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Author(s): Robert U. Ayres and Allen V. Kneese
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# Production, Consumption, and Externalities

*By* ROBERT U. AYRES AND ALLEN V. KNEESE*

> "For all that, welfare economics can no more reach conclusions applicable to the real world without some knowledge of the real world than can positive economics" [21].

Despite tremendous public and governmental concern with problems such as environmental pollution, there has been a tendency in the economics literature to view externalities as exceptional cases. They may distort the allocation of resources but can be dealt with adequately through simple *ad hoc* arrangements. To quote Pigou:

> When it was urged above, that in certain industries a wrong amount of resources is being invested because the value of the marginal social net product there differs from the value of the marginal private net product, it was tacitly assumed that in the main body of industries these two values are equal [22][1].

And Scitovsky, after having described his cases two and four which deal with technological externalities affecting consumers and producers respectively, says:

> The second case seems exceptional, because most instances of it can be and usually are eliminated by zoning ordinances and industrial regulations concerned with public health and safety. The fourth case seems unimportant, simply because examples of it seem to be few and exceptional [25].

We believe that at least one class of externalities—those associated with the disposal of residuals resulting from the consumption and production process—must be viewed quite differently.[2] They are a normal, indeed, inevitable part of these processes. Their economic significance tends to increase as economic development proceeds, and the ability of the ambient environment to receive and assimilate them is an important natural resource of increasing value.[3] We will argue below that

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* The authors are respectively visiting scholar and director, Quality of the Environment Program, Resources for the Future, Inc. We are indebted to our colleagues Blair Bower, Orris Herfindahl, Charles Howe, John Krutilla, and Robert Steinberg for comments on an earlier draft. We have also benefited from comments by James Buchanan, Paul Davidson, Robert Dorfman, Otto Eckstein, Myrick Freeman, Mason Gaffney, Lester Lave, Herbert Mohring, and Gordon Tullock.

[1] Even Baumol who saw externalities as a rather pervasive feature of the economy tends to discuss external diseconomies like "smoke nuisance" entirely in terms of particular examples [3]. A perspective more like that of the present paper is found in Kapp [16].

[2] We by no means wish to imply that this is the only important class of externalities associated with production and consumption. Also, we do not wish to imply that there has been a lack of theoretical attention to the externalities problem. In fact, the past few years have seen the publication of several excellent articles which have gone far toward systematizing definitions and illuminating certain policy issues. Of special note are Coase [9], Davis and Whinston [12], Buchanan and Stubblebine [6], and Turvey [27]. However, all these contributions deal with externality as a comparatively minor aberration from Pareto optimality in competitive markets and focus upon externalities between two parties. Mishan, after a careful review of the literature, has commented on this as follows: "The form in which external effects have been presented in the literature is that of partial equilibrium analysis; a situation in which a single industry produces an equilibrium output, usually under conditions of perfect competition, some form of intervention being required in order to induce the industry to produce an "ideal" or "optimal" output. If the point is not made explicitly, it is tacitly understood that unless the rest of the economy remains organized in conformity with optimum conditions, one runs smack into Second Best problems" [21].

[3] That external diseconomies are integrally related to economic development and increasing congestion has been noted in passing in the literature. Mishan has commented: "The attention given to external effects in

the common failure to recognize these facts may result from viewing the production and consumption processes in a manner that is somewhat at variance with the fundamental law of conservation of mass.

Modern welfare economics concludes that if (1) preference orderings of consumers and production functions of producers are independent and their shapes appropriately constrained, (2) consumers maximize utility subject to given income and price parameters, and (3) producers maximize profits subject to the price parameters; a set of prices exists such that no individual can be made better off without making some other individual worse off. For a given distribution of income this is an efficient state. Given certain further assumptions concerning the structure of markets, this "Pareto optimum" can be achieved via a pricing mechanism and voluntary decentralized exchange.

If waste assimilative capacity of the environment is scarce, the decentralized voluntary exchange process cannot be free of uncompensated technological external diseconomies unless (1) all inputs are fully converted into outputs, with no unwanted material residuals along the way,[4] and all final outputs are utterly destroyed in the process of consumption, or (2) property rights are so arranged that all relevant environmental attributes are in private ownership and these rights are exchanged in competitive markets. Neither of these conditions can be expected to hold in an actual economy and they do not.

Nature does not permit the destruction of matter except by annihilation with antimatter, and the means of disposal of unwanted residuals which maximizes the internal return of decentralized decision units is by discharge to the environment, principally, watercourses and the atmosphere. Water and air are traditionally examples of free goods in economics. But in reality, in developed economies they are common property resources of great and increasing value presenting society with important and difficult allocation problems which exchange in private markets cannot resolve. These problems loom larger as increased population and industrial production put more pressure on the environment's ability to dilute and chemically degrade waste products. Only the crudest estimates of present external costs associated with residuals discharge exist but it would not be surprising if these costs were in the tens of billions of dollars annually.[5] Moreover, as we shall emphasize again, technological means for processing or purifying one or another type of waste discharge do not destroy the residuals but only alter their form. Thus, given the level, patterns, and technology of production and consumption, recycle of materials into productive uses or discharge into an alternative medium are the only general options for protecting a particular environmental medium such as water. Residual problems must be seen in a broad regional or economy-wide context rather

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the recent literature is, I think, fully justified by the unfortunate, albeit inescapable, fact that as societies grow in material wealth, the incidence of these effects grows rapidly . . . " [21]; and Buchanan and Tullock have stated that as economic development proceeds, "congestion" tends to replace "co-operation" as the underlying motive force behind collective action, i.e., controlling external diseconomies tends to become more important than cooperation to realize external economies [7].

[4] Or any residuals which occur must be stored on the producer's premises.

[5] It is interesting to compare this with estimates of the cost of another well known misallocation of resources that has occupied a central place in economic theory and research. In 1954, Harberger published an estimate of the welfare cost of monopoly which indicated that it amounted to about .07 percent of GNP [15]. In a later study, Schwartzman calculated the allocative cost at only .01 percent of GNP [24]. Leibenstein generalized studies such as these to the statement that " . . . in a great many instances the amount to be gained by increasing allocative efficiency is trivial . . . " [19]. But Leibenstein did not consider the allocative costs associated with environmental pollution.

than as separate and isolated problems of disposal of gas, liquid, and solid wastes.

