rowing in the same direction.

First, we will have to assess the form that our actions have taken in the absence of a coherent plan. Next, we address challenges inherent in devising and adopting a roadmap for our future. Finally, a possible target is presented that bears consideration as we grapple with possible modes of human society going forward.

19.1 No Master Plan

The "adults" of this world have not established a global plan for peace and prosperity. This has perhaps worked okay so far: a plan hasn't been necessary. But as the world changes from an "empty" state in which humans were a small part of the planet with little influence to a new "full" regime[1] where human impacts are many and global in scale, perhaps the "no plan" approach is the wrong framework going forward.[2]

Most decisions are made based on whether money can be made or saved in the short or intermediate term. The market then becomes the primary arbiter of what transpires, constrained only by a light touch of legal regulations and public sentiment. Earth and its ecosystems have little voice[3] in our artificially-constructed societal framework—at least in the short term.

Perhaps we are structuring our world *exactly* backwards. An attempt to put a monetary value on the earth and its intricate biological web—a web that by construction[4] is exactly the foundation humans rely upon for survival—produces absurdly large numbers in the sextillions of dollars

1: 96% of mammal mass on Earth is now humans and their livestock [121].

2: In rare cases, small islands like Tikopia operated under plans to live within finite bounds. Now, Earth is effectively a small island and needs to shift to a "small-island" plan.

3: Consider whether a tree or a polar bear can sue a lumber or oil company.

4: . . . that humans evolved into

Uh. Shouldn't I have more pieces? Photo credit: Tom Murphy

(Box 19.1). In this context, the $100 trillion global annual economy is such a minuscule fraction of the value of the earth. Yet reflect on the question: which valuation drives almost all of our decisions?

Box 19.1: Earth's Dollar Value

How much would it cost to purchase a barren planet and then to layer atop it a complete, functioning ecosystem?[5] Starting with the basics, the cost of rough rock, sand, or dirt in the U.S. bottoms out at about $5 per cubic yard.[6] It is the very definition of "dirt cheap." We'll upgrade the volume to a cubic meter[7] for ease. The earth is roughly 10^{21} cubic meters in volume, so even if given a smoking deal on the materials at $1 per cubic meter, the price tag is in the sextillion dollar regime (10^{21}). The high central density of the earth makes the price tag even higher under the more sensible cost per ton,[8] considering the 6 sextillion ton mass of the earth. This is an admittedly naïve way to price a planet, but it puts a scale on things.

A similarly simple calculation applies to minerals. Ignoring the material in the molten mantle, using crustal abundances and only the stuff in the 30 km crust under dry land,[9] the continental crust contains $0.6 quintillion ($0.6 \times 10^{18}$) of silver, $3 quintillion in gold, $5 quintillion in copper, and $20 quintillion in nickel.[10] Aluminum leaps up to $2 sextillion, but probably reflects the energy-intensive extraction process.

By these estimates, the earth is already worth something in excess of 10^{21}, and that's before adding biology, whose billions of years of tuning under evolution is not something we even have the skill to replicate, let alone affix a price tag. Perhaps an evolved biology is more valuable than the raw materials. Given all the barren planets in the universe, an argument can be made that a biologically diverse planet would fetch a premium price. Comparing this to the global 10^{14} budget, the economy registers at less than a millionth the worth of the planet, yet all our decisions are made based on what is good for the tiny flea, ignoring the essential and much larger canine host.

5: . . . actually many connected ecosystems

6: . . . not including delivery

7: . . . would cost 30% more, but we'll ignore this small adjustment

8: . . . rather than by volume

9: . . . just 0.4% of Earth's volume

10: For instance, gold is about $60,000 per kilogram, and according to crustal abundances in Table 15.9 (p. 258), gold is 4 parts per billion of the crust by mass. The crust in question has a volume of 4.4×10^{18} m^3 and a mass around 1.3×10^{22} kg. The expected mass of gold is then about 5×10^{13} kg, and would cost 3×10^{18}. Notice that the total values are amazingly close for these metals: rarer is more expensive in inverse proportion and thus in rough balance.

As human actions on this planet close the door on one species after another, it is important to realize that we are losing an investment of millions upon millions of years of evolutionary fine-tuning that led to this splendid place we call home. The human race has set about to negligently, unwittingly destroy its home, showing essentially no regard for its worth.

Box 19.2: Clueless Cat Analogy

In analogy, domestic cats cannot possibly comprehend why they should *not* be allowed to claw the sofa. To their minds, the sofa is

---

> there, has always been there, feels "right" and good on their claws, and surely can serve no other purpose than to satisfy their urges. How could they possibly understand what it would cost to replace, or why we even care about the appearance to begin with? It would be an even better analogy if the cats' very survival depended[11] on maintaining an unspoiled sofa, in ways the cats could never grasp.

11: . . . for instance, if their food source depended on a pristine sofa for some reason

Maybe humans as a species are as clueless[12] as the cats in Box 19.2 about their present actions. In some sense, this possibility provides a compelling reason to stop. If we can't understand the consequences of our actions, maybe that signals a tremendous risk and we should cease until we have a better grasp: do no more harm until we know what we are about. Unfortunately, there's no money in that idea.

12: Why would evolution have resulted in a being smart enough to fully grasp this exceedingly complex reality? Maybe humans are smart enough to ruin things, but not smart enough to refrain from the ruining. Sec. D.6 (p. 408) explores this further.

19.1.1 The Growth Imperative

Lacking a master plan, the current situation can be described as operating on "autopilot," guided—rather cleverly and impressively—by market forces. In a world far from environmental limits, this model effectively maximizes growth, development, innovation, and prosperity.[13]

13: . . . although resources are not typically well-distributed, leading to inequality

Much as it is in the case of fossil fuels, it is hard to fault growth for all the good it has brought to this world. Yet, as with fossil fuels, nature will not allow us to carry the model indefinitely into the future, as was emphasized earlier in this book. Growth must be viewed as a *temporary phase*,[14] emphasizing the need to identify a path beyond the growth phase. Before discussing how this might manifest, the list below illustrates the dominance of growth in our current society.

14: Recall that Chapter 2 made the argument that economic growth *can't* continue indefinitely.

1. If a politician or activist calls your phone during an election cycle, ask if their platform supports growth.[15] Of course it does. It is hard to find mainstream politicians opposed to growth, and this is fundamentally a reflection of attitudes among the populace.

15: . . . in terms of the economy, jobs, community, housing, you name it

2. Communities make plans predicated on growth. Most seek ways to promote growth: more people, more jobs, more housing, more stores, more everything.

3. Financial markets certainly want growth. Recessions are the scariest prospect for banks and investors. What would interest rates or investment even *mean* without growth? What role would banks play?

4. Social safety net systems[16] are predicated on growth both in the workforce (as population grows) and the economy (so that interest accumulates). In this way, post-retirement pay can be greater than the cumulative contributions that an individual pays during their career. A retiree today is benefiting not only by accumulated interest on their past contributions to the fund, but on a greater workforce today paying into the fund. If growth falters in either or both (workforce/interest), the institution is at risk. It is essentially a

16: . . . pensions, retirement investment, social security, government health care

>   slow-moving pyramid scheme that cannot be sustained long-term,
>   given limits to growth. It was a neat idea for the growth period,
>   but its time will come to an end.
> 5. Various figures from this text (Fig. 1.2; p. 7, Fig. 3.2; p. 31, Fig. 7.7;
>   p. 109, Fig. 8.2; p. 118, Fig. 9.1; p. 139) show a relentless growth
>   in people and resources—often looking like exponential growth.
>   Growth has been a central feature of our regime for many genera-
>   tions.

In other words, human society is deeply entrenched in a growth-based
model for the world. This does not augur well when the finite planet
dictates the impossibility of indefinite growth.

19.2 No Prospect for a Plan

Not only do we lack a plan for how to live within planetary limits, we
may not even have the *capacity* to arrive at a consensus long-term plan.
Even within a country, it can be hard to converge on a plan for alternative
energy, a different economic model, a conservation plan for natural
resources, and possibly even different political structures. These can
represent extremely big changes. Political polarization leaves little room
for united political action. The powerful and wealthy have little interest
in substantial structural changes that may imperil their current status.
And given peoples' reluctance to embrace austerity and take personal
responsibility for their actions, it is hard to understand why a politician
in a democracy would feel much political pressure[17] to make long-term
decisions that may result in short-term hardship—real or perceived.

17: . . . discussed in Sec. 18.2 (p. 309)

Globally, the prospects may be even worse: competition between coun-
tries stymies collective decision-making. The leaders of a country are
charged with optimizing the prosperity of their own country—not that
of the whole world, and even less Earth's ecosystems. If a number of
countries *did* act in the global interest, perhaps by voluntarily reduc-
ing their fossil fuel purchases in an effort to reduce global fossil fuel
use, it stands to reason that other countries may take advantage of the
resulting price drops[18] to acquire more fossil fuels than they would
have otherwise—defeating the original purpose. Then the participating
countries will feel that they self-penalized for no good reason. Unless
all relevant nations are on board and execute a plan, it will be hard to
succeed at global initiatives. The great human experiment has never
before faced this daunting a set of global, inter-related problems (see Box
19.3 for an underwhelming counterpoint). The lack of a global authority
to whom countries must answer may make global challenges almost
impossible to mitigate. Right now, it is a free-for-all, sort-of like ~200
kids lacking any adult supervision.

18: . . . from lowered demand

Box 19.3: What About Ozone?

Scientists discovered an alarming decrease in stratospheric ozone (O_3) in the latter part of the twentieth century—particularly acute aver the Antarctic, earning the title "ozone hole." A global agreement in 1989 called the Montreal Protocol banned the use of chlorofluorocarbons.[19] Substitutes largely—but not entirely—mitigated the ozone problem. Ozone depletion has improved by 20% since 2005 [122]. While the problem is not yet gone, or solved, it is encouraging that global policy can at least reverse and possibly fix a problem.

On the scale of things, this was an easy problem to solve. Climate change and fossil fuel dependence are *much* harder, making the ozone comparison a false equivalency. Getting energy out of fossil fuels *demands* the release of CO_2. We can't "just" switch[20] to some other liquid fuel that doesn't have this problem, as this book makes clear.

Problems are not all the same size. Switching to alternate refrigerants was painful, but not so much that countries and industries could not absorb the cost. Asking to abandon primary energy sources is a *much* bigger ask. Witness the fact that the *rate* of CO_2 emissions grows every year, despite global awareness of the problem.

19: ...often found in refrigerant fluids and aerosol cans

[122]: Nunez (2019), "Climate 101: Ozone Depletion"

20: An analogy is if your doctor told you to avoid monosodium-glutimate (MSG) in your food, you'd be able to find substitutes and still do fine. If your doctor asked you to avoid carbohydrates, protein, and fat—sort-of like the three fossil fuels that are the staple of our diet—we'd be down to what, exactly? Progress in eliminating MSG says little about prospects for addressing the much larger problem.

Many residents of the U.S. also remember great concern over acid rain in the 1970s and 1980s, along with other environmental damages that seem to have been fixed. Part of this is real, and part is illusory. The real part is that coal-fired power plants did adopt technology to scrub sulfur and other trace pollutants out of the emissions stream. This is relatively easy compared to dealing with CO_2, which is not a tiny fraction of emissions, but practically *all* of it.[21] The illusory part of reduced acid rain impact on the U.S. environment has to do with *moving* much of the manufacturing capacity overseas. What happened to environmental quality in Asia as a result? Local solutions are not global ones.

21: It's one thing to rinse off (scrub) cans before putting them in the trash. It's another thing entirely to eliminate the production of trash (CO_2) altogether.

Whether trying to bring about change on a national or global level, the associated political decisions are especially fraught if any form of sacrifice is involved. Examples may be reduced travel, less "comfortable" thermostat settings, taxes or other cost structures making energy and resources more expensive, or more responsible diets. Imposing any such hardships may be politically untenable. Yet, if constrained to operate under a condition of no sacrifice in solving our problems, the only viable paths forward may be closed off, thus setting the stage for failure. In an attempt to have everything, we risk winding up with nothing.

Box 19.4: Paris Agreement and Kyoto Protocol

The United Nations is the closest thing to a global government, but in practice only has as much authority as member nations wish it to

have. Occasionally, the U.N. sponsors international pacts to set limits on CO_2 emissions in a quest to limit the harm of climate change. The Kyoto Protocol of 1997 and Paris Agreement of 2015 are the most notable of these.

Despite the best of intentions, the agreements have not yet shown effectiveness, in that CO_2 rises faster than ever (Fig. 9.1; p. 139). Countries fail to meet their target reductions, and suffer no penalties: what authority would enforce the targets, and how? Until people are willing to voluntarily use less and/or pay more for energy, these scientifically solid and well-meaning international agreements will lose out to political pressures for cheap comfort at a national level.

Author's note: while this box downplays the impact of international agreements, I still would rather have them than not, in that they *do* have *some* impact on emissions and serve as a very public symbolic statement of concern. I'm just not sure they are nearly enough, lacking enforcement and focusing only on climate change: one evident symptom of a much deeper disease about planetary limits and ecosystems. These agreements do not address fundamentals of growth and resource exploitation, and so are band-aids at best. Sometimes band-aids are the appropriate choice for minor wounds, but don't expect them to cure potentially fatal diseases.

19.2.1 Who Makes the Plan?

So how might a viable plan emerge? Who might produce one? Corporations cannot be expected to lay out a responsible plan for our future. Their interest is in company health and profits.[22] In fact, corporations have a fiduciary obligation to their shareholders to maximize profit. Demonstrable failure to do so is technically illegal, and could result in damaging lawsuits.

22: ... usually confined to short-term: quarterly, annual

Governments are in a better position, presumably interested in the long-term health and viability of the country. Many governments, however, are constrained by election cycles that in the U.S. are every 2, 4, or 6 years. Decades-long planning is not natural in such high-turnover[23] systems. Authoritarian governments may be in a better position to effect long-term planning, and even have the ability to impose sacrifice for longer-term goals. Yet, here again the goals are not aimed at achieving global peace and prosperity, but rather securing that particular country's fortunes and survival.

23: ... or even the *threat* of high-turnover

Similar limitations apply to the military sector. Military bodies *do* have the luxury to form long-term strategic plans, and can recognize real threats[24] to the global order. The only problem is that their charge is to win the day: in the event of a global resource competition, they vie to end up in control, generally through use of force or strategic superiority.

24: The author has been visited by U.S. military strategists concerned about the repercussions of diminishing petroleum supply.

The job of formulating a plan may be best suited to the academic world, as academics have the freedom to pursue research in any direction of their choosing, can spend entire careers focused on the effort, and can afford to think over longer timescales than their own lifetimes. An academic agenda *can* be global in scope, rather than fighting for the interests of a single corporation or country. As long as an academic is able to demonstrate impact—typically via substantive and original contributions to published literature—they may uphold their end of the tenure pact.[25] Ideally, academics of all stripes would gather to formulate a viable future plan for how human civilization might carry forward in a way that respects realities from the realms of physical sciences, engineering,

25: Contrary to a common misconception, it is rare for tenured professors to rest on their laurels. Tenured professors by-and-large are intensely driven to push the boundaries of knowledge (which is what earns tenure), and are unlikely to change character upon being granted tenure. In fact, tenure should be looked at not as a reward for past accomplishments, but an investment in a future that (based on past accomplishment) promises to continue bearing fruit.

economics, political science, sociology, psychology and cognitive science, history, anthropology, industrial studies, communications, and really all other academic fields. Every field of study has a stake in the fate of human civilization and has meaningful understandings and insights to contribute.

Of course, any such plan that might emerge will be attacked from all sides, derided as alarmist academic fantasy. It is exceedingly difficult to imagine that the entrenched world will just decide to get started rebuilding the world according to "the plan." But the hope is that if conditions eventually deteriorate to the point that continuation of business as usual is clearly not viable, enough people may remember the plan and dust it off to see what insights it might contain.[26] In such a scenario, hopefully it is not too late to salvage a satisfying future.

26: In this sense, we might view such a master plan as a "break glass in case of emergency" safeguard.

A vital group has been left out of the discussion thus far: people. The vast majority of people are not on corporate boards, in positions of government or military power, or in academic roles. Any effective adaptation to a different future plan will need people to be on board, which means educating them on the choices ahead and the consequences of our actions. Broad support will likely be crucial in redesigning our world to gracefully adapt to the realities of planetary limits.

19.3 Economic Regimes

A very nice metaphor presented in the 2003 documentary *The Corporation* is that early attempts at mechanized flight were doomed to fail because the contraptions were not built on aerodynamic principles of sustainable flight. All the same, the would-be pilots launched off cliff edges and momentarily felt the thrill of flying: the wind was in their hair. Meanwhile, the ground was rushing up. Likewise, our economy and society are not built on principles of sustainable steady-state operation. Even though it feels like quite the amazing rush,[27] it is not hard to see evidence that the ground rushing up. Our only chance is to develop a steady-state economic model—one that is based on principles of long-term sustainability in *partnership* with Earth's ecosystems.

27: The wind is in our collective hair; this is fun!

Paying heed to true sustainability is challenging. Firstly, it is difficult to define what it means. Much depends on the lifestyle imagined. The earth can support fewer people if resource consumption per capita is high, for instance.

A story illustrates the challenge. An economist named Herman Daly worked at the World Bank in a division focused on the interaction between the economy and the environment. When his division was asked to issue a report on this interaction, an early draft had a standard depiction of the economy showing firms and households (Figure 19.1). Firms supplied goods and services to households, while households
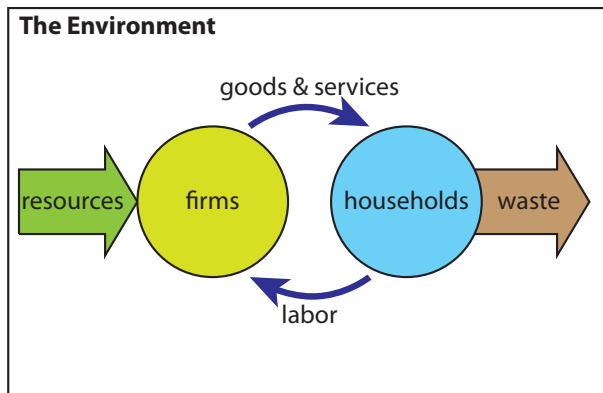
Figure 19.1: Standard concept of the economy, but importantly surrounded by a finite box labeled "The Environment." Most instances of this figure avoid the insinuation that the economy is contained in a finite domain, in part because it raises the uncomfortable question of how close we are to filling up the box.

provided labor for the firms. Resources fed the firms and waste was emitted from the households (and firms). Dr. Daly said: "great, now draw a box around this and label it: The Environment. The obvious point is that all economic activity takes place *inside* the environment. The next draft came back sporting a box drawn around the figure, but no label. Dr. Daly's response: "It *looks* nice, but unless the box is labeled The Environment, it's only a decorative frame." The next draft eliminated the figure altogether.

Once a box is drawn around the economy, many uncomfortable questions arise: how big should the box be (Figure 19.2)? Are we running out of room? What happens when the box fills up? Economists and governments are not prepared to answer such questions.

A nascent field called Ecological Economics [123], of which Herman Daly is a pioneer, has emerged from deep concerns about interactions between human activity and natural systems. Unlike the more established branch called Environmental Economics, which preserves the basic foundations of neo-classical economics and attempts to put prices[28] on environmental factors, ecological economics abandons the growth paradigm and tries to establish rules for maintaining an indefinite relationship with our planet's resources and natural services.

Herman Daly described the different philosophies by analogy to loading a boat. Macro-economics concerns itself with the overall distribution of products within the boat. It is unwise to load all the cargo into the front or back, or all on one side: much better to uniformly load the boat so that it stays level. In analogy, an economy wants to strike a balance across the wide variety of goods and services offered, so that it is not riddled by giant surpluses in one area and deficits in another. Micro-economics deals with the details of how to efficiently manufacture and sell the contents of each box: materials; supply chains; labor; marketing; distribution. But traditional economics has no concept for *how much* cargo the boat should hold—much like we have not established the size of the "box" within which the economy operates (as in Figure 19.2).
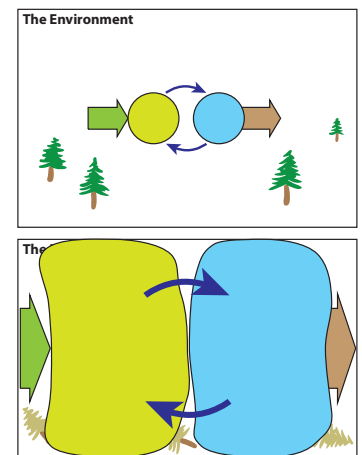
Figure 19.2: Does our economy have ample room, or is it straining the limits of the environment?

28: Economics lingo would call this "internalizing externalities." Typically, the price additions are too minor to be disruptive or fully capture the cost to nature, which is very hard to assess objectively.

In effect, our "boat" has no "waterline" painted on its side to indicate when it is fully loaded. The word "macro" in macro-economics makes it sound like the "big picture" view, but it's really just intermediate. By analogy, we might say that micro-economics is about understanding all the complex workings within a house or building. Macro-economics concerns itself with the distribution of various building types and functions within a city. Missing is a branch evaluating how many cities can fit on Earth and be supplied by the environment. Ecological Economics attempts to address this shortcoming, which is not important in an "empty" earth but becomes crucial as the human scale begins to dominate the planet.

19.3.1 Steady State Economy

Chapter 2 demonstrated that economic growth cannot go on forever. Continuing to operate as if growth can—and should—persist risks irrevocable damage to that from which all value ultimately depends and derives: a healthy natural environment. The sooner we can jump ship to a new economic model that can survive the long haul, the better.

A few key principles will help flesh out aspects of how a steady state economy might work. A critical goal is to reduce the flow (or demand) of resources into the economy, and reduce the waste (pollution, CO_2, for instance) back into the environment. This would be akin to diminishing the sizes of the two thick arrows in Figure 19.1. One approach would be to levy substantial taxes on every tree that is cut, mineral that is mined, drop of oil that is extracted, or wild animal that is unnaturally removed from the environment. Likewise, a heavy tax would accompany disposal of waste and emissions of pollutants. Meanwhile, labor would no longer be taxed. Labor can *add* value to resources already in hand. The idea is to tax the damaging things, not the beneficial things.

Think about what happens under these conditions. Buying a newly manufactured item becomes expensive. Throwing away an old device becomes expensive. Repair (labor) becomes cheaper. Say goodbye to the disposable economy, or "planned obsolescence." Durable goods and lifetime warranties become popular. Items are designed to facilitate upgrade or repair. For instance, once you own a large display at high resolution, good contrast, and good color representation, it should satisfy for a lifetime.[29] Human visual acuity is static, and modern displays are effectively perfect, relative to our biological hardware. If a small electronic component fails, the environmental cost of manufacturing a whole new unit and disposing of the old one is enormous—especially compared to the environmental cost of replacing the small failed component. At present, repair cost often exceeds the cost of replacement and disposal.[30] Under the new arrangement, we begin to place greater value in craftsmanship, community resources, and high-quality goods.[31]

29: Functional upgrades could potentially be modularized to small inserts.

30: . . . disposal is essentially free now

31: . . . less plastic junk!

Consider now the effect on our consumer treadmill.[32] Let's say you find the perfect toothbrush. When it is worn out, you try to get the same one again. But the company has changed its style, so the one you like is no longer available. Why does this happen? The company has a standing army of designers and marketers that must continually "improve" the product to stay competitive in the market. If we all bought less stuff, or more durable items that lasted far longer, the demand for manufacturing would wane. Markets and politicians shudder to contemplate this, as the result would be recession and loss of jobs.

But a widespread cessation of constant disposal and replacement of low-quality goods would mean that not as much income would be *required* to satisfy basic needs and to enjoy a quality life. Maybe all those jobs are not really *necessary*. Maybe a lot of what presently occupies society is a bunch of wasted effort in service of growth[33] and not serving ourselves or the planet well in the process. What if it only takes 10 hours of work per week to live comfortably, having reduced the flow and expense of low-quality stuff once planned obsolescence is—rather poetically—rendered obsolete? Perhaps we could spend more of our time enjoying life, community, family, friends, the natural world, while still retaining scientific literacy and basic technology standards.

It seems that humanity got stuck in a frenetic lifestyle because money and an unrealistic vision of our future trajectory told us to do so.[34] Maybe we need to rethink what we want life to be about, and not simply accept that productivity and profit are the drivers that matter. Are we the boss of money, or is money the boss of us?

Careful thought [124, 125] has been put into how to modify the present financial system toward a steady state. The process has been compared to converting an airplane, which must keep moving forward in order to stay up—as the current economy must grow to survive—into a helicopter that can remain stationary. And this transformation would ideally happen mid-flight. It's a difficult prospect. Moreover, none of the necessary steps would ever spontaneously happen without the population first embracing the ultimate goal of a steady-state economy. Therefore, a collective push to abandon our current economic model must initiate the process, and it is unclear how this ground swell might materialize.

It is possible that a steady-state economic framework—for all its merits and careful thought—is ultimately naïve and infeasible. Individuals may be naturally driven to work very hard to build an empire and improve their own lot. It is not obvious that human nature is suited to a steady state existence: competition and acquisition of power may be built in.[35] Imposing rules to prevent outsized accumulation of wealth or power may seem oppressive and would be hard to sustain, unless societal values uniformly shunned excessive wealth, power, and consumption. But for how many generations could such a state of affairs be maintained? It seems like an unstable scenario.

32: It's like a treadmill in that we never stop consuming and disposing, even if it's somewhat pointless. See the Story of Stuff video.

33: Is growth our master?

34: Ask yourself: to what end?

[124]: Daly (2008), "A Steady-State Economy"
[125]: Chang (2010), *Moving Toward a Steady State Economy*

35: ...as a product of evolutionary pressures

19.4 Upshot on the Plan

Humanity has historically not needed a master plan. Plenty of space, resources, and natural services allowed unwitting expansion. Yes, wars occasionally broke out over contested resources, but generally in localized regions. This state of affairs will continue to be true until it isn't any more. That is to say, just because something has never happened before does not mean it cannot. Earth has never hosted 8 billion high-demand humans, yet here we are. Human imagination is not the ultimate limit in this physical world. At this juncture, it would be prudent to heed the warning signs and attempt to make a plan for survival/prosperity.

Currently, it is hard to imagine any global consensus arising around a plan. Even if able to maintain the current level of global resource demand,[36] those who use resources at a much higher rate than average[37] may have to scale back as the world equilibrates, and this will not easily be agreed upon. Academic circles may be the only place from which a credible plan might emerge,[38] but any such plan would likely be ridiculed and discarded as impractical.

36: It is not at all clear that we could.

37: Ahem, America.

38: Who else would pay for it?

The silver lining is that some folks *have* thought about alternative ways to structure the economy allowing abandonment of growth and living within ecological limits. Some of the elements of this plan are very appealing. Only if embraced on a large scale would it be feasible to migrate in this direction, and it is unclear what circumstances might bring about such an attitude change, if it is possible at all.

19.5 Problems

1. To visualize the scale of the $100 trillion global economy relative to the value of the earth—conservatively one million times larger— let's think in terms of animal volume. Volume scales like the cube of linear dimension. How much would you have to shrink a dog in linear scale for its volume to be one-millionth its original size? If the typical scale of a dog is 0.5 m, how large is the shrunken version, and what animal is about this size?

2. Subjectively, how much more do you think a planet teeming with biodiversity is worth compared to a comparable planet harboring no life at all? Express as a factor: 1.2 times as much, 2 times as much, 10 times as much, 100, 1 million etc.

The value likely goes up if your own biology is adapted to that same life-filled planet: it becomes special to you as almost a part of you.

3. In analogy to the clueless cats in Box 19.2 and the paragraph following, describe how we might plausibly destroy something valuable on our planet without understanding it fully enough to repair the damage.

4. What institutions can you think of that are prevalent now but will be rendered obsolete—or at least radically diminished—if the economy stopped growing permanently?

5. Why do you think we have not yet formulated a master plan for how humans can live on the earth indefinitely without exceeding limits?

6. Do you see a route to global acceptance of a plan? What would it take to get there? Would we first need a global government having authority over all nations?

7. What is your major in college, and what insights or contributions do you imagine may be offered by this field of study in formulating a workable plan for the future of human life on this planet?

8. Presently, the American tendency is to buy a missing tool for a job that may not be needed again for a very long time, if at all. It is likely that some nearby neighbor already has the same tool collecting dust. What do you imagine would be advantages and disadvantages of pooling resources in a lending arrangement?

9. Why do you think the field of traditional economics does not recognize limits to growth, and does not have a macro-macro branch looking at the whole planet and its finite nature?

10. A central question in mapping a comfortable future has to do with how large our economy is with respect to physical boundaries.[39] One solution would be to set aside some fraction of the planet (land and ocean) off limits to human extraction of any plants, animals, or other resources. What fraction seems tolerable to you? If it turns out that half needs to be protected to guarantee long-term survival, do you think that's possible/practical? How many generations do you think could maintain the discipline to preserve that rich world in a pristine state?

39: It is even possible we have exceeded those steady-state bounds and are spending down an irreplaceable inheritance right now.

11. What do you find appealing about a steady-state economic model? What do you find worrisome?

12. Section 19.3.1 contains four instances in which the economy is compared to analogous systems. What are these four, and which do you find most impressionable/memorable?

13. Do you think that human nature—the desire to improve one's lot and expand empire—is compatible with a steady state economic model? Can you see a way that it might work?

14. Are you left thinking that we are likely to establish and implement a viable global plan for how humans might live prosperously on the planet indefinitely, or do you think it is more likely we will fail to do so and "wing it" into whatever fate awaits?

20 Adaptation Strategies

We made it to the final chapter. How are we doing? Anxious? Excited? Alarmed? Inspired? This book has compressed a perspective that took the author many years to absorb. Exposure in one short sprint would likely be overwhelming, and might even generate an impulse to reject the message as both unfamiliar and grim—thus hopefully wrong?

Up to this point, the theme of the book might be characterized as one of closing lots of exits. Growth can't continue indefinitely—requiring a whole different economic model. Space is not a realistic escape hatch. Fossil fuels are what made this life possible, but will not last and are causing disruptive climate change. The alternative options all have their own practical limitations, and offer no drop-in replacements for fossil fuels. At least sunshine offers a ray of light as a super-abundant energy flow. But when it comes to making collectively smart decisions about a future path, more obstacles surface on the human side. Success requires long-term planning and not the more common crisis response.

This chapter changes gears a bit to touch on individual actions and values that could amount to big things in aggregate. At the very least, it may provide individual-scale escape hatches allowing some peace of mind about personal contributions to the problem.

20.1 Awareness

How many people do you know who are concerned about a legitimate threat of collapse of our civilization? It is an extreme outcome, and one without modern precedent. It seems like a fringe, alarmist position that is uncomfortable to even talk about in respectable company. Yet the evidence on the ground points to many real concerns:

Growing your own food is a great way to lower your impact and be closer to nature. Photo credit: Irina Fischer.

1. The earth has never had to accommodate 8 billion people at this level of resource demand;
2. Humankind has never run out of a resource as vital as fossil fuels;
3. Humans have never until now altered the atmosphere to the point of changing the planet's thermal equilibrium;
4. We have never before witnessed species extinction at this rate, or seen such dramatic changes to wild spaces and to the ocean.

Just because something big has not happened yet does not constitute strong evidence that it cannot or will not. But more important than that argument—which is always true—is the number of credible causes for concern that are evident today.

Also important to recognize is that a challenge cannot be effectively mitigated unless it is first identified and acknowledged. The very lack of collective awareness about a credible risk of collapse is *itself* unsettling. If open discussion of the possibility of collapse were not so uncomfortable and off-putting, we would stand a better chance of preventing its unfolding. It would be a huge relief to be wrong about the concern. But not taking it seriously represents a colossal risk.

Box 20.1: The Y2K Scare

The Y2K[1] scare in 1999 offers a good template for how to mitigate a potential disaster. Computer systems became the dominant means for managing financial and government records, transactions, and accounts in the 1960s through 1980s. A two-digit code was used for the year in many records, not anticipating the roll-over to 2000 decades away. The early programmers either doubted that their code would still be functioning in 2000[2] or assumed it would be cleaned up in time. In the year or two leading up to Y2K, the issue got tons of coverage and predictions of mayhem, as peoples' digital lives—a new phenomenon—were tossed into any number of unknown upheavals. But the very fact that the issue dominated public consciousness was exactly what ensured a smooth transition. Every bank and agency got on the job and Y2K came and went without a ripple. It would be great to see a repeat in the case of potential collapse. The lack of a specific time prediction is one barrier, unlike Y2K. Without a firm deadline or a clear-and-present danger, the temptation to delay serious attention is strong.

1: Y2K is short for year-2000.

2: Surprise—it was!

A key contributor to awareness is in how information sources and activities shape opinions and views. A world overflowing with information can be difficult to navigate, and has a tendency to coagulate into isolated domains that cater to predispositions. The result can be disagreement on basic facts, making coordinated progress difficult. Luckily, attentive individuals can perform an assessment of the trustworthiness of various information outlets. The process is to watch an entire live event, like a

debate or a hearing, and see how the event is covered by various news sources. Does the coverage[3] reflect the event as you experienced it? Did it focus on the key developments or on distractions that might be emotionally "triggering?"

Entertainment is another source/activity that can influence mindsets in subtle ways. For example, the grossly simplified and virtual world of video games promotes a false sense of what is possible—rather than helping model responses to real-world challenges, constrained by many layers of practical considerations. Reaching level 42 without suffering too many damage points is a fairly empty accomplishment[4] that just means having followed some game designer's artificial and arbitrary rule set pretty well, combined with some skill in pressing buttons. More impressive is building or creating something, repairing something, or having some beneficial impact in the external world that would not otherwise have happened.

Likewise for movies and shows, which can provide healthy joy and social bonding. But because the industry is not constrained to follow rules of nature, it is easy to form dangerous misconceptions about what humans are capable of doing.[5] As a result, not only does the likelihood of disappointment increase, but the necessary sort of deliberate and unglamorous work that must be initiated well before crisis becomes apparent is less likely to materialize if the populace is trained to hold out for unrealistic and spectacularly successful outcomes.[6]

20.2 Communication

In a democracy, collective public awareness drives the issues politicians serve. Only by having voters demand action will progress follow. Conversations with friends and family then become necessary to raise awareness among others. Effective communication that accomplishes this goal without turning people off is tricky. When the message contains bad news or suggests sacrifice, the effort can easily backfire.

It is important not to polarize the conversation by "bossing" people or projecting a sense of authority. An effective strategy is to fairly represent uncertainty, while still conveying credible concern. Caveats like "it seems that," or "it appears to me that," or "I may be wrong, but" go a long way to taking the edge off of the message and inviting the listener into a constructive conversation. It is possible to couch the language in uncertainty while still hitting the main points. Words like "possible," "likely," "plausible," and "risk" can be useful to soften the tone but still express concern.

Division in the U.S. is frighteningly high right now, so that distrust is a real barrier to sharing a common factual basis. The communication needs to be "we," not "you." For instance, "we really should be concerned

about X" rather than "I think you should X." It is best to try to convey a sense that we are all in this together. Expressions like "I worry that," or "Do you also feel that. . ." bring a human touch and invite a sense of inclusion and collaboration.

One potentially interesting approach is to appeal to the fundamentally conservative nature of most people. This is conservative with a small "c," rather than the Conservative (right-leaning) political party. In this sense, conservative means:

1. low risk: let's not gamble the future on speculative notions;
2. conservation[7] of resources and quality of the earth environment;
3. laying the groundwork for future generations (e.g., grandchildren) to have a livable world.

7: It's right there in the name!

By these traditional definitions of conservative, many in the Conservative wing are more fairly labeled as free market radicals—willing to risk future stability and damage our environment in exchange for short term financial gain. This approach is not inherently conservative.[8] Political identities in the world may, in fact, be ripe for a massive realignment wherein many traditionally conservative values pair more naturally with the political "left" (liberal wing) than with the "right." In recent decades, the left has been more concerned about environmental issues and ecological damage. It would make sense that the most conservative— or low risk—proposals for future paths would emerge from the political left.■

8: Ask yourself: what conservation efforts have Conservatives championed lately?

An apt analogy is that our society is, metaphorically, barreling toward a cliff. Faced with credible warnings, the low-risk (conservative) approach would be to alter course: get serious about a non-fossil infrastructure and transition away from growth. At the very least, let off the gas pedal: reduce resource use while we learn more. Keeping the foot down on the pedal and seeking to accelerate as fast as possible is probably the least wise[9] decision, yet best characterizes the current approach.

9: This decision would be okay if we *knew* that the cliff is not there, but we can't know that, and plenty of evidence suggests reason for concern.

Unfortunately, a common tendency of people on the receiving end of the kind of message this book advances is to get frustrated when the story is *not* tied up neatly into a happy ending. Perhaps our story-telling culture has irreversibly conditioned people to expect resolution at the end of every movie or show. Nature and the world are under no such obligations to satisfy our psychological need for closure, so it is unfair to blame the messenger for accurately conveying the perils and uncertainty of our times. Perhaps people seek a hopeful conclusion so that they can walk away unencumbered—satisfied that everything is under control and that somebody will figure something out. But this tendency is perhaps counter-productive, in that only by internalizing and burdening oneself with the daunting nature of our challenge will it be possible to mount a collective and effective response. If human nature is such that unpalatable stories don't gain traction, it is another way to say that we are not built to overcome a global dilemma of this scale. So push back

on any criticism demanding that you need to supply a happy picture at the end of the story. Challenge the listener to deal with the tension, as this book has attempted to do. Tough love.

