Multivariate Crash Risk

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Research Question

Do investors receive compensation for multivariate crash risk, defined as **exposure to extreme realizations of multiple systematic factors**, in the cross-section of expected stock returns?

Multivariate Crash Risk: Our Main Measure

We propose a new measure of **an asset's systematic exposure to crash events** that is suitable for **models with multiple factors**.

- notation: R_i return of asset i, $\boldsymbol{X}$ vector with N factor returns
- We measure the multivariate crash risk (**MCRASH**) of asset i by

$$\text{MCRASH}_i^{\mathbf{X}} := \mathbb{P}\left[R_i \leq Q_p[R_i] \,\middle|\, \bigcup_{j=1}^{N} \{X_j \leq Q_p[X_j]\} \right] \tag{1}$$

  with p as a small probability level (e.g. $p = 0.05$) and Q_p as p-quantile.

- MCRASH is the conditional probability that asset i realizes a left-tail event given that *at least one* of the systematic factors realizes a left tail event.

Theoretical Framework

- starting point: **projected Stochastic Discount Factor (SDF)**

$$M^{\boldsymbol{X}} = \mathbb{E}[M \mid \boldsymbol{X}] \tag{2}$$

- Eq. (2) defines a measurable function $m : \mathbb{R}^N \to \mathbb{R}$ with $M^{\boldsymbol{X}} = m(\boldsymbol{X})$
- additional assumptions: m is differentiable, decreasing in each argument and convex
- well-known pricing result in terms of m and $\boldsymbol{X}$

$$\mathbb{E}[R_i - R_f] = -(1 + R_f)\,\mathrm{cov}[m(\boldsymbol{X}), R_i] \tag{3}$$

with R_i denoting the return of asset i, R_f as the risk-free return and $\boldsymbol{X}$ as a vector of factor returns

Our SDF Approximation

- standard argument to derive a pricing model: first-order Taylor expansion of m around $\boldsymbol{x}_c = \mathbb{E}[\boldsymbol{X}]$

$$m_L(\boldsymbol{X}) = m(\boldsymbol{x}_c) + \nabla m(\boldsymbol{x}_c) \cdot (\boldsymbol{X} - \boldsymbol{x}_c), \tag{4}$$

- **our main idea:** use the **piecewise linear approximation**:

$$m_{L,e}(\boldsymbol{X}) = m_L(\boldsymbol{X}) + \mathbb{1}(T_p[\boldsymbol{X}])\, d_{\text{tail}}(\boldsymbol{X}), \tag{5}$$

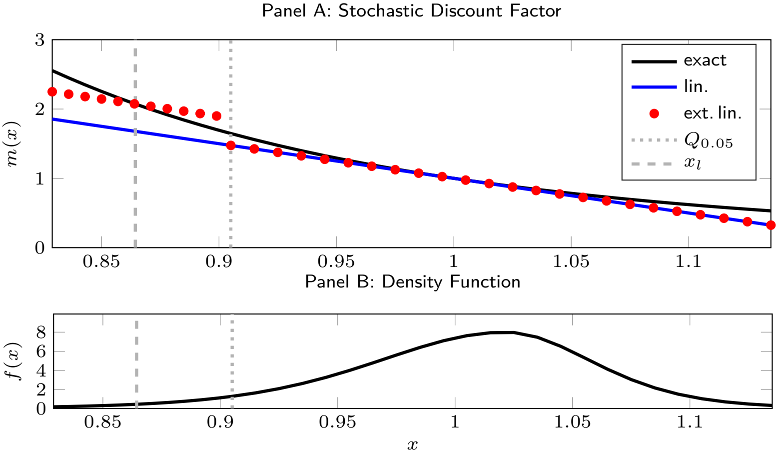
  which allows for an improved approximation quality over the tail area

- simple choice for the tail adjustment $d_{\text{tail}}(\boldsymbol{X})$

$$d_{\text{tail}} \equiv m(\boldsymbol{x}_l) - m_L(\boldsymbol{x}_l) \tag{6}$$

  with $\boldsymbol{x}_l := \mathbb{E}[\boldsymbol{X} \mid T_p[\boldsymbol{X}]]$

Our SDF Approximation: Illustration



(unscaled) power utility with $RRA = 5$, skewed-t distribution for the factor return X.