Frank Knight perhaps provides a key to why these elementary facts have played so small a role in economic theorizing and empirical research.

> The next heading to be mentioned ties up with the question of dimensions from another angle, and relates to the second main error mentioned earlier as connected with taking food and eating as the type of economic activity. The basic economic magnitude (value or utility) is service, not good. It is inherently a stream or flow in time . . . [18].[6]

Almost all of standard economic theory is in reality concerned with services. Material objects are merely the vehicles which carry some of these services, and they are exchanged because of consumer preferences for the services associated with their use or because they can help to add value in the manufacturing process. Yet we persist in referring to the "final consumption" of goods as though material objects such as fuels, materials, and finished goods somehow disappeared into the void—a practice which was comparatively harmless so long as air and water were almost literally free goods.[7] Of course, residuals from both the production and consumption processes remain and they usually render disservices (like killing fish, increasing the difficulty of water treatment, reducing public health, soiling and deteriorating buildings, etc.) rather than services. Control efforts are aimed at eliminating or reducing those disservices which flow to consumers and producers whether they want them or not and which, except in unusual cases, they cannot control by engaging in individual exchanges.[8]

## I. *The Flow of Materials*

To elaborate on these points, we find it useful initially to view environmental pollution and its control as a materials balance problem for the entire economy.[9] The inputs to the system are fuels, foods, and raw materials which are partly converted into final goods and partly become waste residuals. Except for increases in inventory, final goods also ultimately enter the waste stream. Thus goods which are "consumed" really only render certain services. Their material substance remains in existence and must either be reused or discharged to the ambient environment.

In an economy which is closed (no imports or exports) and where there is no net accumulation of stocks (plant, equipment, inventories, consumer durables, or residential buildings), the amount of residuals inserted into the natural environment must be approximately equal to the weight of basic fuels, food, and raw materials entering the processing and production system, plus oxygen taken from the atmosphere.[10] This result, while obvious

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[6] The point was also clearly made by Fisher: "The only true method, in our view, is to regard uniformly as income the *service* of a dwelling to its owner (shelter or money rental), the *service* of a piano (music), and the *service* of food (nourishment) . . . " (emphasis in original) [14].

[7] We are tempted to suggest that the word consumption be dropped entirely from the economist's vocabulary as being basically deceptive. It is difficult to think of a suitable substitute, however. At least, the word consumption should not be used in connection with goods, but only with regard to services or flows of "utility."

[8] There is a substantial literature dealing with the question of under what conditions individual exchanges can optimally control technological external diseconomies. A discussion of this literature, as it relates to waterborne residuals, is found in Kneese and Bower [17].

[9] As far as we know, the idea of applying materials balance concepts to waste disposal problems was first expressed by Smith [26]. We also benefitted from an unpublished paper by Joseph Headley in which a pollution "matrix" is suggested. We have also found references by Boulding to a "spaceship economy" suggestive [4]. One of the authors has previously used a similar approach in ecological studies of nutrient interchange among plants and animals; see [1].

[10] To simplify our language, we will not repeat this essential qualification at each opportunity, but assume it applies throughout the following discussion. In addition, we must include residuals such as $NO$ and $NO_2$ arising from reactions between components of the air itself but occurring as combustion by-products.

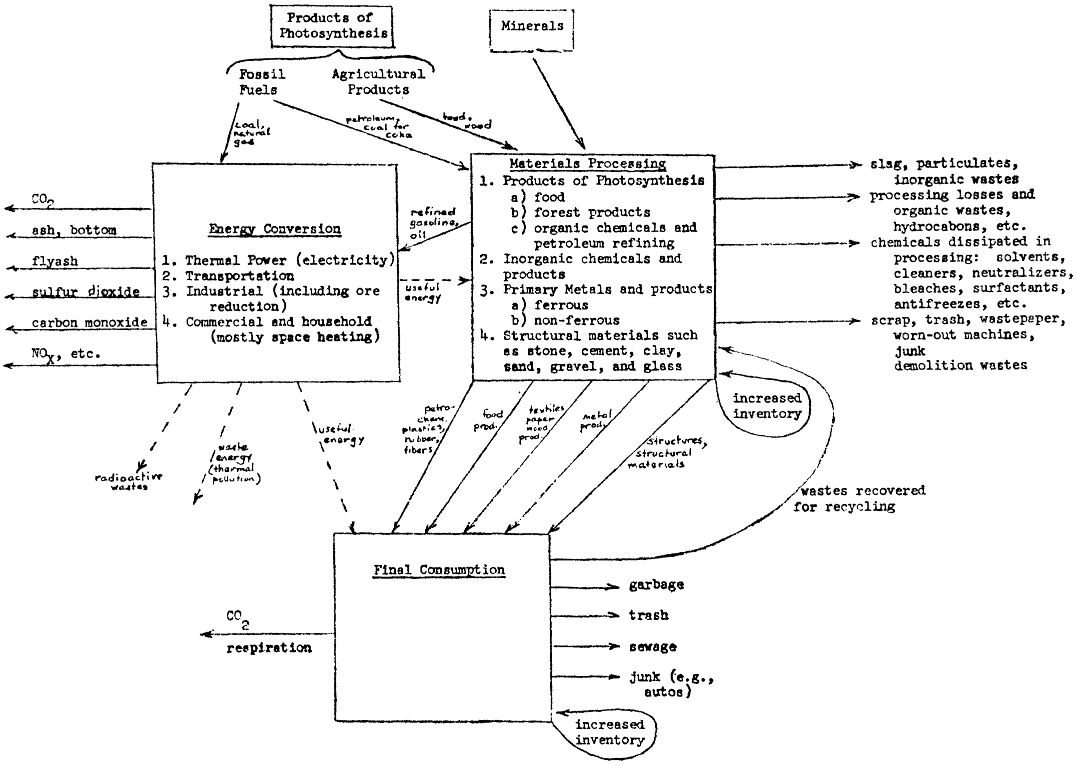

FIGURE 1.—MATERIALS FLOW

upon reflection, leads to the, at first rather surprising, corollary that residuals disposal involves a greater tonnage of materials than basic materials processing, although many of the residuals, being gaseous, require no physical "handling."