20.2.1 Predicament, not Problems

It may also be advisable to avoid characterizing the set of interconnected global challenges as "problems," because the word *problem* implies a *solution*. It implicitly isolates the issue at hand into a stand-alone simple issue, promoting "what if we just..." proposed solutions. The real story is far more intricate, and more like a game of whack-a-mole.[10] A simple "fix" to one corner of the problem makes something worse elsewhere. A better word is predicament, intoning a more serious and possibly intractable situation. Perhaps a predicament can be viewed as an *interconnected set of thorny challenges* rather than a collection of isolated problems.

10: ... a game where heads pop up out of holes to be hammered down, only to see other heads pop up elsewhere

Predicaments don't have solutions, but responses. Piecemeal fixes are unlikely to "solve" the current predicament in a way that permits moving on and relegating the problems to the past.[11] But we *can* imagine re-crafting our world, *responding* to the challenges by adapting our mode of living to be compatible with planetary limits. Problems can be faced head-on and be defeated, whereas predicaments call for stepping around and finding a different path.

11: ... as was the case for curing polio, for instance

20.3 Personal Adaptation and Guidelines

Previous chapters have discussed the challenges democratic governments have in imposing any form of reduction that feels like sacrifice to the population. Since political power in democracies starts with individuals, we focus here on what individuals can do to reduce their demand on energy resources. If enough individuals are *not* willing to make such adjustments, it is doubtful that the U.S. government, for instance, would exert authority over this sensitive domain of personal freedom.

This section addresses ways to take personal control of energy expenditures.[12] The presupposition is that the reader is *interested* in ways to reduce their impact, or footprint, on the planet and its resources. Collective progress on this front would reduce the current 18 TW demand on energy, making it that much easier for fossil fuel replacements to satisfy a more modest demand. A voluntary course of reduction on a broad enough scale would reduce vulnerability to forced reduction that would ultimately accompany declining fossil resource availability or climate-motivated reduction targets. It is also good preparation for potential scarcity.

12: Energy usage correlates strongly with resource usage, in general.

We will first look at attitudes and framing, then some overarching guidelines, followed by specific quantitative assessment of energy expenditures. Readers can identify for themselves areas for potential change in their own habits and expectations.

20.3.1 Overall Framing

In the absence of a major shift in public attitudes toward energy and resource usage, motivated individuals can control their own footprints via personal decisions. This can be a fraught landscape, as some people may try to out-woke each other and others will resist any notion of giving up freedoms or comforts—only exacerbated by a sense of righteous alienation from the "do-gooders."

Some basic guidelines on effective adaptation:

1. Choose actions based on some analysis of impact: don't bother with superficial stuff, even if it's trendy.
2. Don't simply follow a list of actions or impart a list on others: choose a more personalized adventure[13] based on quantitative assessment.

   13: . . . resulting in a mindful pursuit and not an impersonal set of imposed chores

3. Avoid showing off. It is almost better to treat personal actions as secrets. Others may simply notice those choices and ask about them, rather than you bringing them up.[14]

   14: A joke illustrates the usual pitfall: "How will you know if a new acquaintance is vegan? Oh, don't worry, they'll tell you within 10 minutes."

4. Resist the impulse to ask: "what should I *buy* to signal that I'm environmentally responsible?" Consumerism and conspicuous consumption are a large part of the problem. Buying new stuff is perhaps counterproductive and may not be the best path.
5. Be flexible. Allow deviations. Rigid adherence makes life more difficult and might inconvenience others, which can be an unwelcome imposition. Such behavior makes your choices less palatable to others, and therefore less likely to be adopted or replicated.
6. Somewhat related to the last point, chill out a bit. Every corner of your life does not have to be perfect. We live in a deeply imperfect world, so that exercising a 30% footprint compared to average is pretty darned good, and not *that* much different than a "more perfect" 25%. Doing a few big things means more than doing a lot of little things that may drive you (and others) crazy.
7. In the end, it has to matter to *you* what you're doing and why. It's not for the benefit of others.[15]

   15: . . . except, of course, in the broadest collective sense: it's for people you will never meet who are not even alive yet, and for other life on Earth you will never see.

The first two items on the list are not easy: most people are not themselves equipped to quantitatively evaluate the impact of their choices. But some simple guidelines can help.

20.3.2 Energy Assessment Principles

This section contains a number of key insights that can guide actions. Each starts with a simple statement in bold font, followed by elaboration and then an example or two[16] for most.

Heat is costly. Anything whose job it is to create thermal energy (heat) is a power-hog: clothes dryer; home heating; hot water heater; space heater. A small device called the Kill-A-Watt is handy for assessing power draw by plug-in appliances.

> **Example 20.3.1** How much energy does it take to dry a load of clothes using a 5,000 W clothes dryer?
>
> Assuming it takes about an hour to run, this is 5 kWh, or 18 MJ.

> **Example 20.3.2** How much energy does it take to heat all the water in a 40 gal (150 L) tank from 10°C to 50°C?
>
> Recalling Def. 5.5.1 (p. 73) or the definition of the kilocalorie, heating 150 L (150 kg) by $\Delta T = 40°C$ will take 6,000 kcal, which converts to 25 MJ or 7 kWh of energy.

How often is it on? Duty cycle matters a lot: how often it's on. A microwave oven uses a lot of power, but not so much energy, because it is hardly ever running. The Kill-A-Watt mentioned above accumulates kWh and allows determination of the average power of a device.

> **Example 20.3.3** How much energy is a 1,500 W microwave oven at home likely to use in a day, compared to a 25 W television tuner box running 100% of the time?
>
> The microwave might be on for 12 minutes per day, or 0.2 hours. That makes 0.3 kWh[17] for the microwave and 0.6 kWh for the tuner box. Time matters.

Large ΔT is costly. The power it takes to maintain a temperature difference is *proportional to the temperature difference*.[18] For related reasons, a refrigerator in a hot garage has to work especially hard[19] to maintain a large ΔT.

> **Example 20.3.4** How much more daily energy does it take to keep a home at 25°C inside when it is 5°C outside versus keeping it at 15°C inside?
>
> In the first case, ΔT is 20°C, while it's just 10°C in the second case. So it will take twice as much energy to keep the interior at 25°C compared to 15°C.

Use common units. Cross-comparison of energy usage is made more difficult by different units. Table 20.1 provides conversions to kWh as a

standard. In terms of power, many appliances are rated in Btu/hr, which is 0.293 W. So a hot water heater at 30,000 Btu/hr is equivalent to about 10 kW and will consume 5 kWh if running for half-an-hour, for instance. Putting everything in the same units (kWh as a suggestion here) allows useful comparisons of choices.

Table 20.1: Conversions to kWh.

Energy Quantity	kWh
1,000 Btu	0.293
2,000 kcal diet	2.3
1 L gasoline	9.7
1 kg gasoline	13
1 gal. propane	26.8
1 Therm (gas)	29.3
1 gal. gasoline	36.6

> **Example 20.3.5** In a month, the utility bill for a house shows 600 kWh, 20 Therms, and the two cars of the household used a total of 60 gallons of gasoline. How do these stack up, when assessed in the same units?
>
> Using Table 20.1, the gas amounts to 586 kWh—almost identical to electricity—and the gasoline totals about 2,200 kWh, far outweighing the other two.

Electricity source matters. Your local source for electricity[20] can impact choices. It should be possible to determine your local mix via online sources [126]. The fact that conventional power plants tend to convert chemical energy into delivered electricity at 30–40% efficiency needs to be considered in comparing direct use of a fossil fuel against electrical solutions based on fossil fuel. A heat pump design for a water heater can compensate for this loss, and then some.[21]

20: ... coal vs. natural gas vs. hydroelectric, for example

[126]: Nuclear Energy institute (2019), *State Electricity Generation Fuel Shares*

21: ... if the COP is higher than 2.5, for instance, which it usually will be

> **Example 20.3.6** A hot water heater using natural gas is likely about 85% efficient at transferring the heat of combustion into the water (enclosed, insulated), while an electric hot water heater manages to get 100% of the delivered energy into the water via a heating coil immersed in the water. If the source of electricity is also natural gas form a power plant achieving 40% efficiency at converting thermal energy into electricity and then transmitting it to the house at 95% efficiency, which method uses more total fossil fuel energy, and by what factor?
>
> We compare 85% efficient for the direct usage to 40% times 95% times 100%.[22] The ratio of 85% to 38% is 2.2, so it will take 2.2 times more gas at the power plant than in the home to produce the same result in heated water.

22: This last one is for the immersed coil, and does nothing to the answer.

Weight is a guide. A rough rule of thumb is that the energy cost of consumer goods is not too far from the energy contained in the *equivalent weight*[23] *in gasoline*, meaning 13 kWh/kg (Table 20.1). Should you use paper or plastic bags? The one that weighs more probably required greater energy and resource use. Should you drive back home if you forgot your reusable bag? Compare the amount (weight) of gasoline you'll use to the weight of the disposable bags the store uses.[24] High-tech gadgets, like smart phones, almost certainly break this rule and cost far more energy to produce than their gas-equivalent weight—as can be approximated in the next point.

23: ... really we mean mass

24: ... almost certainly *not* worth it to drive back; can you manage without any bags at all and not risk dropping anything?

Example 20.3.7 Should you buy a new, more efficient refrigerator that will use 1.8 kWh per day (75 W average) instead of your current one that uses 2.4 kWh/day (100 W average)?

At a mass around 150 kg, the refrigerator's manufacture might require ~2,000 kWh,[25] taking about 9 years to pay back at the 0.6 kWh/day saving. This is long enough that considerations such as material resources and disposal might tip the scale against replacement.

25: ... 150 kg times 13 kWh/kg

Cost is a guide. A secondary approach to figuring energy content is to suspect that the item's cost is appreciably greater than the cost of the energy that went in. Perhaps a reasonable number is that 15% of the total cost goes toward energy.[26] Conveniently, a typical retail price of electricity of $0.15/kWh then translates to 1 kWh for each $1 of consumer spending. When results from the two approaches (by mass or by price) differ, the higher energy cost number may be the safer bet.

26: This is not a capricious estimate, as it is approximately representative of energy costs in our society as a whole—stacked a little higher here to better reflect manufacturing activities, which are bound to be more energy-intensive than the economy as a whole. Also note that energy intensity, as seen in Fig. 2.2 (p. 19), is characteristically around 5 MJ/$, which is 1.4 kWh/$ and not far from our rule of thumb here.

Example 20.3.8 What do the two methods say about a 1,500 kg car that costs $25,000 and a smart phone that costs $1,000 and has 200 g of mass?

The car estimates are 1,500 kg times 13 kWh/kg for about 20,000 kWh or $25,000 times 1 kWh/$ for 25,000 kWh. In this case, they're pretty close and it hardly matters which one we favor.

For the phone, the mass estimate is just 2.6 kWh, but by price it would be 1,000 kWh. In this case, for reasons argued above, the larger one is more likely on target.[27]

27: We would not go so far as to say that either method is "right." They should be viewed as very approximate guidelines that at least can help differentiate big deals from insignificant things.

Focus on the big. Keep your eye on the big impacts. We are not actually under threat of running out of landfill space, for instance. So while recycling is a preferred approach,[28] very visible in society, and should be practiced when possible, the impact is not dramatic: it still takes a lot of energy to process recycled goods. Metal recycling (especially aluminum) is most effective from energy and resource standpoints, and paper from a resource standpoint (trees), but plastic is less clear on both energy and resource bases. Reducing its use may be best.

28: Better yet is to try getting by without purchasing items that require later disposal.

Example 20.3.9 How effective is it to buy a water bottle for my daily needs?

Compare the weight and cost of the water bottle to the weight and cost of all the plastic cups it displaces[29] as a reasonable guide to the relative impact.

The best of all worlds is not buying something for the purpose, but finding something you *already have* that will get the job done.

29: Consider the duration of ownership or of usage and how many disposable cups are avoided.

30: A side benefit to these actions is saving money, maybe working less hard and retiring earlier.

Reduction rules. Reduction is by far the action with the biggest impact. Buy less stuff. Live more simply. Travel less often and less far.[30] Adapt

yourself better to the climate.[31] Eat more responsibly. The next section digs into related actions in more quantitative detail.

20.3.3 Quantitative Footprint

A useful exercise is to compare your own energy footprint to national averages. How much more or less are you using? For some categories, information is hard to assess. For instance, how much oil is used to transport the goods you buy and the food you eat? How much energy is used in the industrial and commercial sectors on your behalf?[32] In part, your level of consumerism is a good clue, but it still may be hard to compare to others. The following items offer some guidance. The first two entries can be derived from Fig. 7.2 (p. 105), after unit conversions and dividing by the U.S. population.

Electricity: A typical American uses 12 kWh of electricity per day in their residence. To get your own share, look at an electricity bill for your residence and divide by the number of people living in the place and by the number of days[33] in the billing period.

> **Example 20.3.10** In 2019, the author's utility bills[34] indicate total use was 3,152 kWh for a household of two. What is the daily average per person and how does it compare to the national average?
>
> 3,152 kWh divided by 365 days and 2 people is 4.3 kWh per person per day, about one-third of the national average.

Natural Gas: A typical American uses about 13 kWh of natural gas per day in their residence, amounting to 0.44 Therms per day.[35] To get your own share, look at a gas bill for your residence, if applicable, and divide by the number of people living in the place and by the number of days in the billing period.

> **Example 20.3.11** In 2019, the author's utility bills[36] indicate total use was 61 Therms for a household of two. What is the daily average per person and how does it compare to the national average?
>
> 61 Therms divided by 365 days and 2 people is 0.084 Therms (2.4 kWh) per person per day, about 20% of the national average.

Gasoline: A typical American buys about 400 gallons of gasoline[37] per year for personal transportation, amounting to a daily equivalent of 41 kWh[38] of energy use. Keep track of your fuel purchases[39] and compare how much you use. In the case of multiple occupancy in the car, your share can be computed by dividing how many gallons were used in the trip by the number of people. Knowing an approximate fuel economy[40] for the car and distance traveled is enough to estimate fuel usage.

31: It is okay to put on more clothes and sit under blankets in a cooler winter house.

32: Wouldn't it be great if consumer goods had labels revealing embedded energy and resulting CO_2?

33: ...usually a month: about 30 days

34: See the banner image on page 68 for a one-month sample.

35: ...typical billing unit; one Therm is 29.3 kWh; see Table 20.1

36: See the banner image on page 68 for a one-month sample.

37: Personal transportation accounts for about 65% of gasoline in the transportation sector.

38: ...36.6 kWh per gallon, or 9.7 kWh/L

39: This practice is good for tracking fuel economy as well.

40: ...e.g., miles per gallon or L/100 km

Example 20.3.12 The author's household has two vehicles,[41] one of which drove 400 miles and used 22 gallons of gasoline in 2019, and the other covered 8,660 miles using 69 gallons. What is the daily average use per person in the household, and how does this compare to the national average?

A total of 91 gallons for two people is about 45 gallons per person, equivalent to 4.5 kWh/day, and 11% of the national average.

41: ... a non-commuting truck and a commuting plug-in hybrid that mostly uses electric drive, charged at home (the electrical demand for which is represented in Example 20.3.10)

Air travel: Expressing an average in this case is inappropriate, as many Americans do not fly at all, while all use some combination of electricity, gas, and gasoline in some capacity. The average works out to 2,300 miles (3,700 km) per year when averaging all people, but among those for whom air travel is a utilized, the number is generally a good bit higher. To put it in context and enable useful comparisons, we will compare it to ground transportation.

Typical passenger jets get approximately 90 miles per gallon (m.p.g.) *per seat*[42] (2.6 L/100 km) for a fully-occupied plane—worse if seats are empty: down to 45 m.p.g. per passenger if half full, for instance. So traveling 1,000 km in a full airplane uses the same amount of fossil fuel energy per person as driving the same 1,000 km in an efficient doubly-occupied car that gets 45 m.p.g. (5.2 L/100 km). For an 80% full airplane,[43] the effective per-passenger mileage is about 70 m.p.g., coming to an energy cost of about **0.5 kWh per mile (0.32 kWh/km) per passenger**. Because air travel tends to involve *long* trips, the energy used (thus CO_2 emissions) for air travel can easily exceed that for personal car usage, as is seen in the next example.

42: The airplane as a whole gets less than *one* mile per gallon, but each passenger's share of gallons used makes it better on a *per-passenger* basis. It takes almost the same amount of energy to fly a plane from point A to point B independent of passenger load. Most of the energy is used to fight air resistance, which is related to the size and speed of the airplane, essentially independent of the number of passengers inside.

43: ... guessing this to be typical

Example 20.3.13 The author, in 2019, flew about 4,200 miles for personal travel and 9,600 miles work-related. How many kWh per day does this translate to in the two categories, and how does it compare to expenditures in electricity, gas, and personal gasoline?

For personal air travel, 4,200 miles times 0.5 kWh per mile is 2,100 kWh or 5.8 kWh/day, which is slightly larger than the 4.3, 2.4, and 4.5 kWh/-day from electricity, natural gas, and personal gasoline computed in previous examples, but still really in the same ballpark. Business travel[44] accounts for 13 kWh/day, *by itself* exceeding the sum of household expenditures.

44: Ugh. Wish I didn't have to.

Example 20.3.14 If three people are traveling from San Diego to San Francisco at a distance of 700 km, how good does the car's gas mileage need to be to beat an 80% full airplane that would get 90 miles per gallon per passenger if full?

Being 80% full knocks the effective fuel economy down to 72 m.p.g. per passenger. For the three people in question, a car achieving 24 m.p.g. (9.8 L/100 km) will match the airplane's energy expenditure, so

Note that we didn't need the distance. This may seem like a "trick," but consider that life is even trickier: real-world problems have no (or maybe *all available*) information provided, and it's up to us to sort out what's relevant.

anything getting better performance will deliver the three people at a lower energy cost.

Diet Impacts: Modern agricultural practices result in a 10:1 energy expenditure on the production, distribution, and waste of food—so that each kilocalorie of food eaten requires 10 kcal of energy input [97]. A typical 2,100 kcal/day diet translates into 2.4 kWh/day, and applying the 10:1 ratio means that about 24 kWh of energy input is required to cover a typical American's diet—which is substantial on the scale of residential/personal energy use. Because food is also grown for livestock and poultry, then those animals convert the food to meat at some low efficiency, raising animals for meat is a net energy drain: directly eating the grown food ourselves[45] would use less energy and fewer resources.

45: ... preferably in not exactly the same form!

20.3.4 Dietary Energy

This last point on food energy deserves some elaboration, setting the stage for a quantitative evaluation of diet choices. For any food type, it is possible to characterize the amount of energy spent producing the food as a ratio to the metabolic energy contained in the food.[46] Key results of some such studies ([127] and [128]) are provided in Table 20.2. Treat these as *rough* guides rather than absolutely definitive numbers, since specific agricultural, feeding, or fishing practices play a huge role in the energy requirements: large variations can be expected, in practice. All the same, fruits and vegetables consistently require small energy expenditures relative to meat and dairy products.

46: In this sense, it is the inverse of EROEI: energy *invested* to extract the food divided by energy *delivered*.

Category	Type	Ratio	Distrib.	Category	Type	Ratio
Red Meat	Lamb	83	1.8%	Plant-based	Tomatoes	1.67
	Pork	27	62.6%		Apples	0.91
	Beef	16	35.6%		Potatoes	0.83
Poultry	Chicken	5.5			Peanuts	0.71
Fish	Shrimp	110			Dry Beans	0.65
	Salmon	18			Rice	0.48
	Tuna	17			Wheat	0.45
	Herring	0.9			Corn	0.40
Dairy/Egg	Eggs	8.9	11%		Soy	0.24
	Milk	4.9	89%		Oats	0.20

Table 20.2: The ratio of energy invested in producing various common foods to the metabolic energy delivered by the food (sort-of an inverse EROEI), broken into five categories. High ratios indicate large energy costs. When known, the distribution *within* the category is given for standard American diets. Beef is grain-fed, salmon is farmed, and milk is a stand-in for dairy products more generally. Data synthesized from [127, 128].

Let's be clear about what Table 20.2 says. The production of 100 kcal of rice requires an input of 48 kcal, making it a net energy gain. Meanwhile, 100 kcal from beef takes 1,600 kcal of energy to produce, as an energy loser. Lamb and shrimp are very costly, while herring is a steal. It may seem surprising that eggs require more energy input than chicken,[47] but consider that it takes longer for a chicken to produce its weight in eggs than for a chicken to get large enough to be processed for meat.

47: Owning egg-laying chickens and feeding them scraps is a delightful win, however.

Armed with this information, it is possible to assess a dietary energy factor[48] for various dietary choices.

48: "Dietary energy factor" is a term used in this textbook; not likely to be found elsewhere.

Definition 20.3.1 *The **dietary energy factor** is a weighted sum of individual energy ratios for food categories:*

$$\text{d.e.f.} = f_v \cdot R_v + f_{rm} \cdot R_{rm} + f_f \cdot R_f + f_p \cdot R_p + f_d \cdot R_d, \qquad (20.1)$$

where f_x factors are the fraction of one's diet in form "x," in energy terms (calories; kcal), and R_x values are the aggregated relative energy ratios for food category "x," as found in Table 20.3. Subscripts indicate vegetables, red meat, fish, poultry, and dairy/eggs, respectively. Note that care must be exercised to insure that all five f_x factors add to one.

Category	Energy Ratio	Relative Ratio, R_x	American Diet, f_x	Lacto/Ovo Diet, f_x	Vegan Diet, f_x	Poultry Diet, f_x
Plants	0.65	1	0.72	0.80	1.0	0.72
Red Meat	24	37	0.09			
Fish	36	55	0.01			
Poultry	5.5	8.5	0.05			0.15
Dairy/Egg	5.3	8	0.13	0.20		0.13
d.e.f.			6.1	2.4	1.0	3.0

Table 20.3: Dietary energy factor computations for various diets. Energy factors are aggregations over categories from Table 20.2, assuming equal distributions when unknown (e.g., each fish type is 25% and each plant type is 10% of that category's intake). The net effect, at bottom, is a weighted sum of the individual energy ratios, and spans large factors in terms of energy impact.

In Table 20.3, the first column of numbers is a weighted average of factors from Table 20.2, using the distribution weights listed where available, and assuming equal spread otherwise. The next column scales the energy ratios so that the vegetable category has $R_v = 1$,[49] making the dietary energy factor a measure of energy requirements *relative to* a strictly plant-based diet. For instance, red meat requires 37 times as much energy as vegetable matter, for the same metabolic energy content.

49: The second column of numbers is the first column divided by 0.65.

What follows in the table are four diet types, reflecting the average American diet and three variants, each having its own set of f_x factors.[50]

50: Note: contrived to add to 1 in each case.

Example 20.3.15 Let's replicate the American diet result in Table 20.3 using Eq. 20.1.

Using $f_v = 0.72$, $f_{rm} = 0.09$, $f_f = 0.01$, $f_p = 0.05$, and $f_d = 0.13$, then $R_v = 1$, $R_{rm} = 37$, $R_f = 55$, $R_p = 8.5$, and $R_d = 8$, the dietary energy factor computes to $0.72 + 3.33 + 0.55 + 0.425 + 1.04 = 6.07$, confirming the final row. By breaking things out this way, the red meat category stands out as contributing more[51] than any other category.

51: Red meat is 3.33, which is 55% of the total energy cost while providing only 9% of the dietary benefit.

Compared to a strictly plant-based (vegan) diet, the typical American diet requires about six times the energy. Since the average American diet accounts for 24 kWh per day, a vegan diet is therefore down to 4 kWh/day. A vegetarian diet partaking of dairy and eggs (lacto-ovo diet) is 2.4 times[52] the vegan diet, or a little less than 40% of the American diet (about 9 kWh/day). Just replacing all meat consumption with chicken (final column) cuts energy demand in half. These are just a few of the countless examples that may be explored using Eq. 20.1 or variants thereof to evaluate the energy impact of dietary choices.

52: The actual number depends on the fraction of calories coming from dairy/eggs (f_d), and can be dialed at will: it's not stuck at exactly 2.4.

Get on it! Evaluate your own diet and how you might modify it.

Example 20.3.16 What is the dietary energy factor for a diet in which one-third of caloric intake is from red meat, 10% is from dairy/eggs, and the rest is plant matter?

Setting $f_{rm} = 0.33$ and $f_d = 0.10$, we require that $f_v = 0.57$ in order that all three sum to 1.0. Now using $R_{rm} = 37$, $R_d = 8$, and $R_v = 1$, the dietary energy factor computes to $12.2 + 0.8 + 0.57 = 13.6$ for red meat, dairy, and vegetable matter, respectively. This diet requires more than twice the production energy as a standard American diet.

It is possible to abandon Eq. 20.1 and roll your own formulation following similar principles. Rather than adopt the distributions from Table 20.2, the technique can be customized to any diet for which energy factors can be found.

Example 20.3.17 A diet that is 35% rice, 35% wheat, 15% corn, 10% milk, and 5% chicken has an energy cost of $0.35 \cdot 0.48 + 0.35 \cdot 0.45 + 0.15 \cdot 0.40 + 0.10 \cdot 4.9 + 0.05 \cdot 5.5 = 0.17 + 0.16 + 0.06 + 0.49 + 0.28 = 1.15$. This has not been normalized to $R_v = 1$ yet,[53] so we divide by the aggregate 0.65 value for the plant energy ratio found in Table 20.3 to get a dietary energy factor 1.8 times that of a strictly plant-based diet. Note from the sum that milk and chicken are the largest two contributors, despite being a small fraction of the diet.

53: In other words, if performing the same sort of calculation for 10% contributions from each of the ten plant-based foods in Table 20.2, the raw result would be 0.65.

The 10:1 input:output energy ratio mentioned at the beginning of this diet segment may at first glance not square with the whole-diet energy factors computed here (e.g., a factor of 6 for the typical American diet). Missing is **food waste**. The U.S. produces 1.8 kcal of food value for every 1 kcal consumed [127]. This amount of waste may be hard to fathom, but consider waste at restaurants, cafeterias, and grocery stores when perishable items are not consumed before health standards suggest or require disposal. Still, this is an area ripe for improvement.

[127]: Eshel et al. (2006), "Diet, Energy, and Global Warming"

20.3.5 Flexitarianism

Echoing Point #5 in the list in Section 20.3.1, it is worth pointing out that energy and resource concerns are a largely *quantitative* game. One need not become a strict vegan to affect energy demands substantively. For instance, eating meat one meal a week,[54] and tending to stick to poultry when doing so would drop the energy factor of Eq. 20.1 to a value so near to 1.0 that the difference is of little consequence.

54: ... out of about 40 meals

Example 20.3.18 For instance, if one meal per week, or about one in 40 of your meals looks like the last column in Table 20.3—72% plant-based and the rest poultry and dairy—what is the dietary energy factor for this diet?

Since only one in 40 meals is of this type, multiply the poultry and

dairy contributions by $\frac{1}{40}$ and adjust f_v to bring the total to 1.0. Doing so yields $f_v = 0.993$, $f_p = 0.00375$, and $f_d = 0.00325$. Multiplying by the respective R_x values and summing produces 1.05.

Thus, the one meal of poultry/dairy per week achieves 99% of the journey from normal-American (6.1) to full vegan (1.0), from an energy perspective.

The result of Example 20.3.18 is so nearly 1.0 that it is essentially indistinguishable from a purely plant-based diet, quantitatively. This is especially true in the context that the rule-of-thumb factors are themselves not to be taken literally as high-precision numbers. All pork will not have an energy ratio of 27.0. All tuna will not be 17.0. All wheat will not be 0.45. The methods of producing the food—of all types—become important at this stage. Note that gardening (and canning) one's own food is a way to nourish ourselves at a super-low resource burden—undercutting the nominal vegan energy factor even further.

The quantitative focus suggests an approach best called flexitarianism. If energy and resources are the primary concern, rather than ethical issues around eating meat,[55] then the occasional meat treat is no big deal. Under this scheme, it is still possible to enjoy traditional foods on special occasions like holidays.[56] If a friend serves meat at a dinner party, just do the quick calculation and realize that you can easily offset later[57] and make this special-occasion meal disappear into the quantitative noise. The perception you generate is therefore more likely to be as a grateful friend, rather than as a person whose needs are difficult to satisfy.

55: ... valid in its own domain

56: ... arguably making them *more* special

57: ... or note that you have already offset it by prior actions

More people are likely to be attracted to join in responsible behaviors if they are not too rigid or strict. Imagine ordering a bean, rice, and cheese burrito only to take a bite and discover a morsel of meat inside. Score! Meat Treat! It doesn't have to be a bad thing, if resource cost is what matters most. This flexibility can also apply to waste food. Before watching meat get thrown into the trash, intercept with your mouth. From a resource point of view, *wasting* meat—or any food, really—is also something we should strive to avoid: better that the energy investment produce metabolic benefit than be utterly wasted.

20.3.6 Discretionary Summary

We don't have direct and immediate control over all the energy expenditures made on our behalf in the same way that we have control over our own light switches and thermostats. Yet, we must accept our communal share of energy and resources used by governmental, military, industrial, agricultural and commercial sectors providing us with structure, protection, goods, and services. The 10,000 W average American power frequently used as a benchmark throughout this book—and mapping to 240 kWh per day—is not all in our direct control. Individuals can make

political, consumer, and dietary choices that exercise limited control over these distant activities, but effects are small and gradual.

Sector	American (kWh)	Author (kWh)
Electricity	12	4.3
Natural gas	13	2.4
Gasoline	41	4.5
Air travel	3.2	5.8
Diet	24	9
Total	93	26

Table 20.4: American average and author's 2019 expenditures energy on a daily average basis expressed in kilowatt-hours.

Of the things that *are* under our discretion, as discussed in the sections above, Table 20.4 summarizes the average American values and those of the author in 2019.[58] Recall that the average American air travel corresponds to just 2,300 miles (3,700 km) per year. If adding consumerism to the personally-controlled energy toll, perhaps an average American spends $10–20,000 per year[59] on "stuff," which would amount to another 25–50 kWh per day if using the rule-of-thumb 1 kWh/$ from Section 20.3.2.

58: . . . only counting personal travel, and a mostly vegetarian (though not vegan) diet.

59: The author might guess $5,000 for himself as an upper limit, or another 13 kWh per day in this mode.

Combining the discretionary factors in Table 20.4 and a consumerism estimate, Americans have direct control over about half of their total energy footprint.[60] As the author demonstrates, it is possible to make drastic cuts to this portion—in this case a factor of three lower than average. Mostly, this comes about by a combination of awareness, caring, and tolerance for a simpler life without every possible comfort.

60: Recall: 240 kWh per day total.

Box 20.2: Out of Our Control

Many energy expenditures are part of a consensus social contract that individuals cannot easily control. Examples would be lighting and interior temperature control policies for large common spaces like office buildings, campuses, libraries, and airports, for instance. Likewise for street lighting in neighborhoods and along highways. Only by large scale shifts in values would the community potentially prioritize energy and resource costs over financial cost or public health and safety.

20.4 Values Shifts

In the end, a bold reformulation of the human approach to living on this planet will only succeed if societal values change from where they are now. Imagine if the following activities were frowned upon—found distasteful and against social norms:

1. keeping a house warm enough in winter to wear shorts inside;

2. keeping a house so cool in summer that people's feet get cold;
3. having 5 cars in an oversized garage;
4. accumulating enough air miles to be in a special "elite" club;
5. taking frequent, long, hot showers;
6. using a clothes dryer during a non-rainy period;
7. having a constant stream of delivery vehicles arrive at the door;
8. a full waste bin each week marking high consumption;
9. having a high-energy-demand diet (frequent meat consumption);
10. upgrading a serviceable appliance, disposing of the old;
11. wasteful lighting.

At present, many of these activities connote success and are part of a culture of "conspicuous consumption." If such things ran counter to the sensibilities of the community, the behaviors would no longer carry social value and would be abandoned. The social norms in some Scandinavian countries praise egalitarianism and find public displays of being "better" or of having more money/stuff to be in poor taste. Abandonment of consumerist norms could possibly work, but only if it stems from a genuine understanding of the negative consequences. If curtailment of resource-heavy activities is imposed by some authority or is otherwise reluctantly adopted, it will not be as likely to transform societal values.

While it may seem objectionable, it is worth recognizing that public shame carries surprising power.[61] A recent experiment in Bolivia put traffic monitors on the streets wearing zebra costumes[62] to combat irresponsible driving habits endangering pedestrians. The zebras would "call out" violators by making a show and pointing to the offenders. That simple action has been effective. Other cultures have required perpetrators of unsavory acts to stand in a public place for a day wearing a sign announcing their transgression, recognizing the power of public shame. It is difficult to imagine similar remedies today, and for many good reasons.[63]

Yet our society has perhaps gotten too far away from personal ownership of actions. Anonymity in our modern world promotes rude behavior: on roads, on the internet, and in heavily-populated urban areas, where often no one within sight is familiar. If environmentally costly activities were to acquire taboo status, it is pretty certain we would see far less of it, for fear of shame.

61: The author became aware of this power in the context of student project demonstrations open to the whole department to watch. The prospect of public failure provided supplemental motivation for students to work super-hard—harder than the author typically observed in other courses.

62: The costumes served to simultaneously maintain anonymity and allude to black-and-white crosswalk patterns.

63: ...e.g., not a very nurturing approach to promote change

20.5 Flexibility as an Answer to Uncertainty

No one has a crystal ball. No one can credibly say what the future holds. Anyone claiming that we're heading for certain complete collapse should not be trusted. But neither should someone who says everything will be glorious. It is not hard to find either sort of message in this world, yet we cannot discern with confidence which is ultimately correct.

While this text may seem more aligned toward a grim outlook, it is somewhat intentional as a means to raise awareness toward what seems like a minority view—without crossing the line and claiming any certainty on the possible perils ahead.

	right about threat	wrong about threat
plan A	utter disaster	see, we were fine
plan B	fighting chance to avert danger	minor inconvenience

Figure 20.1: Asymmetric risk in the face of a potential devastating threat. Plan A is the natural response if the threat is not believed to be real, and Plan B is appropriate for mitigating the threat. The downside of the threat being real but sticking to Plan A is catastrophic, whereas pursuing Plan B unnecessarily is not ideal, but not nearly as bad. We don't get to choose reality (column), but we do get to chose the plan (row). Are we feeling lucky, or conservative?

Another rationale for this book's tone relates to asymmetric risk (Figure 20.1). If we take potentially catastrophic threats seriously and at least formulate plans to mitigate them, then little harm is done if the threat does not materialize: just "wasted" time and effort being careful.[64] But ignoring the threat could mean "game over." Even if the probability of the threat is low, like 10%, it is worthy of attention if the consequences of ignoring it could be devastating. People routinely buy insurance for similar reasons: to mitigate low-probability but potentially debilitating events.

64: . . . and oh darn—we might end up with a renewable energy infrastructure earlier than we really *had* to have it! After all, it's pretty clear that we need to get there *eventually*.

That said, how does a person navigate their own life choices under a cloud of existential uncertainty? One answer is to pick avenues that can be useful in either eventuality. Choosing a route that only makes sense if things hum along as they have done for the past many decades could be risky. So put some thought into directions that are likely to be valued whether or not the human endeavor suffers large setbacks. Be flexible. Mostly, this involves imagining a more difficult future and asking what paths still work in that scenario, while still having a place in today's world.

What skills or functions will likely always be valuable? One approach is to think about what elements of human existence are likely to always be present. We will always need food, shelter, health care, transportation, fabrication capability, resource utilization, wisdom, and entertainment. The exact form ranges from primitive to high-tech. But not every profession supported today has an obvious place on this list. In the face of this, it seems worthwhile to learn the *fundamentals* of any vocation you elect to pursue, so that if deprived of all the technological assistance available today, you can fall back on the basics and still achieve some worthwhile results.

A first step may be to become less reliant on technology for simple tasks. Use brains more and devices less. The practice will lead to greater mental capacity—in any outcome. Do math in your head. Learn and retain[65] important facts and concepts, so that Google is not required to form full thoughts.

65: . . . internalize; own

Try modes of living that are less cushy than normal, even if temporarily. A week-long backpacking trip is a great way to feel like a part of nature, and come to understand that some level of discomfort or hardship is tolerable—or even confidence-building.[66] The first few days may be a difficult adjustment, but it is surprising how well and how quickly adaptation can happen—given time, decent weather, and a constructive attitude. After doing this a few times, minor inconveniences or discomforts that arise might be met with far greater grace. The person who remains stable in the face of adversity will be more resilient than many of their peers, and can help others hold things together in crisis.

66: Ideally, start small on a one- or two-night trip, and accompany someone experienced at backpacking to learn how to avoid rookie mistakes that could turn you off of the activity forever.

Keep in mind that humans evolved outdoors, dealt with seasons, and often went many days without food. When did we become so fragile that we need to live within a narrow temperature range to be comfortable, and lose our heads if we don't get three square meals a day? What would our ancestors think of us? By learning to toughen up, the future—whatever way it goes—will have less power over us. We can strive to be less vulnerable and at the mercy of events beyond our control. We will have some agency to cope and help others cope with any variety of outcomes. Just by having some confidence in our ability to deal with a bit of adversity or discomfort will allow us to keep cooler heads and be able to recognize important opportunities if and when they come along, rather then being paralyzed by distress.