Pricing Implications

- we derive the **extended linear factor model**

$$\mathbb{E}[R_i - R_f] = \alpha_i + \underbrace{\sum_{j=1}^{N} \beta_i^{(j)} \lambda^{(j)}}_{\text{linear factor model}} + \underbrace{(\mathrm{MCRASH}_i^{\boldsymbol{X}} - p)\, \lambda_{\mathrm{tail}}^{\boldsymbol{X}}}_{\text{crash risk premium}} \quad (7)$$

  - $\beta_i^{(j)}$ standard linear beta
  - $\lambda^{(j)}$ (standard) price of risk for factor j
  - $\lambda_{\mathrm{tail}}^{\boldsymbol{X}} \geq 0$ price of (multivariate) crash risk

- main hypothesis:
  The expected excess return of asset i is increasing in its exposure to multivariate crash risk as measured by $\mathrm{MCRASH}_i^{\boldsymbol{X}}$.

Data

- stock data from CRSP and Compustat
  - all common stocks trading on the NYSE, AMEX, and NASDAQ
  - sample period: January 1964 until December 2018
  - liquidity filters: at least 200 non-zero returns over the past 250 trading days, price of at least USD 2
- factors in our baseline analysis:
  - Fama and French (2015) model (MKT, SMB, HML, RMW, CMA)
  - + Carhart (1997) momentum factor (UMD)
  - + Frazzini and Pedersen (2014) betting-against-beta factor (BAB)
- daily returns (risk estimation) and monthly returns (asset pricing tests)

Baseline MCRASH Estimator

- rolling window with 250 days
- $p = 0.05$, i.e. 5%-quantiles as cut-off points for left tail events
- semiparametric estimation:
  - step 1: estimate GARCH-skewed t models and apply the corresponding conditional cdfs to obtain a sample of probability integral transforms ("copula sample")
  - step 2: calculate MCRASH using the empirical distribution of this sample

$$\widehat{\text{MCRASH}}_{i|t}^{\boldsymbol{X}} = \frac{\#\text{ joint tail realizations of } R_i \text{ and } \boldsymbol{X}}{\#\text{ tail realizations of } \boldsymbol{X}} \quad (8)$$

- robustness checks: alternative volatility model, non-parametric estimation, fully parametric estimation

Univariate Portfolio Sorts

	1	2	3	4	5	6	7	8	9	10	10-1
exret	0.38	0.48	0.53	0.59	0.65	0.68	0.68	0.69	0.73	0.77	0.39
	(1.59)	(1.98)	(2.17)	(2.42)	(2.71)	(2.80)	(2.80)	(2.81)	(2.90)	(3.01)	(3.69)
α 7F	-0.28	-0.17	-0.13	-0.07	-0.02	0.01	0.02	0.05	0.11	0.16	0.44
	(-4.73)	(-3.28)	(-2.56)	(-1.65)	(-0.33)	(0.12)	(0.42)	(0.99)	(1.71)	(2.06)	(4.79)
α 5F	-0.33	-0.25	-0.20	-0.14	-0.08	-0.06	-0.04	-0.01	0.04	0.10	0.43
	(-4.55)	(-3.88)	(-3.43)	(-2.73)	(-1.51)	(-1.09)	(-0.75)	(-0.15)	(0.65)	(1.31)	(4.40)
α 7F+LIQ	-0.27	-0.16	-0.12	-0.07	-0.02	0.00	0.02	0.06	0.11	0.15	0.41
	(-4.39)	(-2.88)	(-2.23)	(-1.50)	(-0.44)	(-0.07)	(0.42)	(1.10)	(1.66)	(1.86)	(4.48)
α 7F+QMJ	-0.20	-0.09	-0.06	-0.02	0.04	0.07	0.09	0.12	0.19	0.26	0.47
	(-3.15)	(-1.78)	(-1.24)	(-0.45)	(0.86)	(1.34)	(1.62)	(2.07)	(2.68)	(3.06)	(4.34)
α M4	-0.15	-0.09	-0.07	-0.03	0.03	0.05	0.06	0.10	0.16	0.23	0.39
	(-1.96)	(-1.33)	(-1.14)	(-0.48)	(0.67)	(1.05)	(1.16)	(1.64)	(2.41)	(2.74)	(3.03)
α q5	-0.11	-0.04	-0.01	0.03	0.07	0.08	0.09	0.13	0.18	0.24	0.35
	(-1.38)	(-0.65)	(-0.24)	(0.51)	(1.17)	(1.15)	(1.24)	(1.63)	(2.00)	(2.22)	(2.70)

risk-adjusted return spread for stocks with high MCRASH minus stocks with low MCRASH is significantly positive.