Figure 1 shows a materials flow of the type we have in mind in greater detail and relates it to a broad classification of economic sectors for convenience in our later discussion, and for general consistency with the Standard Industrial Classification. In an open (regional or national) economy, it would be necessary to add flows representing imports and exports. In an economy undergoing stock or capital accumulation, the production of residuals in any given year would be less by that amount than the basic inputs. In the entire U.S. economy, accumulation accounts for about 10–15 percent of basic annual inputs, mostly in the form of construction materials, and there is some net importation of raw and partially processed materials amounting to 4 or 5 percent of domestic production. Table 1 shows estimates of the weight of raw materials produced in the United States in several recent years, plus net imports of raw and partially processed materials.

Of the active inputs,[11] perhaps three-quarters of the overall weight is eventually discharged to the atmosphere as carbon (combined with atmospheric oxygen in the form of CO or $CO_2$) and hydrogen (combined with atmospheric oxygen as $H_2O$) under current conditions. This results from combustion of fossil fuels and from animal respiration. Discharge of carbon dioxide can be considered harmless in the short run. There are large "sinks" (in the form of vegetation and large water bodies,

[11] See footnote to Table 1.

TABLE 1—WEIGHT OF BASIC MATERIALS PRODUCTION IN THE UNITED STATES PLUS NET IMPORTS, 1963 ($10^6$ tons)

| | 1963 | 1964 | 1965 |
|---|---|---|---|
| *Agricultural (incl. fishery and wildlife and forest) products* | | | |
| Food Crops (excl. livestock feed) | 125 | 128 | 130 |
| Livestock | 100 | 103 | 102 |
| Other products | 5 | 6 | 6 |
| Fishery | 3 | 3 | 3 |
| Forestry products (85 per cent dry wt. basis) | | | |
| Sawlogs | 53 | 55 | 56 |
| Pulpwood | 107 | 116 | 120 |
| Other | 41 | 41 | 42 |
| Total | 434 | 452 | 459 |
| *Mineral fuels* | 1,337 | 1,399 | 1,448 |
| *Other minerals* | | | |
| Iron ore | 204 | 237 | 245 |
| Other metal ores | 161 | 171 | 191 |
| Other nonmetals | 125 | 133 | 149 |
| Total | 490 | 541 | 585 |
| Grand total[a] | 2,261 | 2,392 | 2,492 |

[a] Excluding construction materials, stone, sand, gravel, and other minerals used for structural purposes, ballast, fillers, insulation, etc. Gangue and mine tailings are also excluded from this total. These materials account for enormous tonnages but undergo essentially no chemical change. Hence, their use is more or less tantamount to physically moving them from one location to another. If this were to be included, there is no logical reason to exclude material shifted in highway cut and fill operations, harbor dredging, land-fill, plowing, and even silt moved by rivers. Since a line must be drawn somewhere, we chose to draw it as indicated above.

*Source*: R. U. Ayres and A. V. Kneese [2, p. 630].

mainly the oceans) which reabsorb this gas, although there is evidence of net accumulation of $CO_2$ in the atmosphere. Some experts believe that the latter is likely to show a large relative increase, as much as 50 per cent by the end of the century, possibly giving rise to significant —and probably, on balance, adverse— weather changes.[12] Thus continued combustion of fossil fuels at a high rate could produce externalities affecting the entire world. The effects associated with most residuals will normally be more confined, however, usually limited to regional air and water sheds.

The remaining residuals are either gases (like carbon monoxide, nitrogen dioxide, and sulfur dioxide—all potentially harmful even in the short run), dry solids (like rubbish and scrap), or wet solids (like garbage, sewage, and industrial wastes suspended or dissolved in water). In a sense, the dry solids are an irreducible, limiting form of waste. By the application of appropriate equipment and energy, most undesirable substances can, in principle, be removed from water and air streams[13]—but what is left must be disposed of in solid form, transformed, or reused. Looking at the matter in this way clearly reveals a primary interdependence between the various waste streams which casts into doubt the traditional classification of air, water, and land pollution as individual categories for purposes of planning and control policy.

Residuals do not necessarily have to be discharged to the environment. In many instances, it is possible to recycle them back into the productive system. The materials balance view underlines the fact that the throughput of new materials necessary to maintain a given level of production and consumption decreases as the technical efficiency of energy conversion and materials utilization increases. Similarly, other things being equal, the longer that cars, buildings, machinery, and other durables remain in service, the fewer new materials are required to compensate for loss, wear, and obsolescence—although the use of old or worn machinery (e.g., automobiles) tends to increase other residuals problems. Technically efficient combustion of (desulfurized) fossil fuels

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[12] See [30]. There is strong evidence that discharge of residuals has already affected the climate of individual cities; see Lowry [20].

[13] Except $CO_2$, which may be harmful in the long run, as noted.

would leave only water, ash, and carbon dioxide as residuals, while nuclear energy conversion need leave only negligible quantities of material residuals (although thermal pollution and radiation hazards cannot be dismissed by any means).

Given the population, industrial production, and transport service in an economy (a regional rather than a national economy would normally be the relevant unit), it is possible to visualize combinations of social policy which could lead to quite different relative burdens placed on the various residuals-receiving environmental media; or, given the possibilities for recycle and less residual-generating production processes, the overall burden to be placed upon the environment as a whole. To take one extreme, a region which went in heavily for electric space heating and wet scrubbing of stack gases (from steam plants and industries), which ground up its garbage and delivered it to the sewers and then discharged the raw sewage to watercourses, would protect its air resources to an exceptional degree. But this would come at the sacrifice of placing a heavy residuals load upon water resources. On the other hand, a region which treated municipal and industrial waste water streams to a high level and relied heavily on the incineration of sludges and solid wastes would protect its water and land resources at the expense of discharging waste residuals predominantly to the air. Finally, a region which practiced high level recovery and recycle of waste materials and fostered low residual production processes to a far reaching extent in each of the economic sectors might discharge very little residual waste to any of the environmental media.