Hopefully, such preparedness will never be truly needed. And how bad would it be if we "built some character" along the way for no reason.

20.6 Upshot on Strategies

No one can know what fate awaits us, or control the timing of whatever unfolds. But individuals *can* take matters into their own hands and adopt practices that are more likely to be compatible with a future defined by reduced resource availability. We can learn to communicate future concerns constructively, without being required to paint an artificial picture of hope.[67] Our actions and choices, even if not showcased, can serve as inspiration for others—or at least can be personally rewarding as an impactful adventure. Quantitative assessment of energy and resource demands empowers individuals to make personal choices carrying large impacts. Reductions of factors of 2 and 3 and 4 are not out of reach. Maybe the world does not *need* 18 TW to be happy. Maybe we don't *have* to work so hard to maintain a peaceful and rewarding lifestyle once growth is not the driver. Maybe we can re-learn how to adapt to the seasons and be fulfilled by a more intimate connection with nature. The value of psychological preparedness should not underestimated. By staring unblinking into the abyss, we are ready to cope with disruption, should it come. And if it never does in our lifetimes, what loss do we

67: The hope lies in how we *react* to the challenges, not necessarily in eliminating, conquering, or denying them.

really suffer if we have chosen our adventure and lived our personal values?

In this sense, the best adaptation comes in the form of a mental shift. Letting go of humanity's self-image as a growth juggernaut, and finding an "off-ramp" to a more rewarding lifestyle in close partnership with nature is the main goal. The guidelines provided in this chapter for quantifying and reducing resource demands then simply become the initial outward expressions of this fresh vision. Ignore the potentially counterproductive allure of fusion, teleportation, and warp drive. Embrace instead a humbler, slower, more feasible future that stresses natural harmony over conquest and celebrates life in all forms—while preserving and advancing the knowledge and understanding of the universe we have worked so hard to achieve. Picture a future citizen of this happier world looking back at the present age as embarrassingly misguided and inexplicably delusional. Earth is a partner, not a possession to be exploited. Figuratively throwing Earth under the bus precludes our own chances for long-term success. A common phrasing[68] of this sentiment is that humans are *a part* of nature, not apart from nature. Let's not lose the path in a flight of fossil-fueled fantasy.

> Continuing the freeway metaphor, the current path has us hurtling forward to certain involuntary termination of growth (a dead end, or cliff, or brick wall), very probably resulting in overshoot and/or crash.

> 68: ... attributed to Marc Bekoff, 2002

20.7 Problems (Predicaments?)

1. What barriers do you sense that suppress open communication about collapse possibilities?

2. Why is the lack of open acknowledgment that collapse is a distinct possibility itself more likely to facilitate exactly that outcome?

3. Try constructing a statement that communicates the grave risk we create for ourselves as we flirt with potential collapse without being too off-putting or unjustifiably certain.

4. Try crafting a diplomatic and persuasive response to someone who says that the problem with your story—concerning possible bad outcomes—is that it's a real downer and lacks a message of hope.

5. Come up with an example in life of a predicament that can't be directly solved, but perhaps side-stepped to get around it without eliminating or "solving" the problem's origin.

6. A person loses their snazzy stainless steel water bottle that they carry everywhere—again. So they go to the store to replace it. What elements of the list in Section 20.3.1 are in danger of being violated?

7. Which of the following devices is likely to consume a lot of power, when it is on/running? Explain your selection.

   a) laptop computer
   b) phone charging
   c) toaster oven
   d) television/display
   e) lighting for a room

8. A microwave oven and a space heater might each draw 1,500 W of electrical power. What determines which one uses more *energy*, and describe a realistic scenario in which one uses a lot more energy than the other.

9. If the temperature outside is steady at 10°C, how much more energy must you expend over some period of time[69] in order to keep the inside at a shorts-and-short-sleeves temperature of 22°C vs. a dress-more-warmly 13°C inside?

        69: … effectively power, then

10. A utility bill for April indicates that your household used 480 kWh of electricity and 20 Therms of natural gas. In addition, your household has two cars, each using an average of one gallon of gasoline per day. Convert everything to kWh per day for direct comparison, and also express in W or kW of average power to put in context against the 10 kW American overall average.[70]

        70: *(i)* Most is outside the home.

11. Following the setup for Problem 10, if the household consists of two people, what is the per-person footprint in terms of kWh per day for electricity, natural gas, and gasoline, and how does this compare to U.S. national averages per person?

12. In looking at the utility bill referenced in Problem 10, and converting Therms to kWh, it may seem that natural gas is a larger energy expenditure. But if your local electricity is primarily sourced from natural gas, converting the combustion energy into electricity at 40% efficiency (via a heat engine), what is the *effective* amount of kWh used in the form of natural gas for the electricity, and now how does this compare to the gas used directly in the household?

13. Use the two rule-of-thumb approaches (by mass and by cost) to estimate the energy investment in a farm tractor, whose mass is 2,400 kg and cost is $25,000. If the results are even within a factor-of-two of each other, we can conclude that the estimate is probably pretty decent as a rough guide.

14. Use the two rule-of-thumb approaches (by mass and by cost) to estimate the energy investment in a laptop computer, whose mass is 1.4 kg and cost is $1,300. If the estimates are strikingly different, which do you suspect is more representative of the truth?

15. You and three friends want to take a trip together and are debating whether to fly or drive in a gasoline-powered car. In the context of fossil fuel energy use (and thus CO_2 emission), how good does the car's fuel efficiency need to be (in miles per gallon) in order for

the driving option to use less fuel (per person) than would a fully-occupied airplane, if the airplane gets 90 m.p.g. per passenger? Is it easy to find cars whose performance is at least this good?

16. What does the dietary energy factor become for a person who gets one-quarter of their energy from meat, but only in the form of chicken—the rest from plant matter? How much of the way[71] from the standard American dietary energy factor (6.1 times vegan) to a purely plant-based diet is this?

71: For example, going from 6.1 to 3.8 is a move of 2.3 of the 5.1 total distance to get to 1.0, or 45% of the way "home."

17. What does the dietary energy factor become for a person who is mostly vegan, but eats like a standard American one day a week—on that day getting 28% of their energy as outlined in the American column of Table 20.3? How much of the way[72] from the standard American dietary energy factor (6.1) to a purely plant-based diet is this?

72: See margin note for Prob. 16.

18. What fraction of caloric intake in a Lacto-ovo diet (dairy/eggs but no meat or fish) would allow a dietary energy factor of 2.0, which is 80% of the way from the American 6.1 to the vegan 1.0?

19. How might you imagine our society managing to change values to shun heavy resource usage: what might transpire to make this happen? Do you think this would be a desirable outcome?

20. List three professions around today that will be very unlikely to exist if we revert to a lower energy/resource, less high-tech state.

21. Describe the circumstances and outcomes of the four boxes in Figure 20.1 in the context of a tornado reported heading for your town—which may or may not hit your house if it even hits your town at all. Plan A would be to do nothing in preparation. Plan B would be to board up the windows on your house and take cover in a tornado shelter. Describe the relative costs and feelings about your decisions in the aftermath for all four scenarios.

22. Comparing the human body to a car with a gas tank, and recognizing that a human can *live*[73] for about two weeks without food, provided adequate water and shelter. How many skipped meals does this represent, on the standard of three meals per day? If the body had a fuel gauge indicating how close to "empty" (death) we are, what percentage of "full" does our gauge typically read when we "pull into the station" for another fill-up (meal) in the standard case when we get three meals per day?

73: . . . not comfortably, and this is *not* recommended!

Note that, unlike a car, the body does not function just as well at 25% "full" as it does when it is 100% full. Thus the analogy is very flawed. Yet, in primitive times, it was surely routine to dip well below hunger levels not often tolerated today.

Epilogue

This book may be distressing for some: a body-slam to hope. The message can be more than some are ready to take in, or of an unacceptable flavor. I myself first approached this subject—when assigned to teach a general-education course on energy and the environment—with great enthusiasm, intending to sort out to my own satisfaction how I thought our gleaming future would migrate to renewable energy.

As I "ran the numbers" on various sources, I came to appreciate the tremendous quantitative advantage that solar power has over the alternatives. Being a hands–on person, I started cobbling together various off-grid photovoltaic systems, learning the practical ins and outs of stand-alone solar power[1] coupled with storage as a crucial means to mitigate intermittency of the solar resource. My wife and I also bought a plug-in hybrid vehicle in 2013 to learn the pros and cons[2] of electric cars, while preserving the ability to do occasional longer trips on gasoline. My commute to work is via an electric-assist bicycle charged by my off-grid solar system for a fossil-free transportation option.[3] I have found adventure and delight in challenging myself to live a lower–energy lifestyle, and know from personal experience that dialing down demand does not have to be a crushing defeat for the human race. Our ambitions[4] might suffer, but our spirits need not.

This book takes an approach that deliberately asks the wrong question, chapter after chapter: how can we keep going in a manner resembling the present form in the face of declining fossil fuel resources and/or a commitment to wean ourselves from fossil fuels as a mitigation strategy for climate change? This approach manifested itself as: can we get 18 TW of power[5] from this or that alternative resource? In most cases, the answer was no. Solar is the glaring exception. Also, nuclear breeders— bringing a tangle of tough problems—and the perpetually intractable nuclear fusion could offer long term provision of electricity. But none of the abundant resources easily replace liquid fuels for transportation, and effective utilization of the abundant yet intermittent solar resource depends critically on storage capabilities.

Thus, pretending that the goal is to keep 18 TW and carry on—business as usual—after a tidy substitution of energy turns out to be misguided. The real question becomes one of adapting to a new landscape: one in which our ambitions are checked by planetary limits. Indeed, if energy became essentially unlimited by some technology, I shudder to think what it would mean for the rest of the planet.[6] An age-old saying goes: *With great power comes great responsibility.* Humans have achieved great power, but have not yet demonstrated a respectable degree of responsibility in prioritizing the protection of plants, animals, and ecosystems.

1: . . . as a miniature model of what society at large may one day hope to do in a solar–dominated energy landscape

2: We have watched battery capacity drift down to about 65% presently.

3: Charging the car from the off-grid system would require a substantial upgrade in system size and battery capacity, costing approximately \$10,000.

4: So far, human ambitions have been for the most part unconstrained by physical limitations. Just as a child must eventually shed fantastical dreams and beliefs, so too might the human race need to reign in unrealistic hopes for our future.

5: . . . or some sizable fraction thereof

6: A children's story called *Don't Let the Pigeon Drive the Bus* echoes the sense that great power in the hands of incompetence can be bad news. We would not put a toddler in command of an arsenal.

Should attention to planetary limits turn out to be a crucial element in the assessment of our situation, then we owe it to ourselves to get it straight. Imagine that you are running across a rooftop and have to make a quick decision about whether to jump a large gap between buildings.[7] Would you appreciate a lightning-quick analysis of physics concluding that a successful jump is impossible? Certainly, such insight would be valuable, permitting the formation of an alternate plan, and saving yourself from the unfortunate fate of misplaced faith in your jumping abilities or in some fanciful notion of gravity's weak grip over the chasm.[8]

Humanity is staring at a leap unlike anything history has prepared us to face, having accelerated ourselves to previously unimaginable speeds by the grace of fossil fuels, but now confronting their inevitable removal from the menu. The past offers little guidance on how to navigate such a situation, so we need to do our level best to soberly assess the challenges and recognize what *is* and what is *not* within the realm of practical expectations. I would love to be wrong about the numerous concerns raised in the book, but the asymmetric risk of trying the leap and failing could lead to a devastation that frightens me. Please, let us not risk it all on unfounded hopes or magical thinking.[9]

The situation reminds me of the housing bubble in the U.S. in the early 2000s. My wife and I bought a house when we moved to San Diego in 2003, and soon became worried about a potential crash[10] leaving us "underwater"—owing more than the reduced worth of the house. I pored over articles on the matter, and found two camps. One camp provided rafts of alarming quantitative analysis of the peril: sub-prime lending, soaring price-to-income ratios, unprecedented unaffordability by average families, vulnerability to any weakness in other sectors. The other camp said that the housing market was manifesting a new normal, that San Diego's universal appeal would prevent a price drop, that scary lending practices were easily skirted by re-financing before interest payments ballooned. I chose to go with the quantitative analysis over the hand-wavy platitude-based set of beliefs, and am glad that I did.

Now consider the quality and nature of common counter-arguments to the core message in this book. Humans are smart, innovative, and will figure out something. People 200 years ago could not have possibly predicted our capabilities today, so we are likewise ill-equipped to predict how amazing the future will be. I get the appeal. I really do. But does that mean we get to dismiss the difficulties exposed by careful analysis? Can we ignore the fact that we are pushing planetary boundaries for the first time ever? I would argue that this time really *is* different.[11] The facts are inescapable:

▶ The world has never before been strained with 8 billion people.[12]
▶ Fossil fuels bear tremendous responsibility for our recent climb.

7: ...as might happen in a movie

8: *The Matrix*, while an excellent movie, encourages physics-breaking thinking. Not to be a kill-joy, but it probably *is* air you're breathing, and that probably *is* a spoon.

9: More often, *lack* of critical thought is to blame: unconscious assumptions about how the future will unfold based on recent, anomalous history.

10: Aha! Am I just a recurrent predictor of crashes? Not at all. Numerous times, I had faith in rising markets and prospects for achieving success in difficult endeavors. Meanwhile, I blew off concerns of Y2K (Box 20.1; p. 329) and even downplayed the COVID-19 pandemic in early 2020—in both cases on the premise that high levels of awareness and fear would trigger massive attention and mitigating actions by responsible governments and stakeholders. In the COVID case, my faith in competence was sadly misplaced: I was too optimistic.

11: The wolf *did* come in the apocryphal tale. The adults should accept some responsibility for their failures, rather than throwing the "boy who cried" under the bus.

12: ...and growing

▶ Fossil fuels are a one-time resource—an inheritance—that will not continue propelling the future, and nature does not guarantee a superior substitute.

▶ Wild spaces on the planet are rapidly diminishing as development spreads and resources are culled. Permanent extinction of species accompanies pollution and habitat loss.

▶ Climate change and habitat destruction threaten a mass extinction and environmental disruption whose full consequences are unpredictable.

▶ Modern human constructs[13] have not stood the test of time, and are unlikely to do so given that they have not been founded on principles of sustainable harmony within planetary limits.

13: ...economic, political, societal, agricultural, provision of energy and material resources, manufacturing, buildings, and virtually everything else

Any convincing counter-argument about why we need *not* take this seemingly perilous position seriously would itself need to be serious—relying less on general faith in human abilities and more on nuts-and-bolts details: *How* are we going to supply energy needs without fossil fuels? It isn't good enough to say "solar and wind," without specifying how we deal with the glaring mismatch between demand and intermittent energy availability. What would we use to provide sufficient storage? Do we have the materials and means to make enough battery capacity? What is our strategy for battery upkeep and replacement? How will we afford the new scheme and its prohibitive up-front costs? What about agriculture: how do we permanently fix soil degradation; aquifer depletion? How do we halt deforestation, habitat loss, and resulting permanent extinctions? What is the specific global governance plan to protect planetary resources and deal with the consequences of climate change? How do we structure economies to be complacent and functional without a foundation in growth?

Those who tend to dismiss collapse perils are unlikely to detail a cogent plan of the sort solicited here, because to do so would require some acknowledgment that the exercise is necessary and valuable. No one yet has produced a comprehensive and widely accepted plan for just how all this is supposed to work out!

As it is, **we have no credible global plan**[14] to deal with these foundational global problems. We owe it to ourselves to do a better job than *imagine* that the future may work out just fine.[15] We need to face the challenges, put pencil to paper, and craft a plan that could work—even if it involves some compromise or sacrifice. Let us not forget that we do not have the authority to conjure any reality we might dream: we have no choice but to *adapt* to the physical world as we find it.

14: ...largely due to lack of consensus that we even *need* a non-growth plan

15: The burden of proof is on those who argue for continued modernity: ample evidence indicates that humans can live in primitive relation to nature for tens of thousands of years, but *no* evidence demonstrates that it can survive in anything like the mode of the last century or two for very long. Even the universe fails to deliver evidence (see Sec. 18.4; p. 312).

Returning to the analogy of receiving quantitative analysis on a contemplated leap across a chasm—having indicated serious shortcomings in the notion of maintaining current luxuries—please think twice about trying to carry our resource-heavy ways into the future in heroic fashion. Ignominious failure, not glory, may lie there. But this does not mean the human endeavor has been all for naught, and that we should just sit down and cry about dashed dreams.[16] Let's be smart about this: heed the warning signs; alter course; re-imagine the future; design a new adventure.

16: If the dreams were never realistic, then good riddance!

Contrary to what the tone of the book might suggest, I am a fundamentally optimistic person, which has fueled a lifetime of pursuing tough

challenges and succeeding at (some of) them.[17] Indeed, my irrational hope is that a textbook like this may help get people thinking proactively about changing the course of humanity. In that spirit of wild-eyed optimism, I leave you with the following upbeat-adjacent thoughts about the world into which we may endeavor to gracefully adapt:

▶ Crisis is opportunity: we have a chance to transform the human relationship with this planet.

▶ Imagine the relief in shedding an old narrative of growth and faulty ambitions that only seems to be creating increasingly intractable problems—instead side-stepping to make a fresh start under a whole new conception of humanity's future. It's liberating!

▶ People alive today get to witness and shape what may turn out to be the most pivotal moment in human history, as we confront the realities of planetary limits.

▶ Committed pursuit of steady-state principles could set up rewarding lives for countless generations.

▶ Nature is truly amazing, and making it a *larger* part of our world[18] could be very rewarding.

▶ We, as individuals, are privileged to witness and celebrate the much grander phenomenon of life in this universe: let's be humble participants and value this role over some misguided, ill-considered, hubristic, and perhaps juvenile attempt at dominance.

▶ We have learned so much about how the universe works, and have the opportunity for greater insights still if we can find a glide-path to a long-term sustainable existence. We have built much of value that bears preservation. Posterity[19] relies on a successful embrace of a new vision.

▶ We may yet learn to value nature above ourselves, to the enduring benefit of us all.

That would be a fine way to end the book, so pause for a moment to take in that last point. Richard Feynman once mused about what one, compact sentence transmitted to the future would best help a derailed, post-apocalyptic society get back on track.[20] He decided: "Everything is made of atoms." Personally, I think that misses the mark—putting too much emphasis on the values of (some small subset of) our present civilization. Think about what *you* would want to communicate. For me, the final point above might suggest something along the lines of: Treat nature at least as well as we treat ourselves.

17: This is harder for pessimists to pull off: one has to believe in what is *possible* to get started on years-long ambitious undertakings.

18: . . . as opposed to hacking it down to ever-smaller parcels

19: Billionaires who strive for immortal recognition by launching the human race into space are likely to fail and be forgotten, while those who set us on a truly sustainable path have—by design—a better shot at long-term respect.

20: . . . as if the present track would even be deemed desirable: if such a catastrophic derailment comes to pass, maybe our wisdom is not worth so much.

Image Attributions

1. Cover Photos: Space Shuttle photo courtesy of NASA/Jerry Cannon & George Roberts; Alpine lake on the Olympic Peninsula photo by Tom Murphy.

2. Chapter 1 banner image (page 2): NGC 253 by Dylan O'Donnell; This file is made available under the Creative Commons CC0 1.0 Universal Public Domain Dedication.

3. Chapter 4 banner image (page 54): NASA/Bill Anders from Apollo 8 [22]; manipulated to—ironically—remove empty space; Public Domain.

4. Fig. 4.3 (p. 57): Millennium Simulation, produced by Volker Springel et al. at the Max Plank Institute [25], https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/.

5. Chapter 6 banner image (page 84): State Government Photographer—The History Trust of South Australian, South Australian Government Photo; This file is made available under the Creative Commons CC0 1.0 Universal Public Domain Dedication.

6. Fig. 6.6 (p. 97): originates from U.S. Department of Energy; Public Domain.

7. Fig. 7.1 (p. 103): Source: U.S. Energy Information Administration (2019); Public Domain.

8. Fig. 7.2 (p. 105): Source: U.S. Energy Information Administration (2019); Public Domain.

9. Fig. 8.5 (p. 120): From University of Calgary *Energy Education*: https://energyeducation.ca/encyclopedia/Cap_rock; citation within says: Source unknown. Calgary site materials under Creative Commons license.

10. Chapter 9 banner image (page 138): © C. O'Connell & T. Rodwell, by permission.

11. Fig. 9.7 (p. 145): Created by Robert A. Rohde for Global Warming Art; Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2.

12. Fig. 9.8 (p. 145): Created by Robert A. Rohde, posted on NASA site: https://earthobservatory.nasa.gov/features/EnergyBalance/page7.php.

13. Fig. 9.16 (p. 155): From the National Oceanic and Atmospheric Administration's Climate.gov site; Public Domain.

14. Fig. 12.2 (p. 186): Sourced from U.S. Department of Energy and National Renewable Energy Lab; Public Domain.

15. Fig. 12.4 (p. 188): Adapted from Figure 3 in [74]; Courtesy of WIT Press from *WIT Transactions on State of the Art in Science and Engineering*, Volume 44 (2010), page 92 (reproduced with permission).

16. Fig. 12.6 (p. 190): Adapted from [76]; Reprinted by permission from Springer Nature: *Handbook of Power Systems*, Vol. 2, 2010, "Short-termForecasting in Power Systems: A Guided Tour," by A. Muñoz, E. Sanchez, A. Cruz, Alberto & J. Marín, ©2010.

17. Fig. 12.7 (p. 190): Adapted from Figure 1 in [77]; Copyright © 2017 John Wiley & Sons, Ltd.; Reproduced with permission.

18. Fig. 13.8 (p. 207): Source: © The World Bank; adapted to hide banner, footer and some labels;

adding the blue square and numbers in W/m^2. This file is licensed under the Creative Commons Attribution 4.0 International license (CC-BY).

19. Fig. 13.9 (p. 207): Sourced from U.S. Department of Energy and National Renewable Energy Lab; Public Domain.

20. Fig. 13.11 (p. 208): Sourced from U.S. Department of Energy and National Renewable Energy Lab; Public Domain.

21. Fig. 13.13 (p. 209): Sourced from U.S. Department of Energy and National Renewable Energy Lab; Public Domain.

22. Fig. 13.16 (p. 215): Excerpted graphic from [91]; From U.S. Department of Energy and National Renewable Energy Lab; Public Domain.

23. Fig. 13.24 (p. 220): From U.S. Department of Energy; Public Domain.

24. Fig. 13.25 (p. 220): Credit: Craig Dietrich; This file is licensed under the Creative Commons Attribution 2.0 Generic license.

25. Chapter 14 banner image (page 227): courtesy Rudy and Peter Skitterians; unrestricted.

26. Fig. 15.8 (p. 245): Source: U.S. Department of Energy and Brookhaven National Laboratory; Public Domain.

27. Fig. 15.12 (p. 250): Source: Tennessee Valley Authority; Public Domain.

28. Fig. 15.20 (p. 261): Source: U.S. Department of Energy; Public Domain.

29. Fig. 15.21 (p. 263): Source: U.S. Department of Energy; Public Domain.

30. Fig. 15.23 (p. 267): Credit: © ITER Organization; https://www.iter.org/; Permitted for educational and informational use.

31. Chapter 17 banner image (page 289): courtesy J. Rue; unrestricted.

32. Chapter 18 banner image (page 304): By Sophie Hurel; This file is licensed under the Creative Commons Attribution 3.0 Unported license (CC-BY).

33. Chapter 20 banner image (page 328): Photo by Irina Fischer; obtained under Standard Image License from Shutterstock as royalty-free stock photo.

Changes and Corrections

An electronic textbook has the luxury of being able to correct inevitable typos and mistakes prior to the release of an updated edition. This page reports such corrections. Page ii has information on when this PDF file was produced relative to its initial release on eScholarship.

Changes in the text are marked by a red square ■ which is hyperlinked (in electronic versions) to an entry below. The page number in the entry is also hyperlinked for easy return.

1. Page viii: Demonstration of correction scheme. The page reference returns to the invocation.

2. Page 18: Corrected erroneous energy value for Taiwan in Figure 2.1 (error in Wikipedia table).

3. Page 19: Corrected erroneous energy value for Taiwan in Figure 2.2 (error in Wikipedia table).

4. Page 61: Added Mt. Everest to Figure 4.5.

5. Page 331: Replaced political musings specific to the U.S. with a more general statement.

This page is intentionally blank to make room for corrections without altering pagination of this edition of the book.

Part V

APPENDICES

Math and Equations | A

Depending on background, math and equations may be an intimidating "foreign language" to some students. This brief appendix aims to offer a refresher on techniques, and hopefully inspire a more peaceful relationship for students.

A.1 Relax on the Decimals

First, we can form a more natural, forgiving relationship with numbers. Like your friends, they need not be held to exacting standards: they are simply trying to tell you something useful. Remembering that π is roughly 3 is *far* more important than committing any further decimals to memory. If a friend traced out a circle in the sand and asked how much area it had,[1] the poorly-defined and irregular boundary defies precise measurement, so why carry extra digits. Maybe just recognize that the radius is roughly one meter, so the area is about 3 square meters. Done.[2] The message here is to give yourself a break and just not over-represent the precision (number of decimals) in your answer.

1: Now that's a quality friend!

2: Also notice that the circle fits within a square 2 meters on a side, so the area should be less than 4 m^2: it hangs together.

Part of the reason students have a rigid relationship with numbers is because homework and test problems tend to come pre-loaded with numbers assumed to be exactly known. But the real world is seldom so generous, leaving us to forage for approximate numbers and estimations.

Some classes formalize the concept of significant digits, which is all well and good. But such systems can *add* to the stress of students learning the material (one more thing that can be wrong!).

By being approximate in our use of numbers, we are liberated to do math in our heads more readily. Practice can make this into a life-long skill that becomes second nature. It is helpful to know some shortcuts.

Example A.1.1 To explore the flavor of approximate math, let's consider the statement:
$$\pi \sim \sqrt{10} \sim \frac{10}{3} \sim 3.$$

Your calculator will disagree, but that's why we use the $\sim$ symbol (similar) instead of = (equal). Another common option is $\approx$ (approximately). Your calculator is not as clever as you, and can't appreciate when things are close. It's pedantic. Literal. You can be better.

How do we use this loose association? We saw one example of using $\pi \sim 3$ before, so will not repeat here.

What about $\sqrt{10} \sim 3$?[3] This implies that $3 \times 3 \sim 10$, which is true enough (because 9 and 10 are very close; only 10% different). This means if you pay me \$30 per day for a month, I know immediately that's about \$1,000. Is the month 30 days, or 31? Who cares? Knowing I'll have an extra ~\$1,000 is a good enough basis to make reasonable plans, so it's very *useful*, if not precise.

How about $\frac{10}{3} \sim 3$?[4] This one is actually pretty similar to saying that $\sqrt{10} \sim 3$, since both imply $3 \times 3 \sim 10$. To use another example, let's say you land a \$100,000/yr job, but can only work for 4 months (a third of a year).[5] If $\frac{10}{3} \sim 3$, then you'd expect to get about \$30,000. Why mess around being more precise? Taxes will be larger than the imprecision anyway. Again, it's good enough to have a sense, and make plans.

Much like we have multiplication tables stamped into our heads, it is often very useful to have a few *reciprocals* floating around to help us do quick mental math. Some examples are given in Table A.1 that multiply to 10. Students are encouraged to add more examples to the table, filling in the gaps with their favorite numbers.

The values in Table A.1 are selected to multiply to 10, which is an arbitrary but convenient choice. This lets us "wrap around" the table and continue past three down to 2.5, 2, etc. and learn that the entry for 1.5 would be 6.67. To make effective use of the table, *forget where the decimal point is located*! Think of the reciprocal of 8 as being "1.25–like," meaning it might be 0.125, 12.5, or some other cousin. The essential feature is 125. Likewise, the reciprocal of 2.5 is going to start with a 4.

Example A.1.2 How is Table A.1 useful to us? We can turn division problems, which tend to be mentally challenging, into more intuitive multiplication problems. Several examples may highlight their usefulness.

What is one eighth of 1,000? Rather than carry out division, just multiply 1,000 by the reciprocal—a "125–like" number. In this case, the answer is 125. We can use common sense and intuition to reject 1,250 as $\frac{1}{8}$ of 1,000, as we know the answer should be significantly *smaller* than 1,000. But 12.5 goes too far. Also, we can recognize that $\frac{1}{8}$ is not too far from one-tenth,[6] one-tenth of 1,000 would be 100, close to 125.

How many hours is one minute? Now we are looking for one-sixtieth ($\frac{1}{60}$) of an hour, so we pull out the "167–like" reciprocal and weigh the choices 0.167, 0.0167, 0.00167, etc. Well, $\frac{1}{60}$ can't be *too* far from $\frac{1}{100}$, which would be 0.01. We expect the result to be *a little bigger* than $\frac{1}{100}$, leaving us to have one minute as 0.0167 of an hour.

Now we do a few quick statements that may not match our table *exactly* for all cases, but you should be able to "read between the lines" using blurry numbers to reconcile the statements. One out of seven

3: Really $\sqrt{10} \approx 3.162$.

4: Really it's about 3.333.

5: Money examples often seem easier to mentally grasp because we deal with money all the time. To the extent that money examples are easier, it says that the math itself isn't hard: the unfamiliar *context* is often what trips students up.

Table A.1: Reciprocals, multiplying to 10.

Number	Reciprocal
8	1.25
6.67	1.5
6	1.67
5	2
4	2.5
3	~3

6: This is where "blurry" numbers are useful: $8 \sim 10$ if you squint.

Study Table A.1 for each of these statements to see how it might fit in.

Americans is roughly 15%. A month is about 8% (0.08) of a year. Six goes into 100 a little more than 16 times. Four quarters make up a dollar. π fits into 10 about π times. Each student in a 30-student class represents about 3% of the class population.

Some students view math and numbers as dangerous, unwelcoming territory—maybe like deep water in which they might drown. But think about dolphins, who not only are not afraid to immerse themselves in deep water (numbers), but frolic and play on its surface. Numbers can be that way too: flinging them this way and that to see that a calculation makes sense in lots of different ways. Humans are not natural-born swimmers, but we can learn to be comfortable in water. Likewise, we can learn to be quantitatively comfortable and even have fun messing around. So get your floaties on and jump in!

A.2 Forget the Rules

Math, in some ways, is just an expression of truth—a logic about relationships between numbers and their manipulations. It is easy to be overwhelmed by all the rules taught to us in math classes through the years, and students[7] can lose sight of the simple and verifiable logic underneath it all. Most math mistakes come from faulty or deteriorating memorization. The good news is that we can usually do simple tests to make sure we're getting it right. The lesson is *not to memorize math*! Math makes logical sense, and we can create the right rules by understanding a few core concepts. This section *attempts* to teach this skill.

7: ... teachers, too!

Consider for a moment the concept of language (and see Box A.1 for fun examples). Language is riddled with rules of grammar and spelling, yet we learn to speak without needing to know what adjectives or prepositions are. We learn the rules later, after speaking is second-nature. And unlike math, the rules of language can defy logic and have many exceptions. In this sense, math is much easier and more natural. It is the language of the universe. We would likely share no words in common with an alien species[8], but we can be sure that we would agree on the integers, how they add and multiply, all the way up through calculus and other advanced mathematical concepts. We can use its innate nature to expose the rules for ourselves.

8: Except, perhaps we would learn that we inexplicably share the word "sock."

Box A.1: Rules of Language

Let's step aside for a moment to explore how rules of language are obvious to us without explicit thought.[9] Consider the following constructions: trlaqtoef; flort; aoipw; squeet; yparumd. None of these are English words, but only two of them are even worthy of consideration as viable words. The others violate "unspoken" rules

9: ... acknowledging that this exercise may be less intuitively obvious to non-native English speakers

about how letters might be arranged in relation to each other. We recognize the nonsense without being able to cite specific rules. Math can work the same way: we can rely on intuition to rule out nonsense.

How about this collection of four words: that, how, happen, did. Now arrange them into a single sentence, ignoring punctuation. Notwithstanding how Yoda might arrange things, only one of the 24 possible arrangements[10] makes a single valid sentence. Have you intuited what it is? How did that happen? Without conscious thought, you understand the underlying rules of grammar well enough to see the answer without having to sift through all the combinations. Math *can* work like that, too.

10: The number of combinations is 4! (four-factorial), or $4 \cdot 3 \cdot 2 \cdot 1$. It is a worthy exercise to write out all 24 combinations, not only to verify the result but to give practice in how to systematically shuffle the words in an orderly manner—inventing your own functional rules as you go. You might even stumble on *why* $4 \cdot 3 \cdot 2 \cdot 1$ is the right way to count the combinations based on your method of systematizing.

A.3 Areas and Volumes

This book, and the problems within, often assume facility in computing areas or volumes of some basic shapes. Students who have focused on *memorizing* formulas may see a jumble of π, r to various powers, and some hard-to-remember numerical coefficients. For circles and spheres, how do we bring order to the mess?

A helpful trick is to turn the circle into a square, or the sphere into a cube, where our footing is more secure. Hopefully it is clear that the perimeter (length around) of a square whose side length is a will be $4a$. The area will be $a \cdot a = a^2$. Units can help us, too: if $a = 3$ m, then the perimeter should also be a length with units of meters and the area should be in square meters. It would never do to have something like a^2 describe a perimeter (wrong units) or to have the area *not* contain something like a^2. The cube version has volume a^3.

About those circles and spheres: The task is to fit a circle or sphere inside of a square or cube, so that $a = 2r$. In other words, the *diameter* ($2r$, where r is radius) fits neatly across the side length of the square. The perimeter of the circle should be smaller than the $4a$ perimeter of the square,[11] but a good deal larger than $2a$, which would represent a round trip directly across the square, through its center.[12] So the circle perimeter is between $2a$ and $4a$, probably not far from $3a$. Since $a = 2r$, the perimeter should be somewhat close to $6r$. Suspecting that $\pi \sim 3$ shows up somewhere, the leap is not far to the perimeter being $2\pi r$.

11: . . . literally cutting corners

12: This path would look like a line across the square, traversed twice as a there-and-back trip.

Likewise for the area: a circle within the square has an area smaller than that of the surrounding square (a^2), but surely larger than half the square area—maybe around three-quarters. In terms of radius, the whole square has area $a^2 = 4r^2$, and three-quarters of this is $3r^2 \sim \pi r^2$. Correct again!

Volume is a little harder to visualize, but again the sphere will have a volume smaller than that of the cube: $a^3 = 8r^3$. Maybe the sphere's

volume is about half that of the cube, so $4r^3$. But where would a π go? It's always a multiplier in these situations, so we can harmlessly throw in a $pi/3 \sim 1$ factor to get a volume $\frac{4}{3}\pi r^3$.

The point is that forgetting the exact formulas is not fatal: just back up to a more familiar setting and build out from there. For cylinders, just combine elements of circular and rectangular geometries to realize that the volume is the area of the circle times the height[13] of the cylinder. External surface area is twice the areas of the end-caps (each πr^2) plus the perimeter of the circle times the height—as if rolling out the skin into a rectangle and calculating its area.

13: . . . or length, if on its side

A.4 Fractions

Stressed about fractions? Do you have an intuitive sense for what half a pie ($\frac{1}{2}$) would look like? A fifth of a pie ($\frac{1}{5}$) ? Which is larger: one-third ($\frac{1}{3}$) of a pie or one-quarter ($\frac{1}{4}$) ? Do you have an immediate sense of how many quarters are in a dollar? Back to the pie: if a friend hands you two pie plates, each containing a third ($\frac{1}{3}$) of a pie, do you now have less than half of a pie in total, more than half, or exactly half? If you had little trouble picturing and answering these questions, then you're all set!

But isn't there a lot more to fractions: rules of adding, subtracting, multiplying, and dividing? What about common denominators and all that business? The point of this section is that you can build on your natural intuition[14] to verify and construct the right rules for how the mechanics of fractions should work. You're not in the dark!

14: . . . as expressed in the first paragraph

First, representation. What does $\frac{1}{5}$ mean? Literally, we can say that we divide *something* (a pie, for instance) into 5 pieces (denominator) and extract 1 (numerator). Implicitly, we are *multiplying* the *something* (pie) by the fraction $\frac{1}{5}$. Now what about something like $\frac{3}{5}$? We can interpret this multiple ways,[15] which we will express several ways:

15: . . . all correct, and depends on context of the problem at hand

$$\frac{3}{5}pie = \frac{1}{5}(3\,pies) = \frac{6}{10}pie = \frac{3}{10}(2\,pies) \tag{A.1}$$

Figure A.1 shows the first two options in Eq. A.1 graphically. We can either slice one pie into five slices and take three of them, or slice three pies into five equal pieces and take one. Either way, we end up with the same amount. The possibilities are endless, and it is worth concocting your own variants. The last steps in Eq. A.1 hint at one type of freedom: we could split one pie into 10 pieces and select 6. Or we could split two pies into ten pieces and take 3 of those to end up with the same amount of pie.