Fama/MacBeth Regressions: Betas

	future excess returns							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
MCRASH	4.33	5.56	5.00	5.41	5.04	4.80	4.45	4.37
	(3.58)	(5.77)	(5.42)	(5.76)	(5.90)	(5.82)	(5.90)	(5.89)
β^{MKT}		-0.25	-0.22	0.04	0.16	0.20	-0.01	0.05
		(-1.54)	(-1.19)	(0.18)	(0.71)	(0.87)	(-0.06)	(0.21)
β^{SMB}			-0.03	-0.04	0.11	0.12	0.15	0.09
			(-0.30)	(-0.32)	(0.87)	(0.97)	(1.21)	(0.73)
β^{HML}				0.20	0.24	0.09	0.00	0.02
				(1.84)	(1.77)	(0.75)	(0.02)	(0.15)
β^{RMW}					0.18	0.21	0.24	0.26
					(2.34)	(2.71)	(3.52)	(3.55)
β^{CMA}						0.14	0.11	0.09
						(1.47)	(1.36)	(1.13)
β^{UMD}							-0.09	0.01
							(-0.36)	(0.05)
β^{BAB}								-0.01
								(-0.09)
Intercept	0.22	0.38	0.42	0.33	0.34	0.34	0.36	0.36
	(0.88)	(1.81)	(2.04)	(1.69)	(1.73)	(1.79)	(1.91)	(2.00)

effect of MCRASH on future returns is stable when controlling for factor betas.

Fama/MacBeth Regressions: Characteristics

	future excess returns							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
MCRASH	4.33	5.56	4.80	4.26	3.46	3.37	3.36	2.69
	(3.58)	(5.77)	(6.70)	(5.80)	(4.85)	(4.76)	(4.68)	(3.96)
β^{MKT}		-0.25	-0.27	-0.19	-0.28	-0.30	-0.32	-0.11
		(-1.54)	(-1.56)	(-1.13)	(-1.83)	(-1.90)	(-2.10)	(-0.78)
size			-0.01	-0.01	-0.01	0.01	-0.02	-0.09
			(-0.15)	(-0.14)	(-0.22)	(0.24)	(-0.55)	(-2.27)
bm				0.27	0.25	0.28	0.27	0.23
				(3.40)	(3.30)	(3.49)	(3.24)	(2.80)
mom					0.01	0.01	0.01	0.01
					(5.72)	(5.23)	(4.87)	(4.80)
rev						-0.03	-0.03	-0.02
						(-8.00)	(-7.90)	(-4.96)
illiq							0.01	0.05
							(0.13)	(0.93)
max								-9.39
								(-10.46)
Intercept	0.22	0.38	0.44	0.23	0.22	0.12	0.40	1.16
	(0.88)	(1.81)	(1.21)	(0.62)	(0.63)	(0.33)	(1.06)	(3.34)

effect of MCRASH on future returns is stable when controlling for firm characteristics.

Fama/MacBeth Regressions: Downside Risk Measures

	future excess returns								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
MCRASH	2.69 (3.96)	3.20 (4.60)	2.46 (3.60)	1.49 (2.48)	2.73 (4.01)	2.94 (4.47)	1.95 (3.21)	1.57 (2.56)	3.57 (2.76)
β^{down}		-0.19 (-2.55)							
β^{tail}			0.16 (1.92)						
idiovol				-0.02 (-5.74)					
idioskew					-0.02 (-0.64)				
coskew						0.14 (0.71)			
cokurt							0.28 (3.75)		
VaR								-26.97 (-5.37)	
β^{bear}									-0.29 (-2.65)
Characteristics	yes	yes	yes	yes	yes	yes	yes	yes	yes

effect of MCRASH on future returns is stable when controlling for downside risk measures.

Conclusion

- We propose **MCRASH** as a measure for a stock's sensitivity to **crashes of all risk factors** in an asset pricing model.
- We investigate the pricing of MCRASH using **a new tail-related expansion of the projected SDF**.
- MCRASH shows a **significantly positive impact on average future stock returns** that is not explained by linear risk exposure and market-based downside risk measures.

⇒ capturing extreme dependence with well-known risk factors helps to improve our understanding of the cross-section of expected stock returns