Further complexities are added by the fact that sometimes it is possible to modify an environmental medium through investment in control facilities so as to improve its assimilative capacity. The clearest, but far from only, example is with respect to watercourses where reservoir storage can be used to augment low river flows that ordinarily are associated with critical pollution (high external cost situations).[14] Thus internalization of external costs associated with particular discharges, by means of taxes or other restrictions, even if done perfectly, cannot guarantee Pareto optimality. Investments involving public good aspects must enter into an optimal solution.[15]

To recapitulate our main points briefly: (1) Technological external diseconomies are not freakish anomalies in the processes of production and consumption but an inherent and normal part of them. (2) These external diseconomies are quantitatively negligible in a low-population or economically undeveloped setting, but they become progressively (nonlinearly) more important as the population rises and the level of output increases (i.e., as the natural reservoirs of dilution and assimilative capacity become exhausted).[16] (3) They cannot be properly dealt with by considering environmental media such as air and water in isolation. (4) Isolated and *ad hoc* taxes and other restrictions are not sufficient for their optimum control, although they are essential elements in a more systematic and coherent program of environmental quality management. (5) Public investment programs, particularly including transportation systems, sewage disposal, and river flow regulation, are intimately related to the amounts and

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[14] Careful empirical work has shown that this technique can fit efficiently into water quality management systems. See Davis [11].

[15] A discussion of the theory of such public investments with respect to water quality management is found in Boyd [5].

[16] Externalities associated with residuals discharge may appear only at certain threshold values which are relevant only at some stage of economic development and industrial and population concentrations. This may account for their general treatment as "exceptional" cases in the economics literature. These threshold values truly were exceptional cases for less developed economies.

effects of residuals and must be planned in light of them.

It is important to develop not only improved measures of the external costs resulting from differing concentrations and duration of residuals in the environment but more systematic methods for forecasting emissions of external-cost-producing residuals, technical and economic trade-offs between them, and the effects of recycle on environmental quality.

In the hope of contributing to this effort and of revealing more clearly the types of information which would be needed to implement such a program, we set forth a more formal model of the materials balance approach in the following sections and relate it to some conventional economic models of production and consumption. The main objective is to make some progress toward defining a system in which flows of services and materials are simultaneously accounted for and related to welfare.

## II. *Basic Model*

The take off point for our discussion is the Walras-Cassel general equilibrium model,[17] extended to include intermediate consumption, which involve the following quantities:

$$\overbrace{r_1, \cdots\cdots\cdots\cdots, r_M}^{\text{resources and services}}$$

$$\overbrace{X_1, \cdots\cdots\cdots\cdots, X_N}^{\text{products or commodities}}$$

$$\overbrace{v_1, \cdots\cdots\cdots\cdots, v_M}^{\text{resource prices}}$$

$$\overbrace{p_1, \cdots\cdots\cdots\cdots, p_N}^{\text{product or commodity prices}}$$

$$\overbrace{Y_1, \cdots\cdots\cdots\cdots, Y_N}^{\text{final demands}}$$

The $M$ basic resources are allocated among the $N$ sectors as follows:

$$\begin{aligned} r_1 &= a_{11}X_1 + a_{12}X_2 + \cdots + a_{1N}X_N \\ r_2 &= a_{21}X_1 + a_{22}X_2 + \cdots + a_{2N}X_N \\ &\vdots \\ r_M &= a_{M1}X_1 + a_{M2}X_2 + \cdots + a_{MN}X_N \end{aligned}$$

(1a) or

$$r_j = \sum_{k=1}^{N} a_{jk}X_k \qquad j = 1, \cdots, M$$

In (1a) we have implicitly assumed that there is no possibility of factor or process substitution and no joint production. These conditions will be discussed later. In matrix notation we can write:

$$\text{(1b)} \qquad [r_{j1}]_{M,1} = [a_{jk}]_{M,N} \cdot [X_{k1}]_{N,1}$$

where $[a]$ is an $M \times N$ matrix.

A similar set of equations describes the relations between commodity production and final demand:

$$\text{(2a)} \qquad X_k = \sum_{l=1}^{N} A_{kl}Y_l \qquad k = 1, \cdots, N$$

$$\text{(2b)} \qquad [X_{k1}]_{N,1} = [A_{kl}]_{N,N} \cdot [Y_{l1}]_{N,1}$$

and the matrix $[A]$ is given by

$$\text{(3)} \qquad [A] = [I - C]^{-1}$$

where $[I]$ is the unit diagonal matrix and the elements $C_{ij}$ of the matrix $[C]$ are essentially the well known Leontief input coefficients. In principle these are functions of the existing technology and, therefore, are fixed for any given situation.

By combining (1) and (2), we obtain a set of equations relating resource inputs directly to final demand, viz.,

$$\text{(4a)} \qquad \begin{aligned} r_j &= \sum_{k=1}^{N} a_{jk} \sum_{l=1}^{N} A_{kl}Y_l = \sum_{k,l=1}^{N} a_{jk}A_{kl}Y_l \\ &= \sum_{l=1}^{N} b_{jl}Y_l \qquad j = 1, \cdots, M \end{aligned}$$

or, of course, in matrix notation (4b).

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[17] The original references are Walras [28] and Cassel [8]. Our own treatment is largely based on Dorfman *et al.* [13].

$$\text{(4b)} \quad \begin{aligned} [r_{jl}]_{M,1} &= [a_{jk}]_{M,N} \cdot [A_{kl}]_{N,N} \cdot [Y_{l1}]_{N,1} \\ &= [b_{jl}]_{M,N} \cdot [Y_{l1}]_{N,1} \end{aligned}$$

We can also impute the prices of $N$ intermediate goods and commodities to the prices of the $M$ basic resources, as follows:

$$\text{(5a)} \quad p_k = \sum_{j=1}^{M} v_j b_{jk} \qquad k = 1, \cdots, N$$

$$\text{(5b)} \quad [p_{1k}]_{1,N} = [v_{1j}]_{1,M} \cdot [b_{jk}]_{M,N}$$

To complete the system, it may be supposed that demand and supply relationships are given, a priori, by Pareto-type preference functions:

$$\text{(6)} \quad \text{Demand:} \quad Y_k = F_k(p_1, \cdots, p_N) \qquad k = 1, \cdots, N$$

$$\text{(7)} \quad \text{Supply:} \quad r_k = G_k(v_1, \cdots . v_M) \qquad k = 1, \cdots, M$$

where, of course, the $p_j$ are functions of the $v_j$ as in (5b).