Let's formulate rules about multiplication of fractions based on stuff we know (intuition). What is the rule for the general multiplication of two fractions, expressed symbolically so we can substitute *any* number
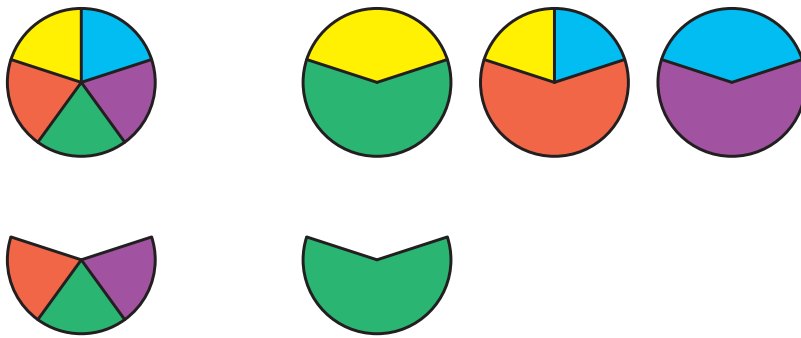
for any symbol (placeholder) and get the right answer? In other words, what should the question marks be in the following:

$$\frac{a}{b} \cdot \frac{x}{y} = \frac{?}{?} \tag{A.2}$$

To answer, pick a scenario you *already know* and back-out the answer. You know that one half of one-half is one quarter. You also know that one half of $\frac{4}{5}$ must be $\frac{2}{5}$, or that three thirds must be a whole "one." In math terms:

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}; \quad \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}; \quad \frac{3}{1} \cdot \frac{1}{3} = 1. \tag{A.3}$$

From these examples—and others that can be fabricated as wished or needed—it is possible to arrive at the conclusion that

$$\frac{a}{b} \cdot \frac{x}{y} = \frac{a \cdot x}{b \cdot y}. \tag{A.4}$$

In other words, just multiply the numerators together and multiply the denominators together, simplifying by common factors as needed.

> **Example A.4.1** What is $\frac{2}{5}$ of the fraction $\frac{15}{24}$? By the straight rules, we get 30 in the numerator and 120 in the denominator.[16] Many common factors appear in the numerator and denominator (even the original $\frac{15}{24}$ could have been reduced to $\frac{5}{8}$) to give the final answer of $\frac{1}{4}$.

16: We can apply our lesson on reciprocals (see Example A.1.2) and realize that multiplying 24 by 5 is a lot like dividing it by 2, up to a decimal place. This gives us something 12–like and thus 120.

One more framing of fractions and their relationship to multiplication and division: dividing by 8 is the same as multiplying by $\frac{1}{8}$. Multiplying by $\frac{2}{3}$ is the same as dividing by $\frac{3}{2}$. Multiplication and division are thus essentially the same, only having to flip the number or fraction upside-down into its reciprocal.

How can our intuition assist us in figuring out addition and subtraction of fractions? Use what you know:

$$\frac{1}{2} + \frac{1}{2} = 1; \quad \frac{1}{2} + \frac{1}{4} = \frac{3}{4}; \quad \frac{3}{4} < \frac{1}{2} + \frac{1}{3} < 1. \tag{A.5}$$

Hopefully, the first two statements in Eq. A.5 are apparent enough. The

last one bounds the answer by what you already know. Since $\frac{1}{3}$ is larger than $\frac{1}{4}$.[17] So adding $\frac{1}{2} + \frac{1}{3}$ must be larger than $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. By similar logic, since one-third is smaller than one-half,[18] their sum must be smaller than 1.

Adding fractions like $\frac{1}{2}$ and $\frac{1}{3}$ is where common denominators come in. We can add numerators *only if the fractions share the same denominator*. We **never add denominators**. We can't replicate the middle example in Eq. A.5 by adding numerators *and* denominators, or we would get the nonsense answer $\frac{2}{6} = \frac{1}{3}$, rather than $\frac{3}{4}$.

So how would we ever recreate the whole common denominator scheme based on intuition? Let's return to the case of $\frac{1}{2} + \frac{1}{3}$. We have already bounded it to be between 0.75 and 1.0 (in Eq. A.5), which is already useful as a check to whatever rule we might try. Looking at the problem graphically, as in Figure A.2, we see that overlaying $\frac{1}{2}$ and $\frac{1}{3}$ naturally creates a missing gap of $\frac{1}{6}$. How do 2 and 3 in the denominators conspire to form 6? Via multiplication, of course. Re-expressing $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{1}{3}$ as $\frac{2}{6}$,[19] allows us to add the numerators directly, having made the denominators the same: $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

To some students, this may seem like an unnecessary and elementary review,[20] but the main point is that when in doubt, use what you already know to test your technique and verify that you are doing things right. If the rules you are trying to apply seem to work for a few different known cases, then you're probably golden. Approaching math this way makes you the boss of the formulas, rather than the other way around.

A.5 Integer Powers

Raising a number to a power, like 4^3, is just a mathematical shorthand for $4 \cdot 4 \cdot 4$. Think of all the room we save in the case of 4^{23}!

So what are the rules for dealing with exponents when we raise the whole thing to another power, or when we multiply two exponentiated pieces together, or if we divide by (or invert) the thing? In other words, what are:

$$(x^a)^b =? \quad x^p \cdot x^q =? \quad \frac{1}{x^n} =? \tag{A.6}$$

The theme of this appendix is: discover the rule through your own experimentation. Tackling in stages, what is $(7^4)^3$? We can write out 7^4 as $7 \cdot 7 \cdot 7 \cdot 7$ easily enough. If we cube this number, it's the same as

17: Splitting a pie into three parts will surely leave larger pieces than splitting it into more (4) parts, so $(\frac{1}{3} > \frac{1}{4})$.

18: If such statements are less than intuitive, think about pie or money, where the natural context lends itself to better intuition.

The point of this section (and appendix) is that you can use *what you already know* to check whether you are applying the right rules, and even re-create the rules that work—verifying that you get the answers you expect for cases you know and trust. Discover for yourself! Doing so gives you full ownership of the math. It's no longer something someone taught you to do: you've taught it to yourself, and that is far more powerful.

19: ... by multiplying top and bottom by the missing factor—or the "other" denominator value

Figure A.2: Graphically, it is easy to see that $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. You can always concoct similar/familiar scenarios to verify (and re-invent) the rules.

20: ... which is why it is relegated to an appendix

Notice that the the symbols used in this equation are just stand-ins for numbers, and have no intrinsic significance—whether we use n or p or a for an exponent is irrelevant. For that matter, x is not special either and we could have used a hexagon to stand in for the base in these relations, as a symbolic placeholder.

writing this set three times, all multiplied together, or $(7 \cdot 7 \cdot 7 \cdot 7) \times (7 \cdot 7 \cdot 7 \cdot 7) \times (7 \cdot 7 \cdot 7 \cdot 7)$, which is just 12 sevens multiplied, or 7^{12}. So we have discovered/formulated the rule:

$$(x^a)^b = x^{a \cdot b}. \tag{A.7}$$

We *multiply* the exponents when raising the inner exponent to an outer one.

How about $3^2 \cdot 3^5$? What is the rule there? The process[21] is similar to before, expanding out to $(3 \cdot 3) \times (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$, which just looks like seven threes multiplied together, or 3^7. Therefore, our rule is:

$$x^p \cdot x^q = x^{(p+q)}. \tag{A.8}$$

We *add* exponents when multiplying two pieces, each having their own exponent. Note that this does *not* work when the bases are unequal, as you could verify yourself for $3^2 \cdot 5^4$.

Finally, what about inversion, or dividing by x^n? As a preview, a negative power is equivalent to putting the item in the denominator, so that $x^{-1} = \frac{1}{x}$. To see this, consider Eq. A.8 in the case where p and q are opposite sign but the same magnitude. For instance, following the "add the exponents rule" we get that $3^4 \cdot 3^{-4} = 3^{4-4} = 3^0 = 1$, because anything raised to the zero power is 1.[22] The only thing we can multiply into $3 \cdot 3 \cdot 3 \cdot 3$ in order to get 1 is $\frac{1}{3 \cdot 3 \cdot 3 \cdot 3}$. This means that 3^{-4} is the same as $1/3^4$, or more generally:

$$\frac{1}{x^n} = x^{-n}. \tag{A.9}$$

Negative exponents therefore flip the construction to the denominator, or denote a division rather than multiplication.

21: Note that some care must be exercised in selecting the example. For instance, picking both exponents as 2 would leave some ambiguity: is the result of 4 2 + 2, 2 × 2, or 2^2?

22: Think of the exponent as how many instances of a number are multiplied together in a chain, implicitly all multiplied by 1. If we have zero instances of the number, then the implicit 1 is all we have left in the multiplication. In other words, 1 is the starting point for all multiplications, just like zero is the starting point for all additions.

A.6 Fractional Powers

In the previous section, we only dealt with integer powers, so that we could write out 3^4 as $3 \cdot 3 \cdot 3 \cdot 3$. How would we possibly write $3^{1.7}$? Yet it is mathematically well defined. A calculator has no trouble.

We can get a hint from Eq. A.8. Consider, for example, $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$. We know that we can just add the exponents, which in this case add to a tidy 1, meaning that the answer is just 5. Therefore we interpret $5^{\frac{1}{2}}$ as the square root of 5, since multiplying it by itself yields 5. So we can re-express our familiar friend as a fractional power:

$$x^{\frac{1}{2}} = \sqrt{x}. \tag{A.10}$$

In principle, then, we could approach $3^{1.7}$ by taking the tenth-root of 3 and raising it to 17th power: $3^{1.7} = (3^{\frac{1}{10}})^{17} = 3^{\frac{17}{10}}$.

More generally, in Chapter 1, we saw that we can represent any base, b, raised to some arbitrary number, n, as:

$$b^n = e^{n \cdot \ln b} = 10^{n \cdot \log_{10} b}, \qquad (A.11)$$

where we use the exponential function and its inverse function (natural log, ln), or alternatively the base-10 equivalents. If, for some reason, we lacked a y^x calculator button, these approaches allow more fundamental ways to get at the same thing.

A.7 Scientific Notation

The single-biggest mistake students make when it comes to scientific notation is easily remedied by understanding it not as a set of rules, but for what it's actually doing.

Most of the time, students get it right: they see 1.6×10^2 and move the decimal to the right two times to get 160. A little harder is negative exponents, like 2.4×10^{-2}. Moving the decimal point twice to the left results in the correct 0.024 answer.

The hangup can come about if the process is misconstrued as simply "counting zeros." Ironically, a student might correctly convert 6×10^3 by adding three zeros to the 6 to get 6,000, but then mistake 10^3 for 10,000—thinking: start with 10 and add three zeros.

The sure-fire way is to connect to the concept of integer powers, so that 10^3 is simply $10 \cdot 10 \cdot 10$, which is unmistakably 1,000. Likewise, 10^{-4} is four repeated (multiplied) instances of 10^{-1}, each one representing $\frac{1}{10}$, or 0.1. String four together, and we have $\frac{1}{10,000}$, or 0.0001. So fall back on the basics.

Example A.7.1 We can also apply the rule of multiplying exponentiated quantities covered in Eq. A.8. So 3.2×10^3 times 2×10^2 can be written out as $3.2 \cdot 2 \cdot 10^3 \cdot 10^2$ (order does not matter), which we can recognize as 6.4×10^5.

What about division: 2.4×10^{13} divided by 8×10^7? Several ways to approach this might be instructive. Let's ignore the pre-factors (2.4 and 8) for now and focus on the powers of ten. The standard practice is to subtract the exponent in the denominator from that in the numerator: $13 - 7 = 6$ in this case, so that we are left with $\frac{2.4}{8} \times 10^6$. We could also represent the 10^7 in the denominator as 10^{-7} in the numerator, as per Eq. A.9. Now we just add the exponents to get the same result. Or we *could* invert the 8×10^7 to become 0.125×10^{-7} and multiply this by 2.4×10^{13}.

But I want to present the way I would do it to make it easy enough

to perform in my head. Recognizing that 24 is divisible by 8, I am strongly tempted to re-express the first number as 24×10^{12}. See what I did there? I multiplied the prefactor by 10, and decreased the exponent by one accordingly to end up at the same place. Now I have $\frac{24}{8} \times 10^{12-7}$, which reduces to 3×10^5. All methods get the same answer,[23] which turns into another lesson that math provides many paths to the same answer, which can be used to check and reinforce.

23: The frolicking dolphin tries several and revels in the reinforcement that comes from consistency.

A.8 Equation Hunting

Students often form a counter-productive dependency on formulas. Experts focus on learning the *concept* expressed by an equation, since an equation is very much like a sentence that speaks some truth.[24] Once the fundamental principle is mastered, the equation or formula is automatic, and can be *generated* from a place of understanding—which is more permanent than memorization.

24: ... perhaps within some context or set of assumptions

The practice is more common and natural than it might seem at first. Let's say a person has a take-home pay of $50,000 per year. Rent is $2,000 per month, groceries and other bills come to $1,000 per month. How much is left per month for discretionary spending? Where is the formula for *that* problem? Of course, you wouldn't bother hunting for a formula in this case and would instead build your own math. You essentially create your own formula on the fly. Whether you first divide the annual figure by 12 and then subtract the monthly expenses, or multiply monthly expenses by 12 before subtracting from the annual amount and *then* dividing by 12, the result is the same: a little more than $1,000 per month.

It is also clear in this context that it makes little sense to perform math down to the penny, since the grocery and other expenses are not going to be exactly the same each month. The lesson is that most people are expert enough in managing money that they don't scramble to find printed formulas whenever they want to figure something out, and they are also forgiving on precision because they know from context not to take it all too literally.

This book *tries* to foster a more expert-like approach to the material. For instance, Def. 5.3.1 (p. 71) introduces the concept of power without explicitly saying $P = \Delta E / \Delta t$. It just says that power is how much energy is expended in how much time. If a student internalizes *that* idea, then why print a formula? By doing so, a student may bypass real understanding[25] and rely on the formula as a crutch, never planting the core idea firmly in the brain. Shortcuts can end up disadvantaging students, as attractive as they may look in the moment. The student who masters the concepts will be in a far better position to deploy them in a wider variety of circumstances—including unfamiliar test questions.

25: ... which in this case is not a heavy lift

A.9 Equation Manipulation

Physics instructors often joke that they teach students the "three Ohm's Laws." The joke is that only one is needed: $V = I \cdot R$. The other forms: $I = \frac{V}{R}$ and $R = \frac{V}{I}$ can be derived from the first. Rigid memorization leads some students to remember all three forms, rather than simply move things around in a way that maintains the relationship.

The rules are easy enough to generate on your own. Think of an equation as a perfectly balanced see-saw—maybe an elephant sitting on both sides. The equation is only valid if it remains balanced. You may add a chicken, but do it to both sides. You may multiply or divide the number of elephants, as long as it is done the same way to both sides. Dividing both sides of (the first) Ohm's Law above by R leads to the second form, for instance.

Example A.9.1 Let's say you are given something you consider to be an ugly equation, or you simply want to solve for one variable. Using symbols for everything,[26] we might have

26: It is easy to substitute numbers anywhere you wish at any time

$$\frac{a}{b} + c = \frac{x + y}{z}$$

Let's say we hate the appearance of fractions. Multiply both sides by b:

$$a + b \cdot c = b\frac{x + y}{z}$$

Now multiply both sides by z to eliminate the remaining fraction:

$$a \cdot z + b \cdot c \cdot z = b \cdot (x + y)$$

What if you wanted a solution for y? Guess we're going to have to return b to denominator status, as we need to divide both sides by b.

$$x + y = \frac{a}{b} \cdot z + c \cdot z$$

And finally we subtract x from both sides to get

$$y = \frac{a}{b} \cdot z + c \cdot z - x$$

Whatever you want to do, just do it to both sides.

It's not always so straightforward. Sometimes we have to "undo" or "invert" a mathematical function. Consider for instance a familiar problem: find the side length, a, of a right triangle whose other side is b and hypotenuse c. We know from the Pythagorean Theorem that $a^2 + b^2 = c^2$, so that $a^2 = c^2 - b^2$. But we want a, not a^2. How do we "undo" the square? Take a page from Eq. A.7. We want a to the power of 1, so we want to raise a^2 to whatever power will neutralize the 2 via multiplication.

Looks like $\frac{1}{2}$ (square root) will do the trick. But we need to treat both sides:

$$a = \left(a^2\right)^{\frac{1}{2}} = \sqrt{a^2} = \sqrt{c^2 - b^2} \qquad (A.12)$$

In this case, the power $\frac{1}{n}$ can be said to perform the inverse function of the power n. In more familiar contexts: subtraction is the inverse of addition; division is the inverse of multiplication. Less familiarly, but in similar veins: the sine is "undone" by arcsine;[27] the exponential e^x is undone by the natural log ($\ln x$); 10^x is undone by $\log_{10} x$, etc.

If you have not done it before (or recently), mess around on a calculator, starting with a custom number you make up that is pleasing to you and recognizable.[28] Square it and then take the square root. Calculate the sine and then inverse sine (ASIN). Take the exponential and then natural log—or other way around. Get to know these things on your own terms!

A.10 Units Manipulation

In the real world, numbers often are packaged with associated units. The radius of the earth is 6,378 km. If we change the unit, we change the number, too. Earth's radius becomes 6,378,000 m or 3,963 miles. Most generally, we aim to quantify something in nature, and the numeric value is utterly dependent on the units we choose to represent the physical reality.

Because the numbers are often *meaningless* without the accompanying units, we should[29] carry around the units in all manipulations. Any time we do something to the number, we need to do the same thing to the unit.

> **Example A.10.1** If we travel 4 meters in 2 seconds, we have
>
> $$\frac{4\,\text{m}}{2\,\text{s}} = \frac{4}{2} \cdot \frac{\text{m}}{\text{s}} = 2\,\text{m/s}.$$
>
> Dividing the numbers alone to get 2 is not the whole story. We also divided the units to create a new one that was not in the initial set (m and s).
>
> If we fill a room whose floor area is 10 square meters with water one meter deep, the volume is
>
> $$10\,\text{m}^2 \cdot 1\,\text{m} = 10 \cdot 1\,\text{m}^2 \cdot \text{m} = 10\,\text{m}^3.$$
>
> So we multiply the meters together just like we do the numbers, following the same rules but acting as if they are variables and keeping it symbolic.

More complicated arrangements follow the same rules. For example, the force of drag[30] on an object moving at speed v through a medium of density ρ is $F_{\text{drag}} = \frac{1}{2}c_{\text{D}}A\rho v^2$, where A is the frontal (cross-sectional) area of the object and c_{D} is the dimensionless drag coefficient.[31] The dimensions of area are m^2; density is kg/m^3 (mass per volume), and velocity is m/s (distance over time). The whole arrangement therefore has dimensions:

$$\text{m}^2 \cdot \frac{\text{kg}}{\text{m}^3} \cdot \left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{m}^2 \cdot \text{kg} \cdot \text{m}^2}{\text{m}^3 \cdot \text{s}^2} = \frac{\text{kg} \cdot \text{m}^4}{\text{m}^3 \cdot \text{s}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}.$$

The end result matches the definition of Newtons, and can be verified by the (possibly familiar) $F = ma$ form of Newton's Second Law,[32] whereby we have mass in kg times acceleration in m/s^2 making kg $\cdot$ m/s^2.

When performing a chain of multiplications or divisions, we can carry the units around and multiply, divide, or (hopefully) cancel them as we go.

Example A.10.2 Let's say we want to know how much energy, in Joules, the U.S. uses in a year based on knowledge that the average citizen accounts for 10,000 W (a Watt is a Joule per second) and the U.S. has 330 million people. First, let's work with the 10,000 W per person metric:

$$\frac{10^4\,\text{J/s}}{\text{person}} = \frac{10^4\,\text{J}}{\text{person} \cdot \text{s}},$$

where we have just moved the seconds into the denominator to multiply "person" (order doesn't matter). Most of the problem is in going from seconds to years. It would like like this:

$$\frac{10^4\,\text{J}}{\text{person} \cdot \text{s}} \cdot \frac{60\,\text{s}}{1\,\text{min}} \cdot \frac{60\,\text{min}}{1\,\text{hour}} \cdot \frac{24\,\text{hour}}{1\,\text{day}} \cdot \frac{365\,\text{day}}{1\,\text{year}}$$

Notice that each of the factors we multiply, even though they carry a non-unity numeric value, are *essentially* identities that describe equal intervals on top and bottom, in differing units.[33] So we are effectively multiplying by 1 repeatedly in a **unit conversion** process.

Also note that the chain we construct allows a boatload of cancellations, as almost all units present appear in both the numerator and denominator once. The only ones that do not are J in the numerator and year and person in the denominator. When we carry out the multiplication above and cancel units, we find that we are left with:

$$3.15 \times 10^{11}\,\frac{\text{J}}{\text{year} \cdot \text{person}}.$$

Oops, the units are helping us here by reminding us that we need to multiply by the population (3.3×10^8 persons) to get the answer

we sought.[34] In this case, we end up with 1.04×10^{20} J/year, which is what we were after.

We just carried out unit conversions (in time) in Example A.10.2, when we multiplied by constructs like 60 s/1 min. The key to unit conversions is to arrange a fraction expressing *the same physical thing* in both the numerator and denominator, just using different units. So we're looking for equivalent measures. *Most* of the time, one of them will be 1, numerically, as in the following example.

Example A.10.3 We might want to convert the 1.04×10^{20} J/year from Example A.10.2 into quadrillion Btu per year. We know that 1 Btu is 1,055 J, and that a quadrillion is 10^{15}. So we arrange the following:

$$1.04 \times 10^{20} \, \frac{J}{year} \cdot \frac{1 \, Btu}{1,055 \, J} \cdot \frac{1 \, quadrillion}{10^{15}} \approx 100 \, quadrillion \, Btu/year.$$

Finally, units can help guide correct usage of factors in a problem. In Example A.10.3, what if we did not know whether to divide or multiply by 1,055? The fact that we wanted to eliminate Joules told us we needed the Joules in the denominator, and so the relation 1 Btu = 1,055 J told us the 1,055 travels with Joules and must be in the denominator.

But what if we are faced with a problem whose application is not as apparent?[35] Let's explore how this might go in a less familiar setting.

Example A.10.4 For some inexplicable reason, you put a brick in the refrigerator, which is 20°C colder than the brick is, initially, and want to find out how long it will take for the brick to cool off. The brick has a mass of 2.5 kg, will dump its heat into the refrigerator at a rate of 10 W (10 J/s), and has a specific heat capacity of 1,000 J/kg/°C.[36] So you feel overwhelmed by lack of familiarity, right? Good, because the units are here to help.

You want an answer in time units, and see one instance in the 10 J/s rate at which heat leaves the brick. To get seconds "up top," you want to make sure the 10 W value is in the denominator. This puts Joules in the denominator, and we don't want it to survive to the final answer. We notice Joules in the specific heat capacity thing in the numerator, so that thing must go in the numerator. Let's take stock of where that leaves us.

$$\frac{1}{10 \, J/s} \cdot \frac{1,000 \, J}{kg \cdot °C} = 100 \, \frac{s}{kg \cdot °C}$$

It looks like if we multiply by the mass in kg and multiply by the temperature difference, we're home free. Doing so results in 5,000 seconds. Whenever possible, try to extract the most context/intuition out of an answer as you can. Does 5,000 seconds mean a lot to you? Divide by 60 (or multiply by $\frac{1 \, min}{60 \, s}$) to get 83 minutes. Better. Or another factor of 60 and we're at 1.4 hours. That seems like the most natural

way to express the answer.

It is also useful to pause and reflect on our operations and whether they made sense. For instance, since we multiplied by mass to get the cooling time, it implies that a larger brick would take longer, which is sensible. If heat left at a rate faster than 10 W, it would cool down faster, again making sense.

We will do one more example in an unfamiliar context, this time involving some ambiguity that your wits can help resolve.

At this point, we could create our own formula based on requiring the units to work! See: formulas are not sacred tablets to be memorized—they are just statements that make logical sense and can be created *by* you to accomplish a task.

Example A.10.5 An outer wall on a sealed brick building measures 5 m long and 2.5 m high, having a thickness of 0.1 m. You are asked how fast heat (thermal energy) is being lost through the wall.[37] It is 20°C warmer inside the wall than it is outside, and you are told that brick has a thermal conductivity of 0.6 W/m/°C.[38] Thermal what? But don't panic.

37: The units would be energy per time, or J/s, which is a power (W).

38: This construction means that m and °C are *both* in the denominator together.

We want an answer in J/s, or W. We see Watts in the numerator of the thermal conductivity, so we want that in the numerator of our answer. We would need to multiply by meters and by °C to cross the finish line. Seems simple enough. But meters shows up three times in the dimensions of the wall. Which should we choose? Or is it a combination?

Engage the intuition. Put yourself in the building next to the wall, mentally. It's warm inside, cold outside. Will I need more heaters (power) or fewer if the wall is taller? If the wall is wider? If the wall is thicker? What does your intuition say?

You might reason that a thicker wall will require less power to keep warm, but that a larger area (increasing width or height or both) would make the job of maintaining temperature more challenging. This suggests that the power will *increase* if width or height increase, and *decrease* if thickness increases. So we should multiply by width and height (or equivalently, by area) and divide by thickness. Area over thickness indeed has units of meters, which we already concluded we needed as a multiplier to get our desired outcome. Putting things together, we have

$$0.6\,\frac{W}{m\cdot°C}\cdot\frac{5\,m\cdot2.5\,m}{0.1\,m}\cdot20°C = 1,500\,W,$$

which is about the output of one space heater.

A.11 Just the Start

It is well beyond the scope of this book to engage in an exhaustive review of math concepts. Hopefully what has been covered provides a

useful foundation. The key lesson is that the knowledge and intuition students already hold in their heads can be leveraged effectively to recreate forgotten rules of math. Just remember: it all makes sense and hangs together. Creating customized simple problems[39] allows a way to make sure the math rules being applied replicate the right answer. If not, a few tests can often get things back on the right track. By doing so, students can claim greater personal ownership of the math, and have a better internal mastery of its workings.

39: . . . whose answers are already known or can be figured out

Chemistry Primer/Refresher | B

This book does not rely heavily on past knowledge of chemistry, but it is helpful to know a few basic elements that play a role in fossil fuels, biological energy, and climate change. This section could act as a refresher, or a first exposure to the fundamentals.

B.1 Moles

Chemistry deals with atoms and molecules and the interactions between them. Atoms and molecules[1] are irreducible nuggets of a substance—the minimum unit that carries the essential properties of that substance. Water, for instance, is comprised of two hydrogen atoms bonded to a single oxygen atom, which we denote as H_2O.

1: Molecules are made from a handful of atoms.

Figure B.1: Periodic Table of the Elements. This version is too small to permit names, so that only symbols are given. The more familiar elements are highlighted. Numbers represent the number of protons in the nucleus of the associated atoms.

Figure B.1 presents a stripped-down version of the periodic table. Additional exploration of more fully-featured versions is encouraged.[2]

2: Perhaps at least identifying the highlighted elements would be worthwhile.

It is natural to imagine that a first step in dealing with piles of atoms and/or molecules is being able to *count* them. But since individual atoms are fantastically small, the numbers can be overwhelmingly large. This is where the mole[3] comes in. A mole is *just a number*, and that number is called Avogadro's number, having a value of $N_A = 6.022 \times 10^{23}$, or 602,214,076,000,000,000,000,000, if written out.[4]

3: . . . the word *molecule* begins with *mole*.

4: Unfortunately, it can be hard to remember if it is supposed to be 6.022×10^{23} or 6.023×10^{22}. For this reason, it may be wise to forget about the 22 and just remember 6.0×10^{23}, or even 6.023×10^{23} as something that is *very slightly* wrong but much better than being 10 times off!

The way the mole is defined, essentially, is that 12.000000 grams of neutral carbon atoms (of the isotope having 6 protons and 6 neutrons in its nucleus) will constitute one mole of atoms. In this way, the masses of

other elements—being comprised of an integer number of protons and neutrons[5]—will tend to be close to an integer number of grams.[6] For instance, a mole of hydrogen atoms is very close to 1.00 grams. A mole of helium is very nearly 4.00 grams, nitrogen 14, oxygen 16, etc.

So the concept of the mole is pretty straightforward: just a number—albeit a very large one.

Box B.1: Moles to Mass

Incidentally, the inverse of Avogadro's number becomes the definition of the atomic mass unit (a.m.u.). The a.m.u. can be thought of as the average mass of an atom per nucleon.[7] In other words, carbon-12 (6 protons, 6 neutrons) has a mass of 12 a.m.u. In fact, this is how the a.m.u. is defined. This means that hydrogen (a single proton) has a mass very close to 1.00 a.m.u., and oxygen-16 (8 protons, 8 neutrons) has a mass close to 16.00 a.m.u. Chapter 15 delves into the subtle reason why these are not exactly 1.00000 and 16.00000 in these cases.

Since one mole of 12.00000 a.m.u. carbon-12 atoms is defined to have a mass of 12.00000 g, one mole of 1.000000 a.m.u. particles[8] would have a mass of 1.000000 g. Therefore, a *single* 1.000000 a.m.u. particle would have a mass of 1.000000 g divided by Avogadro's number, $N_A = 6.02214076 \times 10^{23}$, which turns out to be $1.66053907 \times 10^{-24}$ g, or $1.66053907 \times 10^{-27}$ kg. This is the number you will find if looking up the atomic mass unit (also called a Dalton).

B.2 Stoichiometry

Chemistry starts by counting atoms and molecules. Since molecules are comprised of integer numbers of atoms of specific types, the counting fun does not stop there. When atoms and molecules react chemically, the atoms themselves are never created or destroyed—only rearranged. This means that an accurate count of how many of each atom type are present at the start, a proper count at the end should yield *exactly* the same results.

Before we get into balancing chemical reactions, we need to know something about the scheme for labeling chemical compounds. A compound is an arrangement of atoms (representing pure elements) into a molecule. For instance, water is made of three atoms drawn from two elements: hydrogen and oxygen. Two atoms of hydrogen are bonded to an atom of oxygen to make a molecule of water. We denote this as H_2O.

Examples of a few familiar atoms and molecules are presented in Figure B.2. Each one is named at the top. Below each one appears the bond structure in the case of molecules and the chemical "formula" in all cases. Notice that hydrogen atoms always have a single bond (single
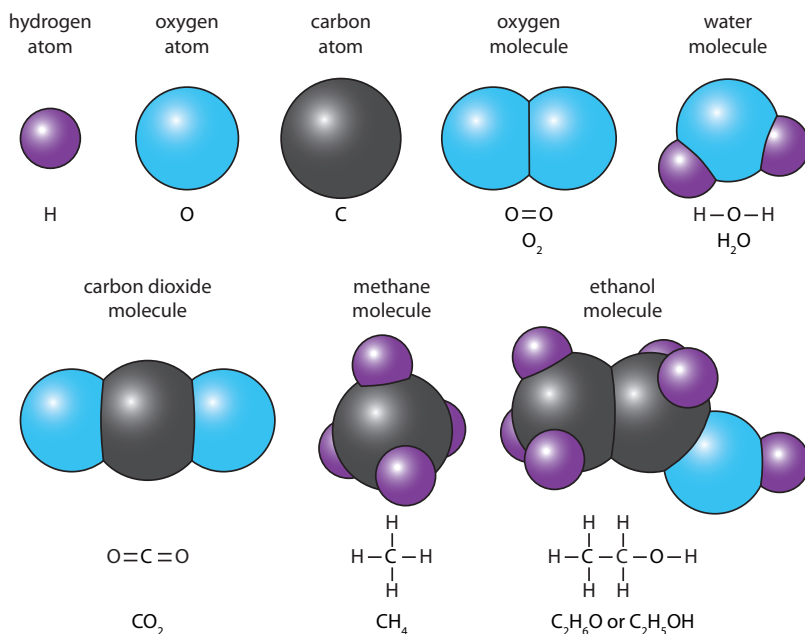
Figure B.2: Representing atoms as colored spheres for schematic purposes, we can depict the general appearance of molecules as bonded collections of atoms. Here, we have three elements—hydrogen, oxygen, and carbon—combined into familiar molecules. Oxygen in the air we breathe is self-bonded into a "diatomic" molecule. Two representations appear below each molecule: a diagram indicating bonds (including double-bonds in some cases), and the chemical formula.

electron to share), oxygen has two (wants to "borrow" two electrons to feel good about itself), and carbon tends to have four (either donating four in the case of CO_2, or accepting four when bonding to hydrogen). The chemical formula for each uses elemental symbols to denote the participants and **subscripts** to *count how many are present*.[9]

Now we come to a bedrock practice in chemistry called stoichiometry—which boils down to counting atoms in a reaction to make sure no atoms are missing or spontaneously appear. To get a sense of this, see Figure B.3 for two examples. The graphical version captures the physical reality, so that simply counting the number of spheres of each color on the left and right had better match. Below each graphical reaction is the associated chemical formula. Each formula contains an arrow indicating the direction of the reaction (separating "before" and "after"). Numerical factors (coefficients, or prefactors) in front of a molecule indicate how many molecules are present in the reaction. To get the total number of atoms represented, we must multiply the subscript for that atom (implicitly 1 if not present) by the prefactor.[10]

9: Two variants are shown for ethanol. The first is a no-nonsense census of the atoms, while the second pulls one of the H symbols to the end to call attention to the OH (hydroxyl) tagged onto the end of the molecule. In either case, the formula specifies 2 carbons, 6 hydrogens, and 1 oxygen, in total.

10: For example, $2H_2O$ has a total of 4 hydrogen atoms and 2 oxygens.

Example B.2.1 Let's figure out a tougher formula, pertaining to the combustion of ethanol (depicted in Figure B.2). In this situation, we combine a C_2H_6O molecule with some number of oxygen molecules (O_2), and the reaction products will be CO_2 and H_2O (carbon dioxide and water). Our job is to figure out how many molecules are needed to balance the reaction:

$$C_2H_6O + ?O_2 \rightarrow ?CO_2 + ?H_2O$$

where question marks indicate what we need to figure out. Three unknowns and one equation? It may seem hopeless, but the formula is *not* the equation. The equations are that the total number of carbons on each side are equal, the total number of oxygens are equal, and the total number of hydrogens are equal. So we actually have three equations.[11]

11: Equations are just statements of truth that we can create on our own. They are just a way to express what we know about a problem.

Start by noticing that the left side has 2 carbons and 6 hydrogens. We don't know how many oxygens yet, but it's good enough to start. On the right, carbon only shows up in CO_2, so getting 2 carbons on the right requires $2CO_2$. Likewise, hydrogen only shows up in water, and ethanol has 6 hydrogen atoms to stuff into water molecules that hold 2 hydrogens apiece. It will obviously take 3 water molecules to account for 6 hydrogens.[12] So now the right side is hammered out:

12: What if the starting point had an odd number of hydrogens on the left? We'd need to double the number of hydrogen-containing molecules on the left to produce an even number and start over.

$$C_2H_6O + ?O_2 \rightarrow 2CO_2 + 3H_2O$$
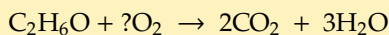
The only thing left to figure out is how many oxygens are on the left. To balance the reaction, count the number of oxygen atoms on the right. Four come from the two CO_2 molecules, and 3 from the water for a total of 7. One oxygen was already present in the ethanol molecule on the left, so only need 6 in the form of O_2, thus requiring three of these:

$$C_2H_6O + 3O_2 \rightarrow 2CO_2 + 3H_2O$$
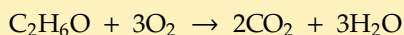
The job is done: the reaction is now balanced. That's stoichiometry.

The treatment above cast chemical reactions at the most fundamental level of individual molecules reacting. In practice, reactions involve great numbers of interacting particles, so it is often more convenient to think in moles. In fact, common practice is to look at the prefactors[13] in chemical reaction formulas as specifying the number of *moles* rather than the number of individual molecules. Either way, the formula looks exactly the same,[14] and it's just a matter of interpretation.

13: ...also called coefficients

14: To be explicit, if a formula is balanced for individual molecules, then it should also be balanced if doubling the "recipe," or tripling, multiplying by 10, or even by 6×10^{23}.

Thinking of the chemical formulas in terms of moles makes assessment of the masses involved more intuitive. Recall that one mole of carbon

atoms is exactly 12 grams, that hydrogen is 1 g, and oxygen is 16 g. That means one mole of water molecules (H_2O) will be 18 g (16 + 1 + 1), one mole of carbon dioxide (CO_2) is 44 g (12 + 16 + 16), and one mole of ethanol (C_2H_6O) is 46 g (12 + 12 + 1 + 1 + 1 + 1 + 1 + 1 + 16). We refer to this figure as the molar mass, and standard periodic tables display the molar masses for each element: the mass of one mole of the substance. The unit is typically grams per mole, or g/mol.

Example B.2.2 How much mass of CO_2 will emerge from the burning of 1 kg of ethanol? We start with the formula we worked out in Example B.2.1:

$$C_2H_6O \; + \; 3O_2 \; \rightarrow \; 2CO_2 \; + \; 3H_2O$$
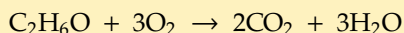
This problem can be approached in Two equivalent ways: either figure out how many moles of ethanol it takes to amount to 1 kg and then scale the formula accordingly; or just work it out for one mole to get a ratio and then apply to 1 kg. We'll do it both ways.