In order to interpret the $X$'s as physical production, it is necessary for the sake of consistency to arrange that outputs and inputs always balance, which implies that the $C_{ij}$ must comprise *all* materials exchanges including residuals. To complete the system so that there is no net gain or loss of physical substances, it is also convenient to introduce two additional sectors, viz., an "environmental" sector whose (physical) output is $X_0$ and a "final consumption" sector whose output is denoted $X_f$. The system is then easily balanced by explicitly including flows both to and from these sectors.

To implement this further modification of the Walras-Cassel model, it is convenient to subdivide and relabel the resource category into tangible raw materials $\{r^m\}$ and services $\{r^s\}$:

$$\begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_L \end{bmatrix} \quad \left.\begin{bmatrix} \overset{m}{r}_1 \\ \overset{m}{r}_2 \\ \cdot \\ \cdot \\ \cdot \\ \overset{m}{r}_L \end{bmatrix}\right\} \begin{matrix}\text{raw materials} \\ \text{(units)}\end{matrix}$$

becomes

$$\begin{bmatrix} r_{L+1} \\ \cdot \\ \cdot \\ r_M \end{bmatrix} \quad \left.\begin{bmatrix} \overset{s}{r}_1 \\ \cdot \\ \cdot \\ \overset{s}{r}_p \end{bmatrix}\right\} \begin{matrix}\text{service} \\ \text{(units)}\end{matrix}$$

becomes

where, of course,

$$\text{(8)} \qquad L + P = M$$

It is understood that services, while not counted in tons, can be measured in meaningful units, such as man-days, with well defined prices. Thus, we similarly relabel the price variables as follows:

$$\begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_L \end{bmatrix} \quad \left.\begin{bmatrix} \overset{m}{V}_1 \\ \cdot \\ \cdot \\ \cdot \\ \overset{m}{V}_L \end{bmatrix}\right\} \begin{matrix}\text{raw material} \\ \text{(prices)}\end{matrix}$$

becomes

$$\begin{bmatrix} V_{L+1} \\ \cdot \\ \cdot \\ \cdot \\ V_M \end{bmatrix} \quad \left.\begin{bmatrix} \overset{s}{V}_1 \\ \cdot \\ \cdot \\ \cdot \\ \overset{s}{V}_p \end{bmatrix}\right\} \begin{matrix}\text{labor and service} \\ \text{(prices)}\end{matrix}$$

The coefficients $\{a_{ij}\}$, $\{b_{ij}\}$ are similarly partitioned into two groups,

$$\begin{array}{llcl} \text{e.g.,} & b_{1j} & & \overset{m}{b}_{1j} \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ & b_{Lj} & \text{becomes} & \overset{m}{b}_{Lj} \\ & b_{L+1,j} & & \overset{s}{b}_{1j} \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ & b_{Mj} & & \overset{s}{b}_{Pj} \end{array}$$

These notational changes have no effect whatever on the substance of the model, although the equations become somewhat more cumbersome. The partitioned matrix notation simplifies the restatement of the basic equations. Thus (1b) becomes (9), while (5b) becomes (10).

$$M\left\{\begin{bmatrix}\vdots\\ r\\ \vdots\end{bmatrix}\right. \equiv \begin{bmatrix} r^m \\ \cdots \\ r^s\end{bmatrix}\begin{matrix}\}L\\ \\ \}P\end{matrix} = M\left\{\overbrace{\begin{bmatrix}\}L & b^m\\ & \cdots \\ \}P & b^s\end{bmatrix}}^{N}\begin{bmatrix} Y \end{bmatrix}\right\}N \tag{9}$$

$$\underset{N}{[p_1,\cdots,p_N]} = \underbrace{\underset{L\quad P}{[v^m\vdots\; v^s]}}_{M}\,\underbrace{\begin{bmatrix} b^m\\ \cdots\\ b^s \end{bmatrix}}_{N}\Big\}M \tag{10}$$

$$= [\cdots v^m \cdots]\begin{bmatrix}\vdots & b^m & \vdots\\ \vdots & \cdots & \vdots\end{bmatrix} + [\cdots v^s\cdots]\begin{bmatrix}\vdots & b^s & \vdots\\ \vdots & \cdots & \vdots\end{bmatrix}$$

The equivalent of (5a) is:

$$p_k = \underbrace{\sum_{j=1}^{L} \overset{m}{b}_{jk}\overset{m}{v}_j}_{\substack{\text{prices imputed}\\ \text{to cost of raw}\\ \text{materials}}} + \underbrace{\sum_{j=1}^{P} \overset{s}{b}_{jk}\overset{s}{v}_j}_{\substack{\text{prices imputed}\\ \text{to cost of}\\ \text{services}}} \tag{11}$$

where $k = 1, \cdots, N$

We wish to focus attention explicitly on the flow of materials through the economy. By definition of the Leontief input coefficients (now related to materials flow), we have:

$C_{kj}X_j$ (physical) quantity transferred from $k$ to $j$

$C_{jk}X_k$ quantity transferred from $j$ to $k$

Hence, material flows *from* the environment to all other sectors are given by:

$$\begin{aligned}\sum_{k=1}^{N} C_{0k}X_k = \sum_{j=1}^{L} \overset{m}{r}_j &= \sum_{j=1}^{L}\sum_{k=1}^{N} \overset{m}{a}_{jk}X_k\\ &= \sum_{j=1}^{L}\sum_{k=1}^{N} \overset{m}{b}_{jk}Y_k\end{aligned} \tag{12}$$

using equation (1), as modified.[18] Obviously, comparing the first and third terms,

$$\underbrace{C_{0k}}_{\substack{\text{total material}\\ \text{flow (0 to } k)}} = \underbrace{\sum_{j=1}^{L} \overset{m}{a}_{jk}}_{\substack{\text{all raw materials}\\ \text{(0 to } k)}} \tag{13}$$

Flows into and out of the environmental sector must be in balance:

$$\underbrace{\sum_{k=1}^{N} C_{0k}X_k}_{\substack{\text{sum of all raw}\\ \text{material flows}}} = \underbrace{\sum_{k=1}^{N} C_{k0}X_0 + C_{f0}X_0}_{\substack{\text{sum of all return}\\ \text{(waste) flows}}} \tag{14}$$

Material flows to and from the final sector must also balance:

$$\begin{aligned}&\underbrace{\sum_{k=1}^{N} C_{kf}X_f}_{\substack{\text{sum of all}\\ \text{final goods}}}\\ &= \underbrace{\sum_{k=1}^{N} C_{fk}X_k}_{\substack{\text{sum of all}\\ \text{materials}\\ \text{recycled}}} + \underbrace{C_{f0}X_0}_{\substack{\text{waste residuals}\\ \text{(plus accumulation[19])}}}\end{aligned} \tag{15}$$

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[18] Ignoring, for convenience, any materials flow from the environment *directly* to the final consumption sector.

[19] For convenience, we can treat accumulation in the final sector as a return flow to the environment. In truth, structures actually *become* part of our environment, although certain disposal costs may be deferred.

Of course, by definition, $X_f$ is the sum of the final demands:

$$X_f = \sum_{j=1}^{N} Y_j \tag{16}$$

Substituting (16) into the left side of (15) and (2a) into the right side of (15), we obtain an expression for the waste flow in terms of final demands:

$$C_{f0}X_0 = \sum_{j=1}^{N}\sum_{k=1}^{N} (C_{jf} - C_{fj}A_{jk})Y_k \tag{17}$$

The treatment could be simplified slightly if we assumed that there is no recycling per se. Thus, in the context of the model, we could suppose that all residuals return to the environmental sector,[20] where some of them (e.g., waste paper) become "raw materials." They would then be indistinguishable from new raw materials, however, and price differentials between the two would be washed out. In principle, this is an important distinction to retain.

### III. *Inclusion of Externalities*

The physical flow of materials between various intermediate (production) sectors and the final (consumption) sector tends to be accompanied by, and correlated with, a (reverse) flow of dollars.[21] However, the physical flow of materials from and back to the environment is only partly reflected by actual dollar flows, namely, land rents and payments for raw materials. There are three classes of physical exchange for which there exist no counterpart economic transactions. These are: (1) private use for production inputs of "common property" resources, notably air, streams, lakes, and the ocean; (2) private use of the assimilative capacity of the environment to "dispose of" or dilute wastes and residuals; (3) inadvertent or unwanted material inputs to productive processes—diluents and pollutants.

All these goods (or "bads") are physically transferred at zero price, not because they are not scarce relative to demand—they often are in developed economies—or because they confer no service or disservice on the user—since they demonstrably do so—but because there exist no social institutions that permit the resources in question to be "owned," and exchanged in the market.

The allocation of resources corresponding to a Pareto optimum cannot be attained without subjecting the above-mentioned nonmarket and involuntary exchanges to the moderation of a market or a surrogate thereof. In principle, the influence of a market might be simulated, to a first approximation, by introducing a set of shadow (or virtual) prices.[22] These may well be zero, where supply truly exceeds demand, or negative (i.e., costs) in some instances; they will be positive in others. The exchanges are, of course, real.

The Walras-Cassel model can be generalized to handle these effects in the following way:

1. One can introduce a set of $R$ common-property resources or services of raw materials $\{r_1^{cp}, \cdots, r_R^{cp}\}$ as a subset of the set $\{r_j\}$; these will have corresponding virtual prices $\{v_j^{cp}\}$, which would constitute an "income" from the environment. Such resources include the atmosphere; streams, lakes, and oceans; landscape; wildlife and biological diversity; and the indispensable assimilative capacity of the environment (its ability to accept and neutralize or recycle residuals).[23]

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[20] In calculating actual quantities, we would (by convention) ignore the weight of oxygen taken free from the atmosphere in combustion and return as $CO^2$. However, such inputs will be treated explicitly later.

[21] To be precise, the dollar flow governs and is governed by a combined flow of materials and services (value added).

[22] A similar concept exists in mechanics where the forces producing "reaction" (to balance action and reaction) are commonly described as "virtual forces."

[23] Economists have previously suggested generalization of the Walras-Cassel model to take account of public goods. One of the earliest appears to be Schles-

2. One can introduce a set of $S$ environmental *disservices* imposed on consumers of material resources, by forcing them to accept unwanted inputs $\{r_1^u, \cdots, r_s^u\}$ (pollutants, contaminants, etc.); these disservices would have negative value, giving rise to *negative* virtual prices $\{u_j\}$.[24]

The matrix coefficients $\{a_{ij}\}$ and $\{b_{ij}\}$ can be further partitioned to take account of this additional refinement, and equations analogous to (9), (10), and (11) can be generalized in the obvious way. Equation (6) carries over unchanged, but (7) must be appropriately generalized to take account of the altered situation. Actually, (7) breaks up into several groups of equations:

$$r_k^m = G_k^m(p_1, \cdots, p_N) \qquad k = 1, \cdots, L \tag{18}$$

$$r_k^s = G_k^s(p_1, \cdots, p_N) \qquad k = 1, \cdots, P \tag{19}$$

However, as we have noted at the outset, the supplies of common-property resources and environmental services or disservices are *not* regulated directly by market prices of other goods and services. In the case of common-property resources, the supplies are simply constants fixed by nature or otherwise determined by accident or noneconomic factors.

The total value of these services performed by the environment cannot be calculated but it is suggestive to consider the situation if the natural reservoir of air, water, minerals, etc., were very much smaller, as if the earth were a submarine or "spaceship" (i.e., a vehicle with no assimilative and/or regenerative capacity). In such a case, all material resources would have to be recycled,[25] and the cost of all goods would necessarily reflect the cost of reprocessing residuals and wastes for reuse. In this case, incidentally, the ambient level of unrecovered wastes continuously circulating through the resource inventory of the system (i.e., the spaceship) would in general be nonzero because of the difficulty of 100 percent efficient waste-removal of air and water. However, although the quantitity of waste products in constant circulation may fluctuate within limits, it cannot be allowed to increase monotonically with time, which means that as much material must be recycled, on the average, as is discarded. The value of common resources plus the assimilation services performed by the environment, then, is only indirectly a function of the ambient level of untreated residuals per se, or the disutility caused thereby, which depend on the cost efficiency of the available treatment technology. Be this as it may, of course, the bill of goods produced in a spaceship economy would certainly be radically different from that we are familiar with. For this reason, no standard economic comparison between the two situations is meaningful. The measure of worth we are seeking is actually the difference between the total welfare "produced" by a spaceship economy, where 100 percent of all residuals are promptly recycled, vis-à-vis the existing welfare output on earth, where resource inventories are substantial and

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inger [23]. We are indebted to Otto Eckstein for calling our attention to this key reference.