Since ethanol has a molar mass of 46 g, one kilogram corresponds to 21.7 moles. So we could re-write the formula as:

$$21.7C_2H_6O \; + \; 65.2O_2 \; \rightarrow \; 43.5CO_2 \; + \; 65.2H_2O$$
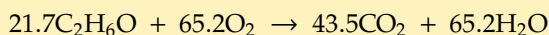
where we have multiplied each prefactor (coefficient) by 21.7. CO_2 has a molar mass of 44 g/mol, so 43.5 moles will come to 1.91 kg.

The other approach is to note that 2 moles of CO_2 are produced for every one mole of ethanol combusted. So 88 g of CO_2 (44 g/mol) results for every 46 g of ethanol supplied. This ratio is 1.91. So 1 kg of ethanol input will make 1.91 kg of CO_2 out, as before.

B.3 Chemical Energy

Atoms (elements) can bond together to make molecules (compounds). The bond—formed by outer electrons within the atoms—can be strong or weak. It takes energy[15] to pull apart bonded atoms. It stands to reason that when two atoms form a new bond, energy is released—usually as vibrations that we know as heat. In a typical reaction, some bonds are broken and other new ones formed. If the balance is that the new bonds are stronger than the broken bonds, energy will be released. Otherwise, energy will have to be put into the reaction to allow it to happen.

In the context of this book, chemical energy is typically associated with combustion (burning) a substance in the presence of oxygen. This is true for burning coal, oil, gas, biofuels, and firewood. In a chemistry class, one learns to look up the energetic properties of various compounds in tables, combining them according to the stoichiometric reaction formula

15: Recall that energy is a measure of work, or a force times a distance.

to ascertain a net energy value. We're going to take a shortcut to all that, by introducing the following *approximate* formula for combustion energy.

The *approximate* energy available from the compound $C_cH_hO_oN_n$—where the subscripts represent the number of each atom in the molecule to be burned—is:

$$100\frac{c + 0.3h - 0.5o}{12c + h + 16o + 14n} \text{ kcal/g.} \tag{B.1}$$

For instance, sucrose has the formula $C_{12}H_{22}O_{11}$, so that $c = 12$, $h = 22$, $o = 11$, and $n = 0$. The denominator in the formula is just the molar mass,[16] or 342 in this case. The numerator adds to 13.1, so that the result is 3.8 kcal/g—very close to the expected value around 4 kcal/g for a carbohydrate like sugar.

The numerator of Eq. B.1 tells us that we get the most energy from each carbon atom, 30% as much from each hydrogen atom, and take a 50% hit (deduction) for each oxygen atom. Nitrogen is energetically inert and does not contribute to the numerator—while degrading the energy density by adding mass in the denominator. The negative coefficient for oxygen tells us something important. Since combustion is a process of joining oxygen to atoms in the fuel, the presence of oxygen *already* in the fuel means it is already partly "reacted" and has less to offer in the way of new oxygen bonds.

We can explore the sensibility of Eq. B.1 by testing it on some known boundary cases.[17] Since one ubiquitous end-product of combustion is CO_2, calculating for CO_2 should offer no energy to us, since it's a "waste" product at the end of the energy process. H_2O, as another common combustion product, is likewise effectively neutralized in the formula (the result is at least made to be very small). Table B.1 provides some examples of what Eq. B.1 delivers for familiar carbon-based substances. Note that oxygen content (last column) drives energy down, while hydrogen offers a boost.

substance	formula	Eq. B.1 kcal/g	true kcal/g	% C	% H	% O
glucose	$C_6H_{12}O_6$	3.7	3.7	40	7	53
typ. protein	$C_5H_{10}O_3N_2$	4.4	~ 4	41	7	52
coal	C	8.3	7.8	100	0	0
typ. fat	$C_{58}H_{112}O_6$	9.8	~ 9	77	12	11
octane	C_8H_{18}	11.8	11.5	84	16	0
methane	CH_4	13.8	13.3	75	25	0

The resulting calculated energies are definitely in the right (expected) ranges. Notice that the "winners" have little or no oxygen as a percentage of the total molecular mass. The lower-energy entries in Table B.1 are more than half oxygen, by mass.

This empirical formula can serve as a general guide, but should not be taken as a literal truth from some profound derivation. It captures the main energy features and produces a useful, approximate result.

16: The coefficients in the denominator reflect the fact that carbon is 12 units of mass, oxygen is 16, etc.

17: This is a generically useful practice: it helps integrate new knowledge into your brain by validating the behavior in known contexts. Does it make sense? Can you accept it, or does it seem wrong/suspect? Experts often apply new tools first to familiar situations whose answers are known to build trust and competence using the new tool before applying it more broadly.

Try it out, using $c = 1$ and $o = 2$.

Try this one, too, coming up with your own values for h and o.

Table B.1: Example approximate chemical energies. The results of the approximate formula are compared to true values (favorably). Fractional mass in carbon, hydrogen, and oxygen also appear—emphasizing the penalty for molecules already carrying oxygen.

B.4 Ideal Gas Law

Another topic covered in chemistry classes that strongly overlaps physics is the ideal gas law. This relationship describes the interactions between pressure, volume and temperature of a gas. In chemistry class, it is learned as

$$PV = nRT, \tag{B.2}$$

where P stands for pressure (in Pascals[18]), V is volume (cubic meters), n is the number of moles, T is temperature (in Kelvin), and R is called the gas constant, having the value

$$R = 8.314 \; \frac{\text{J}}{\text{mol} \cdot \text{K}}. \tag{B.3}$$

To get degrees in Kelvin, add 273.15 (273 among friends) to the temperature in Celsius.[19] Standard atmospheric pressure is about 10^5 Pa.[20]

[18]: A Pascal (Pa) is also a Newton of force per square meter, which reduces to more fundamental units of J/m^3 (Joules of energy per cubic meter).

[19]: And $T(°\text{F}) = 1.8 \cdot T(°\text{C}) + 32$.

[20]: 1 atmosphere is 101,325 Pa.

Example B.4.1 Let's say we have a gas at "standard temperature and pressure" (STP), meaning 0°C (273 K) and 1.013×10^5 Pa. How much volume would one mole of gas[21] occupy?

We have everything we need to solve for volume, so

$$V = \frac{nRT}{P} = \frac{(1 \text{ mol})(8.314 \text{ J/K/mol})(273 \text{ K})}{1.013 \times 10^5 \text{ Pa}} \approx 0.0224 \text{ m}^3 = 22.4 \text{ L}.$$

Okay; lots going on here. After the three values in the numerator are multiplied, the only surviving unit is J (Joules of energy). The unit in the denominator is Pascals, but this is equivalent to Joules per cubic meter. So the answer emerges in cubic meters, as a volume should. Since a cubic meter is 1,000 liters, we find that a mole of gas at STP occupies 22.4 L—a number memorized by many a chemistry student!

[21]: It may be surprising, but the ideal gas law does not care what element or molecule we are considering!

Physicists prefer a variant of the ideal gas law that derives from the study of "statistical mechanics," which is practically synonymous with thermodynamics and relates to the study of interactions between large ensembles of particles. The form looks pretty familiar, still:

$$PV = Nk_\text{B}T. \tag{B.4}$$

Pressure, volume, and temperature are all unchanged, and expressed in the same units as before. Now, N describes the *number* of particles (quite large, usually), and k_B is called the Boltzmann constant, having a value

$$k_\text{B} = 1.3806 \times 10^{-23} \; \frac{\text{J}}{\text{K}}. \tag{B.5}$$

Notice that N, the number of particles, and n, the number of moles, differs simply by a factor of Avogadro's number, $N_\text{A} = 6.022 \times 10^{23}$. Indeed, if we multiply N_A by k_B, we get 8.314, and are back to R.[22]

[22]: The units work, too, since N_A effectively has units of a number (of particles) per mole.

Example B.4.2 Gas is stored at high pressure at room temperature in a metal cylinder, at a pressure of about 200 atmospheres.[23] The cylinder is designed to meet a safety factor of 2, meaning that it likely will not fail until pressure reaches 400 atmospheres. If a fire breaks out and the cylinder heats up, the pressure will rise. How hot must the gas get before the cylinder may no longer be able to hold the pressure (assuming no fire damage to the cylinder itself)?

We *could* start throwing numbers into the ideal gas law, but we don't know the volume or number of moles (or particles). Heck, we're not even given a temperature. Ack! Students hate this sort of problem, because it does not appear to be algorithmic in nature. No plug and chug (an activity that does not engage the brain heavily, and thus its appeal).

But we're okay. What is room temperature? Something like 20–25°C, so that's 293–298 K. Whatever the volume is, or the amount of gas in the cylinder, those things don't change as the temperature rises.[24] What we're left with is a straightforward scaling between temperature and pressure (because the numerical factors are all constant for our problem). Therefore, if temperature doubles, pressure doubles.[25]

Hey, it's doubling pressure that we are interested in, which will happen if the temperature doubles. So if the temperature goes up to about 600 K, we may be in trouble. It is easy to imagine that a fire could create such conditions. Notice that we are not bothering to say 586–596 K, but just said about 600 K. Do you want a precise temperature when the thing will rupture? Good luck. The point at which it explodes may be 405 atmospheres or it may hold on until 453. Also, how likely is it that all the gas throughout the cylinder is at exactly the same temperature when being heated by a nearby fire? So let's give ourselves a break and not pretend we're totally dialed in. There's a fire, after all.

23: ... means 200 times atmospheric pressure

This is an example where internalizing the ideal gas law for what it *means*, or what it *says* is more important than treating it like a recipe for cranking out problems. Don't just treat equations as mechanical objects: learn what it is they have to *say*!

24: The gas is not leaking out, and the cylinder does not change size—at least not significantly—as it warms.

25: That's one of the things Eq. B.4 is trying to say, beneath all the bluster.

Numerical answers are given as ranges or other hints, meant to facilitate checking for gross departures from the right track, without revealing the precise answer so that shortcuts are discouraged. Questions for which the answer is already known (questions asking to verify an answer), easily validated in the text, or that are a matter of original thought or opinion may not be included here.

The ranges sometimes may be annoyingly large, but think of them as guard rails to prevent a tragic miscalculation or to catch a fundamental misunderstanding of the underlying concepts. It can help catch errors like dividing the wrong things or swapping numerator and denominator, or multiplying when division is called for. In many cases, intuition, or guessing, might lead you already to similar answers or ranges. With practice, students may be able to anticipate what they think are reasonable ranges for answers. In fact, it is a great practice to think about expectations before working on the problem.

This appendix, then, might be thought of as an "intuition implant" that simulates how problems are for experts. Real life does not provide "answers at the back of the book," so experts rely on experience, intuition, and a sense for "reasonable" results to help them understand when they've taken a wrong turn. A successful use of this appendix would help train students to develop their own "common sense" guard rails.

Chapter 1

4. Between 250 and 300 years

5. Between 250 and 300 years

6. Between 10^{18} and 10^{20}

7. Between 100 and 150

8. More than 10^{40}

9. Between 5 and 500

10. Later than 23:50

11. Before 12:10 AM

12. Between 10 and 40 years

14. Between 75 and 100 years

15. Between 300 and 500 billion

16. It's not 100 times longer

17. A few millennia

19. Between 50 and 100 W

20. A smidge higher then boiling

21. Between 200 and 250 K

22. Between 150 and 250 K

23. Between 100 and 275 K

Chapter 2

1. Nearly $100 billion

2. 4%: ~$1 trillion; 5%: more than $100 trillion

4. On the low end of advanced countries

5. Between 25 and 100 MJ/$

6. Between 5% and 50%

7. The text had trading art, singing lessons, therapy, and financial planning

8. Between 100 and 1,000; Less than 10 to go

9. Between 20 and 100 years

10. May help to think of something once prevalent, now rare

11. Especially fruitful might be biological dependencies

14. It can't all be free of material substance

Chapter 3

1. Less than a third

3. Comparable to U.S. population today

4. Comparable to world population 200 years ago

5. Table 3.2 offers a rough check

6. Over 16 billion; less than half the time we now experience

7. Pretty close to Table 3.2 except for first two entries

8. Answer *must* be less than 14 billion; whereas Problem 6 was in excess of 15 billion

9. Two are negative; three are positive

12. Between 1 and 5%

13. Add almost a half million; more than half million born; less than half million died

14. Answers should round to the table values

16. Only one country in the table creates more total demand, and only two have higher per-citizen contributions

17. Correct results are in the table

19. See Figure 3.15

20. This is why Africa gets attention, while North America is perhaps a greater concern.

21. It nearly triples

23. Area is key

25. Lesotho is relevant

Chapter 4

1. Earth: smaller than peppercorn and basketball-court distant; Moon: sand grain a hand's width away

3. Comparable to the actual Earth radius

5. 1 AU = 1 km; Earth 1/12,000 km

6. A fast walk or slow jog

7. Think about the subtended angle

8. Multiply sets to get accumulated scale factors

9. A good deal farther than the moon, but still well short of the sun/Mars

10. Ratio is more than a billion, and would take more than 4 lifetimes

16. Think in terms of area as fraction of plot space

17. Text has climbing Mt. Everest, supersonic commercial flight, squirrel obstacle course, and economic decoupling

18. Will take 15–20 tanks of gas, and achieve a fuel economy a factor of 30 or so below typical cars

19. Double the gasoline from previous problem; gasoline mass almost as much as the car itself

Chapter 5

1. Several inches

2. About the length of a typical room

3. A few kJ total, most in sliding

4. You've got this

6. Nearly 1 GJ

7. Two of the points in Example 5.2.1 offer guidance

8. Figure 5.1 offers hints

9. Roughly half human metabolic power

10. Less than 5 seconds

12. Between 50–100 kcal (200–400 kJ)

13. Sensibly, a little less than 2 minutes

14. Results should be roughly consistent with Figure 5.2

15. A bit less than 10% of household electricity

16. They're actually close, within 10%

17. More than two

18. Several kWh; less than $1

19. On the low end of the human metabolism range; the equivalent cost of 10–20 burritos

20. A few hundred W

21. Comparable to running a clothes dryer (Fig. 5.2)

22. Several hundred MJ

23. Over 100 kWh; 2–4 burritos-worth

24. Several Therms; cost of fast-food lunch

25. Several gallons; cost of fast-food lunch for two

26. One is about twice the other

27. A little in excess of 10 kW

28. Less than a quarter of estimated

29. Between 1 and 2 hours per day

30. The largest number is near 10^8

31. Six of the entries are inverses of six others

32. A little over an Amp

33. Nearly 10 kW; will cost over $1,000; don't do this!

34. Will last 2–3 hours

35. In line with most chemical reactions, in the 50–200 kJ/mol range

36. Between 3–5×10^{-19} J per photon; get more than 10^{18}/sec

37. In the neighborhood of 1 μm or 1 eV

Chapter 6

1. Approx. 200 kJ, depending on mass

2. Several minutes

3. Several minutes

4. About 5 minutes

5. A few hours

6. Not below freezing

7. Not quite up to "room" temperature

8. Not quite half the time

9. The cost of two burritos per day

10. Instances of heat/flame causing movement

11. See Table 6.2

12. Roughly 30 kJ and 100 J/K

13. Between 5–10%

14. A couple dozen percent, roughly

15. Pushing 100%, but not quite there

16. Achieves about 1/3 of theoretical

17. $\Delta T > 50°C$; environment not that cold

18. Close to a dozen kJ

19. Twice, twice

20. Just short of 5 years

Chapter 7

1. a) between 30–40%; b) almost all; c) close to 2/3; d) roughly a quarter

2. Coal is near 12 qBtu, for instance

3. Nuclear is about 22%, for instance

4. Residential is about 5 qBtu, for instance

5. Industry is a little over 30%, for instance

6. About 14% is renewable, for instance

7. Less than 10%

8. Between 5 and 10%

9. It is one of the fossil fuels

10. Well over 100 years

11. Surprisingly soon: maybe before student loans paid off

12. Nothing to see here

13. Nothing to see here

14. Pay attention to the dashed line

15. Pay attention to the dashed line

Chapter 8

1. All lines overlap the up-slope

2. Likely vs. hopeful?

3. Many features unchanged

4. Won't be zero into future

5. What enabled, then disappeared?

6. Opposite of ideal

7. Did not behave like U.S.

8. Based on energy density

9. Roughly one-third

10. 2 H per C plus 2 more

11. In the neighborhood of 20 bbl/yr

12. Should be appropriate fraction of 10,000 W total

13. A little over 100 MJ and a few dozen kWh

14. Sum to about 15 kg, which would fill a refrigerator shelf in the water-bottle equivalent.

15. drinking glass

16. A few dozen times more volume, and about 10^2 in mass

17. $> 1,000\times$ more expensive

18. Will cost nearly $1,000

19. Between 10–15%

23. Approximately half-century

24. Roughly a third

25. If the rate of production increases. . .

26. What have you wanted that was all gone?

27. Shorter than R/P suggests

28. Opposite of virtual

29. Can't have what's not there

30. Reasons could fill a book

Chapter 9

1. A single integer works okay for all three

2. Nearly 100 kg

3. Between 10–15 kg

4. Approaching 1 GJ, and human-mass scale

5. Total is like small adult or large child

6. More than a factor of two

7. Get about 50 years; rate not constant

8. The numbers basically match

9. Between 1–2 ppm_v, in agreement with Figure 9.3

10. What is it we *know*?

11. Seems deserving of high marks

12. Historical vs. current activity levels

13. About 10°C cooler than actual

14. Two pure cases and one partial

15. Several degrees warmer

16. Very good for us at the right level

17. Numbers are not far from realistic

18. Triple pre-industrial and almost 5°C

19. End ~3°C high; almost linear, but not quite

20. No need to balance: Nature doesn't bother

21. It's no game-changer

22. Student's choice

23. E.g., $390 - 152 = 238$ for a match

24. Use 290.6 K; looks like continuation of panel progression

25. A few millimeters

26. A little over a century

27. A year or two

28. A couple of degrees

29. Sum to about 700 years; almost all in ice and ocean

30. A few hundred meters

31. A finger's breadth per year

32. Keen to hear your thoughts

33. Keen to hear your thoughts

Chapter 10

1. Mostly clean; not all, though

2. Nothing is free

3. What would unlimited mean?

4. Table 10.2 has some help

5. Can't rely on any sun-driven energy

6. Between 200–250 W/m^2

7. Photosynthesis supports essentially all life

8. Comparing numbers in TW

9. More than half

10. A little less than 1%

11. Not far from 1,000 W/m^2

12. Nearly 10 degrees

13. Look for crazy-big input

14. Between 0.5–1 gallon

15. More than 4,000×

Chapter 11

1. Roughly 20 kJ

2. About 10 stories of a building

3. Close to 0.1 kJ

4. About 4 times higher than airliners travel

5. About two-thirds Earth radius

6. Try using half the mass and half the energy

7. Cube is roughly as big as height from ground

8. About 6 times typical nuclear plant

9. Nearly 200 m

10. A little shy of 500 m^3/s

11. Between 50–75%

12. Roughly 50%

13. About a million homes

14. Approaching 10,000 cubic meters per second

15. You've got a little over an hour

16. Less than 1 TW in the end

17. Between 1–2 meters

Chapter 12

1. A few Joules

2. Roughly 1°C

3. Something like 10 m/s

4. Mass shows up in both mgh and $\frac{1}{2}mv^2$

5. In the neighborhood of 1,500 m/s

6. About 5–10 humans–worth of mass!

7. Comparable to the height of Mt. Everest

8. Around about 8 times

9. Follow the cube. . .

10. Runs approximately 10 kW to 1 MW

11. Roughly two-thirds the original speed

12. Close to 10 MW

13. Closer to 10 m/s than to 15 m/s

14. Almost double freeway speeds

15. Between 5–10 m/s

16. In the ballpark of 70 kW

17. Recover 0.65%

18. Unpack W/m^2 to confirm kg/s^3

19. Outer box area corresponds to running at 100%, full time

20. Definitely less than 50%

21. Looks like a factor of 8

22. Approaching (American) football field length

23. Approximately 1 MW

24. They may not have equivalent energy needs

Chapter 13

1. How big are the packages?

2. Something times 10^{21}

3. Roughly 10^{16}

4. About 1,000

5. About 4,000 times

6. Use Eq. 13.3 to guide your reasoning

7. Should match Figure 13.1

8. One micron for each finger?

9. Think about spill-over into UV and/or IR

10. Peak around 2.5×10^8, about 1 μm wide; matches up well

11. Think energetics and depth

12. Is the answer transparent?

13. Just comparing two energies

14. Several hundred km/s

15. Condense the saga to that of a winner

16. Answer might involve physics, biology, rooftops

17. Inversely: larger in one means smaller in the other

18. Already extremely similar

19. Think of current as a rate of electron flow in the circuit

20. Get very close to 1,360 W/m^2

21. Sweltering is not preferred

22. Between 5–6 kWh/m^2/day; between 200–250 W/m^2

23. Involves interpreting kWh/m^2/day as full-sun-hours

24. Not far from 200 W/m^2

25. Range straddles 200 W/m^2, varying about 10%

26. Best at latitude; almost 15% better than flat

27. Approaches 6 kWh/m^2/day

28. Large house (and just the PV for one person)

29. Square is about as wide as Arizona or California east-to-west

30. Cost, surely—but other challenges and mismatches as well

31. A little over 200 W

32. Roughly the size of a bedroom

33. Will spend a little over $4,000

34. A little over a decade

35. Even lower than ~20% from insolation vs. overhead

36. About $2-worth of sun

37. Hint: study Figures 13.23 and 13.24

38. In absolute terms. . .

Chapter 14

1. About a dozen tons of CO_2

2. Between 0.1–0.5%

3. Almost 100 logs per person per year

4. In the neighborhood of half-dozen logs per day

5. Won't be exactly 15 years, but close

6. Almost 1.5 L of ethanol

7. Roughly consistent with Table 14.1 for coal

8. Net is one-third production

9. Extra land is twice the yield-land

10. A bit longer than a U.S. Presidential term

11. Nothing to spare

12. Corn now approximately 15% as much as this; still more than total arable land

13. Box barely fits north–south in U.S.

14. Personal preferences play a role

Chapter 15

1. A blueberry

2. $N = 8$

3. Use $Z = 26$ to get there

4. Should match quite well

5. Two diagonals have no gray squares

6. One has a half-life longer than a million years

7. Roughly twice as old as agriculture

8. Between 1–2%

9. One is about 3% of the other (both decay)

10. Step right

11. That last step might take a while

12. Two decays do it

14. Sand does the job

15. Somewhere between a car and a bus?

16. Close to 1 kg

17. Around a couple-dozen micrograms

18. Table should match

19. Energy has a mass, via $E = mc^2$

20. Adds about 1% to the mass

21. Not far from 1,500 MeV

22. Not much

23. Figure 15.14 is relevant

24. It's a strontium isotope

25. A is twice a prime number

26. Stick to $80 < A < 110$ and $125 < A < 155$ to respect distributions

27. Mid-20s of MeV

28. From steam onwards, it's basically the same

29. Between 3 and 5 cents per kWh

30. A few per week!

31. Around 20 tons per year (more in reality)

32. Almost 2 million tons

33. A few hundred tons

34. Less than a decade

37. Two stand out

38. Centuries

39. More often than once every two years

41. A nearly exact match!

42. Worked out in text: no calculation necessary—just interpretation

43. Energy jump size

44. Like a milk jug

Chapter 16

1. Shortfall is more than a factor of 200

2. A bit farther than the record

3. About as thick as a six-story building is tall

6. Ranges about 55–85%

7. Geothermal is a bit less than 1% of alternative electricity

8. A little shy of the 8 m design height, sensibly

9. Works out

10. Diameter like a small house's footprint

11. Comparable to human metabolism; 1% of American demand

12. A little more than 6 times that in Example 16.4.1

Chapter 17

3. Algae who?

5. Two words almost say it all

6. Fine if it is a little shy: transfer rates vary

7. Think about what a house *can* access, and steam plants

10. May be up there with solar (4 to 6, likely)

Chapter 18

1. 16 equal portions

2. Predicts largest well; not *too* far on smallest

4. Gaping disparities on opposite poles is no random fluke

6. Brilliant future if you can figure out effective ways

7. How else will change happen? (but elaborate. . .)

8. Will contribute 2–3% of the annual total

9. A bit over half the global energy budget!

10. Two approaches: cynical or hopeful; make either pitch

11. In the hundreds

12. I was hoping you had some ideas

Chapter 19

1. Still could be a parasite, even if larger than a flea

3. Easier to break than make

4. What things are dependent on growth to operate normally?

5. What limits?

7. Does it bear on humanity in some way?

8. Wait; who has my. . .

9. Focus on what has mattered until now

10. What's the alternative?

13. Please figure out how it can work!

14. Is this the movie version, or the real-life one?

Chapter 20

2. What needs to happen to avert?

3. Focus on demonstrable new conditions that likely push limits

4. Obligations of reality?

5. Some things are out of our control

7. What type of activity tends to consume a lot of power?

8. Duty cycle

9. Proportional to ΔT

10. Gasoline is about 4 times the other two

11. Just a bit less than average in all categories

12. Close to twice the gas is used in the form of electricity

13. Both in the same neighborhood

14. Big disparity; which is more likely?

15. S.U.V. might not make the cut, but smaller cars will

16. Surprisingly far: almost two-thirds of the way

17. Is six-sevenths a coincidence?

18. As if one day a week is all dairy/eggs

19. Is it directed or emergent?

20. Think frivolous or huge resource demand

21. Do your best: might prevent the worst

22. Can you even tell the needle isn't at full?

Alluring Tangents | D

This Appendix contains tangential information that may be of interest to students, but too far removed from the main thread of material to warrant placement within chapters. Many of these items were prompted by student feedback on the first draft of the textbook, wanting to know more about some tantalizing piece mentioned in the text. Pick and choose according to your interests.

D.1 Edge of the Universe

Sec. 4.1 (p. 54) built a step-wise scale out to the edge of the visible universe, which a margin note clarifies as the visible horizon of our universe. This fascinating and deep concept deserves elaboration.

Two foundations of experimental physics and cosmology are that the speed of light is finite, and the universe began in a Big Bang 13.8 billion years ago. Ample evidence supports of both claims. It should be noted that these notions were not at all accepted by scientists until the preponderance of evidence left little choice but to adopt them as how the world really appears to work.

The finite speed of light means that looking into the distance amounts to looking back in time. Here, Imperial units have a brief moment of glory, in that every foot of distance (0.3 m) is one nanosecond of time. We see the moon as it was 1.25 seconds "in the past," the sun as it was 500 seconds (8.3 minutes) ago, and the nearest star 4.2 years back. The "nearby" Andromeda Galaxy is 2.5 million years in the past, and as we peer farther into the universe we look ever farther back in time. Indeed, at great distances we see infant galaxies in the process of forming as gravitational vacuum cleaners collecting materials from the diffuse gas that came before.

So what happens when we look 13.8 billion years into the past, when the Big Bang is alleged to have happened? Shouldn't we see the explosion? And shouldn't it—perhaps confusingly—be visible in all directions?

The answer is a resounding, though qualified, YES. Yes, we see evidence of the Big Bang in all directions, as a glow that appears in the microwave region of the electromagnetic spectrum. The Cosmic Microwave Background, or CMB, as it is called, represents the glowing plasma when the universe was just 380,000 years old and about 1,100 times smaller than it is today. We cannot see earlier than this because the hot ionized plasma

that existed before this time is opaque[1] to light travel. The universe only became "clear" after this time, when the plasma cooled into neutral (mostly) hydrogen atoms. So we can see *almost* back to the Big Bang, at least 99.997% of the way before the scene becomes opaque.

So that's the limit to our vision, based on the idea that light has not had time to travel farther since the universe began. This is what we mean by the edge of the *visible* universe.

But is it a real edge? All indications are that it is certainly not. When we look 13.8 billion light years away, we just see this glowing plasma (the CMB). But in the intervening years, galaxies and stars and planets have formed in that region of space, and would appear "normal," or mature today. So imagine a being on such a planet looking at us today, 13.8 billion light years distant. But they see us 13.8 billion years ago, when our neighborhood was still a glowing plasma well before the formation of galaxies, stars, and planets.

Let's say the distant being is directly behind you and you are both looking off in the same direction—the alien essentially looking over your shoulder as you look directly opposite the direction to the alien. You (or the primordial gas that will someday become you) are at the limit of their vision, and they can't see anything beyond you. You sit at their edge. But to you it's no edge. You have no trouble seeing more "normal" universe stretching another 13.8 billion light years beyond what our distant friend can see. It's only a perceived edge, based on the limit of light travel time.

A nice way to think of it is familiar scenes of limited vision, like in a fog or in the ocean, or even on the curved surface of Earth. All cases have a horizon: a limit to the distance visible. Yet moving to the edge of vision reveals a whole new region that was before invisible. Keep going and your starting region will no longer be visible, or within your horizon. But it has not ceased to exist.

Similarly, the universe would seem to be much larger than our visible horizon. Measurements of the "flatness" of the geometry of space suggest a universe that is *at least* 100 times larger than our horizon, and may in fact be *unfathomably* larger. We may never know for sure, as limits to light travel seal us off from direct observation of most of the universe.

1: ... much as the sun, another plasma is opaque: we only see light from its outer surface

D.2 Cosmic Energy Conservation

Sec. 5.2 (p. 69) discussed the foundational principle of the conservation of energy, claiming that the principle is never violated except on cosmic scales. Besides elaborating on that point, this section follows the story of energy across vast spans of time as our sun forms and ultimately delivers energy to propel a car. We also clarify what it means for energy to be "lost" to heat.

D.2.1 Cosmological Exception

Emmy Noether was a leading mathematician in the early twentieth century who also dabbled in physics. In a very profound insight, she recognized a deep connection between symmetries in nature and conservation laws. A *symmetry*, in this context, is a property that looks the same from multiple vantage points. For instance, a sphere is symmetric in that it looks the same from any angle. A cylinder or vase also has symmetry about one axis, but more limited than the sphere.

The symmetries Noether considered are more subtle symmetries in time, space, and direction.

> **Definition D.2.1** *Symmetry in time means that physics behaves the same at all times: that the laws and constants are the same, and an experiment cannot be devised that would be able to determine absolute time.*
>
> *Symmetry in space means that the laws of physics are the same no matter where one goes: fundamental experiments will not differ as a function of location.*
>
> *Symmetry in direction is closely related to the previous one. It says that the universe (physical law) is the same in every direction.*

The insight is that these symmetries imply conservation laws. Time symmetry dictates conservation of energy. Space symmetry leads to conservation of momentum. Directional symmetry results in conservation of angular momentum.

Great. As far as we know, the latter two are satisfied by our universe. To the best of our observational capabilities, the universe appears to be *homogeneous* (the same everywhere) and *isotropic* (the same in all directions). Yes, it's clumpy with galaxies, but by "same," we mean that physics appears to act the same way. Therefore, substantial observational evidence supports our adopting conservation of momentum and conservation of angular momentum as a fact of our reality.

But time symmetry is a problem, because the universe does not appear to be the same for all time. It appears to have emerged from a Big Bang (see Section D.1), and was therefore much different in the past than it is now, and continues to change/evolve. An experiment to measure the effective temperature of the Cosmic Microwave Background is enough to establish one's place on the timeline of the cosmic unfolding.

As a consequence, conservation of energy is not strictly enforced over cosmological timescales. When a photon travels across the universe, it "redshifts," as if its wavelength were being stretched along with the expansion of the universe. Longer wavelengths correspond to lower energy. Where did the photon's energy go? Because time symmetry is broken in the universe, the energy of the photon is under no obligation to remain constant over such timescales. Deal with it, the universe says.

On timescales relevant to human activities, conservation of energy is extremely reliable. One way to put this is that the universe is 13.8 billion years old, or just over 10^{10} years. So in the course of a year, physics would allow an energy change by one part in 10^{10}, or in the tenth decimal place. Generally, this is beyond our ability to distinguish, in practical circumstances.

But violations are even more restricted than that. A photon streaming across the universe is in the grip of cosmic expansion and bears witness to associated energy changes. But a deposit of oil lying underground for 100 million years is chemically bound and not "grabbable" by universal expansion, so is not "degraded" by cosmic expansion. In the end, while we acknowledge that energy conservation is not strictly obeyed in our universe, it might as well be for all practical purposes. Thus, this section amounts to a tiny asterisk or caveat on the statement that energy is always conserved.

D.2.2 Convoluted Conservation

This section follows a chain of energy conversions that starts before our own Sun was formed, and ends in a car wreck as a way to flesh out the manner in which energy is conserved, in practice. Don't worry about understanding every step, but absorb the overall theme that energy is changing from one form to the other throughout the process.

A gas cloud in space collapses due to gravitational attraction, exchanging gravitational potential energy into kinetic energy as the gas particles race toward the center of the cloud. The cloud collapses into a tight ball and all that kinetic energy in the gas particles "thermalizes" through collisions,[2] generating a hot ball of gas that is to become a star. As the ball of gas contracts more, additional gravitational potential energy is exchanged for thermal energy as the proto-star gets hotter.

Eventually, particles in the core of the about-to-be star are moving so fast as they heat up that the electrical potential barrier[3] is overcome so that protons can get close enough for the strong nuclear force to take over and permit nuclear fusion to occur, at which point we can call this thing a *star*. Four protons[4] bond together, two of which convert to neutrons to form a helium nucleus. The total mass of the result is *less* than the summed mass of the inputs, the balance[5] going into photons, or light energy.

The photons eventually make it out of the opaque plasma of the star, and stream toward Earth, where a leaf absorbs the energy and cleverly converts it to chemical energy by rearranging atoms and electrons into sugars.[6] The leaf falls off and eventually settles at the bottom of a shallow sea to be buried by sediments and ultimately becomes oil, preserving most[7] of its chemical energy as it changes molecular form.

2: Thermal energy is nothing more than kinetic energy—fast motion—of individual particles at the microscopic scale. Thermalizing means transferring energy into heat, or into randomized kinetic energy of particles in the medium.

3: . . . charge repulsion of two protons as their collision course brings them close to each other

4: . . . hydrogen nuclei

5: . . . via $E = mc^2$; see Sec. 15.3 (p. 246)

6: We call this photosynthesis.

7: To the extent that energy is "lost" in any of these exchanges—operating at $< 100\%$ efficiency—we should recognize that the missing energy just flows into other paths, generally into heat.

One day, a silly human digs up the oil and combusts it with oxygen, converting chemical energy to thermal energy in a contained fireball explosion. The thermal energy is used to produce kinetic energy of a piston in a cylinder, transmitted mechanically to wheels that in turn propel a car along a freeway.[8]

8: . . . kinetic energy

The car climbs a mountain, converting chemical energy in the fuel into gravitational potential energy via the same thermal-to-mechanical chain described above. Along the way, kinetic energy is given to the air, thermal energy is given to the environment by the hot engine, and brakes get hot as the kinetic energy of the car is converted to heat via friction as the car comes to a stop. But the car does not fully stop in time before tipping over a cliff and giving up its gravitational potential energy to kinetic energy as the car plummets and picks up speed.

At the bottom, the crunch of the car ends up in bent metal[9] and heat. It does not explode, since this is not a movie. All the heat that was generated along the way ends up radiating to space as infrared radiation (photons), to stream across empty space—probably for all eternity.

9: . . . a form of electric potential energy

Tracing the energy we use for transportation back far enough, passes through oil, photosynthesis, sunlight, and nuclear fusion in the sun's core. Going back further, we recognize nuclear energy as deriving from gravitational energy of the collapsing material. What gave the atoms in the universe gravitational potential energy? The answer would have to be the Big Bang, truly arriving at the end (beginning) of the story.

D.2.3 Lost to Heat

The sequence in Section D.2.2 terminated in heat and infrared radiation. But let's flesh this out a bit, as heat is an almost-universal "endpoint" for energy flows.

Since energy is conserved, whatever goes to heat does not truly disappear: the energy is still quantifiable, measurable energy. It is considered to be "low grade" energy because it is hard to make it do anything useful, unless the resulting temperature is significantly different from the surroundings. Sec. 6.4 (p. 88) discusses notable exceptions, wherein we derive substantial useful work from thermal energy in heat engines.[10] For now, we just note that it is entropy that limits the use of thermal energy.

10: In this way, we can make an explosion or fireball do useful work as in dynamite, internal combustion, or a coal-fired power plant.

When a book slides across a floor, it gives up its kinetic energy to heat caused by friction in the floor–book interface. When a car applies brakes and comes to a stop, a very similar process heats the brake pads and rotors. As a car speeds down the road, it stirs the air and also experiences friction in the axle/bearings and in the constant deformation of the tire as its round shape flattens on the road continuously. The stirred air swirls around in eddies that break up into progressively smaller ones

until at the millimeter scale viscosity (friction) turns even this kinetic motion into randomized motion (heat).

Human metabolism converts chemical energy from food into exportable mechanical work (lifting, moving, digging, etc.) at an efficiency of 20–25%. The rest is heat, which is conveniently used to maintain body temperature in most environments. But even most of the external work performed ends up as heat. The primary exception may be lifting masses to a higher location. Even this is temporary in the very long run,[11] and the stored energy will ultimately flow into heat.

11: The "shelf" we place the mass on eventually collapses or is otherwise disturbed.

Light from our artificial sources and screens survives for a few nanoseconds as photon energy, but eventually is absorbed onto surfaces and turns to heat. Some small fraction of our light escapes to space and carries non-thermal energy away, but this is incidental and could be said to represent poor design (not putting light where it is useful).

Even devices whose job it is to *cool* things are net generators of heat. The air pushed out the back and bottom of a refrigerator is warm, as is the exhaust from an air conditioning unit. Virtually all energy pulled out of an electrical wall socket ends up as heat in the room in some way or another. A fan actually *deposits* a little bit of energy (heat) into the room, but feels cool to us only because the moving air enhances evaporation of water from our skin (perspiration), carrying energy away.

Essentially the only exceptions to the heat fate of our energy expenditures is anything that we launch into space, like electromagnetic radiation (radio, light). This is a very tiny fraction of our energy expenditure, and can be quantitatively ignored. Most of our energy is from burning fossil fuels, which is an inherently thermal process. The part we salvage as useful energy itself tends to end up as heat after serving its intended purpose.

In the end, most of the heat we generate on Earth's surface finds its way back to space as infrared radiation. All objects glow in the infrared, and once the radiation escapes our atmosphere it is gone from Earth forever.[12] At this point, the energy is pretty well spent, so that we would not be able to profit from its use should we try to capture it.[13] The energy that came from the universe returns there, as part of the dull, fading glow that lingers from the Big Bang.

12: ...except for some improbable paths that reflect off the moon, for instance, and return to Earth

13: The temperature of the radiating entities is so close to ambient temperature that its efficiency to perform useful work would be nearly zero.

D.3 Electrified Transport

This section aims to answer the question: Why can't we *just*[14] electrify transportation and be done with fossil fuels? It turns out to be hard. Rather then rely on external studies, this section applies lessons from the book to demonstrate the power of first-principles quantitative assessment.

14: ...beware of the word "just," often hiding lack of familiarity

Box 13.3 (p. 212) indicated that direct drive of cars and airplanes from solar energy is impractical: while it may work in limited applications, solar power is too diffuse to power air and car travel as we know it.

Thus electrified transport becomes all about **storage**, generally in batteries. Several times in the book, the energy density of gasoline was compared to that of battery storage. In rough numbers, gasoline delivers about 11 kcal/g, working out to ~13 kWh/kg in units that will be useful to this discussion. Meanwhile, lithium-ion batteries characteristic of those found in cars[15] have energy densities about one-hundred times smaller.

15: The larger Tesla battery pack, for instance, provides 265 miles (425 km) of range and holds 85 kWh at a mass of 540 kg for an energy density of 0.16 kWh/g.

This section will use the most optimistic energy density for lithium-ion batteries—around 0.2 kWh/kg—which is about 65 times less than for gasoline. Offsetting this somewhat is the fact that electric drive can be as high as 90% efficient at delivering stored energy into mechanical energy, while the thermal conversion of fossil energy in large vehicles is more typically 25%. The net effect is roughly a factor of twenty[16] difference in delivered energy per kilogram of fuel vs. storage.

16: The math goes: 13 kWh/kg divided by 0.2 kWh/kg times 0.25/0.90, yielding a factor of 18. For the sake of estimation, 18 is close enough to a factor of 20 to use the more convenient and memorable 20× scaling factor in what follows.

The enormous mismatch in energy density between liquid fossil fuels and battery storage is the crux of the problem for transportation, the implications of which are explored here. We will start at the hard end, and work toward the easier.

D.3.1 Airplanes

Box 17.1 (p. 290) already did the work to evaluate the feasibility of powering typical passenger planes electrically. The result was a reduction in range by a factor of 20, consistent with the premise above: the best lithium-ion technology—not yet achieved in mass-market—at 90% efficiency delivers about 5% as much mechanical energy per kilogram as do liquid fossil fuels.

Keeping the same 15 ton[17] "fuel" mass, but now at 0.2 kWh/kg results in a 3,000 kWh battery capacity. The factor-of-twenty energy reduction per mass results in ranges down from 4,000 km via jet fuel to 200 km on battery, which is a two-hour drive, effectively. Charging a 3,000 kWh battery in the 30 minutes it typically takes for a plane to turn around—in efficient operations, anyway—would consume 6,000 kW, or 6 MW of power, which is about the same as the average electricity consumption of 5,000 homes.

17: One metric ton is 1,000 kg, and is often spelled tonne. Here, ton is used to mean metric ton, which is only 10% larger than the Imperial "short ton."

We will keep track of kWh per kilometer as a useful metric for transportation efficiency, putting it all together at the end (Section D.3.7). In the case of air travel, it's 3,000 kWh to go 200 km, or 15 kWh/km. On a per-passenger basis, 150 passengers in the airplane results in 0.1 kWh/km/person.

D.3.2 Shipping

Large container ships ply the seas carrying stacks of shipping containers over very long stretches of open ocean. A typical ship operating between Shanghai and Los Angeles travels 10,400 km carrying 10,000 20-foot equivalent[18] containers each bearing an average of something like 10 tons of cargo. Thus, the full (maximum) load is 100,000 tons.[19]

At normal cruising speed, the ship takes 10 days to make the journey, consuming about 325 tons of fuel per day. A battery large enough to replace 3,250 tons of fuel would be 20 times more massive, at 65,000 tons, displacing two-thirds of the cargo capacity, and requiring triple the number of ships to carry the same cargo. The resulting 13,000,000 kWh of storage[20] to travel 10,000 km results in 1,300 kWh/km.

The open ocean has no refueling stations. Even a refueling ship/platform would have to get the electrical energy from *somewhere*. Thus, shipping would be radically changed if electrified. Electric ships may not be able to cross open ocean, instead hugging the coast dotted with power plants[21] to supply frequent and lengthy charge stops for the ships.

D.3.3 Long-haul trucking

Typical "big rigs" on the highway achieve a fuel economy around 6 miles per gallon (40 L/100 km) of fuel, while the most aerodynamic ones achieve 8 mpg (30 L/100 km). Long haul rigs carry two fuel tanks, each holding about 150 gallons (570 L; 425 kg). The range for the more efficient trucks therefore becomes about 2,000 miles (3,200 km).[22] Cargo capacity is about 20 tons.

Total fuel mass is 300 gal times 2.85 kg/gal,[23] or about 850 kg. The same mass of battery would hold 170 kWh and deliver a range of 100 miles (160 km; roughly 1 kWh/km). Ugh. Lots of recharging stops.

But wait, trucks are big, right? Surely a larger battery can be accommodated. Unlike airplanes, where mass is critical, trucks can afford to pack on a larger battery. Some of the cargo space could be devoted to energy storage, surely. What fraction of the space would be acceptable?

To achieve comparable range as is presently realized, the battery mass would need to be about 20 times the gasoline mass, or 17,000 kg. Oh dear—the maximum cargo load was about 20 tons. So 85% of the cargo capacity is taken up by battery, which would seem to be unacceptable.

A solution would be smaller batteries and more frequent charging stops—possibly in the form of forklift-loaded pre-charged modules that are owned by the trucking company and can be interchanged among the fleet. Otherwise a substantial fraction of time would be spent charging: very possibly more time than is spent driving.

18: . . . TEU: 20-ft-equivalent units

19: . . . called DWT: dead-weight tonnage

20: This is 13 GWh, which would take the equivalent of an entire 1 GW power plant 13 hours to charge—or longer considering imperfect charge efficiency.

21: . . . and from what source do *they* get energy? . . . picturing outposts on the remote Aleutian Islands

22: . . . not using 100% of capacity to leave some prudent reserve

23: . . . density, in unusual units; equivalent to ~0.75 kg/L

It is not impossible[24] to electrify long-haul trucking, but neither is it free of significant challenges. Certainly it is not as easy and convenient as fossil fuels.

D.3.4 Buses

Like cargo ships and long-haul trucks, public transit buses are on the go much of the time, favoring solutions that can drive all day and charge overnight. Given the stops and breaks, a typical bus may average 30 km/hour and run 14 hours per day for a daily range of approximately 400 km. At an average fuel economy of 3.5 mpg (70 L/100 km), each day requires about 300 L or 220 kg of fuel—no problem for a fuel tank. The equivalent battery would need to be 4,500 kg (900 kWh; 2.3 kWh/km), occupying about three cubic meters. Size itself is not a problem: the roof of the bus could spread out a 0.15 m high pack covering a 2 m × 10 m patch. Buses typically are 10–15 tons, so adding 4.4 tons in battery is not a killer.

Electrified transit is therefore in the feasible/practical camp. What makes it so—unlike the previous examples—is slow travel, modest daily ranges, and the ability to recharge overnight. Raw range efficiency is low, at 2.3 kWh/km, but this drops to a more respectable 0.2 kWh/km per person for an average occupancy of 10 riders.

For charging overnight, a metropolitan transit system running 50 routes and 8 buses per route[25] and therefore needs to charge 400 buses over 6 hours at an average rate of 150 kW per bus[26] for a total demand of 60 MW—equivalent to the electricity demand of about 50,000 homes.

D.3.5 Passenger Cars

Passenger cars are definitely feasible and practical for some uses. Typically achieving 0.15–0.20 kWh/km, the average American car driving 12,000 miles per year (about 50 km/day, on average) would need at least 10 kWh capacity to satisfy average daily driving, but would need closer to 100 kWh to match typical ~500 km ranges of gasoline cars.

At a current typical cost of $200–300 per kWh, such a battery costs $20,000 to $30,000, without the car.[27] The most basic home charger runs at 120 V and 12 A,[28] multiplying to 1,440 W. A 100 kWh battery actually takes closer to 110–120 kWh of input due to 80–90% charge efficiency. Dividing 115 kWh by 1.44 kW leaves 80 hours[29] as the charge time. Table D.1 provides similar details for this and two other higher-power scenarios.

The middle row of Table D.1 is most typical for home chargers and those found in parking lot charge stations, resulting in an effective *charge speed* of about 10 miles per hour, or 16 km/hr. This is a convenient way to

Volts	Amps	circuit	kW	hours	mi/hr	km/hr
120	12	15 A	1.44	80	4	6
240	16	20 A	3.8	30	10	16
240	40	50 A	9.6	12	25	40

Table D.1: Approximate charge times and effective speeds (in miles per hour and kilometers per hour) for charging a 100 kWh battery at three different household power options. Such a battery delivers a range of about 300 miles, or 500 km.

characterize charge times. Adding enough charge to cover an average day of 30 miles or 50 km will take about 3 hours for the middle-row case, or just over an hour for the high-power charge.

Imagine now making a long road trip, driving at 100 km per hour. Even the fastest charge rate[30] in Table D.1 is 2.5 times slower. Every 400 km driven will take 4 hours on the road plus 10 hours at a charger for an average rate of 28 km/hr,[31] or 18 mi/hr.

30: . . . which is much higher than typical parking lot chargers that are more in line with the middle row

31: . . . 400 km in 14 hours

Special fast-charge stations can provide a staggering 250 kW[32] of power, cutting charge times dramatically. But this is neighborhood-scale energy delivery that households cannot expect to supply themselves. It is also informative to compute the temperature rise of a battery from a fast charge. If charging is 90% efficient, the other 10% turns to heat in the battery. Each kilowatt-hour of battery capacity has an associated mass around 5–10 kg, and receives 0.1 kWh (360 kJ) of thermal energy when charged. At a specific heat capacity around 1,000 J/kg/°C, a 360 kJ deposition increases the cell's temperature by 36–72°C, depending on energy density.[33] This is not a small rise (reaching boiling temperatures on warm days), and can contribute to shorter battery lifetime.

32: . . . like 200 homes

33: . . . higher energy density (better) batteries will experience a larger temperature rise based on less mass to heat up per amount of energy injected

So electric cars are not simple drop-in replacements for the gasoline machines roaming the roads today, that effectively refuel at a rate of 10 MW[34] given the fast delivery of an extremely energy-dense liquid. On performance and convenience measures, it would be hard to characterize them as *superior* substitutes. But they can *certainly* suit well for local travel when given ample time to recharge—overnight, for instance. And in the long run, it seems we will have little choice.

34: . . . the equivalent electricity consumption of 10,000 homes or a medium-sized college campus

For all this, several things still are not clear:

1. Will large scale ownership of electric cars become affordable, or remain cost prohibitive? Battery prices will surely fall, but enough?
2. If widespread, how will residential areas cope with tremendous increases in electrical demand during popular recharge hours?
3. How could night-time charging utilize solar input?
4. Will enough people willingly give up long-range driving capability? Will dual-system cars (like plug-in hybrids) be preferred to maintain gas capability for the occasional longer trip?
5. Will people sour over costly battery decline and replacement?

Electric cars are a growing part of transportation, and will no doubt grow more. It is too early to tell whether they will be able to displace fossil cars in the intermediate term. If not, personal transportation is likely to decline as fossil fuel use inevitably tapers away.

D.3.6 Wired Systems

To finalize the progression of hardest–to–easiest electrified transportation, we leave the problematic element behind: batteries. Vehicles on prescribed routes (trains, buses) can take advantage of wires carrying electricity: either overhead or tucked into a "third rail" on the ground. Most light rail systems use this approach, and some cities have wires over their streets for trolley buses. High-speed trains also tend to be driven electrically, via overhead lines.

The ease with which wired electrical transport is implemented[35] relative to the other modes discussed in this Appendix is another way to emphasize the degree to which storage is the bottleneck.

35: Wired electrified transport has been a steady contributor to transportation for over a century.

D.3.7 Collected Efficiencies

Each transportation mode in the previous sections reported an efficiency, in terms of kilowatt-hours per kilometer. Not surprisingly, mass and speed play a role, making container ships very hard indeed to push along, followed by airplanes. In some cases, it makes sense to express on a per-passenger or per-ton basis, distributing the energy share among its beneficiaries. Table D.2 summarizes the results, sometimes offering multiple options for vehicle occupancy to allow more fruitful comparisons among modes. Note that air travel looks pretty good until realizing that the distances involved are often quite large, making total energy expenditure substantial for air travel.

Mode	context	kWh/km	load	kWh/km/unit
Ship	cargo	1,300	100 kton	~0.01/ton
Air	passenger	15	150 ppl	0.1/psn
	cargo		15 ton	1/ton
Bus	passenger	2.3	10 ppl	~0.2/psn
	passenger		30 ppl	~0.07/psn
Truck	cargo	~1	20 ton	0.05/ton
Car	passenger	0.18	1 psn	~0.18/psn
	passenger		2 ppl	~0.09/psn

Table D.2: Energy requirements for various modes of transportation (lower numbers are more efficient). Total energy is distance times the measure in kWh/km. Loads are expressed contextually either as people (ppl) or tons (1000 kg). Per-passenger/ton efficiency depends on occupancy—expressed as kWh/km per person (psn)—for which multiple instances are offered in some cases. While trucks have a far better kWh/km measure than ships, ships are about four times more efficient per ton, carrying 5,000 times more cargo. Air freight is 100 times more energetically costly than by ship!

D.4 Pushing Out the Moon

Some forms of alternative energy are tagged with asterisks in Table 10.1 (p. 166), indicating that they are not *technically* renewable, but will last a very long time so might as well be considered to be renewable.

Tidal energy, covered in Sec. 16.2 (p. 280), is one such entry that honestly does not deserve much attention. The text mentioned in passing that

aggressive use of tidal energy has the power to push the moon away from Earth, providing the mechanism by which we could "use up" this resource. Curious students demanded an explanation. Even though it's not of any practical importance, the physics is neat enough that the explanation can at least go in an appendix.

Figure D.1: The moon pulls harder on the near side of the earth, and less hard on the back side. Relative to the earth as a whole (medium force), the near side advances toward the moon and the back side lags the rest of the earth, creating a bulge on both sides that is aligned toward the moon. Note that a drawing to scale would put the moon well off the page.
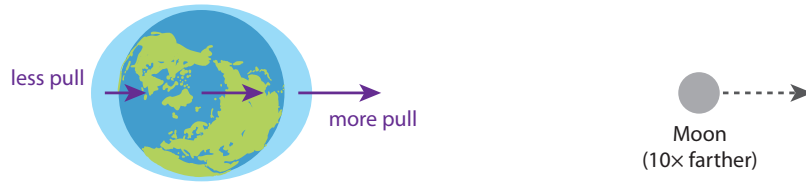
The first step is realizing that Earth and Moon each *pull on each other*[36] via gravitation. Since the strength of gravity decreases in proportion to the square of the distance between objects, the side of the earth closest to the moon is pulled more strongly than the center of the earth, and the side opposite the moon is pulled less strongly. The result is an elongation of the earth into a bulge—mostly manifested in the oceans (Figure D.1).

36: In fact, equally, per Newton's third law.

Figure D.2: The rotation of Earth and it continents "underneath" the tidal bulge creates a friction, or drag, that pulls the bulge around a few degrees (somewhat exaggerated here), so that it no longer points directly at the moon.
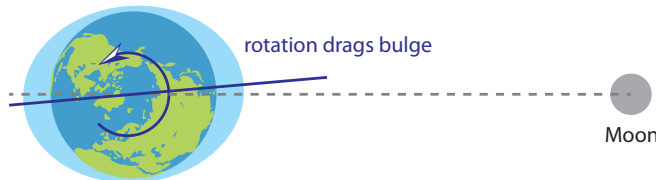
The second step is to appreciate that the earth rotates "underneath" the moon, so that the bulge—pointing at the moon—is not locked in place relative to continents.[37] But friction between land and water "drag" the bulge around, very slightly rotating the bulge to point a *little* ahead[38] of the moon's position (Figure D.2).

37: This is why we experience two high tides per day and two low tides: the earth is spinning underneath the opposite bulges, so that a site on the surface passes under a bulge (high tide) every ~ 12 hours.

38: The angular shift is around 1–2°.

Figure D.3: Gravitationally, the earth looks like a big central mass and two bulge masses displaced from the connecting line. The closer mass pulls harder than the more distant one, so the addition of all the force vectors (not to scale) results in a little asymmetry, leaving a small sideways component of the force along the same direction as the moon's orbital velocity (up in this drawing).

Now think about how the moon sees the earth, gravitationally. It mostly sees a spherical earth, but also a bulge on the front side, slightly displaced, and a bulge on the back side, also displaced in the opposite direction (Figure D.3). While the bulge masses are equal, the closer one has a greater gravitational influence and acts to pull the moon a little forward in its orbit, speeding it up.[39]

39: It may help to think of this bulge as being like a carrot dangled in front of a horse, encouraging it forward.

Accelerating an orbiting object along its trajectory adds energy to the orbit and allows the object to "climb" a little farther away from the

central body. So this displaced tidal bulge on Earth is tugging the moon forward and causing it to climb about 3.8 cm per year away from Earth. That's one-ten-billionth of its orbital radius per year, so it's not going away for a very long time, indeed.[40]

Figure D.4: If we built some inconceivable global-scale tidal capture structure the size of oceans and let them drain for six hours or so, the artificial bulges we created would travel farther around with the earth's rotation, enhancing the sideways "kick" and encouraging the moon to climb away from earth at a faster rate.
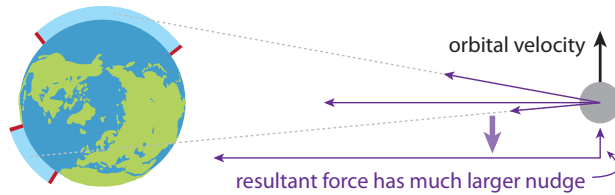
If we built global-scale structures (Figure D.4) to capture tidal energy in a big way,[41] we would effectively increase the lag angle of the tidal bulge. This is because we would likely release the captured stack of water over a period of many hours,[42], rotating this stack of water around the planet farther than it would naturally go. Now the gravitational pull in the forward direction would increase and the egress would speed up. If we managed to extract 18 TW[43] out of tides, this would be six times larger than the current 3 TW of tidal dissipation, and we might expect the egress to increase to about 23 cm per year.

41: Don't count on this happening: it's a truly ludicrous idea plagued by a giant list of practical problems, and all for such a small gain.

42: . . . like the ~6 hours between high tide and the next low-to-high tide cycle

43: . . . our current energy scale; not feasible, but used to illustrate

It's still a slow rate, and would not drive the moon away faster than hundreds of millions of years. So technically tidal energy is a one-time resource whose use diminishes its long term capacity.[44] But the timescales are so ridiculously long that we may as well think of tidal energy as inexhaustible.

44: This is why it is not strictly renewable

D.5 The Long View for Humanity

Sec. 8.1 (p. 114) took a sweeping view of humanity's timeline as a useful lens through which to appreciate the very short age during which fossil fuels impart a substantial energy benefit. This section revisits this time-warping perspective in a slightly different way as a means to reflect on humanity's far future.

D.5.1 Success vs. Failure

We start by noting that human civilization is about 10,000 years old, to the nearest order-of-magnitude.[45] Consider this question:

> Is human civilization still in its infancy, or are we closer to the end than the beginning?

45: This roughly marks the start of agriculture, and in any case is far closer to the truth than the adjacent order-of-magnitude figures of 1,000 years or 100,000 years.

Wow. Heavy question. Of course, we do not *know* the answer, but most of us would prefer to believe the first—that we are only beginning. So let's roll with that and explore the consequences.

In order for human civilization to be in its infancy, it would have to continue for *at least* 10,000 years more, if not far longer. What would it mean for us to still be operating "successfully" 10,000 years from now? Our physics and math approach actually allows us to place constraints!

This discussion is limited to living on Earth. Chapter 4 laid out reasons why imagining a space-faring future may be misguided. But even ignoring these arguments, Chapter 1 illustrated that human growth ambitions would be brought to an end long before 10,000 years pass. In this light, it is most straightforward to concentrate on what it would take to succeed on Earth itself.[46]

If we manage to carry our civilization into the far future,[47] we can comfortably call this **success**. If we don't, well, that would be **failure**. Can we sketch out what success looks like? One easy way to get there is to start enumerating the things that *can't* be carried into the far future.

1. Fossil fuels will not power civilization: a large fraction of the initial inheritance has been spent in a short 200 years,[48] so that 10,000 years in the future it is safe to say they will be long gone.
2. No steady annual decline of natural resources like forests, fisheries, fresh water, or species populations can be brooked. Allowing any component to decline would mean eventually losing that resource, which may be critical to our survival.
3. Human population will not be allowed to grow. Even small growth rates will step up pressure on natural resources, and Earth can only support so much, long-term. Independent of what the "right" number is,[49] once settled, we will not be able to dial it up without imperiling the hard-won success.
4. Even under steady human population, any increase in resource use per person will also not be compatible. In general, growth leads to a dead end: to failure.
5. Mining materials from the Earth will not continue at anything near the current pace. In the last few hundred years, the best deposits of copper, gold, aluminum, etc. have been found and exploited. Even if only 10% of the attainable resource has been consumed thus far,[50] continuing for tens of thousands of years (and beyond) cannot be expected.
6. Ultimately, any activity that draws down a finite natural resource will be impossible to sustain if the extraction rate is modest or high in relation to the initial resource abundance. Anything that can't last for well over 10,000 years is not a viable long-term solution and should not be exploited if success is the goal. Likewise, any pollutant that can build up to dangerous levels on even these very long timescales cannot be tolerated, if failure is to be avoided.
7. We can use the rule of 70 to say that anything having a doubling time (or halving-time in the case of depletion) shorter than 10,000 years is a no-go for success on these timescales, meaning that any activity impacting resources would have to be held to a growth rate

46: Even if extending to other planets, the same logic will apply.

47: A useful definition might be uninterrupted preservation of the knowledge and history gained thus far, without some apocalyptic collapse forcing a start-from-scratch revival—to the extent that's even plausible.

48: ... most of this in the last 50 years

49: ... unlikely as high as 10 billion, and it could even be well less than a billion, depending on living standards

50: ... author's conjecture; it could well be higher

or depletion rate of less than 0.007% per year, which is essentially zero-growth.

It becomes clear that long-term **success** is practically synonymous with the word **sustainable**. Any practice that is not long-term sustainable will fail to continue.[51] We therefore cannot *depend* on any non-sustainable resource if we strive for success.

D.5.2 Sustainable Living

Imagine that you have a stash of $100,000 tucked under your mattress, and that you have figured out a way to live on $20,000 per year. You *could* decide to live on this fund for five years, and then figure out later how to keep going. Perhaps this is not the wisest move. A smarter move would be to figure out how long you expect to live—maybe 50 more years—and ration out the fund, allowing $2,000 per year. You'll still need a job earning $18,000 per year to meet the $20,000 annual goal. Maybe the smartest move would be to *ignore* the money under the mattress and get a job for $20,000 per year.[52] Now you have the safety of resources should you need it, and can even pass it along down the generations to kids and grandkids who have also been taught not to use it, but to survive on their annual income.

The analogy is clear, and perhaps it is also clear why we did not allow interest accumulation, as many of Earth's resources are one-time endowments that do not spontaneously grow larger.[53] If our human civilization succeeds at surviving uninterrupted for 10,000 years, it will necessarily be *because* we figured out how to live on the **annual income**[54] provided by Earth's natural renewable flows, rather than on the **inheritance** in the form of finite resources that are not replenished. In other words, humanity needs to learn to refrain from any dependence on one-time resources (the inheritance).

Success, therefore, puts humans as *a part* of nature, not *apart* from nature. Anything else is failure. The closer we are to nature, the more likely we are to succeed.

Nature prepared a biosphere that has stood the test of time. Natural selection has operated to eliminate non-viable solutions and create inter-dependencies cleverly balanced in a stable equilibrium of sorts. Elements of modern human civilization—our cities, agricultural practices, fossil fuel dependence—have *not* withstood the test of time, nor can they. Which system would be the wiser bet for long-term survival: the well adapted natural world, or the artificial world humans have erected and operated for a few dozen decades—without attention to sustainable principles? The answer seems obvious.

51: Any activity today not geared to contribute to ultimate success (true sustainability) is therefore likely only contributing to failure. Most activities today are in the latter category, alarmingly.

52: . . . hardly different from $18,000 per year, so if you're going this far already, why not?

53: Interest is an artificial construct made possble by accelerating resource use.

54: Annual income would be in the form of solar energy delivered and biomass that has grown in the course of the year, for instance.

D.5.3 Time to Grow Up

In a sense, humanity is going through an awkward adolescent phase: growth spurts, a factory (pimple) strewn landscape, an attitude that we have all the answers—adults[55] can't possibly understand or tell us what to do. Conversely, nature is mature.[56] Ignoring recent human influences, it had already forged its complex, never perfect, but functional interdependencies and had settled into something resembling a steady state. Adolescents new to the scene may be hugely disruptive and destructive, and unless they change their ways, civilization drives straight into the jaws of failure. The adolescents lack the wisdom to build lasting systems that will have the privilege of co-existing with nature for very long.

In human society, most adolescents become adults who learn to live within their means. Sometimes this involves sacrifices or perhaps selecting a diet based on nutrition and health rather than what might be most tasty.[57] Likewise, humankind needs to define a scale for its activities that fits within nature's capacity to replenish, so that each subsequent generation is not deprived of resources the previous ones enjoyed. At present, civilization is *nowhere close* to this operating principle.

D.5.4 Frameworks

Humanity needs to develop a framework by which to evaluate its activities and ask whether each helps or hurts ultimate human success. Sometimes this might produce jarring results. Consider, for instance, a cure for cancer or other advances that might extend human lifetimes. Only if balanced against a smaller population or a smaller resource utilization per capita could such seemingly positive developments be accommodated once a steady equilibrium is established. Otherwise, total demand on Earth's resources goes up if the same number of people live longer at a fixed annual resource utilization per living person. In a successful world, any proposed new activity would have to demonstrate how it fits within a sustainable framework. Ignoring the issue would irresponsibly imperil overall long-term human happiness.[58]

At some level, humans need to realize that success means the thriving of not only themselves, but all of Earth's precious irreplaceable species and ecosystems. Without them, humans cannot be successful anyway. This is another part of maturing: many adolescents have difficulty considering the impacts of their actions on anyone other than themselves. Humans need to realize that hurting any component of Earth is hurting humans, long-term. Our legal system affords rights to humans, but gives no agency to plants, animals, or even non-living features of our planet. A successful future must give voice to every element of our world, lest we trample it and rue the day.

55: . . . in this case hypothetical wise humans who have managed a successful transition through the adolescent phase

56: Note the similarity of the words!

57: . . . opting out of the plan to eat ice cream for dinner

58: In such cases, the outcome may ultimately settle on longer lives for fewer people.

Can it work? Can humans create the institutions and uncorrupted global authority to regulate the entire biosphere—or at least the human interface—to prevent unsustainable disruption to the rest? Is human nature compatible with such schemes? Do we have the discipline to deny ourselves easily reached resources for the good of the whole? Individual desires for "more" may always work to subvert sustainable practices. Individual lifetimes are so very short compared to the necessarily long-term considerations of success that it will be very hard to universally accept seemingly artificial restrictions generation after generation. Also unclear is whether it is possible to maintain a technological society preserving knowledge and history while living on the annual renewable resources of the planet. We simply have no guiding precedent for that mode of human existence.

It is therefore an open question whether a technological society is even *compatible* with planetary limits. Are modern humans just a passing phase whose creations will crumble into oblivion in a geological blink, or can we stick it out in something other than a primitive state? We again have no evidence[59] one way or another. The current state of *apparent* success cannot be taken as a meaningful proof-of-concept, because it was achieved at the expense of finite resources in a shockingly short time: an extravagant party funded by the great one-time inheritance. The aftermath is only beginning to appear.

59: See Sec. 18.4 (p. 312) on the Fermi paradox for a worrisome—albeit inconclusive—lack of evidence of success in the universe.

We have a choice: work toward success—hoping and assuming that it is indeed possible; or acquiesce to failure. It seems that if we are not wise enough to know whether long-term success is even *possible*, the responsible course of action would be to *assume* that we can succeed, and do what we can to maximize our chances of arriving there. When should we start? Again—without knowing any better—the sooner we start, the more likely we are to succeed. Any delay is another way of driving ourselves toward a more likely failure.

D.6 Too Smart to Succeed?

This section pairs nicely with Section D.5, taking a slightly different perspective on the prospect of future success.

Evolution works incrementally by random experimentation: mutations that either confer advantages or disadvantages to the organism. Advantages are then naturally selected to propagate to future generations,[60] while disadvantages are phased out by failure of afflicted organisms in competition for resources and mates. Evolution is slow, and hard to spot from one generation to the next. When a common ancestor of the hippo evolved into whales, the nose did not suddenly disappear from the face to end up behind the head as a blow-hole, but took a tortuously long adaptive route to its present configuration.

60: After all, advantages make survival and procreation more likely.

Intelligence confers obvious advantages[61] to organisms, able to "out-smart" competition to find resources, evade dangers, and adapt to new situations. It also has some cost in terms of energy resources devoted to a larger brain. But multiple organisms from across the animal kingdom have taken advantage of the "smart" niche: octopuses, ravens, dolphins, and apes to name a few. Experiments reveal the ability of these species to solve novel, brainy puzzles in order to get at food, for instance.

Like other attributes, intelligence would not be expected to arrive suddenly, but would incrementally improve. Humans are justified in appraising themselves as the most intelligent being yet on the planet.

So here's the thing. The first species smart enough to exploit fossil fuels will do so with reckless abandon. Evolution did not skip steps and create a *wise* being—despite the fact that the *sapiens* in our species name[62] means wise. A wise being would recognize early on the damage inherent in profligate use of fossil fuels[63] and would have refrained from unfettered exploitation.

Put another way, the first species entertaining the notion that they are able to outsmart nature is in for a surprise. Earth's evolutionary web of life is dumb: it has no intelligence at all. But it exists in this universe on the strength of billions of years of tested success. All the random experiments along the way that were unworkable got weeded out. The vast majority of species around today have checked the box for long-term viability.

Modern humans—those who have moved beyond hunter-gatherer lifestyles, anyway—represent an exceedingly short-lived experiment in evolutionary terms. This is especially true for the fossil fuel era of the last few centuries. It would be premature to declare victory. The jury is still out on whether civilization is compatible with nature and planetary limits, as explored in Section D.5.

Evolution does not avoid mistakes. In fact, it is built upon and derives its awesome power precisely *because* of those few mistakes that somehow escape the more likely failed outcomes and find advantage in the mistake.[64] Maybe humans are one of those more typical evolutionary mistakes that will culminate in the usual failure, as so often happens. The fact that we're here and smart says nothing about our chances for long-term success. Indeed, humankind's demonstrated ability to produce unintended global adverse consequences would suggest that success is less than a safe bet.

It seems fairly clear that hunter-gatherer humans could have continued essentially indefinitely on the planet. And the brains of hunter-gatherer *Homo sapiens* are indistinguishable from those of modern humans. So intelligence *by itself* is not enough to cross the line into existential peril, if continuing to operate within and as a part of natural ecosystems. But once that intelligence is applied toward creating artificial environments[65] that no longer adhere to the ways of nature—once we make our own rules

61: Intelligence is not the only sort of advantage, and can easily lose to tooth and claw, or even mindless microscopic threats: nature has devised many ways to "win."

62: ...self-assigned flattery

63: Not only is climate change a problem, but building an entire civilization dependent on a finite energy resource and also enabling a widespread degradation of natural ecosystems seems like an amateur blunder.

64: Since mutations are random mistakes, and some actually, surprisingly, turn out to be advantageous, one might say that life is a giant pile of mistakes that failed to deliver the expected bad outcomes, snatching success from the jaws of failure.

65: ...e.g., agriculture, cities

as we "outsmart" nature—we run a grave risk as nature and evolution cease to protect us. In other words, a species that lives completely within the relationships established by the same evolutionary pressures that *created* that species is operating on firm ground: well adapted and likely to succeed, having stood the test of time.[66]

66: Since evolution is slow, any species has a reasonably long track record of success behind it.

Once we part ways with nature and create our own reality—our own rules—survival is no longer as guaranteed. Even 10,000 years is not enough time to prove the concept, when human evolution works on much longer timescales. This is especially true for the fossil fuel world, being mere centuries old. Nature will be patient while our fate unfolds.

The situation is similar to establishing a habitat on the lunar surface: an artificial environment to provision our survival in an otherwise deadly setting. The resources that were available to construct the habitat are not continually provided by the lunar environment, just as the fossil fuels and mined resources and forests are not continually re-supplied[67] as we deplete them. Just because the habitat *could* be built does not mean it can be maintained indefinitely. Likewise, the world we know today—being rather different from anything that nature prepared—may be a one-off that proves to be unsustainable in the long run.

67: Forests can grow back, but not at the rate of their destruction at present.

Since evolution is incremental, we cannot expect to have been made wise enough to avoid the pitfalls of being *just* smart enough to exploit planetary resources. And being slow, it seems unlikely that wisdom will evolve fast enough to interrupt our devastating shopping spree. It is *possible*[68] that we can install an "artificial" wisdom by using our intelligence to adopt values and global rules by which to ensure a sustainable existence. Probably most smart people assume that we can do so. Maybe. But living in a collective is difficult. Wisdom may exist in a few individuals, but bringing the entire population around to enlightened, nuanced thinking that values nature and the far future more than they value themselves and the present seems like a stretch.

68: What hope we have lies here, and provides the underlying motivation for writing this book. The first step is appreciating in full the gravity of the challenge ahead.

One way to frame the question:

> Are humans collectively capable of leaving most shelves stocked with treats, within easy reach, while refraining from consuming them, generation after generation?

Do we have the discipline to value a distant and unknown future more than we value ourselves and our own time? Successful non-human species have never had to answer this question, but neither has any species been smart enough—until we came along—to develop the capability to steal all the goodies from the future and, in so doing, jeopardize their own success.[69]

69: Success here means preserving civilization. It is far easier—and perhaps more likely—to at least survive as a species in a more primitive, natural state.

D.6.1 Evolution's Biggest Blunder?

As a brief follow-on, we framed evolution as a mistake-machine, sometimes accidentally producing functionally advantageous incremental improvements. Countless species adapt in ways that are not able to survive long term, and die off. So those "blunders" are inconsequential failed experiments. Evolution is indifferent to failure, being a mechanism rather than a sentient entity.

But most of the time, these failures are isolated, bearing little consequence on the wider world. Did anybody notice the three-dotted bark slug[70] disappear? If the human species turns out to be another of evolution's failed experiments—having made a creature too smart to stay within the lanes of nature—is it just another inconsequential blunder?

70: . . . totally made up

Unfortunately, it may turn out to be a rather costly blunder, if the failed species creates a mass extinction as part of its own failure. By changing the climate and habitat on the planet, we have already terminated or imperiled a number of species, and are nowhere near finished yet. Mass extinctions have happened many times through history, but seldom due to an evolutionary blunder. We may yet distinguish ourselves!

It is true that cyanobacteria transformed the climate starting about 2.5 billion years ago by pumping oxygen into the atmosphere. Called the Great Oxygenation Event, this precipitated the first-known mass extinction on the planet—essentially poisoning the simple anaerobic lifeforms that existed until that time. But we would hesitate to call it an unmitigated disaster, as it paved the way for multi-cellular life[71] in all the richness we see today. So accidental? Yes. Blunder? Okay. Disastrous? Let's say no, on balance.[72]

71: . . . although it took over a billion years to get there: no instant gratification

72: The anaerobic life would disagree, but when do we ever listen to them anymore? In any case, the result was tremendous biodiversity, which ultimately may be a decent figure of merit for value in this world.