[24] The notion of introducing the possibility of negative prices in general equilibrium theory has apparently been discussed before, although we are not aware of any systematic development of the idea in the published literature. In this connection, it is worth pointing out the underlying similarity of negative prices and effluent taxes—which have been, and still are being considered as an attractive alternative to subsidies and federal standard-setting as a means of controlling air and water pollution. Such taxes would, of course, be an explicit attempt to rectify an imbalance caused by a market failure.

[25] Any consistent deviation from this 100 per cent rule implies an accumulation of waste products, on the average, which, by definition, is inconsistent with maintaining an equilibrium.

complete recycling need not be contemplated for a very long time to come.

This welfare difference might well be very large, although we possess no methodological tools for quantifying it. In any case, the resource inventory and assimilative capacity of the environment probably contribute very considerably to our standard of living.

If these environmental contributions were paid for, the overall effect on prices would presumably to be push them generally upward. However, the major *differential* effect of undervaluing the environmental contribution is that goods produced by high residual-producing processes, such as papermaking, are substantially underpriced vis-à-vis goods which involve more economical uses of basic resources. This is, however, not socially disadvantageous per se: that is, it causes no misallocation of resources unless, or until, the large resource inventory and/or the assimilative capacity of the environment are used up. When this happens, however, as it now has in most highly industrialized regions, either a market must be allowed to operate or some other form of decision rule must be introduced to permit a rational choice to be made, e.g., between curtailing or controlling the production of residuals or tolerating the effects (disservice) thereof.

It appears that the natural inventory of most common resources used as inputs (e.g., air as an input to combustion and respiratory processes) is still ample,[26] but the assimilative capacity of the environment has already been exceeded in many areas, with important external costs resulting. This suggests a compromise treatment. If an appropriate price could be charged to the producers of the residuals and used to compensate the inadvertent recipients—with the price determined by appropriate Pareto preference criteria—there would be no particular analytic purpose in keeping books on the exchange of the other environmental benefits mentioned, although they are quantitatively massive. We will, therefore, in the remainder of the discussion omit the common-property variables $\{r_j^{cp}\}$ and the corresponding virtual-price variables $\{v_j^{cp}\}$ defined previously, retaining only the terms $\{r_j^u\}$ and $\{u_{jk}\}$. The variable $\{r_j^u\}$ represents a physical quantity of the *j*th unwanted input. There are $S$ such terms, by assumption, whose magnitudes are proportional to the levels of consumption of basic raw materials, subject to the existing technology. However, residuals production is not immutable: it can be increased or decreased by investment, changes in materials processing technology, raw material substitutions, and so forth.

At first glance it might seem entirely reasonable to assert that the *supplies* of unwanted residuals received will be functions of the (negative) prices (i.e., compensation) paid for them, in analogy with (7). Unfortunately, this assertion immediately introduces a theoretical difficulty, since the assumption of unique coefficients $\{a_{ij}\}$ and $\{C_{ij}\}$[27] is not consistent with the possibility of factor or process substitution or joint-production, as stated earlier. To permit such substitutions, one would have to envision a very large collection of alternative sets of coefficients: one complete set of $a$'s and $C$'s for each specific combination of factors and processes. Maximization of any objective function (such as GNP) would involve solving the entire system of equations as many times as there are combinations of factors and pro-

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[26] Water is an exception in arid regions; in humid regions, however, water "shortages" are misnomers: they are really consequences of excessive use of watercourses as cheap means of waste disposal. But some ecologists have claimed that oxygen depletion may be a very serious long-run problem; see Cole [10].

[27] Or $\{b_{ij}\}$ and $\{A_{ij}\}$.

$$[r] = \begin{bmatrix} r^m \\ r^s \\ r^u \end{bmatrix} = M \left\{ \underbrace{\begin{bmatrix} a^m \\ \cdots \\ a^s \\ \cdots \\ a^u \end{bmatrix}}_{N} \begin{bmatrix} \\ X \\ \\ \end{bmatrix} \right\} N = \begin{bmatrix} b^m \\ \cdots \\ b^s \\ \cdots \\ b^u \end{bmatrix} \begin{bmatrix} \\ Y \\ \\ \end{bmatrix} \tag{21}$$

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cesses, and picking out that set of solutions which yields the largest value. Alternatively, if the *a*'s and *C*'s are assumed to be continuously variable functions (of each other), the objective function could also, presumably, be parameterized. However, as long as the *a*'s and *C*'s are uniquely given, the supply of the *k*th unwanted residual is only marginally under the control of the producer, since it will be produced in strict relationship to the composition of the bill of final goods $\{Y_j\}$.

Hence, for the present model it is only correct to assume

$$\overset{u}{r}_k = \overset{u}{G}_k(Y_1, \cdots, Y_N) \tag{20}$$

This limitation does not affect the existence of an equilibrium solution for the system of equations; it merely means that the shadow prices $\{u_{jk}\}$ which would emerge from such a solution for given coefficients $\{a_{ij}\}$, $\{b_{ij}\}$, and $\{C_{ij}\}$ might be considerably higher than the real economic optimum, since the latter could only be achieved by introducing factor and process changes.

Of course, the physical inputs are also related to the physical outputs of goods, as in (21).