The most recent mass extinction, 65 Myr ago, was caused by an asteroid impact, and the two before that appear to be connected to volcanic activity. The two prior to these are mixed: the first appears to have been caused by geological processes, and the next by a changing climate likely connected to diversification of land-based plants. And that's it, since the much earlier cyanobacteria oxygenation event. Only one of the five is likely attributable to evolution itself—and in this case not the fault of a single species.

A human-caused mass extinction *could* pave the way to whole new modes of lifeforms. But it was much easier in the early days to break new ground. It seems much less likely that a human-induced mass extinction will unleash a fantastic evolutionary richness hitherto unexplored. That leaves only downside, and the ignominious distinction of being the one species that evolution would most regret, if ever it could.

Please, please, please—let this tragic fate not come to pass!

Bibliography

References are in order of appearance in the book.

[1] D H Meadows et al. *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind*. Universe Books, 1972 (cited on pages v, 36).

[2] David J. C. MacKay. *Sustainable Energy—without the hot air*. Available at `https://www.withouthotair.com`. UIT Cambridge, 2008 (cited on page vii).

[3] U.S. Energy Inform. Administration. *Annual Energy Review, Appendix E1*. 2011. URL: `https://www.eia.gov/totalenergy/data/monthly/pdf/sec12_24.pdf` (cited on page 7).

[4] G. Kopp and J. L. Lean. "A new, lower value of total solar irradiance: Evidence and climate significance". In: *Geophysical Research Letters* 38.1 (Jan. 2011). DOI: `10.1029/2010GL045777` (cited on pages 11, 144, 167, 206).

[5] T W Murphy. *Exponential Economist Meets Finite Pysicist*. Online. Accessed 2021 Jan. 8. Apr. 2012. URL: `https://dothemath.ucsd.edu/2012/04/economist-meets-physicist/` (cited on page 17).

[6] *List of countries by GDP (nominal)*. Wikipedia. Source: *World Economic Outlook Database*, International Monetary Fund. Accessed 2019 Sept. 3. Oct. 2019. URL: `https://en.wikipedia.org/wiki/List_of_countries_by_GDP_(nominal)` (cited on pages 18, 19, 39).

[7] *List of Countries by Total Primary Energy Consumption and Production*. Wikipedia. Source: *International Energy Statistics*, U.S. Energy Information Administration. Accessed 2017 Feb. 12. 2016. URL: `https://en.wikipedia.org/wiki/List_of_countries_by_total_primary_energy_consumption_and_production` (cited on pages 18, 19, 43, 44).

[8] *List of Countries by Population (United Nations)*. Wikipedia. Accessed 2020 Jul. 18. July 2019. URL: `https://en.wikipedia.org/wiki/List_of_countries_by_population_(United_Nations)` (cited on pages 18, 19, 39, 40, 42–44).

[9] T W Murphy. "Maximum spectral luminous efficacy of white light". In: *Journal of Applied Physics* 111.10 (2012), p. 104909. DOI: `10.1063/1.4721897` (cited on page 21).

[10] Energypedia. *Lighting Technologies*. 2015. URL: `https://energypedia.info/wiki/Lighting_Technologies` (cited on pages 21, 22).

[11] *Luminous Efficacy*. Wikipedia. Sources: various, cited from Wikipedia page. URL: `https://en.wikipedia.org/wiki/Luminous_efficacy` (cited on pages 21, 22).

[12] T J Garret. *Rebound, Backfire, and the Jevons Paradox*. Online. Accessed 2021 Feb. 1. 2014. URL: `https://www.inscc.utah.edu/~tgarrett/Economics/Jevons_Paradox.html` (cited on page 23).

[13] *List of Countries by Total Primary Energy Consumption and Production*. Wikipedia. Sources: various, cited from Wikipedia page. URL: `https://en.wikipedia.org/wiki/List_of_countries_by_total_primary_energy_consumption_and_production` (cited on page 28).

[14] K Klein Goldewijk et al. "The HYDE 3.1 spatially explicit database of human-induced global land-use change over the past 12,000 years". In: *Global Ecology and Biogeography* 20.1 (Jan. 2011). Available via Wikipedia: Estimates of Historical World Population, pp. 73–86. DOI: `10.1111/j.1466-8238.2010.00587.x` (cited on pages 30–32, 37).

[15] *Estimates of Historical World Population*. Wikipedia. Source: U.S. Census Bureau, International Data Base. 2017. URL: `https://en.wikipedia.org/wiki/Estimates_of_historical_world_population` (cited on pages 30–32, 37, 45).

[16] V Smil. *Energy Transitions: Global and National Perspectives*. Also incorporates data from the *BP Statistical Review of World Energy*. 2017. URL: `https://ourworldindata.org/energy` (cited on pages 30, 45, 102, 110, 118, 140).

[17] *Green Revolution*. Wikipedia. Accessed 2020 Jul. 21. URL: `https://en.wikipedia.org/wiki/Green_Revolution` (cited on page 31).

[18] P. R. Ehrlich and Sierra Club. *The Population Bomb*. A Sierra Club-Ballantine book. Ballantine Books, 1971 (cited on page 32).

[19] *List of Sovereign States and Dependent Territories by Birth Rate*. Wikipedia. Source: *The World Factbook*. Retrieved 2014-02-21. 2016. URL: `https://en.wikipedia.org/wiki/List_of_sovereign_states_and_dependent_territories_by_birth_rate` (cited on pages 39–44, 50).

[20] *List of Sovereign States and Dependent Territories by Mortality Rate*. Wikipedia. Source: *Causes of mortality*, OECD Stat Health Status, OECD. 2011. URL: `https://en.wikipedia.org/wiki/List_of_sovereign_states_and_dependent_territories_by_mortality_rate` (cited on pages 39–44, 50).

[21] J Diamond. *Collapse: How Societies Choose to Fail or Succeed*. Penguin Books, 2005, pp. 286–293 (cited on page 47).

[22] Bill Anders and NASA. Online. [available online at Wikimedia Commons as picture ID 81692094; Accessed 2020 Aug. 4. 1968. URL: `https://spaceflight.nasa.gov/gallery/images/apollo/apollo8/html/as08-14-2383.html` (cited on pages 54, 354).

[23] *List of artificial objects leaving the Solar System*. Wikipedia. Accessed 2020 Aug. 9. 2020. URL: `https://en.wikipedia.org/wiki/List_of_artificial_objects_leaving_the_Solar_System` (cited on page 56).

[24] *Saturn V*. Wikipedia. Accessed 2020 Aug. 9. 2020. URL: `https://en.wikipedia.org/wiki/Saturn_V` (cited on page 56).

[25] V Springel et al. "Simulations of the formation, evolution and clustering of galaxies and quasars". In: *Nature* 435 (2005), p. 629. DOI: `10.1038/nature03597` (cited on pages 57, 354).

[26] C Dreier. "A new accounting for Apollo: how much did it really cost?" In: *The Space Review* (June 2019). `https://www.thespacereview.com/article/3737/1` Accessed 2020 Aug. 11 (cited on page 58).

[27] NASA Kennedy Space Center. *Space Shuttle and International Space Station FAQ*. Online. Accessed 2020 Aug. 11. 2017. URL: `https://www.nasa.gov/centers/kennedy/about/information/shuttle_faq.html` (cited on page 58).

[28] M Bartels. "NASA to pay Russia $90 million for a Soyuz seat on a crew launch this fall". In: *Space.com* (May 2020). Accessed 2020 Aug. 11 (cited on page 58).

[29] F Taylor. *The Scientific Exploration of Mars*. Cambridge University Press, 2010, p. 306 (cited on page 58).

[30]   *International Space Station*. Wikipedia. Accessed 2020 Aug. 11. 2020. URL: https://en.wikipedia.org/wiki/International_Space_Station (cited on page 59).

[31]   *Biosphere 2*. Wikipedia. Accessed 2020 Aug. 3. 2020. URL: https://en.wikipedia.org/wiki/Biosphere_2 (cited on pages 60, 66).

[32]   IPAC/NASA. *NASA Exoplanet Archive*. Online catalog. Accessed 2020 Aug. 9. 2020. URL: https://exoplanetarchive.ipac.caltech.edu (cited on page 63).

[33]   M Kleiber. "Body size and metabolic rate". In: *Physiological Reviews* 27.4 (1947), pp. 511–541. DOI: 10.1152/physrev.1947.27.4.511 (cited on page 72).

[34]   U.S. Energy Inform. Administration. *Annual Energy Review*. 2011. URL: https://www.eia.gov/totalenergy/data/annual/index.php (cited on pages 84, 89, 102, 103, 107, 130, 170, 229, 230, 422).

[35]   LLNL. *Energy Flow Charts*. Online. Accessed 2020 Dec. 29. 2019. URL: https://flowcharts.llnl.gov/ (cited on page 105).

[36]   R M Suits, N J Matteson, and E J Moyer. *U.S. Energy History Visualization*. Online. Accessed 2021 Feb. 6. Nov. 2020. URL: https://us-sankey.rcc.uchicago.edu/ (cited on page 105).

[37]   International Energy Admin. *Electricity Information: Overview*. Online. Accessed 2020 Dec. 6. July 2020. URL: https://www.iea.org/reports/electricity-information-overview (cited on page 107).

[38]   M K Hubbert. "Energy resources: a report to the Committee on Natural Resources of the National Academy of Sciences; National Research Council". In: (Dec. 1962). Figure 54, p. 91 (cited on page 116).

[39]   U.S. Energy Inform. Admin. *Coal Explained*. Online. Accessed 2020 Aug. 27. 2019. URL: https://www.eia.gov/energyexplained/coal/ (cited on page 118).

[40]   *Energy Density*. Wikipedia. Accessed 2020 Aug. 27. 2020. URL: https://en.wikipedia.org/wiki/Energy_density (cited on page 118).

[41]   J M K C Donev. *In a barrel of oil*. Online. In *Energy Education*, University of Calgory. Accessed 2020 Aug. 27. 2017. URL: https://energyeducation.ca/encyclopedia/In_a_barrel_of_oil (cited on page 119).

[42]   J M K C Donev et al. *Cap rock*. Online. In *Energy Education*, University of Calgory. Accessed 2020 Nov. 13. 2019. URL: https://energyeducation.ca/encyclopedia/Cap_rock (cited on page 120).

[43]   J Diamond. *Collapse: How Societies Choose to Fail or Succeed*. Penguin Books, 2005 (cited on page 125).

[44]   *List of Countries by Proven Oil Reserves*. Wikipedia. Accessed 2020 Nov. 9. 2020. URL: https://en.wikipedia.org/wiki/List_of_countries_by_proven_oil_reserves (cited on page 126).

[45]   *List of Countries by Proven Natural Gas Reserves*. Wikipedia. Accessed 2020 Nov. 9. 2020. URL: https://en.wikipedia.org/wiki/List_of_countries_by_natural_gas_proven_reserves (cited on page 126).

[46]   *List of Countries by Proven Coal Reserves*. Wikipedia. Accessed 2020 Nov. 9. 2020. URL: https://en.wikipedia.org/wiki/List_of_countries_by_coal_reserves (cited on pages 126, 127).

[47] D Rutledge. "Estimating long-term world coal production with logit and probit transforms". In: *International Journal of Coal Geology* 85 (2011), pp. 23–33. DOI: `10.1016/j.coal.2010.10.012` (cited on page 127).

[48] J Laherrère. *private communication*. Based on several sources and corrected by Jean Laherrère to be 2P back-dated discoveries. 2020 (cited on page 129).

[49] U.S. Energy Inform. Admin. *U.S. Field Production of Crude Oil*. Online. Accessed 2020 Nov. 10. 2020. URL: `https://www.eia.gov/dnav/pet/hist_xls/MCRFPUS2m.xls` (cited on page 130).

[50] National Oceanic & Atmospheric Administration. *Trends in Atmospheric Carbon Dioxide*. Global Monitoring Laboratory, online. Accessed 2020 Dec. 3. 2020. URL: `https://www.esrl.noaa.gov/gmd/ccgg/trends/data.html` (cited on page 139).

[51] NASA. *Global Temperature*. NASA Global Climate Change: online. Accessed 2020 Dec. 3. 2019. URL: `https://climate.nasa.gov/vital-signs/global-temperature/` (cited on page 139).

[52] H Ritchie. *Who has contributed most to global CO_2 emissions*. Online: Our World in Data. Accessed 2020 Dec. 2. Oct. 2019. URL: `https://ourworldindata.org/contributed-most-global-co2` (cited on page 143).

[53] R Rapier. *The World's Top 10 Carbon Dioxide Emitters*. Forbes online. Accessed 2020 Dec. 2. Dec. 2019. URL: `https://www.forbes.com/sites/rrapier/2019/12/04/the-worlds-top-10-carbon-dioxide-emitters/?sh=55dd24ad2d04` (cited on page 143).

[54] R T Pierrehumbert. "Infrared radiation and planetary temperature". In: *Physics Today* 63.1 (Jan. 2011), pp. 33–38. DOI: `10.1063/1.3541943` (cited on page 143).

[55] *Atmospheric Transmission (PNG)*. Wikimedia Commons. Accessed 2020 Nov. 29. 2020. URL: `https://commons.wikimedia.org/w/index.php?curid=2623190` (cited on page 145).

[56] J T Kiehl and K E Trenberth. "Earth's Annual Global Mean Energy Budget". In: *Bulletin of the American Meteorological Society* 78.2 (Feb. 1997), pp. 197–208. DOI: `10.1175/1520-0477(1997)078%3C0197:EAGMEB%3E2.0.CO;2` (cited on page 145).

[57] RealClimate: Climate science from climate scientists. *The CO_2 problem in 6 easy steps*. Online. Primary references within; Accessed 2020 Nov. 22. 2007. URL: `http://www.realclimate.org/index.php/archives/2007/08/the-co2-problem-in-6-easy-steps/` (cited on page 147).

[58] National Oceanic and Atmospheric Administration (NOAA). *Global Climate Report*. Online. Accessed 2020 Nov. 22. 2019. URL: `https://www.ncdc.noaa.gov/sotc/global/201913%5C#gtemp` (cited on page 147).

[59] U.S. Geological Survey. *How much water is there on Earth?* Online. Accessed 2020 Dec. 1. URL: `https://www.usgs.gov/special-topic/water-science-school/science/how-much-water-there-earth?` (cited on page 153).

[60] B Davies. *Calculating glacier ice volumes and sea level equivalents*. Online. Accessed 2020 Dec. 1. 2020. URL: `http://www.antarcticglaciers.org/glaciers-and-climate/estimating-glacier-contribution-to-sea-level-rise/` (cited on pages 153, 156).

[61] R Lindsey. *Climate Change: Global Sea Level*. Online: NOAA. Accessed 2020 Dec. 1. Aug. 2020. URL: `https://www.climate.gov/news-features/understanding-climate/climate-change-global-sea-level` (cited on page 155).

[62] K Lambeck et al. "Sea level and global ice volumes from the Last Glacial Maximum to the Holocene". In: *Proceedings of the National Academy of Sciences of the United States of America* 111.43 (Oct. 2014), pp. 15296–15303. DOI: `10.1073/pnas.1411762111` (cited on page 157).

[63] NASA. *The NASA Earth's Energy Budget Poster*. Online. Accessed 2020 Nov. 16. 2014. URL: https://science-edu.larc.nasa.gov/energy_budget/ (cited on pages 167, 168).

[64] J Barber and P D Tran. "From natural to artificial photosynthesis". In: *J. of the Royal Society Interface* 10 (2013), p. 20120984. DOI: 10.1098/rsif.2012.0984 (cited on pages 167, 168).

[65] A Kleidon. "Physical limits of wind energy within the atmosphere and its use as renewable energy: From the theoretical basis to practical implications". In: *arXiV* 2010.00982 (2020). https://arxiv.org/ftp/arxiv/papers/2010/2010.00982.pdf (cited on pages 167, 168).

[66] *List of largest hydroelectric power stations*. Wikipedia. Accessed 2020 Dec. 4. 2020. URL: https://en.wikipedia.org/wiki/List_of_largest_hydroelectric_power_stations (cited on page 176).

[67] *Study on the Importance of Harnessing the Hydropower Resources of the World*. Online: Eurelectric, Brussels. Accessed 2020 Nov. 24. Apr. 1997. URL: https://silo.tips/download/study-on-the-importance-of-harnessing-the-hydropower-resources-of-the-world (cited on page 178).

[68] *Life Cycle GHG Emissions*. Wikipedia. Accessed 2020 Dec. 20. 2020. URL: https://en.wikipedia.org/wiki/Life-cycle_greenhouse_gas_emissions_of_energy_sources (cited on pages 181, 194, 218, 221, 264).

[69] National Renewable Energy Lab. *U.S. Average Annual Wind Speed at 80 Meters*. Online. Accessed 2021 Jan. 16. 2011. URL: https://windexchange.energy.gov/maps-data/319 (cited on page 186).

[70] C de Castro et al. "Global Wind Power Potential: Physical and Technological Limits". In: *Energy Policy* 39.10 (2011), pp. 6677–6682. DOI: 10.1016/j.enpol.2011.06.027 (cited on pages 186, 193, 295).

[71] M Ragheb and A M Ragheb. "Wind Turbines Theory - The Betz Equation and Optimal Rotor Tip Speed Ratio". In: (2011). In *Fundamental and Advanced Topics in Wind Power*, ed. by R. Carriveau. DOI: 10.5772/21398 (cited on pages 188, 189, 423).

[72] A Betz. "Wind-Energie und ihre Ausnützung durch Windmühlen". In: *Göttingen, Vandenhoeck & Ruprecht* (1926). in German (cited on pages 188, 423).

[73] H Glauert. *The elements of airfoil and airscrew theory*. second ed. Cambridge University Press, UK, 1993 (cited on page 188).

[74] A P Schaffarczyk. "Aerodynamics and Aeroelastics of Wind Turbines". In: *WIT Transactions on State of the Art in Science and Engineering* 44 (2010). https://www.witpress.com/Secure/elibrary/papers/9781845642051/9781845642051003FU1.pdf. DOI: 10.2495/978-1-84564-205-1/03 (cited on pages 188, 354).

[75] J Meyers and C Meneveau. "Optimal turbine spacing in fully developed wind farm boundary layers". In: *Wind Energy* 15 (Mar. 2012), pp. 305–317. DOI: 10.1002/we.469 (cited on page 189).

[76] Antonio Muñoz et al. "Short-term Forecasting in Power Systems: A Guided Tour". In: *Handbook of Power Systems*. Vol. 2. May 2010, pp. 129–160. DOI: 10.1007/978-3-642-12686-4_5 (cited on pages 190, 354).

[77] A Fischer et al. "Statistical learning for windpower: a modeling and stability study towards forecasting". In: *Wind Energy* 20.12 (2017), pp. 2037–2047. DOI: 10.1002/we.2139 (cited on pages 190, 354).

[78] *Wind Power in China*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_China (cited on page 191).

[79] *Wind Power in the United States*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_the_United_States (cited on page 191).

[80] *Wind Power in Germany*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_Germany (cited on page 191).

[81] *Wind Power in India*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_India (cited on page 191).

[82] *Wind Power in Spain*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_Spain (cited on page 191).

[83] *Wind Power in the United Kingdom*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_in_the_United_Kingdom (cited on page 191).

[84] *Wind power by country*. Wikipedia. Accessed 2020 Dec. 6. 2020. URL: https://en.wikipedia.org/wiki/Wind_power_by_country (cited on page 191).

[85] U.S. Energy Inform. Admin. *Electric power monthly*. Online. Accessed 2020 Dec. 5. 2020. URL: https://www.eia.gov/electricity/monthly/ (cited on pages 191, 215, 279, 426).

[86] *Shockley–Queisser Limit*. Wikipedia. Accessed 2021 Jan. 17. 2021. URL: https://en.wikipedia.org/wiki/Shockley-Queisser_limit (cited on page 204).

[87] M Sengupta et al. "The National Solar Radiation Data Base (NSRDB)". In: *Renewable and Sustainable Energy Reviews* 89 (June 2018). https://www.nrel.gov/gis/solar.html, pp. 51–60. DOI: 10.1016/j.rser.2018.03.003 (cited on pages 207–209).

[88] National Renewable Energy Lab. *Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors*. Online. Accessed 2020 Dec. 7. 1994. URL: https://www.nrel.gov/docs/legosti/old/5607.pdf (cited on pages 210, 214, 224).

[89] National Renewable Energy Lab. *Solar Installed System Cost Analysis*. National Renewable Energy Lab: online. Accessed 2021 Feb. 18. 2021. URL: https://www.nrel.gov/solar/solar-installed-system-cost.html (cited on pages 211, 215, 256, 267).

[90] T W Murphy. "Home photovoltaic systems for physicists". In: *Physics Today* 61.7 (July 2008), pp. 42–46. DOI: 10.1063/1.2963010 (cited on page 213).

[91] R Fu, D Feldman, and R Margolis. *U.S. Solar Photovoltaic System Cost Benchmark: Q1 2018*. National Renewable Energy Lab: online. Accessed 2020 Dec. 7. Nov. 2018. URL: https://www.nrel.gov/docs/fy19osti/72399.pdf (cited on pages 215, 355).

[92] *Solar power by country*. Wikipedia. Accessed 2020 Dec. 8. 2020. URL: https://en.wikipedia.org/wiki/Solar_power_by_country (cited on page 217).

[93] International Energy Admin. *Solar PV*. Online. Accessed 2020 Dec. 8. 2019. URL: https://www.iea.org/reports/solar-pv (cited on page 217).

[94] E Elhacham et al. "Global human-made mass exceeds all living biomass". In: *Nature* (Dec. 2020). DOI: 10.1038/s41586-020-3010-5 (cited on page 229).

[95] C A S Hall, J G Lambert, and S B Balogh. "EROI of different fuels and the implications for society". In: *Energy Policy* 64 (2014), pp. 141–152. DOI: 10.1016/j.enpol.2013.05.049 (cited on page 232).

[96] M Inman. "How to Measure the True Cost of Fossil Fuels". In: *Scientific American* (Apr. 2013). https://www.scientificamerican.com/article/how-to-measure-true-cost-fossil-fuels/ (cited on page 232).

[97]   D A Pfeiffer. *Eating Fossil Fuels*. https://newsociety.com/books/e/eating-fossil-fuels. New Society Press, Oct. 2006 (cited on pages 234, 339).

[98]   D Pimentel and T Patzek. "Ethanol production using corn, switchgrass, and wood; biodiesel production using soybean and sunflower". In: *Natural Resources Research* 14 (2005), pp. 65–76. DOI: 10.1007/s11053-005-4679-8 (cited on page 234).

[99]   D J Murphy et al. "Order from Chaos: A Preliminary Protocol for Determining the EROI of Fuels". In: *Sustainability* 3.10 (2011), pp. 1888–1907. DOI: 10.3390/su3101888 (cited on page 234).

[100] M G Saad et al. "Algal Biofuels: Current Status and Key Challenges". In: *Energies* 12.10 (2019), p. 1920. DOI: 10.3390/en12101920 (cited on page 235).

[101] *Nuclear power by country*. Wikipedia. Accessed 2020 Dec. 13. 2020. URL: https://en.wikipedia.org/wiki/Nuclear_power_by_country (cited on page 256).

[102] Union of Concerned Scientists. *The Cost of Nuclear Power*. Online. Accessed 2020 Dec. 13. Dec. 2015. URL: https://www.ucsusa.org/resources/nuclear-power-cost (cited on page 256).

[103] *List of Countries by Uranium Reserves*. Wikipedia. Accessed 2020 Nov. 24. 2020. URL: https://en.wikipedia.org/wiki/List_of_countries_by_uranium_reserves (cited on page 257).

[104] Pew Research. "Elaborating on the Views of AAAS Scientists, Issue by Issue". In: (July 2015). https://www.pewresearch.org/science/2015/07/23/elaborating-on-the-views-of-aaas-scientists-issue-by-issue/ Accessed 2020 Dec. 13 (cited on page 264).

[105] L Zyga. *Why nuclear power will never supply the world's energy needs*. Online: PhysOrg. 2011. URL: https://phys.org/news/2011-05-nuclear-power-world-energy.html (cited on page 269).

[106] H Johnston. *Radioactive decay accounts for half of Earth's heat*. Online. Accessed 2020 Nov. 24. 2011. URL: https://physicsworld.com/a/radioactive-decay-accounts-for-half-of-earths-heat/ (cited on page 275).

[107] Intern'l Renewable Energy Agency. *Renewable Energy Statistics 2018*. Online. Accessed 2020 Dec. 14. July 2018. URL: https://www.irena.org/publications/2018/Jul/Renewable-Energy-Statistics-2018 (cited on page 279).

[108] *Geothermal Heating*. Wikipedia. Accessed 2020 Dec. 14. 2020. URL: https://en.wikipedia.org/wiki/Geothermal_heating (cited on page 279).

[109] *Geothermal Power*. Wikipedia. Accessed 2020 Dec. 14. 2020. URL: https://en.wikipedia.org/wiki/Geothermal_power (cited on page 279).

[110] Volkswagen. *Hydrogen or battery? A clear case, until further notice*. Online. Accessed 2021 Feb. 5. Nov. 2019. URL: https://www.volkswagenag.com/en/news/stories/2019/08/hydrogen-or-battery--that-is-the-question.html (cited on page 286).

[111] T W Murphy. "Beyond Fossil Fuels: Assessing Energy Alternatives". In: *State of the World 2013: Is Sustainability Possible?* Ed. by The Worldwatch Institute. Island Press, 2013. Chap. 15, pp. 172–183 (cited on page 289).

[112] W F Pickard. "A Nation-Sized Battery?" In: *Energy Policy* 45 (June 2012), pp. 263–267. DOI: 10.1016/j.enpol.2012.02.027 (cited on page 294).

[113] E S Andreiadis et al. "Artificial Photosynthesis: From Molecular Catalysts for Light-driven Water Splitting to Photoelectrochemical Cells". In: *Photochemistry and Photobiology* 87.5 (2011), pp. 946–964. DOI: 10.1111/j.1751-1097.2011.00966.x (cited on page 295).

[114] Humanmetrics. *Jung Typology Test*. Online. Accessed 2020 Dec. 17. URL: https://www.humanmetrics.com/cgi-win/jtypes2.asp (cited on page 305).

[115] 16Personalities. *Personality Types; NERIS Analytics Limited*. Online. Accessed 2020 Dec. 17. URL: https://www.16personalities.com/personality-types (cited on page 305).

[116] Truity. *Myers & Briggs 16 Personality Types*. Online. Accessed 2020 Dec. 17. URL: https://www.truity.com/personality-type (cited on page 305).

[117] Myers & Briggs Foundation. *How Frequent Is My Type?* Online. Accessed 2020 Dec. 17. 2002. URL: https://www.myersbriggs.org/my-mbti-personality-type/my-mbti-results/how-frequent-is-my-type.htm (cited on page 306).

[118] C S Weiler, J K Keller, and C Olex. "Personality type differences bet. Ph.D. climate researchers and the general public: implications for effective communication". In: *Climate Change* 112 (May 2012), pp. 232–242. DOI: 10.1007/s10584-011-0205-7 (cited on page 307).

[119] *Fermi Paradox*. Wikipedia. Accessed 2020 Dec. 17. 2020. URL: https://en.wikipedia.org/wiki/Fermi_paradox (cited on page 312).

[120] *Drake Equation*. Wikipedia. Accessed 2020 Dec. 17. 2020. URL: https://en.wikipedia.org/wiki/Drake_equation (cited on page 312).

[121] Y M Bar-On, R Phillips, and R Milo. "The biomass distribution on Earth". In: *Proceedings of the National Academy of Sciences* 115.25 (2018), pp. 6506–6511. DOI: 10.1073/pnas.1711842115 (cited on page 316).

[122] C Nunez. "Climate 101: Ozone Depletion". In: *National Geographic Online* (Apr. 2019). https://www.nationalgeographic.com/environment/global-warming/ozone-depletion/ Accessed 2020 Dec. 18 (cited on page 320).

[123] H E Daly and J Farley. *Ecological Economics: Principles and Applications, Second Edition*. Island Press, 2011 (cited on page 323).

[124] H Daly. "A Steady-State Economy". In: *UK Sustainable Development Commission* (Apr. 2008). http://www.pym.org/eco-justice-collaborative/wp-content/uploads/sites/58/2016/05/DalyteadyStateEconomy.pdf Accesses 2020 Dec. 22 (cited on page 325).

[125] S A Chang. *Moving Toward a Steady State Economy*. New York Society of Security Analysts: online. Accessed 2020 Dec. 18. Apr. 2010. URL: https://post.nyssa.org/nyssa-news/2010/04/moving-toward-a-steadystate-economy.html (cited on page 325).

[126] Nuclear Energy institute. *State Electricity Generation Fuel Shares*. Online. Accessed 2020 Dec. 19. 2019. URL: https://www.nei.org/resources/statistics/state-electricity-generation-fuel-shares (cited on page 335).

[127] G Eshel and P A Martin. "Diet, Energy, and Global Warming". In: *Earth Interactions* 10.9 (Apr. 2006), pp. 1–17. DOI: 10.1175/EI167.1 (cited on pages 339, 341).

[128] D Pimentel and M H Pimentel. *Food, Energy, and Society*. https://www.routledge.com/Food-Energy-and-Society/Pimentel-Pimentel/p/book/9780367388539. CRC Press, 2007 (cited on page 339).

[129] S Sripad and V Viswanathan. "Performance Metrics Required of Next-Generation Batteries to Make a Practical Electric Semi Truck". In: *ACS Energy Letters* 2.7 (2017), pp. 1669–1673. DOI: 10.1021/acsenergylett.7b00432 (cited on page 400).

Notation

This list describes several symbols that are commonly used within the body of the book.

c Speed of light in a vacuum inertial frame: 2.99792458×10^8 m/s $\approx 3 \times 10^8$ m/s

ε Efficiency; typically $0 \leq \varepsilon \leq 1$; unitless

g acceleration due to gravity: 9.8 m/s^2 or ≈ 10 m/s^2

h Planck's constant: 6.626×10^{-34} J $\cdot$ s

k_B Boltzmann's constant: 1.38×10^{-23} J/K

N_A Avogadro's number: 6.022×10^{23} particles per mole

ΔQ Change in thermal energy, in Joules

$R_\oplus$ Radius of Earth: 6,378 km

$R_\odot$ Radius of Sun: 695,700 km

$r_{\oplus\odot}$ Earth–Sun distance (1 AU): 149.6 million km

σ Stefan-Boltzmann constant: 5.67×10^{-8} W/K/m^2

ΔS Change in entropy, in J/K

ΔT Change in temperature, typically in °C or Kelvin (K)

ΔW Change in energy—work performed, in Joules

Scale Factor Prefixes

Factor	Letter	Prefix	Factor	Letter	Prefix
10^{-21}	z	zepto	10^{21}	Z	zetta
10^{-18}	a	atto	10^{18}	E	exa
10^{-15}	f	femto	10^{15}	P	peta
10^{-12}	p	pico	10^{12}	T	tera
10^{-9}	n	nano	10^{9}	G	giga
10^{-6}	μ	micro	10^{6}	M	mega
10^{-3}	m	milli	10^{3}	k	kilo

Greek Letters, with Pronunciation

Character	Name	Character	Name
α, A	alpha *AL-fuh*	ν, N	nu *NEW*
β, B	beta *BAY-tuh*	ξ, Ξ	xi *KSIGH*
γ, Γ	gamma *GAM-muh*	o, O	omicron *OM-uh-CRON*
δ, Δ	delta *DEL-tuh*	π, Π	pi *PIE*
ϵ, ε, E	epsilon *EP-suh-lon*	ρ, P	rho *ROW*
ζ, Z	zeta *ZAY-tuh*	σ, Σ	sigma *SIG-muh*
η, H	eta *AY-tuh*	τ, T	tau *TOW (as in cow)*
θ, Θ	theta *THAY-tuh*	υ, Υ	upsilon *OOP-suh-LON*
ι, I	iota *eye-OH-tuh*	ϕ, Φ	phi *FEE, or FI (as in hi)*
κ, K	kappa *KAP-uh*	χ, X	chi *KI (as in hi)*
λ, Λ	lambda *LAM-duh*	ψ, Ψ	psi *SIGH, or PSIGH*
μ, M	mu *MEW*	ω, Ω	omega *oh-MAY-guh*

Glossary

AC

alternating current. 77, 215

AER

Annual Energy Review. 102, 103, 105–107, 109, 170

alpha decay

(α) happens when a nucleus emits an alpha particle, otherwise known as a ^{4}He nucleus. 243, 422, 435

alpha particle

(α) is a ^{4}He (helium) nucleus, ejected from a larger nucleus in an alpha decay It therefore consists of two protons and two neutrons. 243, 245, 422

Amp

(A) is short for Ampere. 77, 293, 422, 425

Ampere

(A, or Amp) is the SI unit of current, defined such that one Ampere is the same as one Coulomb per second (1 A = 1 C/s). 77, 422

a.m.u.

atomic mass unit. 241, 246–248, 253, 254, 265, 376, 422, 432, 433, 435

Annual Energy Review

is compiled by the U.S. EIA, capturing energy use and trends for all sources and sectors [34]. 102, 170, 422

Astronomical Unit

(AU) is a unit of distance, equal to the average Earth–Sun distance of 149.6 million kilometers (1.496×10^{11} m). 56, 422

asymmetric risk

describes a condition where given the choice to pursue action B for fear of some future condition instead of the normal action A, the downside of being correct about the threat and *not* taking action B is *far* more disastrous than being wrong about the threat and pursuing route B unnecessarily. 345, 351

atomic mass unit

(a.m.u.) is defined so that a single neutral carbon atom, consisting of 6 protons, 6 neutrons, and 6 electrons has exactly 12.00000 a.m.u. In other units, it is 931.4941 MeV or 1.66054×10^{-27} kg. This unit sometimes goes by the name: Dalton. 241, 376, 422, 433

AU

Astronomical Unit. 56, 206, 420, 422

Avogadro's number

is $N_A = 6.022 \times 10^{23}$, pertaining to one mole of particles (e.g., atoms, molecules). 375, 376, 381, 433

band gap

is the energy difference between the conduction band and the valence band, determining how much energy is needed to promote an electron out of an atom and into conduction. 203, 204, 223

barrel

(bbl) is a unit of volume used primarily for petroleum. It is exactly 42 U.S. gallons, amounting to 159 L of volume. A commonly used measure of energy is barrels of oil equivalent (b.o.e.), amounting to 6.1 GJ of combustion energy. 112, 120, 126, 129–131, 135, 141, 211, 266

beta decay

(β) happens when a nucleus emits either an electron (β^-) or a positron (β^+). 243, 245, 265, 435, 438

Betz limit

is a theoretical maximum amount of kinetic power that can be removed from wind without slowing the wind too much. It computes to 19/27, or 59%, and is independent of technology [71, 72]. 188, 189, 195

Big Bang

is the name given to the start of the universe, about 13.8 billion years in the past. 9, 55, 239, 257, 392, 394, 396

biofuel

describes a liquid chemical fuel derived from biologically grown plants: algae, sugar, corn, rapeseed, etc. The two most common forms are ethanol and biodiesel. 165, 227, 230, 231, 428

biomass

is a generic term for biological matter, but in the energy context usually means firewood or dung that may be burned for thermal energy. 170, 227, 229–231

birth rate

quantifies the number of births per 1,000 people per year, typically. Numbers tend to be in the 5–30 range. 38, 426

blackbody

is a term describing the radiative qualities for thermal emission of light (infrared radiation for "normal" temperatures, becoming visible for very hot objects). A perfect blackbody is not reflective (i.e., "black" at the wavelengths of interest) and emits energy as a function of wavelength according to the Planck spectrum. 145, 199–201, 203, 223, 434, 438

boiling water reactor

is a type of nuclear fission reactor in which water surrounding the fuel rods acts both as a moderator and as the means of transporting heat away from the nuclear fuel. 255, 256

Boltzmann constant

is a fundamental constant of nature associated with thermodynamics. In SI units, it has a value of $k_{\mathrm{B}} = 1.38 \times 10^{-23}$ J/K. 89, 199, 381

breeder reactor

is a nuclear fission reactor that transforms non-fissile nuclei into ones that are fissile by means of neutron capture and subsequent radioactive decay. 250, 262, 264, 296, 423

breeding

see breeder reactor. 259

British thermal unit

(Btu) is a unit of energy in the Imperial unit system, defined as the amount of energy required to heat one pound of water by 1°F. It is equivalent to 1,055 Joules. 75, 423, 427, 435, 437

Btu

is short for British thermal unit. 75, 97, 98, 335, 372, 423

Calorie

(Cal, or kcal) is a unit of energy, defined as the amount of energy required to heat one kilogram (1 kg, 1 L, 1,000 cm^3) of water by 1°C. It is equivalent to 4,184 Joules, and is the exact same

thing as a kilocalorie. Note the capital C differentiates it from the calorie, which is 1,000 times smaller, making this the dumbest unit convention around, and strongly favoring the use of the equivalent kcal instead. 73

calorie

(cal) is a unit of energy, defined as the amount of energy required to heat one gram (1 g, 1 mL, 1 cm^3) of water by 1°C. It is equivalent to 4.184 Joules. 73, 85, 177, 194, 424, 431

capacity factor

is the fraction of energy delivered by an installation compared to what it would deliver if operating continuously at peak operating ("nameplate") capacity. 176, 179, 180, 182, 183, 190, 191, 196, 212, 216, 217, 226, 256, 267, 279, 281, 282, 288

caprock

is a geological feature of impermeable rock that can trap oil, gas, or steam below it. 120, 278

carrying capacity

refers to the limiting population that can be supported long-term by the environment. No consensus exists for Earth's carrying capacity for humans, though standards of living have a large influence. 34, 432

CFL

compact fluorescent light. 21

chain reaction

is a self-feeding process that keeps itself going. In the context of nuclear fission, neutrons released by the fission precipitate the next fission event, and so on. 251, 252, 255, 262, 425, 432

charge

is a measure of the degree to which a particle or object is influenced by electromotive forces. Electric charge can be positive or negative, so that like charges repel and opposites attract. The unit for electric charge is the Coulomb. 77, 240, 241, 246, 395, 425, 427, 429, 433

Chart of the Nuclides

is a Periodic Table on steroids, listing the properties of every known nuclide including mass or energy, abundance (if stable), half life (if unstable), decay mode, neutron cross section, nuclear spin, and other salient properties; see https://people.physics.anu.edu.au/~ecs103/chart/. 240–242, 244, 245, 248, 251, 253, 259, 266, 270

chemical energy

is energy stored in chemical bonds, like gasoline or wood that might be burned, or in the food we eat. 70, 117, 121, 182, 227, 228, 234, 379, 395, 396

climate sensitivity parameter

relates a change in radiative forcing to the net temperature change once all the feedback mechanisms have acted. The units are °C per W/m^2, and a typical value is 0.8. 147, 160

coefficient of performance

(COP) refers to the energy gain by a heat pump, usually in the context of heating rather than cooling. It is identical to ε_{heat}, as defined in Eq. 6.11 (p. 95). 97, 425

compound

describes a particular combination of elements that construct a particular molecule. For instance, H$_2$O is the compound we know as water. 376, 379, 432

concentrated solar power

(CSP) refers to a form of solar thermal (ST) energy, employing troughs or "power towers" or any technique that focuses solar power to create high temperatures, often then used to generate electricity. 220, 425, 436

conduction band

is the energy level a step up from that of electrons in the valence band. Electrons in the conduction band are very loosely bound and freely wander about the crystal, hopping from one atom to the next, and therefore able to contribute to a current. 202, 203, 422, 435, 438

confinement

in the context of fusion refers to the trapping and holding of a high-temperature plasma, usually by magnetic means. 265

conservation of energy

says that energy is never created or destroyed, only shifting from one form to another. 70, 91–93, 95, 246, 393, 394, 425

conservation of mass-energy

extends conservation of energy to include mass, so that the *combined* mass-plus-energy of a closed system is never created or destroyed, only shifting from one form to another (mass-energy exchange via $E = mc^2$). 246

control rod

is used in a nuclear fission reactor to absorb neutrons so that the chain reaction does not get out of control and cause a meltdown. 251, 255, 256, 432

COP

coefficient of performance. 97, 99, 335, 424

Coulomb

(C) is the SI unit of electric charge. An electron has a charge of -1.6×10^{-19} C and a proton has a charge of $+1.6 \times 10^{-19}$ C. 77, 422, 424, 427

coupled

refers to the tight connection often seen between energy/resource use and economic scale (as measured, for instance, by GDP). 18, 425

critical mass

is the mass of fissile material (assumed to be in spherical form) above which a self-sustained chain reaction will occur. Below this, the material poses no danger. Right at critical mass, the material will limp along in a slow chain reaction. Above this threshold—super-critical—an exponential runaway detonation will occur, and is the basis of nuclear weapons. For ^{235}U, critical mass is 52 kg (a bit smaller than a volleyball), and for ^{239}Pu, it is 10 kg, and about the size of an American softball. 262

CSP

concentrated solar power. 220, 424, 436

current

is a measure of charge flow, expressed in the SI unit of Amps. 77, 85, 202, 203, 205, 422, 425, 436

D–D fusion

uses deuterons (^{2}H nuclei) as the fuel for fusion, achieving an energy density of 137 million kcal/g. 265

death rate

quantifies the number of deaths per 1,000 people per year, typically. Numbers tend to be in the 5–30 range. 38, 426

decay chain

refers to a consecutive series of radioactive decays. 244, 245

decoupling

is the notion that economic activities need not incur a large energy or resource cost, breaking the tendency for economic scale to be tightly coupled to physical goods. 20

demographic transition

refers to the process in which an undeveloped country initially having high birth rate and high death rate transitions to low death rates followed by low birth rates as medical and resource conditions improve. 39, 44

deuterium

is an isotope of hydrogen, in which the nucleus (called a deuteron) contains one proton and one neutron. 248, 265, 266, 272, 274, 299, 426

deuteron

is the nucleus of deuterium, consisting of one proton and one neutron. 265, 425, 426

dietary energy factor

is the quantitative energy impact of a set of dietary choices compared to a vegetarian diet. A typical American diet has a dietary energy factor around 2, meaning it takes twice as much energy as would a vegetarian diet. This term is not in universal use. 339–341, 349, 428

differential equation

is an equation that relates functions and their derivatives. The subject is often sequenced after calculus within a curriculum. 33, 34

doping

is a process by which deliberate impurities are introduced into a semiconductor in order to change its properties with respect to transport of electrons or holes. 202, 431

doubling time

is how long it takes a system or collection to double its amount under conditions of growth, such as in exponential growth. See also the rule of 70. 2, 6, 23, 31, 32, 405

D–T fusion

combines a deuteron (^{2}H nucleus) and a triton (^{3}H) as the fuel for fusion, achieving an energy density of 81 million kcal/g. 265, 267

duty cycle

refers to the percentage of time something is "active." For example, a refrigerator may be on 40% of the time to maintain internal temperature, in which case its duty cycle is 40%. 88, 334

Ecological Economics

is a field that builds economic theory on top of the notion that the planet offers finite resources and flows. A principle aim is that of a steady-state economy capable of indefinite planetary compatibility. 323, 324

EER

energy efficiency ratio. 97–99, 427, 430

EIA

Energy Information Administration. 7, 75, 102, 103, 106, 107, 131, 170, 215, 422, 426

Electric Power Monthly

(EPM) is compiled by the U.S. EIA, capturing electricity production and usage at the state level from all energy sources [85]. 215, 427

electromagnetic radiation

refers to any transport of energy by electromagnetic waves, which include light, ultraviolet, infrared, X-rays, microwaves, gamma rays, and radio waves. 10, 198, 397, 426, 431, 435

electromagnetic spectrum

refers to the sweep of wavelengths or frequencies of electromagnetic radiation, including light, ultraviolet, infrared, X-rays, microwaves, gamma rays, and radio waves. 79, 392

electron

is a fundamental particle typically found in the outer parts of atoms, surrounding the nucleus. Electrons have negative charge equal and opposite to that of protons, but are 1,836 times lighter than the proton, at 0.511 MeV. 77, 78, 198, 202, 239, 244, 245, 255, 395, 422, 423, 425, 426, 429, 430, 433–435, 438

electron-volt

(eV) is a unit of energy, defined as the energy (work) it takes to push a charge of one fundamental charge unit (see entry for Coulomb) through an electric potential of one Volt. 1 eV is equivalent to 1.6×10^{-19} Joules. 78, 198, 223, 248, 428, 432

element

pertains to a single atom on the Periodic Table. For instance, hydrogen, helium, and carbon are all elements. 376, 379, 424, 432

energy

is defined as the capacity to do work. The SI unit is the Joule. 68, 73, 77, 174, 334, 379, 423–425, 427, 431, 434, 437, 438

energy density

describes how concentrated energy is in a substance, quantified as energy per unit mass. In chemical contexts, anything around 10 kcal/g or higher is considered energy–dense, while substances at about 1 kcal/g or lower are poor. Carbohydrates and proteins are middling, around 4 kcal/g, while fat is 9 kcal/g, and therefore among the more energy–dense substances. 121, 122, 175, 228, 230, 231, 236, 237, 254, 256, 264, 277, 290, 380, 398, 425, 426, 428

energy efficiency ratio

(EER) refers to the energy gain by a heat pump, usually in the context of cooling rather than heating. Its units are odd, defining how many British thermal units (thermal energy) may be moved per Watt-hour of input energy, but relating to $\varepsilon_{\mathrm{cool}}$ (defined in Eq. 6.10 (p. 95)) by a simple numerical factor: EER = $3.41\,\varepsilon_{\mathrm{cool}}$. Sometimes seen as SEER to represent a *seasonal* average EER value. 97, 98, 426

energy intensity

measures the energy use of a society relative to its economic scale. A typical value may be about 5 MJ/$. 19, 336

energy trap

refers to a phenomenon in which energy shortage motivates aggressive pursuit of alternative energy schemes, but that pursuit requires substantial energy investment—forcing an even more acute but voluntary energy shortage, which is politically difficult. 132, 301, 310, 311

enriched

see enrichment. 258, 262

enrichment

refers to the process of increasing the concentration of a particular isotope within a sample of an element. Usually, this term is applied to the concentration of ^{235}U from its natural 0.72% to 3–5% for power plants or >20% (typically ~85%) for weapons. 258, 427

entropy

is a measure of how many ways a system can be configured for some fixed energy level. The entropy of a closed system cannot decrease. 90, 396, 420

Environmental Economics

is an offshoot of neo-classical economics that adds a layer of pricing to capture "externalities," or environmental costs not normally included in market price. 323

EPM

Electric Power Monthly. 215, 426

EROEI

> Energy Returned on Energy Invested: a measure of how profitable an energy source is in terms of energy, expressed as a ratio. For instance, a 9:1 EROEI means 9 units were extracted or produced for an investment of 1 unit, leaving a net gain of 8 units of energy. 1:1 is break-even, deriving no net energy benefit. 231, 235, 236, 278, 295, 297, 301, 302, 310, 311, 315, 339

estimated total resource

> is an educated extrapolation of proven reserves trying to characterize the amount of resource that may be ultimately found and extracted. 127, 131, 258

ethanol

> (C_2H_5OH) is a liquid alcohol frequently produced as a biofuel having an energy density of ~7 kcal/g. 108, 230, 297, 377, 423

eV

> electron-volt. 198, 203, 248, 427, 432

exponential growth

> happens when the rate of growth—as a percentage or fraction—is constant. 2, 4, 31, 33, 61, 319, 426, 434

feedback

> is the response of a system when a change is made that itself influences the change: either counteracting it as in negative feedback or amplifying it as in positive feedback. 145, 147, 424

fill factor

> is a generic term describing the fraction of total area occupied. For instance a polka-dot pattern of circles on a piece of fabric might have a fill factor of 15%. 189

fissile

> describes a nucleus that is prodded into fission by a (slow) thermal neutron. The three fissile nuclides of interest are ^{233}U, ^{235}U, and ^{239}Pu. 255, 259, 262, 423, 425, 428

fission

> is a nuclear process in which a heavy nucleus splits into two lighter nuclei. Only ^{233}U, ^{235}U, and ^{239}Pu are usually considered as accessible nuclides that are fissile in the presence of slow (thermal) neutrons. 85, 239, 249, 264, 289, 296, 423–425, 428, 432

flexitarianism

> is the practice of pursuing dietary choices based on *quantitative* assessment of energy costs in an effort to keep the dietary energy factor low, without enforcing complete strictness, enjoying the occasional deviation on special occasions or just to avoid being a pain to others. 342

fossil fuel

> refers to an energy source buried in the ground, in the form of coal (solid), petroleum (liquid), or natural gas (gaseous). Fossil fuels represent ancient solar energy captured in living matter, processed and stored underground over millions of years. 7, 22, 27, 31, 61, 103, 104

fracking

> is slang for **hydraulic fracturing**, a technique used to extract "tight" oil and gas resources locked up in less permeable rock formations. High-pressure fluids are used to create cracks in the rock that the allow oil and/or gas to flow. 120, 124, 128, 130, 232

frequency

> characterizes the number of cycles per second in a periodic phenomenon (often in wave phenomena). The units are Hertz, or 1/s. 79, 198, 426, 430, 434, 436

fuel rod

> is a long cylinder having a high-enough concentration of fissile material to be used in a nuclear fission reactor. 255, 256, 260, 263, 272, 423

fusion

is a nuclear process in which two light nuclei merge to form a larger nucleus. Repulsion of the charges in the nuclei make it exceedingly hard to achieve, requiring temperatures of many millions of degrees. 85, 239, 249, 265, 289, 299, 395, 425, 426, 437

galaxy

is a collection of stars held together by mutual gravitational attraction, generally numbering in the billions of stars. 9, 54, 55, 312, 394, 438

gamma decay

(γ) is when a nucleus in an energetically excited state emits a high-energy photon. 244, 429, 435

gamma ray

(γ) is a high-energy photon, as may be generated by a gamma decay or by annihilation of an electron and positron. 244, 251, 255, 434, 435

GDP

Gross Domestic Product, effectively representing the total monetary flow of goods and services within a society, typically over a one year period. 18, 24, 39, 425

generator

converts mechanical motion (rotation, typically) into electrical current, generally by the relative motion of wire loops and a strong magnetic field. 89, 99, 164, 165, 175, 184, 185, 190, 250, 279, 280, 282, 285, 430, 436

geothermal

refers to thermal energy within the earth, both from the original heat of formation and from radioactive decay. 85, 99, 108, 166, 275

GHG

greenhouse gas. 146, 151, 152, 155, 160, 161, 429

Gppl

is a short-hand unit for giga-people, or billion people. 32, 37

gravitational potential energy

is the energy stored in a mass, m, lifted a height, h, above some reference in the presence of gravity, $g \approx 10\,\text{m/s}^2$. The energy amounts to mgh, and will be in Joules if the inputs are in kg, m, and s. 66, 69, 70, 77, 89, 167, 173, 174, 177, 184, 275, 280, 283, 395, 396, 438

Green Revolution

refers to the modernization of agricultural practices worldwide beginning around 1950, when fossil fuels transformed both fertilization and mechanization. 31, 37, 123, 124, 234

greenhouse gas

(GHG) absorbs infrared radiation and acts as a thermal blanket in a planetary atmosphere. H_2O, CO_2, O_3, and CH_4 are powerful greenhouse gases. 11, 12, 144–146, 151, 429

grid tied

refers to a photovoltaic system connected to the local electrical utility grid, enabling export of solar production by day and use of utility electricity by night. 213, 222

half life

is the time after which half a sample of radioactive nuclei will have undergone radioactive decay. After N half-life periods, the remaining fraction will be $1/2^N$. 242, 243, 257, 259, 261, 262, 270, 276, 424, 435

heat capacity

is the amount of energy it takes to raise an object's temperature by $1°C$. The *specific heat capacity* is the heat capacity divided by mass, becoming an intrinsic property of the material. Water's

specific heat capacity is 4,184 J/kg/°C, intimately tied to the definition of the kilocalorie. 74, 85, 99, 147, 153, 168, 194, 271, 277, 372, 401

heat engine

is a device that converts thermal energy into another form, usually mechanical motion. Automobile engines are a common example, as are power plants that create steam from a thermal source that itself drives a turbine and generator. 89, 92, 165, 239, 267, 276, 277, 286, 294, 298, 348, 396, 433, 436

heat loss rate

as used in this book is the power per ΔT (in °C) required to maintain a temperature differential. Units are W/°C, and typical houses might be a few hundred W/°C. 87, 99, 100, 334

heat of fusion

is the energy barrier associated with either forming (fusing) or melting a solid from a liquid. In the case of water (ice), the heat of fusion is 334 J per gram. 152, 153

heat of vaporization

is the energy barrier associated with turning a liquid into gas. In the case of water going to water vapor, the heat of vaporization is about 2,250 J per gram. 177

heat pump

is a device that moves thermal energy from a cold environment to a hotter one, against normal flow. Some energy input is required to drive this reverse flow, but thermodynamic principles permit a small amount of input energy to drive a larger amount of thermal energy transfer. 85, 95, 297, 335, 427, 430

heating seasonal performance factor

(HSPF) refers to the energy gain by a heat pump in the context of heating, but in the same units as the EER so that HSPF is COP times 3.41, numerically. 97, 98, 430

heavy oil

refers to oil that is very viscous—closer to tar than to gasoline. Heavy oil is more difficult to extract, process, and obtain gasoline via refinement. 131

Hertz

(Hz) is the SI unit for frequency, and is equivalent to cycles per second, or 1/s. 198, 428

hockey stick

is a term used to describe plots that suddenly shoot up after a very long time of relative inaction. Plots of human population, atmospheric CO_2, energy use, all tend to show this characteristic—which resembles an exponential curve. 31, 115

hole

in the context of semiconductors is the absence of an electron—or an electron vacancy. When another electron fills the hole, it leaves behind another hole, and it is as if the hole moved—effectively like a positive charge able to roam through the crystal. 202, 204, 426, 435

HSPF

heating seasonal performance factor. 97–99, 430

HST

Hubble Space Telescope. 59

hydrocarbon

is a chain of carbon and hydrogen atoms such as the alkanes (methane, ethane, propane, butane, octane, etc.) having chemical formula C_nH_{2n+2}, where $n = 1$ for methane, 2 for ethane, 8 for octane, etc. 119, 121, 131, 229, 436

hydrological cycle

is the solar-driven process by which evaporation of water from the surface (bodies of water or moist land) forms clouds, and the clouds deliver rain back to the surface. 166, 168, 177

infrared radiation

    is the property that all objects glow in light, or electromagnetic radiation. For objects that are not "red hot," the emission is invisible to the human eye, at longer wavelengths than the visible spectrum. The power radiated obeys the Stefan–Boltzmann law. 10, 84, 143, 147, 151, 161, 169, 396, 397, 423, 429, 435, 437

insolation

    is the annual average solar flux reaching flat, level ground for a particular location. A typical number is 200 W/m^2, but can range from half that at high latitudes to about 350 W/m^2 for arid areas at lower latitudes. 168, 178, 189, 206–208, 228, 276, 285

inverse function

    is a mathematical operation that "undoes" its counterpart, like the square root undoes the square, or the natural logarithm undoes the exponential. 5, 32, 367, 370

isotope

    is what we call atoms that have various nuclear configurations for the same element. That is, variants of a nucleus having the same number of protons but differing numbers of neutrons, and therefore differing mass number. See also nuclide. 240, 242, 252, 261, 266, 296, 375, 376, 426, 433, 437

ISS

    International Space Station. 58–60

Jevons paradox

    is named after early economist William Stanley Jevons, and describes the backfire of efficiency improvements leading to *increased* usage of the associated resource due to greater demand for the more attractive, efficient technology. Also called the rebound effect. 23, 435

Joule

    (J) is the SI unit of work or energy, and is equivalent to Newtons times meters (N·m), or kg · m^2/s^2. 19, 69, 159, 309, 371, 420, 423, 424, 427, 429, 431, 432, 434, 435, 437, 438

junction

    describes an interface between two semiconductors that have different doping. Junctions are the basis of photovoltaic, diodes, light emitting diodes (LEDs), transistors, and many light detectors. 202, 204

kcal

    kilocalorie. 74, 85, 99, 118, 121, 122, 136, 140, 228–230, 233, 236, 237, 246, 253, 254, 258, 265, 277, 290, 334, 335, 339, 340, 380, 398, 423–428, 431

Kill-A-Watt

    is the name of a relatively inexpensive device that can measure instantaneous power in Watts and accumulated energy in kWh of electrical appliances. The name is a pun on units. 334

kilocalorie

    (kcal) is a unit of energy, equivalent to 1,000 calories, defined as the amount of energy required to heat one kilogram (1 kg, 1 L, 1,000 cm^3) of water by 1°C. It is equivalent to 4,184 Joules. 73, 84, 254, 334, 339, 424, 430, 431

kilowatt-hour

    (kWh) is a unit of energy, constructed as a power (kilowatts) times time (hours). It is equivalent to 3,600,000 Joules, or 3.6 MJ. 72, 159, 172, 209, 214, 226, 343, 401, 402, 432

kinetic energy

    is the energy of motion, given by $\frac{1}{2}mv^2$ for a mass, m, at velocity, v. If input units are kg and m/s, the resulting unit will be Joules. 69–71, 89, 174, 184, 185, 223, 275, 282, 284, 395, 396, 437

kWh

kilowatt-hour. 76, 159, 214, 272, 334, 335, 337, 338, 342, 348, 398, 431, 438

LED

light emitting diode. 21, 29, 78, 83, 431

life-cycle CO_2 emission

is an assessment of how much CO_2 is released from an energy source when considering the entire enterprise—including manufacture/construction, operation, etc. See the Wikipedia page on List of life-cycle greenhouse gas emissions. 181, 194, 218, 221, 264

liquefied natural gas

(LNG) is cryogenically-cooled natural gas (methane) at −160°C that can be stored much more compactly than the gaseous form, making it suitable to transport. 432

LNG

liquefied natural gas. 121, 432

logistic

describes a mathematical model in which rate of growth depends on how close the population is to the carrying capacity. The resulting population curve over time is called the logistic function, or more informally, an S-curve. 34

macro-economics

concerns itself with the allocation of goods and services across the marketplace, optimizing supply and demand, aiming to minimize surplus or deficits. 323, 324, 433

mass number

(A) is simply the total number count of protons and neutrons (nucleons) in a nucleus. For example, a carbon atom having 6 protons and 6 neutrons has $A = 12$. 240, 431

meltdown

refers to a failure mode of nuclear fission reactors, in which the chain reaction becomes uncontrolled due to too many neutrons triggering new fission events (as may happen if control rods are absent or insufficiently deployed). 262, 263, 425

MeV

is a mega-electron-volt, or 10^6 eV. In Joules, it is equivalent to 1.6×10^{-13} J. Nuclear masses are often expressed in MeV/c^2 terms, where 1 a.m.u. is equivalent to 931.4941 MeV. 78, 246–248, 253, 265, 422, 427, 433, 435

micro-economics

concerns itself with the production of goods, including raw resources, marketing, and distribution. 323, 324, 433

micron

(μm) is 10^{-6} meters, or a micro-meter. 198, 200, 438

moderator

in the context of nuclear fission is a material used to slow down neutrons speeding out from the break-up so that they can become thermal neutrons and stimulate subsequent fission events in a chain reaction. Light atoms like water are a good choice for absorbing the neutron impacts. 251, 255, 256, 423

molar mass

is the mass of one mole of an element or compound. The molar mass for carbon, for instance, is 12 grams. The number is often found on a Periodic Table, in addition to the proton number for the element. 121, 138, 241, 379, 380

mole

is a *number* of atoms or molecules, tuned so that one mole of the carbon-12 isotope is exactly 12.000 grams. It takes 6.022×10^{23} atoms for this to happen, which is called Avogadro's number. 78, 83, 375, 376, 378, 422, 432

negative feedback

involves a reaction to some stimulus in the direction opposite the stimulus, performing a corrective action and leading to stability. Systems in equilibrium must have negative feedback keeping them there. 33, 123, 147, 428, 433

neo-classical economics

is the prevailing economic regime practiced today, driven by supply and demand, fueled by growth, market investment, and focus on micro-economics and macro-economics. 323, 427

neutrino

is a fundamental particle associated with the weak nuclear force that has almost no mass, travels near the speed of light, and interacts so weakly with matter that it could pass through light-years of rock before being likely to hit anything. Neutrinos from the sun stream through our bodies constantly, day and night, since Earth is transparent to them. 243, 244, 438

neutron

is one of two basic building blocks of atomic nuclei, the other being the proton. Neutrons have no electric charge, and a mass of 939.565 MeV, or 1.008665 atomic mass unit (a.m.u.). Neutrons are made up of three quarks: 1 up and 2 down. 240, 243, 244, 255, 299, 375, 376, 395, 422–426, 428, 431–433, 435, 437, 438

Newton

(N) is the SI unit of force, and is equivalent to $kg \cdot m/s^2$. 68, 371, 431, 438

nuclear binding energy

is the energy associated with the strong nuclear force that holds a nucleus together against charge repulsion. Typical levels are 8 MeV per nucleon. 247, 248

nuclear energy

derives from reconfiguring the nuclei of atoms, releasing tremendous thermal energy that can be harnessed in a heat engine. 103, 104, 239

nucleon

is either of the two building blocks of a nucleus, meaning that it is either a proton or a neutron. 240, 247, 248, 251, 376, 432, 433, 437

nucleus

is at the center of an atom, composed of protons and neutrons and spanning $\sim 10^{-15}$ m. The vast majority (99.97%) of an atom's mass is in the positively charged nucleus, which attracts a cloud of negative-charge electrons to complete the neutral atom. 239, 375, 376, 422, 423, 425–429, 431–433, 435, 437

nuclide

is any bound arrangement of protons and neutrons. Every nucleus of every isotope is one of the possible nuclides, designated, for instance as C12, C-12, or ^{12}C. 240, 242, 243, 424, 428, 431, 435

overshoot

occurs when the negative feedback in a system is delayed. After surpassing the equilibrium, oscillation may ensue. 36

parts per million

(ppm) is a unit used to measure small contributions. One ppm is 0.0001%. 61, 139, 141, 434

parts per million by mass

(ppm$_m$) is a parts per million measure in terms of fractional mass. For instance, a gram is is 1 ppm$_m$ of a metric ton (1,000 kg). 141, 258

parts per million by volume

(ppm$_v$) is a parts per million measure in terms of fractional volume occupied. For instance, a cubic millimeter (1 μm, or micro-liter) is 1 ppm$_v$ of a liter. 140–142

payback time

is how long it takes to recuperate an investment by removing a chronic cost. For example, spending $1,000 to no longer pay an annual $100 charge has a payback time of 10 years. 215, 226

photon

is the smallest indivisible particle of light: a minimum quantum packet of energy. Each photon has a well defined energy, which can also be expressed as a wavelength or frequency. 21, 70, 78, 79, 198, 199, 202, 227, 243, 244, 251, 394–396, 429

photosynthesis

is the process by which living matter captures sunlight and stores some of it as chemical energy. Effectively, it takes CO_2 out of the atmosphere, combines the carbon with water to make sugars, releasing oxygen back into the air. 227, 395

photovoltaic

(PV) is a semiconductor technology by which light directly drives an electrical current by interacting with electrons in the material. 165, 197, 201, 217, 218, 239, 267, 289, 292, 315, 350, 429, 431, 435

Planck spectrum

describes a mathematically precise spectrum of light emission from a blackbody, fully defined by the temperature of the blackbody. 145, 199–201, 423, 436, 438

Planck's constant

is a fundamental constant of nature associated with quantum mechanics and the world of the very small. In SI units, its value is $h = 6.626 \times 10^{-34}$ J $\cdot$ s. 79, 198, 199

plasma

is a gas hot enough to strip electrons from atoms to create a highly-ionized medium, such as the gas comprising the sun. 265, 267, 268, 392, 393, 395, 425, 437

positive feedback

involves a reaction to some stimulus in the same direction as the stimulus, thus amplifying the effect. Positive feedback leads to an unstable, runaway process—like exponential growth. 33, 123, 147, 428

positron

is an elementary particle of anti-matter, and specifically an anti-electron, having the same mass and opposite charge as the electron and will annihilate with an electron into gamma rays. 243–245, 255, 423, 429

power

is the *rate* of energy, or change in energy per change in time. The units are Joules per second (J/s), or Watts (W). 7, 10, 11, 30, 43, 71, 73, 77, 86, 110, 118, 146, 151, 171, 176, 187, 205, 280, 334, 335, 368, 373, 431, 435–438

ppm

parts per million. 61, 139, 141, 433, 437

predicament

describes a seemingly intractable situation: more than a *problem*, but possibly a tangled set of interconnected problems. Predicaments require responses rather than tidy *solutions*. 332, 436

proliferation

is used to describe widespread distribution of dangerous nuclear materials, which becomes difficult to control if they exist in abundance due to increased reliance on nuclear energy. 239, 260, 262, 264, 269, 296, 297

proton

is one of two basic building blocks of atomic nuclei, the other being the neutron. Protons have positive charge, equal and opposite to that of the electron. Protons have a mass of 938.272 MeV, or 1.0072765 a.m.u.. Protons are made up of three quarks: 2 up and 1 down. 77, 240, 243, 244, 375, 376, 395, 422, 425–427, 431–433, 437, 438

proven reserve

pertains to the amount of resource known to exist, having been discovered and surveyed to estimate the economically recoverable amount. 126, 127, 131, 257, 258, 267, 269, 428

PV

photovoltaic. 165, 197, 201, 205, 217, 220, 221, 267, 269, 281, 292, 293, 434

qBtu

is short for a quadrillion (10^{15}) British thermal units, and is equivalent to 1.055×10^{18} Joules. 28, 75, 103–105, 170, 214, 229, 230, 234, 236, 277

R/P ratio

or reserves-to-production ratio is a means to assess time remaining for a resource of quantity R units, being used (produced) at a rate of P units per year. The result is years available at the present rate, absent discovery of additional resources or change in rate of use. 126, 129, 131, 133, 136, 267

radiation

is a broad term that can can describe light (e.g., electromagnetic radiation, infrared radiation, gamma rays) or particles from radioactive decay or cosmic origin. High-energy radiation of any form can cause damage to materials and biological tissues (DNA being perhaps most critical). 263

radiative forcing

is used to describe the areal power (in W/m^2) of absorbed solar energy and infrared radiation to space. In equilibrium, a balance exists so that the net radiative forcing is zero. 146, 147, 151–153, 155, 160, 424

radioactive

describes a nucleus, or nuclide that is unstable and will undergo radioactive decay with some half life. 241, 257, 260, 261, 263, 268, 276, 296, 297, 299, 429

radioactive decay

involves a change in the nucleus of an atom, most commonly in the form of alpha decay, beta decay, or gamma decay. 239, 242–244, 262, 275, 297, 423, 425, 429, 435, 437

rebound effect

describes the counterintuitive process by which efficiency improvements lead to *greater* use of the resource as the enhanced appeal and lower cost results in more widespread adoption and use. Also called the Jevons paradox. 23, 431

recombination

is when an electron in the conduction band of a semiconductor finds a vacancy (hole) for it to settle into. By disappearing from the conduction band, it is no longer available to contribute to current, and the energy it had becomes unrecoverable. 202, 204

refinement

is the process by which crude oil—as it comes out of the ground—is separated by approximate hydrocarbon chain length. In order of lighter/shorter to heavier/longer chains, crude oil yields propane and butane, gasoline (around octane), kerosene, diesel, heating oil, lubricating oil, and tar. 119, 430

renewable

forms of energy are not necessarily depleted by their use. In other words, the resource is replenished naturally at some rate. The sun will still shine and wind will still blow even if we harness some of the energy. Firewood will grow back, but at a limited rate. 103, 104, 106

response

is an appropriate reaction to a predicament, which may fall well short of a *solution*, but still represents a reasonable compromise approach. 332, 434

rule of 70

tells us that the time it will take a system or collection to double in size is 70 divided by the percentage growth rate. The time units depend on how the time over which percentage growth is expressed—like 2% *per day* or 2% *per year*, for instance. The rule works most accurately for smaller growth rates, under 10%. 2, 5, 6, 28, 31, 405, 426

R-value

describes the thermal resistance, or insulating quality of a wall or similar barrier. It is an inverse to the U-value, numerically $5.7/U$. Units are $°F \cdot ft^2 \cdot hr/Btu$, and larger numbers translate to better insulation. 87, 438

sea level rise

is one of the inevitable consequences of climate change, as land-bound ice melts and ocean water thermally expands. 151, 155

sector

refers to a domain of activity, typically dividing into residential, commercial, industrial, and transportation. 104, 165, 193, 337

semiconductor

is a material poised between being a good conductor of electrical current and an insulator (not passing current). Silicon is the most commonly used semiconductor. 201, 202, 426, 430, 431, 435

SI

Système International. 68, 71, 85, 88, 422, 423, 425, 427, 430, 431, 433, 434

solar constant

measures 1,360 W/m^2, and is the power flux of the sun at the top of Earth's atmosphere. It is not technically a *constant*, but is very stable. 11, 144, 167, 203, 206, 211

solar system

refers to our own star, the sun, and the planets that surround it, including Earth. 54

solar thermal

(ST), also called concentrated solar power (CSP), typically refers to troughs or "power towers" or any technique that focuses solar power to create high temperatures, often then used to generate electricity via a heat engine and generator. 165, 197, 219, 221, 424, 436

spectrum

describes a distribution, often associated with light. In this context, a light spectrum specifies how much light is present as a function of wavelength or frequency. The Planck spectrum is a good example. 145, 200, 434

ST

solar thermal. 219–221, 269, 424, 436

Stefan–Boltzmann constant

(σ) has a value of 5.67×10^{-8} W/m^2/K^4 and is used in the Stefan–Boltzmann law relating to infrared radiation. 10, 144, 199, 437

Stefan–Boltzmann law

says that the power emitted from a surface of area, A, and temperature, T will be $P = A\sigma T^4$, where σ is the Stefan–Boltzmann constant. 10, 144, 199, 431, 437

stoichiometry

amounts to the counting of atoms and balancing formulas in chemical reactions to reflect the survival of every atom in a reaction: none created or destroyed. 377, 378

strong nuclear force

is the force that binds nucleons together in a nucleus, overcoming the electrical repulsion of protons. 240, 245, 247, 395, 433, 438

substitution

refers to interchangeability between goods and services, so that an unavailable or inferior resource can be replaced by an alternative, possibly superior one. 21

terraforming

is the speculative idea of transforming the atmosphere and environment of a planet hostile to human life into one that is suited to human needs. 60, 61

Therm

is a unit of energy defined as 100,000 British thermal units, and is equivalent to 1.055×10^8 Joules. 76, 335, 337, 348

thermal energy

is the energy of heat, and is really just randomized kinetic energy (motion) of atoms and molecules vibrating and zipping around. 70, 71, 84, 86, 89, 99, 165, 194, 203, 227, 246, 275, 277, 294, 334, 335, 373, 395, 396, 401, 420, 423, 427, 429, 430, 433

thermal equivalent

is a construct used to compare thermal energy sources like coal, oil, and natural gas to sources like solar, wind, and hydroelectricity, which do not derive from thermal sources. Usually in the context of electricity production, multiplying by about 2.7 puts non-thermal sources into thermal-equivalent terms. 106–108, 170, 256, 279

thermal expansion

describes how materials expand, or swell, as temperature increases. Typical rates of expansion are in the range of 5–100 ppm per °C. 155

thermal neutron

is a neutron whose kinetic energy (speed) is no greater than it would naturally possess based on the temperature of its surroundings. Sometimes it is called a "slow" neutron because it is not traveling faster than thermal jostling would establish. 250, 428, 432

tokamak

is the name of a donut-shaped chamber in which high-temperature plasma can be confined, and potentially used to generate fusion. 267

transmutation

describes the transformation of a nucleus into a different one, usually via neutron absorption— possibly followed by radioactive decay. 259, 268

tritium

is an isotope of hydrogen, in which the nucleus (called a triton) contains one proton and two neutrons. 265–268, 299, 438

triton

is the nucleus of tritium, consisting of one proton and two neutrons. 265, 426, 437

turbine

is essentially fan blades on a rotating shaft, which can be compelled to move by a flow of air, water, or steam through the blades. 89, 99, 164, 175, 185, 190, 250, 279, 280, 282, 295, 430

universe

refers to the entirety of our physical realm, including all galaxies. 9, 54, 55, 257, 312, 392, 423

U-value

describes the insulating quality of a wall or similar barrier, in terms of how many Watts move through each square meter of surface area for each 1°C difference in temperature across the barrier. Units are $W/m^2/°C$, and smaller numbers mean better insulation. The U-value is an inverse measure to the R-value, numerically $5.7/R$. 87, 436

valence band

is the energy level of outer electrons bound to an atom. Valance electrons stay home, as opposed to electrons in the conduction band. 203, 422, 425

Volt

(V) is a unit of voltage, or electric potential, and can be thought of as the electrical analog to gravitational potential energy, and is also somewhat like pressure in a fluid system. 77, 198, 427, 438

voltage

is a measure of electric potential energy, expressed in units of Volts. 77, 438

Watt

(W) is a unit of power, defined so that 1 W is 1 J/s (one Joule per second). 7, 18, 21, 43, 71, 77, 118, 169, 196, 267, 309, 371, 431, 434, 438

watt-hour

(Wh) is a unit of energy, constructed as a power (watts) times time (hours). It is equivalent to 3,600 Joules, or 0.001 kWh. 73, 77, 97, 110, 176, 427, 438

wavelength

measures the length of a wave from crest to crest or trough to trough, and can apply to waves in water, air (sound), or electromagnetic waves (light). The symbol λ (lambda) is often used to denote wavelength. The units are length (m), often expressed in microns (μm). 79, 144, 198, 283, 394, 426, 431, 434, 436

weak nuclear force

joins gravity, electromagnetism, and the strong nuclear force as one of nature's four fundamental forces, responsible for beta decays and neutrino interactions. 244, 245, 433

Wh

watt-hour. 73, 97, 98, 110, 175, 176, 438

Wien law

describes the wavelength for which the Planck spectrum is at maximum brightness. It is roughly 2.9 mm divided by the blackbody temperature, in Kelvin. 199

work

is a mechanical expression of energy, defined as a force (Newtons) times distance (meters) through which the force acts (along the same direction). The resulting unit is the Joule. 68, 84, 89, 174, 248, 379, 427

Alphabetical Index