Written out in full detail (21) is equivalent to:

$$\text{(22) raw materials} \quad \overset{m}{r}_k = \sum_{j=1}^{N} \overset{m}{a}_{kj} X_j = \sum_{j=1}^{N} \overset{m}{b}_{kj} Y_j \qquad k = 1, \cdots, L$$

$$\text{(23) labor and technical services} \quad \overset{s}{r}_k = \sum_{j=1}^{N} \overset{s}{a}_{kj} X_j = \sum_{j=1}^{N} \overset{s}{b}_{kj} Y_j \qquad k = 1, \cdots, P$$

$$\text{(24) unwanted inputs} \quad \overset{u}{r}_k = \sum_{j=1}^{N} \overset{u}{a}_{kj} X_j = \sum_{j=1}^{N} \overset{u}{b}_{kj} Y_j \qquad k = 1, \cdots, S$$

where, of course,

$$L + P + S = M \tag{25}$$

The corresponding matrix equation for the prices of goods, in terms of production costs, is

$$[p_1, \cdots, p_N] = [v^m \vdots\, v^s \vdots\, u] \begin{bmatrix} b^m \\ \cdots \\ b^s \\ \cdots \\ b^u \end{bmatrix} \tag{26}$$

Written out in the standard form, we obtain

$$p_k = \underbrace{\sum_{j=1}^{L} \overset{m}{b}_{jk} \overset{m}{v}_j}_{\text{cost of raw materials}} + \underbrace{\sum_{j=1}^{P} \overset{s}{b}_{jk} \overset{s}{v}_j}_{\text{cost of labor and technical services}} + \underbrace{\sum_{j=1}^{S} \overset{u}{b}_{jk} \overset{u}{v}_j}_{\text{cost (compensation) for providing environmental disservices}} \qquad k = 1, \cdots, N \tag{27}$$

Evidently, the coefficients $b_{jk}^{u}$ are empirically determined by the structure of the regional economy and its geography. It is assumed that a single overall (negative) price for each residual has meaning, even though each productive sector—and even each consumer—has his own individual utility function. Much the same assumption is conventionally made, and accepted, in the case of positive real prices.

All of the additional variables now fit into the general framework of the original Walras-Cassel analysis. Indeed, we have $2N+2M-1$ variables $(r_i, Y_i, p_i, v_i)$ (allowing an arbitrary normalization factor for the price level) and $2N+2M-1$ independent equations.[28] If solutions exist for the Walras-Cassel system of equations, the arguments presumably continue to hold true for the generalized model. In any case, a discussion of such mathematical questions would carry us too far from our main theme.

## IV. *Concluding Comments*

The limited economics literature currently available which is devoted to environmental pollution problems has generally taken a partial equilibrium view of the matter, as well as treated the pollution of particular environmental media, such as air and water, as separate problems.[29] This no doubt reflects the propensity of the theoretical literature to view externalities as exceptional and minor. Clearly, the partial equilibrium approach in particular is very convenient theoretically and empirically for it permits external damage and control cost functions to be defined for each particular case without reference to broader interrelationships and adjustments in the economy.

We have argued in this paper that the production of residuals is an inherent and general part of the production and consumption process and, moreover, that there are important trade-offs between the gaseous, liquid, or solid forms that these residuals may take. Further, we have argued that under conditions of intensive economic and population development the environmental media which can receive and assimilate residual wastes are not free goods but natural resources of great value with respect to which voluntary exchange cannot operate because of their common property characteristics. We have also noted, in passing, that the assimilative capacity of environmental media can sometimes be altered and that therefore the problem of achieving Pareto optimality reaches beyond devising appropriate shadow prices and involves the planning and execution of investments with public goods aspects.

We have exhibited a formal mathematical framework for tracing residuals flows in the economy and related it to the general equilibrium model of resources allocation, altered to accommodate recycle and containing unpriced sectors to represent the environment. This formulation, in contrast to the usual partial equilibrium treatments, implies knowledge of all preference and production functions including relations between residuals discharge and external cost and all possible factor and process substitutions. While we feel that it represents reality with greater fidelity than the usual view, it also implies a central planning problem of impossible difficulty, both from the standpoint of data collection and computation.

What, if any, help can the general interdependency approach we have outlined offer in dealing with pollution problems effectively and reasonably efficiently? A minimal contribution is its warning that partial equilibrium approaches, while more

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[28] There is one redundant equation in the system, which expresses the identity between gross product and gross income for the system as a whole (sometimes called "Walras law").

[29] See, for example, the essays in Wolozin [29].

tractable, may lead to serious errors. Second, in projecting waste residuals for an economy—a regional economy would usually be the most relevant for planning and control—the inter-industry materials flow model can provide a much more conceptually satisfying and accurate tool for projecting future residuals production than the normal aggregative extrapolations.[30] The latter not only treat gaseous, liquid, and solid wastes separately, but do not take account of input-output relations and the fact that the materials account for the region must balance.

We think that in the next few years it will be possible to make improved regional projections of residuals along the lines sketched above. Undoubtedly, there will also be further progress in empirically estimating external costs associated with residuals discharge and in estimating control costs via various alternative measures. On the basis of this kind of information, a control policy can be devised. However, this approach will still be partial. Interrelations between the regional and national economy must be treated simplistically and to be manageable, the analysis must confine itself to a specific projected bill of goods.

The basic practical question which remains to be answered is whether an iterated series of partial equilibrium treatments—e.g., focusing on one industry or region at a time, *ceteris paribus*—would converge toward the general equilibrium solution, or not. We know of no theoretical test of convergence which would be applicable in this case but, in the absence of such a criterion, would be willing to admit the possible relevance of an empirical sensitivity test more or less along the following lines: take a major residuals-producing industry (such as electric power) and parametrize its cost structure in terms of emission control levels, allowing all technically feasible permutations of factor (fuel) inputs and processes. It would be a straightforward, but complicated, operations research problem to determine the minimum cost solution as a function of the assumed (negative) price of the residuals produced. If possible industry patterns—factor and process combinations—exist which would permit a high level of emission control at only a small increase in power production cost, then it might be possible to conclude that for a significant range of (negative) residuals prices the effect on power prices —and therefore on the rest of the economy —would not be great. Such a conclusion would support the convergence hypothesis. If, on the other hand, electric power prices are very sensitive to residuals prices, then one would at least have to undertake a deeper study of consumer preference functions to try to determine what residuals prices would actually be if a market mechanism existed. If people prove to have a strong antipathy to soot and sulfur dioxide, for instance, resulting in a high (negative) price for these unwanted inputs, then one would be forced to suspect that the partial equilibrium approach is probably not convergent to the general equilibrium solution and that much more elaborate forms of analysis will be required.

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